CONGRUENCE OF TRIANGLES

7

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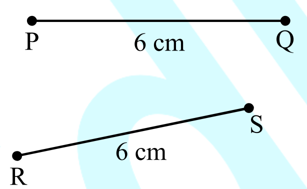
➤ CONGRUENT FIGURES

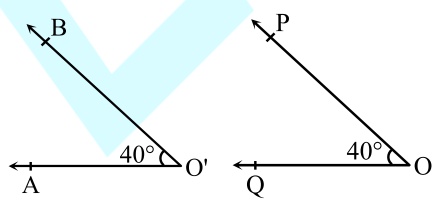
Two figures/objects are said to be congruent if they are exactly of the same shape and size. The relationship between two congruent figures is called congruence. We use the symbol ≅ for 'congruent to'.

1. **Congruence among line segments.** Two line segments are congruent if they have the same length.



Thus, line segment PQ ≅ line segment RS as PQ = RS = 6 cm.

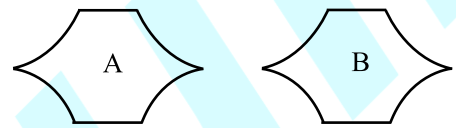
2. **Congruence of Angles.** Two angles are congruent if they have the same measure.



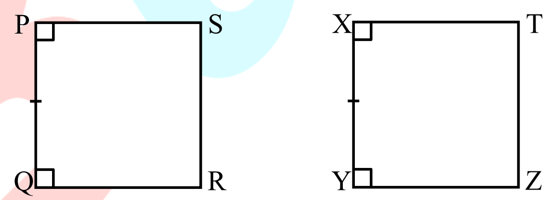
Thus, ∠AO'B ≅ ∠QOP,

as m ∠AO'B = m ∠QOP = 40°.

3. **Congruence of plane figures.** Two plane figures A and B are congruent as they superpose each other. We can write it as figure A ≅ figure B.

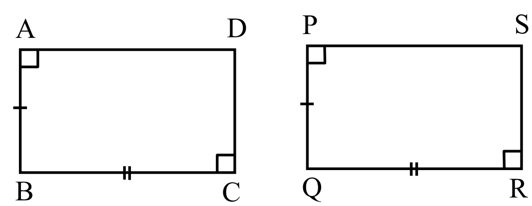


4. **Congruence of squares.** Two squares are congruent if they have same side length.



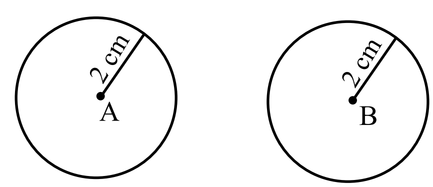
Square PQRS ≅ Square XYZT as PQ = XY.

5. **Congruence of rectangles.** Two rectangles are said to be congruent if they have the same length and breadth.



Rectangle ABCD ≅ Rectangle PQRS as AB = PQ and BC = QR.

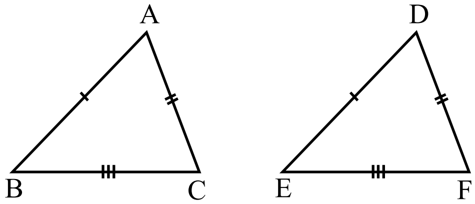
6. **Congruence of circles.** Two circles are congruent if they have the same radius.



Circle A ≅ Circle B, as radius of A = radius of B = 2 cm.

➤ CONGRUENCE OF TRIANGLES

Two triangles are congruent if they are copies of each other, and when superposed they cover each other exactly.



ΔABC and ΔDEF have the same size and shape. They are congruent. So we would express this as ΔABC ≅ ΔDEF. This means that, when we place ΔDEF on ΔABC, D falls on A, E falls on B and F falls on C, also $\overline{DE}$ falls along $\overline{AB}$, $\overline{EF}$ falls along $\overline{BC}$ and $\overline{DF}$ falls along $\overline{AC}$.

