Electromagnetic Induction

6

MAGNETIC FLUX

* The concept of magnetic lines of force was first proposed by Faraday. Faraday tried to provide the lines of force a real form assuming them as stretched rubber bands. In modern physics the concept of magnetic lines of force is used in visualization or explaination of principles only.

* The tangent drawn at any point on a line of force in a magnetic field shows the direction of magnetic field shows the direction of magnetic field at that point and the density of lines of force, i.e., the number of lines of force crossing normally a unit area indicates the intensity of magnetic field.

* The lines of force in a uniform magnetic field are parallel straight lines equidistant from each other. Where the lines of force are near each other, B is higher and where the lines of force are far apart, B is lesser.

* The number of lines of force crossing a given surface is called flux from that surface. If it generally represented by ϕ. Flux is a property of a vector field. If the vector field is a magnetic field, then the flux is called magnetic flux.

* The magnetic flux crossing a certain area is equal to the scalar product of the vector field $(\vec{B})$ and the vector area $(d\vec{A})$, that is

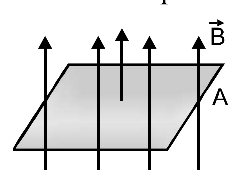
Magnetic flux $d\phi = \vec{B}.d\vec{A} = BdA\cos\theta$

where θ is the angle between the vector field $(\vec{B})$ and the vector area $d\vec{A}$. $\phi = \int \vec{B}.d\vec{A}$

For a uniform magnetic field $\vec{B}$ and plane surface $\vec{A}$ $\quad \phi = \vec{B}.\vec{A} = BA\cos\theta$

(Note : In real sense area is a scalar quantity, but it can be treated as whose direction is in the direction of perpendicular pointing outward from the surface)

* Magnetic flux is a scalar quantity.



* If a plane surface of area A is imagined in a uniform magnetic field $\vec{B}$, then

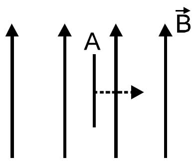
(a) when a surface is perpendicular to the magnetic filed, the lines of force crossing that area, i.e., the magnetic flux is

$\phi = BA$ because $\theta = 0$, $\cos\theta = 1$

(b) if the surface is parallel to the field, then

$\theta = 90^o,\ \cos\theta = 0$

$\therefore\ \phi = BA\cos 90 = 0$

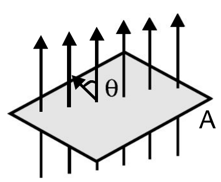


(c) when the normal to the surface makes an angle θ with the magnetic field, the magnetic flux is

$\phi = BA\cos\theta$

* If the magnetic flied is not uniform and the surface in not plane, then the element $\vec{dA}$ of the surface may be assumed as plane and magnetic field $\vec{B}$ may also be assumed as uniform over his element. Thus the magnetic flux coming out from this element is

$$d\phi = \vec{B}.\vec{dA}$$



Hence magnetic flux coming out from the entire surface

$$\phi = \int_S \vec{B}.\vec{dA}$$

* For a closed surface the vector area element pointing outward is positive and the vector area element pointing inward is negative.
* Magnetic lines of force are closed curves because free magnetic poles do not exist. Thus for a closed surface whatever is the number of the lines of force entering it, the same number of lines of force come out from it. As a result for a closed curve

$$\phi = \oint_S \vec{B}.\vec{dA} = 0 \quad \text{or} \quad \nabla.\vec{B} = 0$$

Thus the net magnetic flux coming out of a closed surface is equal to zero.

* For a normal plane surface in a magnetic field

$$\phi = BA \text{ Hence} \qquad B = \frac{\phi}{A}$$

Thus the magnetic flux passing normally from a surface of unit area is equal to magnetic induction B. Therefore $\frac{\phi}{A}$ is also called flux density.

* **Unit of magnetic flux** - In M.K.S. system the unit of magnetic flux is weber (Wb) and in C.G.S. system unit of magnetic flux is maxwell.

  1 weber = 10^8 maxwell

  The M.K.S unit of flux density or magnetic induction is weber/m^2. It is also called tesla.

  1 tesla = 1 weber/m^2

  The C.G.S unit of magnetic flux density is gauss.

  1 gauss = 1 maxwell/cm^2

  1 tesla = 1 weber/m^2 = 10^4 gauss

* **Dimensions of magnetic flux** :

  $\phi = BA$

$$[\phi] = \frac{N}{A-m} \times m^2 = \frac{N-m}{A}$$

$$\frac{(kg-m-s^2)\times m}{A} = kg\text{-}m^2\text{-}s^2\text{-}A^{-1}$$

$$= M^1L^2T^{-2}A^{-1}$$

Solved Examples

Ex.1 The plane of a coil of area $1m^2$ and having 50 turns is perpendicular to a magnetic field of 3×10^{-5} weber/m^2. The magnetic flux linked with it will be -

(1) 1.5×10^{-3} weber (2) 3×10^{-5} weber
(3) 15×10^{-5} weber (4) 150 weber

Sol. $\phi = NBA\cos\theta$

but $N = 50$, $B = 3 \times 10^{-5}$ wb/m^2,

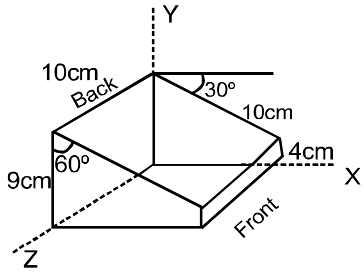
$A = 1m^2$, $\theta = 0$ or $\phi = NBA$

$= 50 \times 3 \times 10^{-5} \times 1$

$= 150 \times 10^{-5}$ weber

$\therefore$ Answer will be (1)

Ex.2 Consider the fig. A uniform magnetic field of 0.2 T is directed along the +x axis. Then what is the magnetic flux through top surface of the figure ?



(1) zero (2) 0.8m Wb
(3) 1.0m Wb (4) –1.8m Wb

Sol. the magnetic flux is

$\phi = BA \cos\theta$

for the top surface, the angle between normal to the surface and the x-axis is

$\theta = 60º$, and

$B = 0.2$ T, $A = 10 \times 10 \times 10^{-4}$ m^2

Thus $\phi = 0.2 \times 10^{-2} \times \cos(60)$

$= 10^{-3}$ Wb.

The correct answer is thus (3)

ELECTROMAGNETIC INDUCTION

* The change in magnetic flux linked with a circuit induces an e.m.f. in it. This effect is called electromagnetic induction.
* Current induced in the circuit due to induced e.m.f. is called induced current.

FARADAY'S LAWS OF ELECTROMAGNETIC INDUCTION

* **When the magnetic field linked with a circuit changes, emf or current is induced in the circuit.**
* As long as the magnetic field linked with the circuit changes, e.m.f or current is induced in the circuit.
* The e.m.f induced in the circuit is directly proportional to the rate of change of magnetic flux. Let the e.m.f induced in the circuit be E and the rate of change of magnetic flux be $\frac{d\phi}{dt}$, then

$$E \propto \frac{d\phi}{dt}$$

If the are N turns in the coil, then

$$E \propto N\frac{d\phi}{dt}$$

* If rate of change of magnetic flux is 1 Wb/s, then the e.m.f induced will be 1 volt.

LENZS LAW

* This law gives the direction of induced e.m.f
* According to this law, the direction of induced emf or current in a circuit is such that it opposes the cause due to which it is produced.

  Since the magnitude of induced e.m.f due to electromagnetic induction is obtained from Faraday's law and its direction is obtained from Lenz's law. By combining these laws the expression for the e.m.f induced by the electromagnetic induction can be written as

$$E = -\frac{d\phi}{dt}$$

  where negative sign shows that induced emf opposes the cause due to which it is produced.
* This law is based on the law of conservation of energy.
* In electromagnetic induction mechanical energy is converted into electrical energy.
* In electromagnetic induction magnetic energy is also converted into electrical energy.
* In a circuit e.m.f is induced due to change of magnetic and mechanical energy.