Corresponding angles are : ∠A and ∠D, ∠B and ∠E, ∠C and ∠F.

Corresponding vertices are : A and D, B and E, C and F.

Corresponding sides are : $\overline{AB}$ and $\overline{DE}$, $\overline{BC}$ and $\overline{EF}$, $\overline{AC}$ and $\overline{DF}$.

Hence, three sides and three angles are the six matching parts for the congruence of triangles.

❖ EXAMPLES ❖

Ex.1 Write the correspondence between the vertices, sides and angles of the triangles XYZ and MLN, if ΔXYZ ≅ ΔMLN.

Sol. By the order of letters, we find that

X ↔ M, Y ↔ L and Z ↔ N

∴ XY = ML, YZ = LN, XZ = MN

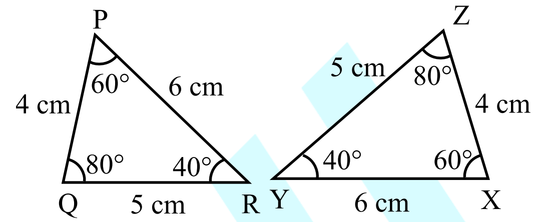
Also ∠X = ∠M, ∠Y = ∠L and ∠Z = ∠N.

Ex.2 In following pairs of triangles, find the correspondence between the triangles so that they are congruent.

In ΔPQR : PQ = 4 cm, QR = 5 cm, PR = 6 cm, ∠P = 60°, ∠Q = 80°, ∠R = 40°.

In ΔXYZ : XY = 6 cm, ZY = 5 cm, XZ = 4 cm, ∠X = 60°, ∠Y = 40°, ∠Z = 80°

Sol. Let us draw the triangles and write the measures of their corresponding parts along with them.



From the above figures, we note that

PQ = XZ, QR = YZ, PR = XY

and ∠P = ∠X, ∠Q = ∠Z, ∠R = ∠Y

∴ P ↔ X, Q ↔ Z and R ↔ Y

Hence, ΔPQR ≅ ΔXZY

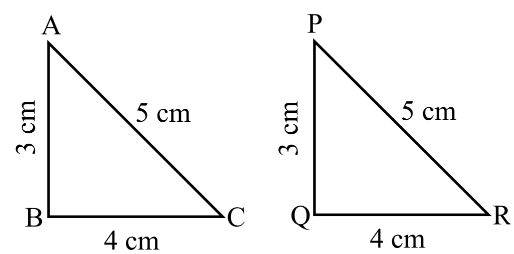
➤ CRITERIA FOR CONGRUENCE OF TRIANGLES

1. SSS Congruence Criteria (Condition)

Two triangles are congruent, if three sides of one triangle are equal to the corresponding three sides of the other triangle.

❖ EXAMPLES ❖

Ex.3 Two triangles, ABC and PQR have been drawn such that AB = 3 cm, BC = 4 cm and AC = 5 cm. Also PR = 5 cm, QR = 4 cm and PQ = 3 cm.



Examine the congruence of triangles by method of superposition. Also verify the congruence by equality of six corresponding elements of the triangles.

Sol. Trace a copy of a ΔABC and super-impose it on ΔPQR. We find that the triangles cover each other exactly, so that $A \leftrightarrow P$, $B \leftrightarrow Q$ and $C \leftrightarrow R$ i.e., $\Delta ABC \cong \Delta PQR$.

Also measure the angles of the triangles and fill the information in the following table :

Triangle ABC	Triangle PQR	Difference
$\angle A =$	$\angle P =$	$\angle A - \angle P =$
$\angle B = 90°$	$\angle Q = 90°$	$\angle B - \angle Q = 0$

We find that in all cases the difference is either zero or very close to zero, which may be treated as zero.

So we have $\angle A = \angle P$, $\angle B = \angle Q$, $\angle C = \angle R$

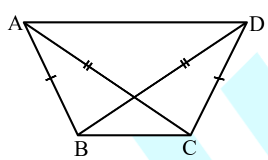
Because all sides (given) and all the angles (observed) of ΔABC are equal to the corresponding sides and angles of triangle PQR. $\therefore \Delta ABC \cong \Delta PQR$

Ex.4 ABC and DBC are two triangles drawn on a common base BC such that AB = DC and DB = AC on the same side of BC. (See figure)

Are ΔADB and ΔDAC congruent ?

If yes, state the corresponding parts. Which condition did you use to establish the congruence ?

Sol. In ΔADB and ΔDAC, we have



AB = DC (Given)

BD = CA (Given)

and AD = AD (Common side)

$\therefore$ $\Delta ADB \cong \Delta DAC$

Also, $A \leftrightarrow D$, $D \leftrightarrow A$ and $B \leftrightarrow C$

Since, the three corresponding equal parts are the sides of the triangles, therefore, SSS congruence condition is used to prove the congruence.

2. SAS Congruence Criteria (Condition)

When two sides and the included angle of one triangle is equal to the corresponding sides and the included angle of another triangle, the two triangles are congruent. This, condition of congruence is known as side-angle-side congruence. In short we write SAS condition.

❖ EXAMPLES ❖

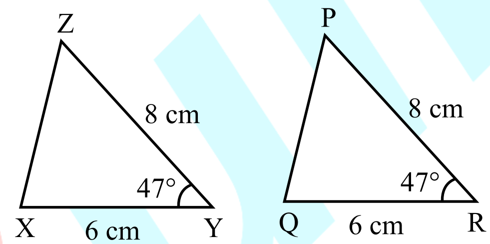
Ex.5 Given below are measures of some parts of two triangles.

Examine whether the two triangles are congruent or not by using the given information.

In ΔXYZ : XY = 6 cm, YZ = 8 cm, $\angle Y = 47°$

In ΔPQR : QR = 6 cm, PR = 8 cm, $\angle R = 47°$

Sol. Let us make a rough sketch of the triangles before examining their congruence.



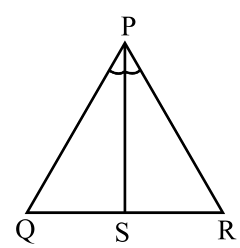
Clearly, here XY = QR = 6 cm, ZY = PR = 8 cm and $\angle Y = \angle R = 47°$ (included angles). Thus, by SAS congruence criteria $\Delta XYZ \cong \Delta QRP$.

Ex.6 Triangle PQR is isosceles with PQ = PR. Line segment PS bisects $\angle P$ and meets the side QR at point S.

(i) Is $\Delta PSQ \cong \Delta PSR$?

(ii) Can we say that QS = SR ?

Sol. In ΔPSR and ΔPSQ, the three pairs of equal parts (two sides and one angle) are as follows:



PQ = PR (given)

PS = PS (common)

and $\angle QPS = \angle RPS$ (PS bisects $\angle P$)

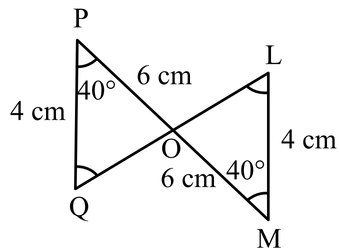
So (i) Yes, $\Delta PSQ \cong \Delta PSR$

(ii) Yes, QS = SR (corresponding sides of congruent triangles).