Solved Examples

Ex.3 When a small piece of wire passes between the magnetic poles of a horse-shoe magnet in 0.1 sec, emf of 4×10^{-3} volt is induced in it. The magnetic flux between the poles is -

(1) 4×10^{-2} weber (2) 4×10^{-3} weber
(3) 4×10^{-4} weber (4) 4×10^{-6} weber

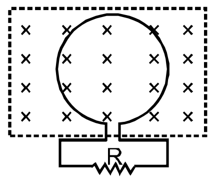
Sol. $E = -\frac{d\phi}{dt}$

or $d\phi = -Edt = (0 - \phi)$ or $\phi = 4 \times 10^{-3} \times 0.1$

$= 4 \times 10^{-4}$ weber

$\therefore$ Answer will be (3)

Ex.4 The magnetic flux passing perpendicular to the plane of the coil and directed into the paper is varying according to the relation. $\phi = 3t^2 + 2t + 3$ where ϕ is in milli webers and t is in seconds. Then the magnitude of emf induced in the loop when t = 2 second.



(1) 31 mV (2) 19 mV

(3) 14 mV (4) 6 mV

Sol. The induced emf

$$E = -\,d\phi/dt = \frac{d}{dt}(3t^2 + 2t + 3) \times 10^{-3}$$

(because given flux is in mWb). This

$E = (-6t - 2) \times 10^{-3}$ at t = 2 sec,

$E = (-6 \times 2 \times -2) \times 10^{-3}$

$= -14$ mV

The correct answer is (3)

Ex.5 There is a window of metallic frame (120 cm × 50cm) in a wall which is parallel to the magnetic meridian. The resistance of the window is 0.01Ω. The induced charge flowing in the window when it is opened through 90º will be, if the horizontal component of earth's magnetic field is H = 0.35 Gauss.

(1) 2.16×10^{-3} coulomb (2) zero

(3) 2.16×10^{-6} coulomb (4) 2.16×10^{-9} coulomb

Sol. $q = \frac{\phi_2 - \phi_1}{R} = \frac{AH}{R}$ $\because$ B = H

$\therefore \phi_1 = 0$ Because the window lies in the magnetic meridian

$\phi_2 = AH$ because on opening through 90º the plane of the window becomes perpendicular to magnetic field

$$\therefore \quad q = \frac{0.6 \times 0.36 \times 10^{-4}}{0.01}$$

coulomb = 2.16×10^{-3} coulomb

The correct answer is (1)

Ex.6 The normal magnetic flux passing through a coil changes with time according to following equation $\phi = 10t^2 + 5t + 1$ where ϕ is in milliweber and t is in second. The value of induced e.m.f produced in the coil at t = 5sec will be -

(1) zero (2) 1V (3) 2 V (4) 0.105 V

Sol. $e = \frac{d\phi}{dt} = -\frac{d}{dt}[10t^2 + 5t + 1] \times 10^{-3}$

$= -[10 \times 10^{-3}\,(2t) + 5 \times 10^{-3}]$

$e = -[2 \times 10^{-2}\,t + 5 \times 10^{-3}]$

at t = 5 second

$e = -[10 \times 10^{-2} + 5 \times 10^{-3}]$

$= -[0.1 + 0.005]$

$= -[0.105]$ V

$|e| = 0.105$ V

Hence the correct answer will be (4)

INDUCED EMF, CURRENT AND CHANGE IN A CIRCUIT

* If e.m.f induced in a circuit is E and rate of change of magnetic flux is $d\phi/dt$, then from Faraday's and Lenz's law

$$E \propto -\left(\frac{d\phi}{dt}\right)$$

or $$E = -K\left(\frac{d\phi}{dt}\right)$$

where K is constant, equal to one.

Thus $E = -\left(\frac{d\phi}{dt}\right)$

* If there are N turns in the coil, then induced e.m.f will be

$$E = -N\left(\frac{d\phi}{dt}\right)$$

* If the magnetic flux linked with the circuit changes from ϕ_1 to ϕ_2, in time t, then induced e.m.f will be

$$E = -N\left(\frac{d\phi}{dt}\right)$$

$$= -N\left(\frac{\phi_2 - \phi_1}{t}\right)$$

* If the resistance of the circuit is R, then the current induced in the circuit will be

$$I = \frac{E}{R} = -\frac{N(\phi_2 - \phi_1)}{tR} \text{ ampere}$$

$$= -\frac{N}{R}\left(\frac{d\phi}{dt}\right) \text{ ampere}$$

* Induced current depends upon

(a) the resistance of the circuit $I \propto \frac{1}{R}$

(b) the rate of change of magnetic flux $I \propto \left(\frac{d\phi}{dt}\right)$

(c) the number of turns (N) ; $I \propto N$

* If $R = \infty$, that is, the circuit is open, then the current will not flow and if the circuit if closed, then current will flow in the circuit.

* If change dq flows in the circuit in time dt, then the induced current will be

$$I = \left(\frac{dq}{dt}\right) \text{ or } dq = I\ dt$$

but $I = \frac{1}{R}\left(\frac{d\phi}{dt}\right)$

$$\therefore \ dq = \frac{1}{R}\left(\frac{d\phi}{dt}\right) dt = \frac{1}{R} d\phi$$

or $q = \int \frac{d\phi}{R} = \frac{\phi_2 - \phi_1}{R}$

If N is the number of turns, then

$$dq = \frac{Nd\phi}{R},\ q = \frac{N(\phi_2 - \phi_1)}{R}$$

* Charge flowing due to induction does not depend upon the time but depends upon the total change in the magnetic flux. It does not depend upon the rate or time interval of the change in magnetic flux. Whether the change in magnetic flux be rapid or slow, the charge induced in the circuit will remain same.

Thus $q \propto d\phi$ or $q \propto (\phi_2 - \phi_1)$

* Induced charge depends upon the resistance of the circuit, i.e., $q \propto 1/R$

If $R = \infty$ or circuit is open, q = 0 that is charge will not flow in the circuit.

If $R \neq \infty$ or circuit is closed, then $q \neq 0$, that is, induced charge will flow in the circuit

* The e.m.f induced in the circuit does not depend upon the resistance of the circuit.
* The e.m.f induced in the circuit depends upon the following factors -

(a) Number of turns (N) in the coil,

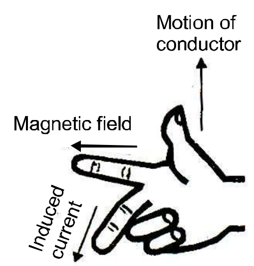
(b) Rate of change of magnetic flux,

(c) Relative motion between the magnet and the coil,

(d) Cross-sectional area of the coil,

(e) Magnetic permeability of the magnetic substance or material placed inside the coil.

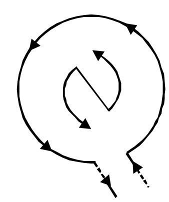
FLEMINGS RIGHT HAND RULE



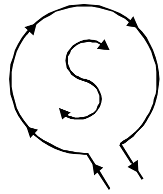
This law is used for finding the direction of the induced e.m.f or current. According to this law, if we stretch the right hand thumb and two nearby fingers perpendicular to one another and first finger points in the direction of magnetic field and the thumb in the direction of motion of the conductor then the central finger will point in the direction of the induced current.

DIRECTION OF INDUCED EMF AND CURRENT (APPLICATIONS OF LENZS LAW)

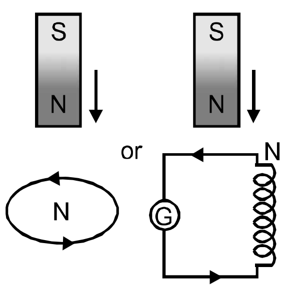
* If current flowing in a coil appears anti-clockwise, then that plane of coil will behave like a N-pole.



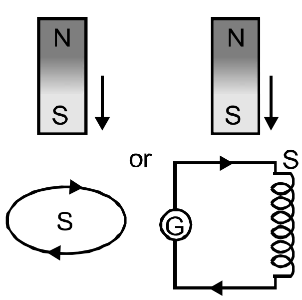
* If current flowing in the coil appears clock-wise, then that plane of coil will behave like a S-pole.



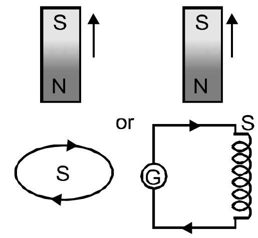
* If the north pole of magnet is moved rapidly towards the coil, then according to Lenz's law the induced current will flow in the coil in such a direction so as to oppose the motion of the magnetic. This will happen only when the face of the coil towards the magnet behaves as a north pole, that is, the induced current will appear flowing in anti-clockwise direction as seen from the side of magnet. Thus a force of repulsion will be produced between the magnet and the coil coming near each other which will oppose the motion of the magnet. Hence some mechanical work has to be done to move the magnet near the coil against this opposing force and this work (mechanical energy) is converted into current (electrical energy)



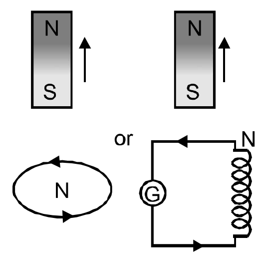
* On bringing a south pole towards a coil the current induced in the coil will appear flowing in clockwise direction as observed from the side of magnet and the face of the coil towards the magnet will behave as a south pole.



* On moving the north pole of a magnet away from the coil the current induced in the coil will appear flowing in the clockwise direction as seen from the side of magnet and the face of the coil towards the magnet will behave as a south pole.

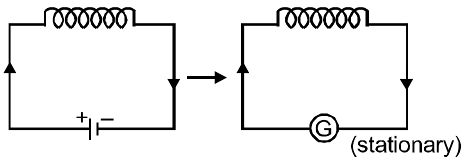


* On moving the south pole of a magnet away from the coil the current induced in the coil will appear flowing in the anticlockwise direction as seen from the side of magnet and the face of the coil towards the magnet will behave like a north pole.

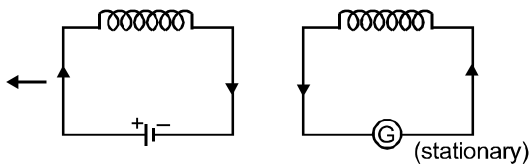


* If a magnet is allowed to drop freely through a copper coil, then an induced current will be produced in the coil. This current will oppose the motion of the magnet, as a result the acceleration of the falling magnet due to gravity will be less than 'g'. If coil is cut somewhere, then the emf will be induced in the coil only but current will not be induced. In absence of induced current the coil will not oppose the motion of magnet and the magnet will fall through the coil with the acceleration equal to g.
* If a magnet is dropped freely in a hollow long metal cylinder, then the acceleration of falling magnet will be less than gravitational acceleration. As the magnet keeps on falling inside a tube, its acceleration will continue to decrease and after traversing a certain distance the acceleration will become zero. Now the magnet will fall with constant velocity. This constant velocity is called terminal velocity.

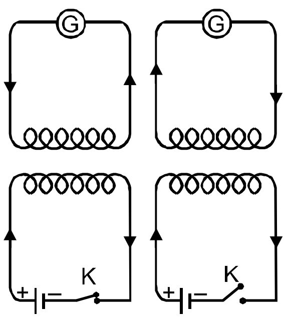
* If a current carrying coil is brought near another stationary coil, then the direction of induced current in the second coil will be in the direction of current in the moving coil.



* If a current carrying coil is taken away from a stationary coil, then the direction of induced current in the second coil will be opposite to the direction of current in the moving coil.

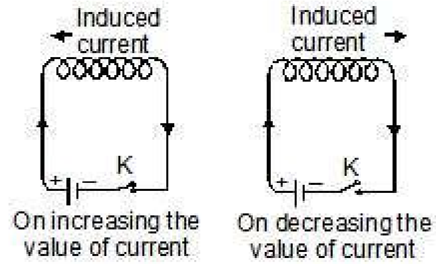


* In the coils arranged in the following way, when the key K connected in the circuit of primary coil, is pressed, an induced current is produced in the secondary coil. The direction of induced current in the secondary coil is opposite to the direction of the current in the primary coil. (From lenz's law)

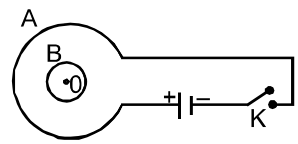


When the key is opened, then the current in the primary coil is reduced to zero but current is induced in the secondary coil. The direction of this induced current is same as the direction of current in the primary coil. (Form Lenz's law).

* When current is passed through a coil, the current flowing through the coil changes. As a result the magnetic flux linked with the coil changes. Due to this a current is induced in the coil. If the current induced in the coil flows in the opposite direction of the applied current. If the current flowing in the coil is decreased, then the current induced in the coil flows in the direction of the applied current so as to oppose the decrement of the applied current.

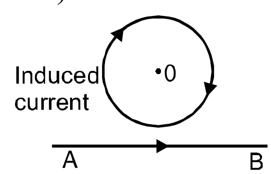


* Two coil A and B are arranged as shown in the figure. On pressing the key K current flows through the coil A in the clockwise direction and the current induced in the coil B will flow in the anticlockwise direction . (From Lenz's law)

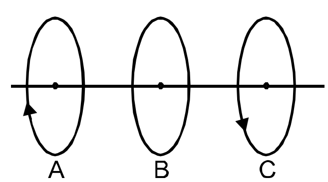


On opening the key K the current flowing through the coil A will go on decreasing. Thus the current induced in the coil B will flow in the clockwise direction.

* If current flows in a straight conductor from A to B as shown in figure, then the direction of current induced in the loop placed near it will be clockwise. (From Lenz's law).

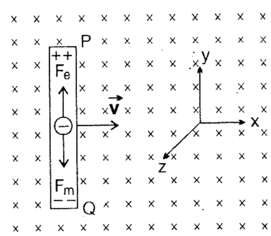


* Three indentical circular coils A, B and C are arranged coaxially as shown in figure. The coils A and C carry equal currents as shown. Coils B and C are fixed in position. If coil A is moved towards B, then the current induced in coil B will be in clockwise wise direction because the direction of current induced in the coil B will be to oppose the motion of coil A. (The face of A towards B is south pole, then the face of B towards A is south pole). There is no relative motion between B and C so current will not be induced in coil B due to coil C.



INDUCED EMF OF A CONDUCTING ROD IN A UNIFORM MAGNETIC FIELD

* If a conducting rod of length ℓ is in the plane of paper, magnetic field $\vec{B}$ is pointing into the plane of paper and velocity $\vec{v}$ of the rod is pointing towards +x-axis, then the force $\vec{F} = q(\vec{v} \times \vec{B})$ acts downwards $(-\hat{j})$ on the free electrons present in the conductor due to the magnetic field. As a result electrons are concentrated at the Q end of the conductor due to which Q end of the conductor becomes negatively charged and P end positively charged.



* The emf induced between the ends of the rod is

$$E = -(\vec{v} \times \vec{B}).\vec{\ell}$$

If $\vec{v}$ and $\vec{B}$ are perpendicular to each other, then induced e.m.f will be

$$E = -B\ell v$$

* If the direction of motion of the conductor makes an angle θ with the magnetic field, then induced e.m.f (potential difference) between the both ends of the conductor will be

$$E = -B\ell v \sin\theta$$

(iv) If the direction of motion of the conductor is in the direction of magnetic field, that is, $\theta = 0$, then no e.m.f will be induced between the ends of the moving conductor, i.e., E = 0

* If a galvanometer is connected between the ends of the conductors, then the current will be

$$i = \frac{E}{R} = \frac{-B\ell v}{R}$$

* Force acting on the conductor

$$\vec{F} = i\,(\vec{\ell} \times \vec{B}) = -\frac{Bv\ell}{R}\ell B\hat{x}$$

* Power dissipated in moving the conductor

$$P = Fv = \frac{B^2\ell^2 v}{R}v = \frac{B^2\ell^2 v^2}{R}$$

* **When a train moves on rails, then a potential difference between the ends of the axle of the wheels is induced** because the axle of the wheels of the train cuts the vertical component B_v of earth's magnetic field and so the magnetic flux linked with it changes and the potential difference or emf is induced.

$$E = B\ell v$$

where ℓ is the length of the axle and v is the speed of the train.

* A potential difference or e.m.f across the wings of an aeroplane flying horizontally at a definite height is also induced because aeroplane cuts the vertical component B_v of earth's magnetic field.

Thus induced emf $E = B_v \ell v$ volt

where ℓ is the length of the wings of an aeroplane and v is the speed of the aeroplane.

(a) If an aeroplane is landing down or taking off and its wings are in the east-west direction, then the potential difference or emf will be induced across the wings.

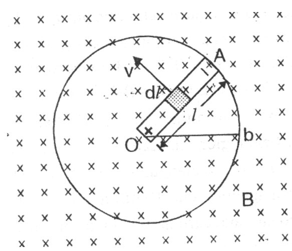
(b) If an aeroplane is landing down or taking off and its wings are in the north-south direction, then no potential difference or emf will be induced.

* Keeping a conducting wire in the east-west direction, if it is allowed to fall freely, then emf will be induced across the wire. If the conducting wire is kept in the north-south and then it is allowed to fall freely, no emf will be induced in the wire.