Ex.7 In adjoining figure, prove that

$\Delta POQ \cong \Delta MOL$

Sol. In ΔPOQ and ΔMOL, we have



$\angle P = \angle M = 40°$

$PO = OM = 6$ cm

and $PQ = ML = 4$ cm (given)

Thus, by SAS congruence criteria

$\Delta POQ \cong \Delta MOL$

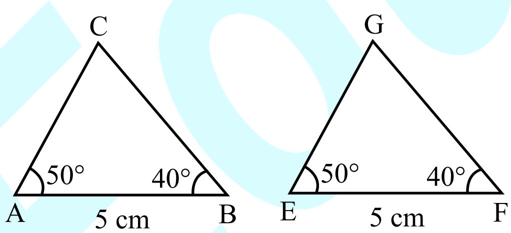
3. ASA Congruence Criteria (Condition)

Two triangles are congruent, if two angles and the included side of one is equal to the corresponding angles and side of the other.

❖ EXAMPLES ❖

Ex.8 In the following pair of triangles figure, the measure of some parts are given. Verify if the two triangles are congruent.

Sol.



In triangles, ABC and EFG.

Given, $AB = EF = 5$ cm

$\angle A = \angle E = 50°$

$\angle B = \angle F = 40°$

Therefore, by ASA congruence condition

$\Delta ABC \cong \Delta EFG$

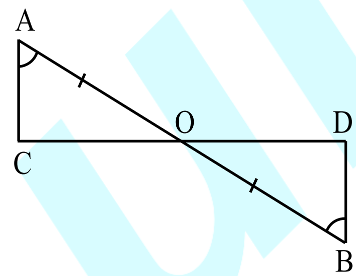
Ex.9 In figure, $AO = BO$ and $\angle A = \angle B$.

(i) Is $\angle AOC = \angle BOD$? Why ?

(ii) Is $\Delta AOC \cong \Delta BOD$ by ASA congruence condition ?

(iii) State the three facts you have used to answer (ii).

(iv) Is $\angle ACO = \angle BDO$?



Sol. (i) Yes, $\angle AOC = \angle BOD$

[Vertically opposite angles]

(ii) In ΔAOC and ΔBOD, we have

$\angle AOC = \angle BOD$

[Vertically opposite angles]

$AO = BO$ [Given]

$\angle OAC = \angle DBO$ [Given]

Therefore, by ASA congruence condition, we have

$\Delta AOC \cong \Delta BOD$

(iii) $AO = BO$, $\angle A = \angle B$ and $\angle AOC = \angle BOD$

(iv) Yes, since $\Delta AOC \cong \Delta BOD$

4. RHS Congruence Criteria (Condition)

Two right triangles are congruent, if the hypotenuse and one side of one triangle are respectively equal to the hypotenuse and a side of the other triangle.

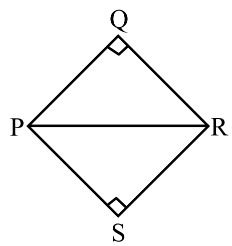
❖ EXAMPLES ❖

Ex.10 In figure, PQ = PS, PQ ⊥ QR and PS ⊥ RS.

(i) Is ΔPQR ≅ ΔPSR ? Why ?

(ii) Is QR = RS ? Why ?

Sol.



In ΔPQR and ΔPSR, we have

PQ = PS (given)

∠PQR = ∠PSR (both are right angles)

PR = PR (common side)

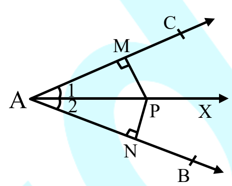
(i) ∴ By RHS congruence condition, we have

ΔPQR ≅ ΔPSR

(ii) Yes, QR = RS, because they are corresponding parts of congruent triangles.

Ex.11 AX is the bisector of ∠BAC, P is any point on AX. Prove that the perpendicular drawn from P to AB and AC are equal.

Sol. **Given :** An angle BAC bisected by AX. From any point P on AX, PM and PN are perpendiculars drawn to AB and AC respectively.



To Prove : PM = PN

Proof : In ΔAMP and ΔANP

∠M = ∠N [Each 90°]

∠1 = ∠2

[AX is bisector of ∠BAC]

AP = AP [Common]

ΔAMP ≅ ΔANP

[By AAS congruence condition]

PM = PN

[Corresponding parts of congruent triangles]

Ex.12 Complete the following statements :

(i) Two line segments are congruent if _____ .