* If the orbital plane of an artificial satellite of metallic surface is coincident with the equatorial plane of the earth, then no emf will be induced. If orbital plane makes an angle with the equatorial plane, then emf will be induced on it.

INDUCED EMF DUE OF ROTATION OF A CONDUCTING ROD IN UNIFORM MAGNETIC FIELD

A conducting rod of length ℓ, whose one end is fixed, is rotated in the acticlockwise direction about an axis passing through the fixed end and perpendicular to the direction of uniform magnetic field $\vec{B}$ (pointing inward normal to the plane of paper) with an angular velocity, then



(a) a force $\vec{F} = q(\vec{v} \times \vec{B})$ acts on the free electrons radially outwards due to which electrons are displaced towards the free end. Thus fixed end of the rod becomes positively charged and free end negatively charged.

(b) The emf induced across the ends of the conductor

$E = \frac{1}{2}\omega B\ell^2$

(c) If the angular frequency of the conducting rod is n, then $\omega = 2\pi n$

$\therefore$ induced emf $E = B\pi n\ell^2$

(d) If the area covered by the moving rod is A, then $A = \pi\ell^2$

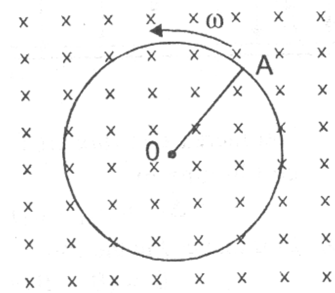
$\therefore \quad E = ABn$

(e) emf induced in the conducting rod depends upon

$E \propto B,\ E \propto n \quad \text{and} \quad E \propto \ell^2 \ \text{or A}$

INDUCED EMF DUE TO ROTATION OF A METALLIC DISC IN A UNIFORM MAGNETIC FIELD

If a metallic disc of radius R is rotating in the anticlockwise direction about its own axis perpendicular to a uniform magnetic field $\vec{B}$ (pointing inward normal to the plane of paper) with an angular velocity ω, then



(a) The force acts on the free electrons due to magnetic field due to which these electrons are displaced and are concentrated at its circumference. In this way the rim of the disc becomes negative charged and its centre positively charged.

(b) The emf induced between the centre and the circumference

$E = \frac{1}{2}B\omega R^2$

If n is the rotational frequency $\quad \omega = 2\pi n$

$\therefore \ E = \frac{1}{2}\omega(2\pi n)R^2 = B\pi R^2 n$

(c) If A is the area of the disc, then E = ABn

Solved Examples

Ex.7 A copper disc of radius 0.1m rotates about its centre with 10 revolutions per second in a uniform magnetic field of 0.1 tesla. The emf induced across the radius of the disc is

(1) $\pi/10$V (2) $2\pi/10$V

(3) 10πmV (4) 20πmV

Sol. The induced emf between centre and rim of the rotating disc is

$E = \frac{1}{2}B\omega R^2$

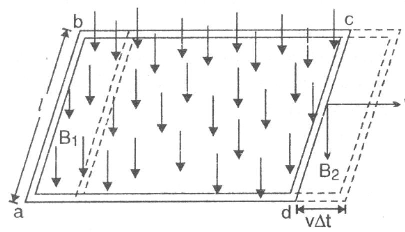
$= \frac{1}{2} \times 0.1 \times 2\pi \times 10 \times (0.1)^2$

$= 10\pi \times 10^{-3}$ volt

The correct answer is (3)

A RECTANGULAR LOOP MOVING IN A NON-UNIFORM MAGNETIC FIELD WITH A CONSTANT VELOCITY

* A rectangular coil abcd is placed in a non-uniform magnetic field perpendicular to it such that the magnetic field at the arm ab is B_1 and at arm cd is B_2 ($B_1 > B_2$). The lengths of the ab and cd arms are ℓ. If coil is moved normal to the magnetic field with a velocity v, then



(a) Net increase in flux crossing through the coil in time Δt

$$\Delta\phi = (B_2 - B_1)\ell v\ \Delta t$$

(b) Emf induced in the coil

$$E = (B_1 - B_2)\ell v$$

* If the resistance of the coil is R, then the current induced in the coil

$$I = \frac{E}{R} = \frac{(B_1 - B_2)}{R}\ell v$$

* Resultant force acting on the coil

$$F = I\ell(B_1 - B_2) \text{ (towards left)}$$

* The work done against the resultant force

$$W = (B_1 - B_2)^2 \frac{\ell^2 v^2}{R}\Delta t \text{ joule}$$

Energy supplied in this work appears in the form of electrical energy in the circuit.

* Energy supplied due to flow of current I in time Δt

$$H = I^2 R\ \Delta t$$

or $H = (B_1 - B_2)^2 \dfrac{\ell^2 v^2}{R}\Delta t$ joule

or $H = W$

* In electromagnetic induction electrical energy is produced by the mechanical energy which is then transformed into heat energy by current flows. As these energies are equal in magnitude it is proved that energy is conserved, i.e., in electromagnetic induction law of conservation of energy is obeyed.

* **When magnetic field is uniform** - If a coil is moved in a uniform magnetic field with constant velocity, then the magnetic flux crossing this coil does not change with time. Hence emf induced in it is zero, i.e., in this case

$$B_1 = B_2$$

$$\therefore\ E = (B_1 - B_2)v\ell = 0$$

* **When magnetic field is uniform and in a limited region** - In this case as long as the moving coil remains completely in the magnetic field $\vec{B}$, induced emf remains zero. But as soon as arm of the coil enters a region of zero magnetic field, that is, $B_2 = 0$, $B_1 = B$, induced emf becomes $E = (B_1 - B_2)v\ell = Bv\ell$

As soon as the coil is totally out of the magnetic field region, induced emf becomes zero again.

* If a rectangular loop is moved in a uniform magnetic field $\vec{B}$ with a velocity $\vec{v}$, then induced emf and current will not be produced because the magnetic flux linked with the coil does not change. But if loop is drawn out of the magnetic field, then emf and current will be induced in it.

Solved Examples

Ex.8 A aeroplane having a distance of 50 metre between the edges of its wings is flying horizontally with a speed of 360 km/hour. If the vertical component of earth's magnetic field is 4×10^{-4} weber/m^2, then the induced emf between the edge of it wings will be -

(1) 2mV (2) 2V (3) 0.2 V (4) 20V

Sol. $E = B\ell v$

$$= 4 \times 10^{-4} \times 50 \times \frac{360 \times 1000}{60 \times 60} = 2V$$

$\therefore$ Answer will be (2)

Ex.9 Two rail tracks, insulated from each other and the ground, are connected to millivoltmeter. What is the reading of the milli voltmeter when train passes at a speed of 180 km/hr along the track, given that the horizontal component of earth's magnetic field is 0.2×10^{-4} Wb/m^2 and rails are separated by 1 metre.

(1) 1 mV (2) 10 mV (3) 100 mV (4) 1 V

Sol. The induced emf

$E = B\ell v$

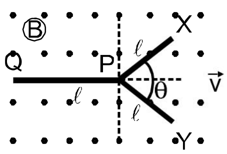
$= 0.2 \times 10^{-4} \times 1 \times 180 \times 1000/3600$

$= 0.2 \times 18/3600$

$= 1 \times 10^{-3}$

The correct answer is thus (1)

Ex.10 A conducting wire in the shape of Y, with each side of length is moving in a uniform magnetic field B, with a uniform speed v as shown in fig. The induced emf at the two ends X and Y of the wire will be -



(a) zero (2) $2B\ell v$

(3) $2B\ell v \sin(\theta/2)$ (4) $2B\ell v \cos(\theta/2)$

Sol. The induced emf $E = -(\vec{v}\times\vec{B}).\vec{\ell}$

for the part PX, $\vec{v}\perp\vec{B}$, and the angle between $-(\vec{v}\perp\vec{B})$ direction (the dotted line in figure) and $\vec{\ell}$ is $(90 - \theta/2)$. Thus,

$E_p - E_x = vB\ell \cos(90 - \theta/2)$

$= vB\ell \sin(\theta/2)$

similarly $E_y - E_p = vB\ell \sin(\theta/2)$

Therfore induced emf between X and Y be

$E_{yx} = 2Bv\ell \sin(\theta/2)$

The correct answer is (3)

Ex.11 A wire of length 4m, lying at right angles of a magnetic field of $(2\hat{i}+4\hat{j})$ Tesla, is moving with a velocity of $(4\hat{i}+6\hat{j}+8\hat{k})$ m/s. The e.m.f induced between the ends of wire will be -