(ii) Among two congruent angles, one has a measure of 70°, the measure of the other angle is _____ .

(iii) When we write ∠A = ∠B, we actually mean ——— .

(iv) Two circles C_1 and C_2 are congruent, then their radii will be _____ .

Sol. (i) Two line segments are congruent if **they have the same length.**

(ii) Among two congruent angles, one has a measure of 70°, the measure of the other angle is 70°.

(iii) When we write ∠A = ∠B, we actually mean **m ∠A = m ∠B.**

(iv) Two circles C_1 and C_2 are congruent, then their radii will be **equal**.

Ex.13 If ΔABC ≅ ΔFED under the correspondence ABC ↔ FED, write all the corresponding congruent parts of the triangles.

Sol. As ΔABC ≅ ΔFED

So, ∠A ↔ ∠F, ∠B → ∠E, ∠C ↔ ∠D.

$\overline{AB} \leftrightarrow \overline{FE}$, $\overline{BC} \leftrightarrow \overline{ED}$, $\overline{AC} \leftrightarrow \overline{FD}$.

Ex.14 If ΔDEF ≅ ΔBCA, write the part(s) of ΔBCA that correspond to

(i) ∠E (ii) $\overline{EF}$

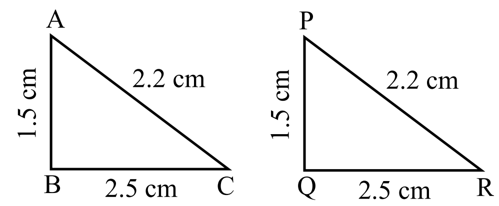
(iii) ∠F (iv) $\overline{DF}$

Sol. If ΔDEF ≅ ΔBCA, then D ↔ B, E ↔ C, F ↔ A

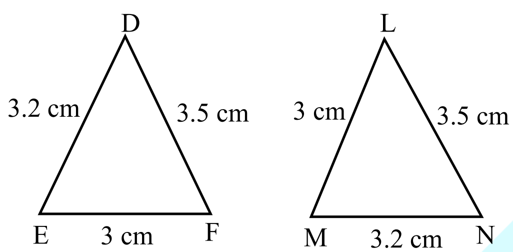
(i) ∠E = ∠C (ii) $\overline{EF} = \overline{CA}$

(iii) ∠F = ∠A (iv) $\overline{DF} = \overline{BA}$

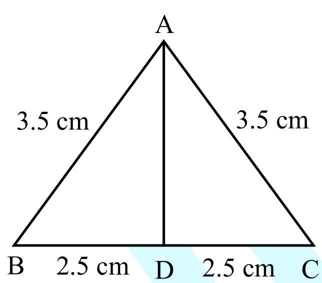
Ex.15 In the figures given below, lengths of the sides of the triangles are indicated. By applying SSS congruence rule, state which pairs of triangles are congruent. In case of congruent triangles, write the result in symbolic form.



(i)



(ii)



(iii)

Sol. (i) In ΔABC and ΔPQR

AB = PQ = 1.5 cm

BC = QR = 2.5 cm

CA = RP = 2.2 cm

∴ ΔABC ≅ ΔPQR (by SSS)

(ii) DE ≠ LM, EF ≠ MN

So, ΔDEF ≇ ΔLMN.

(iii) In ΔADB and ΔADC

AD = AD (common)

AB = AC = 3.5 cm

BD = CD = 2.5 cm

ΔADB ≅ ΔADC (by SSS)

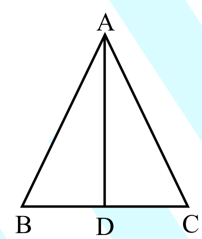
Ex.16 In figure, AB = AC and D is the mid point of $\overline{BC}$.

(i) State the three pairs of equal parts in ΔADB and ΔADC.