(1) 4V (2) 8V (3) 16V (4) 32V

Sol. $e = -\vec{\ell}.(\vec{V}\times\vec{B})$

$$\vec{V}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 6 & 8 \\ 2 & 4 & 0 \end{vmatrix} = -32\hat{i}+16\hat{j}+4\hat{k} \qquad \vec{\ell} = 4\hat{k}$$

$e = -4\hat{k}\,[-32\hat{i}+16\hat{j}+4\hat{k}]$

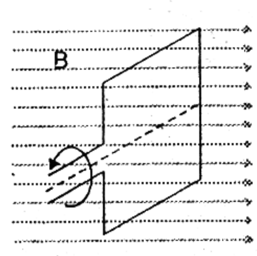
$e = -16V$

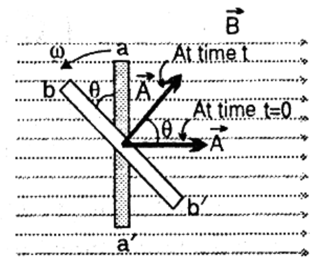
$|e| = 16$ volt

Hence the correct answer will be (3)

ROTATION OF A RECTANGULAR COIL IN A UNIFORM MAGNETIC FIELD

* If the figure a conducting rectangular coil of area A and turns N is shown. It is rotated in a uniform magnetic field B about a horizontal axis perpendicular to the field with an angular velocity ω. The magnetic flux linked with the coil is continuously changing due to rotation.





θ is the angle between the perpendicular to the plane of the coil and the direction of magnetic field.

* The magnetic flux passing through the rectangular coil depends upon the orientation of the plane of the coil about its axis.

* Magnetic flux passing through the coil

$\phi = \vec{B}.\vec{A} = BA \cos\theta = BA \cos \omega t$

If there are N turns in the coil, then the flux linked with the coil $\phi = BAN \cos \omega t$

* Since ϕ depends upon the time t, the rate of change of magnetic flux

$$\frac{d\phi}{dt} = -BAN\omega \sin \omega t$$

* According to Faraday's law, the emf induced in the coil

$$E = -\frac{d\phi}{dt}$$

or $E = BAN\,\omega \sin \omega t$

$BAN\,\omega$ is the maximum value of emf induced,

Thus writing

$BAN\omega = E_0$

$\therefore\; E = E_0 \sin \omega t$

This equation represents the instantaneous value of emf induced at time t.

* If the total resistance of circuit along with the coil is R, then the induced current due to alternating voltage

$$I = \frac{E}{R} = \frac{E_0}{R} \sin \omega t \qquad \text{or} \qquad I = I_0 \sin\omega t$$

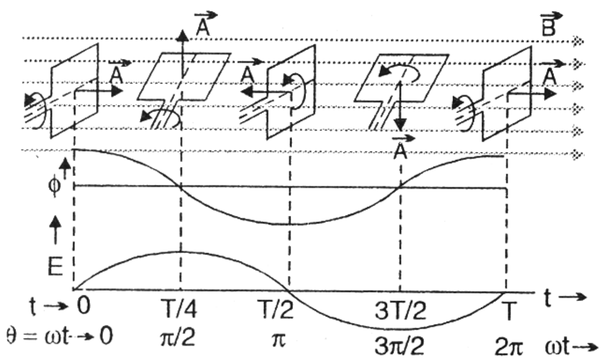
where $I_0 = \frac{E_0}{R}$

is the maximum value of current.

* The magnetic flux linked with coil and the emf induced at different positions of the coil in one rotational cycle are shown in the following table :

Time	Position of coil	Magnetic flux	Induced emf
t = 0	Plane of the coil normal to $\vec{B}(\theta = 0)$	ϕ = NBA = maximum flux	E = 0
t = T/4	Plane of the coil parallel to $\vec{B}(\theta = 90^\circ)$	ϕ = 0	E = NBA ω = maximum
t = T/2	Plane of the coil normal to $\vec{B}$ again (θ = 180º)	ϕ = –NBA	E = 0
t = 3T/4	Plane of the coil parallel to $\vec{B}$ again (θ = 270º)	ϕ = 0	E = –NBA ω
t = T	Plane of the coil normal to $\vec{B}$ (θ = 360º)	ϕ = NBA	E = 0

* The variations of magnetic flux linked with the coil and induced e.m.f at different times given in the above table are shown in the following figure.



* The phase difference between the instantaneous magnetic flux and induced emf is $\pi/2$.
* The ratio of E_{max} and ϕ_{max} is equal to the angular velocity of the coil, Thus

$$\frac{E_{max}}{\phi_{max}} = \frac{NBA\omega}{NBA} = \omega$$

* If $\theta = \frac{\pi}{4} = 45^\circ$, then

$$\phi = \frac{NBA}{\sqrt{2}} \quad \text{and} \quad E = \frac{NBA\ \omega}{\sqrt{2}}$$

In this case the ratio of the induced emf and the magnetic flux is equal to the angular velocity of the coil. Thus

$$\frac{E}{\phi} = \frac{NBA\omega}{\sqrt{2}} / \frac{NAB}{\sqrt{2}} = \omega$$

* The direction of induced emf in the coil changes during one cycle so it is called alternating emf and current induced due to it is called alternating current. This is the principle of AC generator.

Solved Examples

Ex.12 The phase difference between the emf induced in the coil rotating in a uniform magnetic field and the magnetic flux associated with it, is

(1) π (2) $\pi/2$ (3) $\pi/3$ (4) zero

Sol. ϕ = NAB cos ωt and E = NAB ω sin ωt

Hence the phase difference between ϕ and E will be $\pi/2$.

$\therefore$ Answer will be (2)

Ex.13 A coil has 20 turns and area of each turn is 0.2 m^2. If the plane of the coil makes an angle of 60º with the direction of magnetic field of 0.1 tesla, then the magnetic flux associated with the coil will be -

(1) 0.4 weber (2) 0.346 weber

(3) 0.2 weber (4) 0.02 weber

Sol. ϕ = n(B da cosθ)

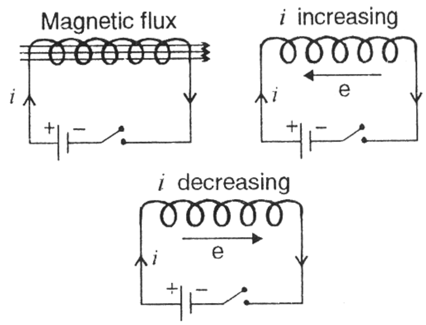
$= 20 \times 0.1 \times 0.2 \cos (90^\circ - 60^\circ)$

$= 20 \times 0.1 \times 0.2 \times \frac{\sqrt{3}}{2} = 0.346$ weber

$\therefore$ Answer will be (2)

SELF INDUCTION AND SELF INDUCTANCE

* On changing the current flowing in a coil, the magnetic flux linked with the coil changes. As a result emf is induced in the coil. This phenomenon is called self induction.



* Self induction not only occurs in current carrying coil but also in each current carrying circuit or its component. The phenomenon is specially important in current carrying coils.
* The magnetic flux linked with the coil is directly proportional to the current flowing in the coil. That is,

$$\alpha \propto I$$

or $\phi = LI$

where L is a constant called self inductance or coefficient of self inductance.

or $L = \dfrac{\phi}{I}$

* If $I = 1$ ampere, than $\phi = L$

Thus the self inductance of the circuit is equal to that magnetic flux which is linked with the circuit when one ampere current flows through it.

* Induced emf $E = -\left(\dfrac{d\phi}{dt}\right) = -\dfrac{d}{dt}(LI)$

or $E = -L\left(\dfrac{dI}{dt}\right)$

* If $\dfrac{dI}{dt} = 1$ ampere/s, then $E = L$

Thus the self inductance of a coil is equal to the emf induced in the coil when the rate of change of current in it is 1 ampere/s.

* Unit of L – henry = weber/ampere

$$1\text{mH} = 10^{-3}\text{H}$$

$$1\mu\text{H} = 10^{-6}\ \text{H}$$

Dimension of self inductance

$$\text{henry} = \frac{\text{volt} \times \text{time}}{\text{ampere}}$$

$$= \frac{(\text{joule}/\text{coulomb}) \times \text{second}}{\text{ampere}} = \text{kg–m}^2\text{–s}^{-2}\text{A}^{-2}$$

Dimensions of $L = ML^2T^{-2}A^{-2}$

* Self inductance of a coil

$$L = \frac{\phi}{I} = \frac{BNA}{I} = \frac{\mu NI}{2R}\frac{NA}{I} = \frac{\mu N^2 \pi R}{2}$$

$$= \mu_r \mu_0 \frac{N^2 \pi R}{2} \text{ henry}$$

* Self inductance of a solenoid

$$L = \frac{\phi}{I} = \frac{BNA}{I} = \frac{\mu NI}{\ell}\frac{NA}{I}$$

$$= \frac{\mu_0 \mu_r N^2 A}{\ell} \text{ henry}$$

* Self inductance of a coil depends upon

(a) the number of turns in the coil,

(b) the radius or cross-section area of the coil,

(c) the magnetic permeability of the material forming to core of the coil,

* If two coils of self inductances L_1 and L_2 are placed at a large distance and connected

(a) in series, then the equivalent self inductance $L = L_1 + L_2$

(b) in parallel, then equivalent self inductance

$$L = \frac{L_1 L_2}{L_1 + L_2}$$

In the above derivation it is assumed that there is no mutual induction between them.

Solved Examples

Ex.14 The current changes in an inductance coil of 100mH from 100 mA to zero in 2 milisecond. The e.m.f induced in the coil will be -

(1) –5V (2) 5V (3) –50V (4) 50V

Sol. $E = -L\frac{dI}{dt} = -100 \times 10^{-3}$

$\frac{(0-100)\times 10^{-3}}{2\times 10^{-3}} = 5.0$ V

$\therefore$ Answer will be (2)

Ex.15 Keeping the number of turns of a coil constant the area of its cross-section is doubled, its self induction will become -

(1) half (2) doulbe

(3) four times (4) $\sqrt{2}$ times

Sol. $L = \frac{\mu_0\mu_r N^2 R}{2}$, so $L \propto R$

$\therefore L \propto R \propto \sqrt{A}$

$\therefore L = \sqrt{2}$ times

$\therefore$ Answer will be (4)

Ex.16 Two inductance coils of inductances L_1 and L_2 are at sufficient distance apart. On connecting them in series the equivalent inductances will be -

(1) L_1L_2 (2) $\sqrt{L_1L_2}$

(3) $L_1 - L_2$ (4) $L_1 + L_2$

Sol. In inductors when connected in series as same current will flow in both and the total magnetic flux linked with them will be equal to the sum of the magnetic fluxes linked with them individually.

$\therefore \phi = \phi_1 + \phi_2$

$Li = L_1i_1 + L_2i_2$

$\therefore L = L_1 + L_2$

$\therefore$ Answer will be (4)

Ex.17 In question (28) if the two coils are connected in parallel, then the equivalent inductance will be -

(1) $\frac{L_1L_2}{L_1+L_2}$ (2) $\frac{L_1+L_2}{L_1L_2}$

(3) $\sqrt{L_1L_2}$ (4) L_1L_2

Sol. If currents i_1 and i_2 are flowing in the coils respectively, then

$i = i_1 + i_2$

or $\frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt}$

$\therefore -\frac{e}{L} = -\frac{e}{L_1} - \frac{e}{L_2}$

or $L = \frac{L_1L_2}{L_1+L_2}$

$\therefore$ Answer will be (1)

Ex.18 A current increases uniformly from zero to one ampere in 0.01 second, in a coil of inductance 10 mH. Find the direction and value of the induced emf.

Sol. $E = -L\ dI/dt$

$= -10 \times 10^{-3} \times \frac{(1-0)}{0.01} = -1$ volt

Ex.19 What will be the self inductance of a coil of 100 turns if a current of 6 ampere produces a magnetic flux of 6×10^3 Maxwell in it ?

(1) 2×10^{-3} H (2) 0.5×10^{-3} H

(3) 10^{-3} H (4) zero

Sol. $L = \frac{N\phi}{I} = \frac{100\times 6\times 10^3\times 10^{-8}}{6} = 10^{-3}$ H

Hence the correct answer will be (3)

Ex.20 The length of solenoid of diameter 0.05 m and number of turns 500 turns/cm is 1m. When 2 ampere current is passed in it then the value of magnetic flux will be -

(1) 12.34 weber (2) 6.7 weber

(3) 25 weber (4) zero

Sol. $L = \mu_0 n^2 \pi r^2 \ell$

$= 4\pi = 10^{-7} \times (50000)^2 \times \pi \times (0.025)^2 \times 1$

$L = 6.17$ H

$\phi = L\ I = 6.17 \times 2 = 12.34$ weber

Hence the correct answer will be (1)

Ex.21 A coil is wound on a rectangular frame. Keeping the number of turns per unit length constant, if the linear dimensions of the frame are doubled then the coefficient of self induction of the coil will become

(1) 4 times (2) 12 times

(3) 16 times (4) 8 times

Sol. $\ell' = 2\ell,\ b' = 2b,\ n' = n$ ......(1)

$L' = \mu A' n'^2 \ell'$

$L' = \mu \ell' b' n'^2 \ell'$ ......(2)

From equation (1) and (2)

$L' = \mu(2\ell)^2 (2b)\ n^2$

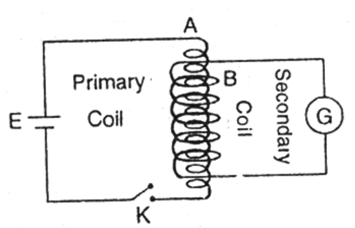
$= 8\mu\ell^2\ bn^2$

$L' = 8L$

Hence the correct answer will be (4)

MUTUAL INDUCTION AND MUTUAL INDUCTANCE

* **On changing the current in one coil if the magnetic flux linked with a second coil changes and as a result emf is induced in that coil, then this phenomenon is called mutual inductance**.



The coil in which current is allowed to flow in called primary coil and in which emf is induced due to mutual induction is called secondary coil. In figure A and B are primary and secondary coils.

* If at any time the current in the coil A is I_A and the magnetic flux linked with the coil B due to magnetic field produced by current I_A is ϕ_B, then

$\phi_B \propto I_A$

or $\phi_B = MI_A$

where M is a constant which gives information about the magnitude of mutual induction between the two coils. It is called mutual inductance.

* If $I_A = 1$, then $\phi_B = M$

Thus the mutual inductance between two coils linked magnetically is numerically equal to the magnetic flux linked with the secondary coil, when unit current flows in the primary coil,

The emf induced in the secondary coil according to Faraday's and Lenz's laws

$$E_B = -\frac{d\phi_B}{dt} = -M\frac{dI_A}{dt}$$

Hence $M = \dfrac{E_B}{-(dI_A/dt)}$

$$= \frac{\text{emf induced in secondary coil}}{\text{rate of decrease of current in primary coil}}$$

If $-\dfrac{dI_A}{dt} = 1$, then $M = E_B$

Thus the mutual inductance between two coils is numerically equal to the emf induced in the secondary coil when the current in the primary coil changes at the rate of one ampere/s.

* **Unit of M** : In M.K.S. system unit of mutual inductance is henry

$$M = \frac{E_B}{-(dI_A/dT)} = \frac{\phi_B}{I_A}$$

$\therefore$ 1 henry $= \dfrac{1\text{ volt}}{1\text{ ampere/s}} = \dfrac{1\text{ weber}}{\text{ampere}}$

$= \dfrac{(\text{joule/coulomb})\text{s}}{\text{ampere}} = J/A^2$

* **Dimensions of M** :

$$M = \frac{J}{A^2} = \frac{\text{joule}}{\text{ampere}^2} = \frac{\text{newton} \times \text{metre}}{\text{ampere}^2}$$

$$= \frac{\text{kg} \times \text{metre} \times \text{sec}^{-2} \times \text{metre}}{\text{ampere}^2}$$

$= ML^2T^{-2}A^{-2}$

* Mutual inductance between the coils depends upon the number of turns in the coils, area and the permeability of the core placed inside the coils. Larger is the magnitude of M, more is the emf induced in the secondary coil.

* Out of the two coils coupled magnetically one coil can be taken as primary and the other coil as secondary. Thus mutual inductance

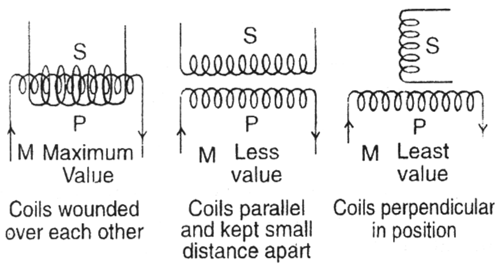
$M_{AB} = M_{BA} = M$

* Mutual inductance between two coaxial solenoids of length ℓ and cross-sectional area A is

$$M = \frac{\mu_0 N_1 N_2 A}{\ell}$$

where N_1 and N_2 are the number of turns in the two coils respectively.