(ii) Is ΔADB = ΔADC ? Give reason

(iii) Is ∠B = ∠C ? Why ?

Sol. (i) In ΔADB and ΔADC



AB = AC (given)

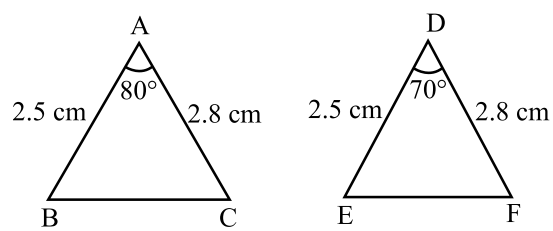
AD = AD (common)

BD = DC (∵ D is mid point of BC)

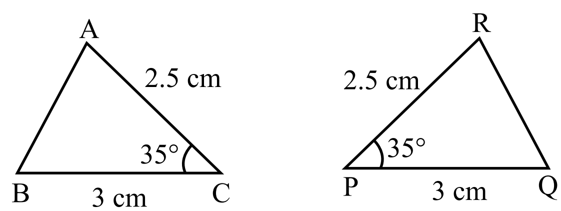
(ii) ΔADB ≅ ΔADC (by SSS property)

(iii) Yes, ∠B = ∠C (by corresponding parts of congruent triangles)

Ex.17 In figures, measures of some parts of the triangles are indicated. By applying SAS congruence rule, state the pairs of congruent triangles, if any, in each case. In case of congruent triangles, write them in symbolic form.



(i)



(ii)

Sol. (i) In ΔABC and ΔDEF

As $AB = DE = 2.5$ cm ($\because 80° \neq 70°$)

$\angle A \neq \angle D$ $AC = DF = 2.8$ cm

So, $\Delta ABC \ncong \Delta DEF$

(ii) In ΔACB and ΔRPQ

$AC = RP = 2.5$ cm

$\angle C = \angle P = 35°$ $CB = PQ = 3$ cm

$\therefore$ $\Delta ACB \cong \Delta RPQ$ (by SAS)

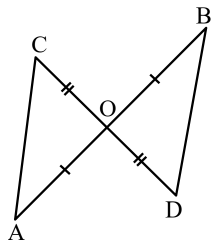
Ex.18 In figure, $\overline{AB}$ and $\overline{CD}$ bisect each other at O.

(i) State the three pairs of equal parts in two triangles AOC and BOD.

(ii) Which of the following statements are true

(a) $\Delta AOC \cong \Delta DOB$ (b) $\Delta AOC \cong \Delta BOD$?

Sol.



(i) $AO = OB$ $CO = OD$

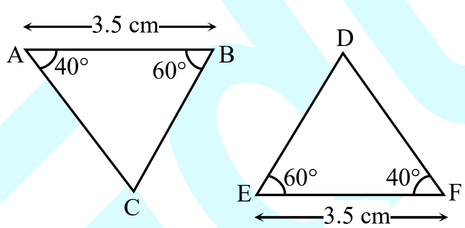
$\angle AOC = \angle BOD$

(vertically opposite angles)

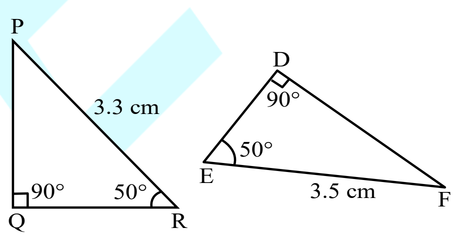
(ii) $\Delta AOC \cong \Delta BOD$ (by SAS)

Hence, (b) is true

Ex.19 In figures, measures of some parts are indicated. By applying ASA congruence rule, state which pairs of triangles are congruent. In case of congruence, write the result in symbolic form.



(i)



(ii)

Sol. (i) In ΔABC and ΔEFD.