* If two coils are wound one over the other, then mutual inductance will be maximum and it will be less in other arrangements.



M and L have the following relation :

$M \propto \sqrt{L_1 L_2} \qquad M = K\sqrt{L_1 L_2}$

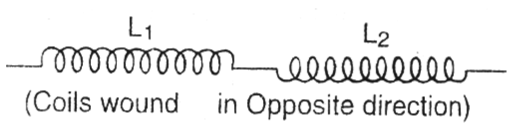
where K is a coupling constant of coils and its value varies from 0 to 1.

(a) If K = 0, then there will be no coupling between the coils, that is magnetic flux produced by the primary coil is not linked with the secondary coil.

(b) If K = 1, then both coils are coupled together with maximum transfer to energy, that is, magnetic flux produced by the primary coil is totally linked with the secondary coil.

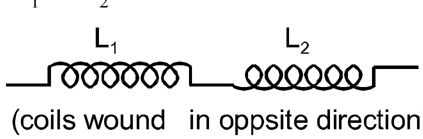
* If two coils of self inductances L_1 and L_2 are coupled in series such that their windings are in the same sense and mutual inductance between them is M, then the equivalent inductance will be

$L = L_1 + L_2 + 2M$



If two coils are coupled in series such that their windings are in opposite sense then equivalent inductance will be

$L = L_1 + L_2 - 2M$



Solved Examples

Ex.22 A 50 Hz a.c. current of crest value 1A flows through the primary of a transformer. If the mutual inductance between the primary and secondary be 1.5 H, the crest voltage induced in secondary is

(1) 75V (2) 150V (3) 225V (4) 300V

Sol. The crest value is attained in T/4 time where T is the time period of A.C.

Thus dI = 1A in dt = T/4 sec.

$T = \frac{1}{50}$ or $dt = \frac{1}{200}$

The induced emf is $|E_2| = M\frac{dI_1}{dt}$

$= 1.5 \times \frac{1}{(1/200)}$

$= 1.5 \times 200 = 300$ V

The correct answer is (4)

Ex.23 A coil of radius 1 cm and 100 turns is placed at the centre of a long solenoid of radius 5 cm and 8 turn/cm. The value of coefficient of mutual induction will be -

(1) 3.15×10^{-5} H (2) 6×10^{-5} H
(3) 9×10^{-5}H (4) zero

Sol. $M = \mu_0 n_1 N_2 \pi r^2$

$= 4\pi \times 10^{-7} \times 800 \times 100\ \pi \times (0.01)^2$

$= 3.15 \times 10^{-5}$ H

Hence the correct answer will be (1)

Ex.24 The coefficients of self induction of two coils are 0.01 H and 0.03 H respectively. If they oppose each other then the resultant self induction will be, if M = 0.01H

(1) 2H (2) 0.02H (3) 0.02H (4) zero

Sol. $L = L_1 + L_2 - 2M$

$= 0.01 + 0.03 - 2 \times 0.01$

Hence the correct answer will be (3)

ENERGY STORED IN AN INDUCTANCE

* At a certain time if current i flows through a coil of emf inductance L the emf induced in the coil due to variation of current

$$E = -L\frac{di}{dt}$$

* The work done by the induced emf against the current flow per second.

$$-iE = Li\frac{di}{dt}$$

* If current in the coil increases from 0 to i_{max}, then the total work done

$$W = \int_0^{i_{max}} Li\frac{di}{dt}\,dt = \frac{1}{2}Li_{max}^2$$

* Total work done by the induced emf against the current is stored in the inductor in the form of magnetic energy.

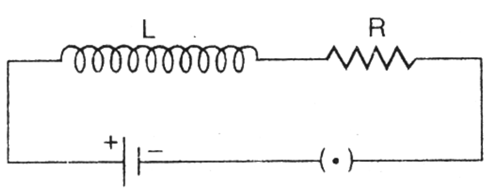
$\therefore$ Energy stored in the inductor $= \frac{1}{2}Li_{max}^2$

* The energy stored in unit volume of magnetic field is called magnetic energy density.

$$u_B = \frac{B^2}{2\mu_0} J/m^3$$

GROWTH AND DECAY OF CURRENT IN L-R CIRCUIT

* When the key K is pressed or closed, the current in a circuit increases but it does not attain the steady state value immediately because emf is induced in the coil which opposes the growth of current.



* The steady state value of current is attained after some time.
* When key is opened, the current in the circuit does not decreases to zero at once but decreases to zero after some time because emf is induced in the coil which opposes the decay of current.
* The time taken by the current to attain the steady value, depends upon (L/R) of the circuit. Growth and decay of current in the circuit are according to the exponential law.

The equation of growth of current is

$$I = I_0\,(1 - e^{-Rt/L})$$

and the equation of decay of current is

$$I = I_0e^{-Rt/L}$$

where I_0 is the maximum or steady value of current.

* Time constant of circuit = L/R
* Lesser the value of L/R more swiftly the current will increase and on opening the key current will decrease.
* At time $t = \left(\frac{L}{R}\right)$

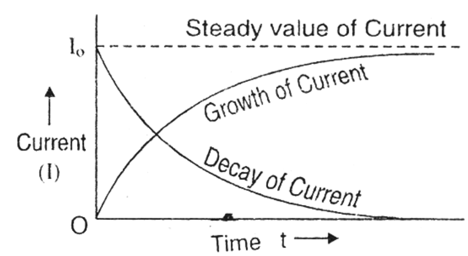
(a) When circuit is closed $I = 0.632\ I_0$

i.e., current is 63.2% of steady state current.

(b) When circuit is open $I = 0.368\ I_0$

i.e., current is 36.8% of steady state current.

* The growth and decay of current is shown in the figure :



* When circuit is opened, then

$$R = \infty \text{ and } \frac{L}{R} = 0$$

In this case the current decays very swiftly and the emf induced in the circuit is large. As a result sparking takes place in the air gap between the terminals of key at the time of break and the current flows for a moment. The sparking can be avoided by increasing L so that L/R becomes more and the current in the circuit decays slowly.

Due to this reason when sometimes a bulb is switched off, it gets fused.

Solved Examples

Ex.25 A solenoid has a self inductance of 50H and a resistance of 25 ohm. If it is connected to a battery of 100 volt, the time during which the current grows from zero to half of its maximum value, will be -

(1) 4s (2) 2s (3) 1.4s (4) 1.2s

Sol. $I = I_0(1 - e^{-t/(L/R)})$

$\frac{1}{2} = I - e^{-tR/L}$

$\therefore \; t = \frac{L}{R}\log_e 2 = \frac{L}{R}(2.3\log_{10} 2)$

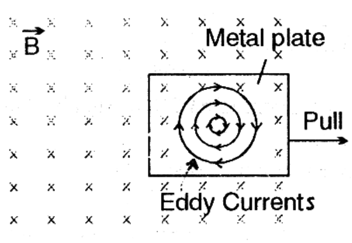
$= \frac{50 \times 2.3 \times 0.3}{25}$

= 1.38 = 1.4 s

$\therefore$ Answer will be (3)\

EDDY CURRENT

* When a conductor is placed in a changing magnetic field, induced emf is produced in it. As a result local currents are produced in the conductor. These local currents are called eddy currents.
* If a conducting material is moved in a magnetic field, then eddy currents are also produced.
* Eddy currents flows in closed paths.
* There is loss of energy due to eddy currents and it appears in the form if heat.
* In order to minimize the energy loss in the form of heat due to eddy currents the core of dynamo, motor or transformer is not taken as a single piece of soft iron but in the form of a peck of thin sheets insulated from each other by a layer of insulating varnish, called laminated core. This device increases the resistance for the eddy currents. In this way eddy currents are considerably reduced and loss of energy becomes less.



* **Uses of eddy currents :**

(a) Moving coil galvanometer

(b) Induction furnace

(c) Dead beat galvanometer

(d) Speedometer

(e) Electric brakes

GENERATOR OR DYNAMO

* Generator or dynamo is an electrical device which converts mechanical energy into electrical energy.
* Working of generators is based on the principle of electromagnetic induction.
* **Generators are of two types :**

(a) A.C. generator : If the current produced by the generator is alternating, then the generator is called A.C. generator.