$\angle A = \angle F$ (40° each)

$AB = EF$ (3.5 cm each)

and $\angle B = \angle E$ (60° each)

$\therefore$ $\Delta ABC \cong \Delta FED$ (by ASA)

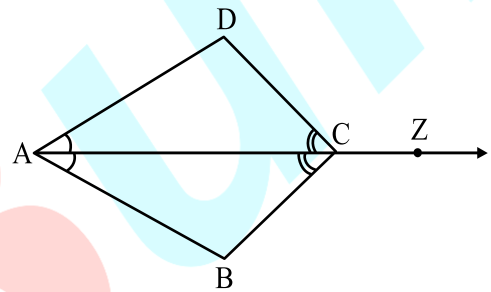
(ii) $\Delta PQR \ncong \Delta DEF$

as $\angle Q = \angle D = 90°$

$\angle E = \angle R = 50°$

$PR \neq EF$ ($\because$ 3.3 cm $\neq$ 3.5 cm)

Ex.20 In figure, ray AZ bisect $\angle DAB$ as well as $\angle DCB$.



(i) State the three pairs of equal parts in ΔBAC and ΔDAC.

(ii) Is $\Delta BAC \cong \Delta DAC$? Give reasons.

(iii) Is $AB = AD$? Justify your answer.

(iv) Is $CD = CB$? Give reasons.

Sol. (i) In ΔBAC and ΔDAC,

$\angle BAC = \angle DAC$ [$\because$ AZ bisects $\angle DAB$]

$AC = AC$ (common)

$\angle BCA = \angle DCA$ [$\because$ AZ bisects $\angle DCB$]

(ii) Yes, $\Delta BAC \cong \Delta DAC$ (by ASA)

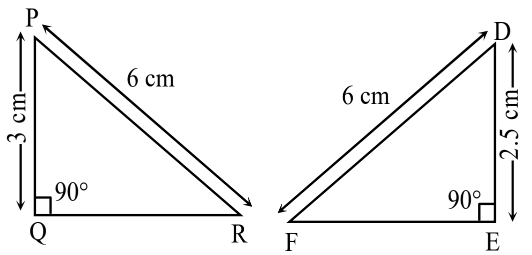
(iii) Yes, $AB = AD$

(corresponding parts of congruent triangles)

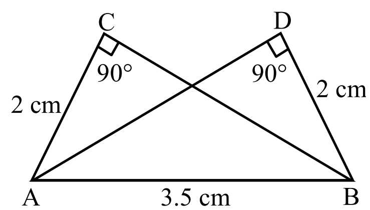
(iv) Yes, $CD = CB$

(corresponding parts of congruent triangles)

Ex.21 In figure, measures of some parts of triangles are given. By applying R.H.S. congruence rule, state which pairs of triangles are congruent. In case of congruent triangles, write the result in symbolic form.



(i)



(ii)

Sol. (i) In ΔPQR and ΔDEF

as PR = DF = 6 cm

∠Q = ∠E = 90°

But PQ ≠ DE (as 3 cm ≠ 2.5 cm)

So, ΔPQR ≇ ΔDEF

(ii) In ΔCAB and ΔDBA

∠C = ∠D = 90° each

AB = AB = 3.5 cm

CA = DB = 2 cm

ΔCAB ≅ ΔDBA (by RHS)

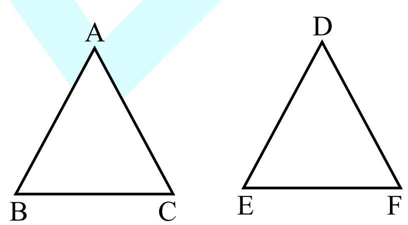
Ex.22 Which congruence criterion do you use in the following ?

(a) Given AC = DF

AB = DE

BC = EF

So, ΔABC ≅ ΔDEF

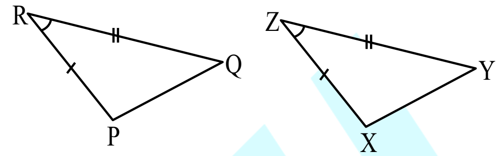


(b) Given RP = ZX

∠PRQ = ∠XZY

RQ = ZY

So, ΔPQR ≅ ΔXYZ

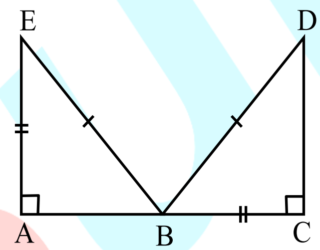


(c) Given EB = DB

AE = BC

∠A = ∠C = 90°

So, ΔABE ≅ ΔCDB



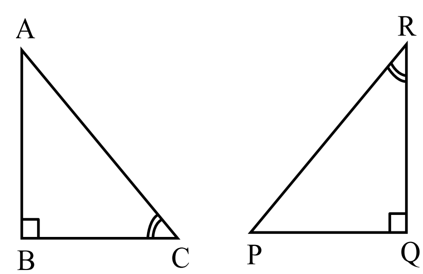
Sol. (a) ΔABC ≅ ΔDEF (by SSS)

(b) ΔPQR ≅ ΔXYZ (by SAS)

(c) ΔEAB ≅ ΔDCB (by RHS)

Ex.23 If ΔABC and ΔPQR are to be congruent, name one additional pair of corresponding parts. What criterion did you use ?

Sol. To prove ΔABC ≅ ΔPQR,



We need one additional pair of corresponding parts which is

BC = QR

As, if ∠ABC = ∠PQR (90° each)

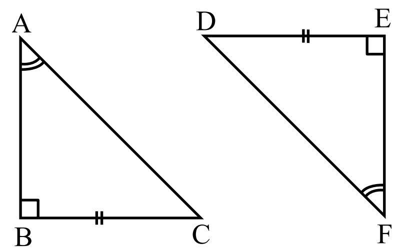
BC = QR

∠ACB = ∠PRQ (given)

ΔABC ≅ ΔPQR (by ASA)

Ex.24 Explain why ΔABC ≅ ΔFED.

Sol. In ΔABC and ΔFED



∠A = ∠F (given)

∠B = ∠E (90° each) (given)

⇒ ∠C = ∠D ….(i)

(third ∠C = third ∠D)

So, now ∠B = ∠E = 90°

BC = DE (given)

∠C = ∠D [From (i)]

ΔABC = ΔFED (by ASA)

IMPORTANT POINTS TO BE REMEMBERED

(1) Two figures are congruent, if they have the same shape and size.

(2) Two line segments say $\overline{AB}$ and $\overline{CD}$ are congruent if they have equal lengths, we write this as AB ≅ CD.

(3) Two squares are congruent if measure of their side is same.

(4) Two rectangles are congruent if they have the same length and breadth.

(5) Two circles are congruent if they have same radius.

(6) Two triangles are congruent if the three sides and three angels of one triangle are equal to the corresponding sides and angles of the other triangle.

(7) Two triangles are congruent if three sides of one triangle are equal to corresponding three sides of another triangle (SSS congruence condition).

(8) Two triangles are congruent if two sides and the included angle of one triangle are equal to corresponding sides and included angle of the other triangle (SAS congruence condition). 'Triangle' can be denoted as 'Δ'.

(9) Two triangles are congruent if two angles and included side of one triangle are equal to the corresponding angles and included side of the other (ASA congruence condition).

(10) Two right triangles are congruent if the hypotenuse and one side of one triangle are equal to the hypotenuse and corresponding side of other triangle.

(11) Two congruent figures are equal in area, but the figures having equal area may not be congruent.

(12) There is no such thing as AAA congruence of two triangles.

(13) Two triangles with equal corresponding angles need not be congruent. In such a correspondence, one of them can be enlarged copy of the other. (They would be congruent only if they are exact copies of one another).