(b) D.C. generator : If the current produced by the generator is direct current, then the generator is called D.C. generator.

* Generator consists of the following parts.

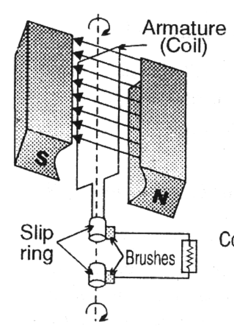
(a) Armature (coil) (b) Magnet

(c) Slip rings (d) Brushes

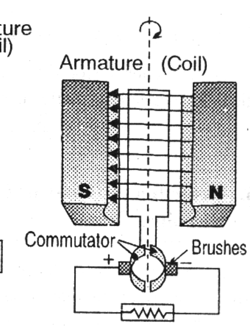
In D.C. generator commutator is used in place of slip rings.

* In order to produce the magnetic field in big generators several magnetic poles are used. In these generators the armature coils are kept stationary and magnetic pole pieces are made to rotate around the armature. The frequency of alternating current produced by generator of multi poles is

$$= \frac{\text{number of poles} \times \text{rotational frequency}}{2} = \frac{Nn}{2}$$



AC Generator



DC Generator

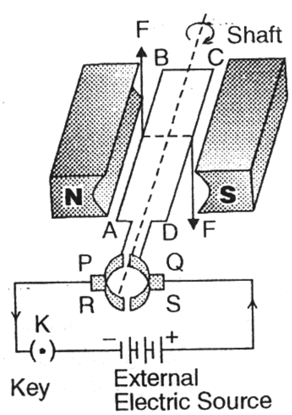
* **Energy loss in generators :** The loss of energy is due to the following reasons :
  (a) Flux leakage, (b) Copper losses,
  (c) Eddy current losses, (d) Hysteresis losses,
  (e) Mechanical losses
* **Efficiency of generator :** Practical efficiency of a generator

$$= \frac{\text{Electrical power generated by the generator}}{\text{Mechanical energy given to the generator}}$$

Practical efficiencies of big generators are about 92% to 95%.

MOTOR

* It converts electrical energy into mechanical energy.
* When a current carrying conductor (coil) is placed in a magnetic field, a couple acts on it which makes the coil to rotate.
* Electric motors are of two types :
  (a) Alternating current motor (AC motor)
  (b) Direct current motor (DC motor)
* D.C. motor consists of the following parts :
  (a) armature (b) magnet
  (c) commutator (d) brushes
* **Back E.M.F :** When current from an external electric source is passed through the armature of the electric motor, armature coil rotates in the magnetic field. In cuts the magnetic lines of force as a result emf is induced in it. According to Lenz's law this induced emf opposes the rotation of the armature i.e., the emf induced works opposite to the emf applied by the external electric source and opposes the motion of the armature. This induced emf is called back emf. Greater is the speed of armature coil, greater is the back emf.



* At the time of start of the motor back emf is almost zero and the current flowing in the motor in maximum. As the speed of the armature coil increases, back emf also increases. When the coil increases, back emf also increases. When the coil attains maximum speed, the induced emf becomes constant and current reduced to minimum.
* Back emf is directly proportional to the angular velocity ω of rotation of armature and the magnetic field B, i.e., for constant magnetic field back emf. $e \propto \omega$ or $e = K\omega$ where K is a constant.
* If E is applied emf, e is the back emf and R is the resistance of the coil (armature), then the current flowing through the coil will be

$$i = \frac{E - e}{R}$$

or $E = e + iR$

but $e = K\omega$

$$\therefore \quad i = \frac{E - K\omega}{R}$$

* In the beginning, i.e. at the time of start of the motor

$$\omega = 0 \quad \therefore i = \frac{E}{R}$$

In this case current will be maximum.

* As armature coil is made from copper wire so its resistance is very small. When motor starts running, a very heavy current passes through the armature coil in the beginning. Due to which motor may get burnt. To prevent the motor from burning at the time of start a special variable resistance is connected in series with the armature, which is called starter.
* High resistance is connected in series with the armature coil with the help of starter at the time of start of the motor. As the motor starts picking up speed, the resistance is gradually reduced till it becomes zero.
* Starter is used in a high power motors but not in the low power motors because its coil starts rotating with a very high speed in a short time
* Power of electric motor = ie

* Efficiency of motor

$$\eta = \frac{\text{Work done by the motor}}{\text{Energy taken from the electric source by the motor}}$$

$$= \frac{W}{P} \times 100\%$$

or $\eta = \frac{\text{Back emf}}{\text{Applied emf}} \times 100\% = \frac{e}{E} \times 100\%$

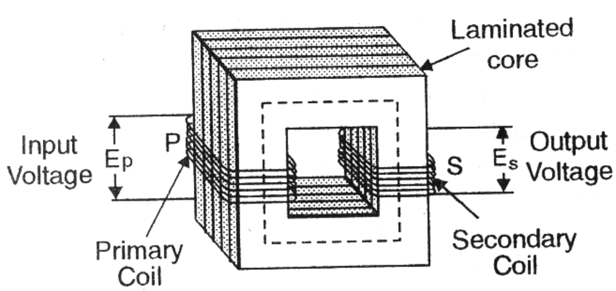
Generally the efficiency of the motor is from 80% to 90%.

TRANSFORMER

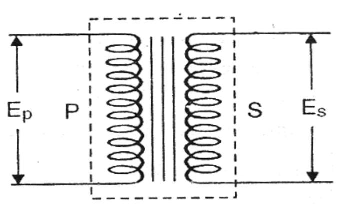
* It is an instrument which changes the magnitude of alternating voltage or current.
* The magnitude of D.C. voltage or current can not be changed by it.
* It works with alternating current but not with direct current.
* It converts magnetic energy into electrical energy.
* It works on the principle of electro-magnetic induction.
* It consists of two coils :

(a) Primary coil : in which input voltage is applied.

(b) Secondary coil : from which output voltage is obtained.



* The frequency of the output voltage produced by the transformer is same as that of input voltage, i.e., frequency remains unchanged.



* Transformer core is laminated and is made of soft iron.
* Let the number of turns in the primary coil be n_p and voltage applied to it be E_p and the number of turns in the secondary coil be n_s and voltage output be E_s, then

$$\frac{E_s}{E_p} = \frac{n_s}{n_p} = K$$

Thus the ratio of voltage obtained in the secondary coil to the voltage applied in the primary coil is equal to the ratio of number of turns of respective coils. This ratio is represented by K and it called transformer ratio.

* If $n_s > n_p$, then $E_s > E_p$ and $K > 1$. The transformer is called step-up transformer.
* If $n_s < n_p$, then $E_s < E_p$ and $K < 1$. The transformer is called step-down transformer.
* In ideal transformer

Input power = output power

$$E_pI_p = E_sI_s$$

where i_p – current in primary coil

I_s – current in secondary coil

or $\frac{I_p}{I_s} = \frac{E_s}{E_p} = \frac{n_s}{n_p} = K$ or $\frac{I_s}{I_p} = \frac{E_p}{E_s} = \frac{1}{K}$

Thus the ratio of currents in the secondary coil and the primary coil is inverse of the ratio of respective voltages.

* As the voltage changes by the transformer, the current changes in the same ratio but in opposite sense, i.e., the current decreases with the increase of voltage and similarly the current increases with the decrease of voltage. Due to this reason the coil in which voltage is lesser, the current will be higher and therefore this coil is thicker in comparison to the other coil so that it can bear the heat due to flow of high current.
* In step-up transformer

$n_s > n_p$, $K > 1$

$\therefore$ $E_s > E_p$ and $I_s < I_p$

and in step down transformer

$n_s < n_p$, $K < 1$

$\therefore$ $E_s < e_p$ and $I_s > I_p$

* If Z_p and Z_s are impedances of primary and secondary coils respectively, then

$$\frac{E_s}{E_p} = \frac{I_p}{I_s} = \frac{n_s}{n_p} = \sqrt{\frac{Z_s}{Z_p}}$$

* Law of conservation of energy is applicable in the transformer.
* Efficiency of tranformer

$$\frac{\text{Power obtained from secondary coil}}{\text{Power applied in primary coil}} \times 100\%$$

Generally the efficiency of transformers is found in between 70% to 95%.

* Energy losses in transformers : Losses of energy are due to following reasons :

(a) Copper losses due to resistance of coils

(b) Eddy current losses in core.

(c) Hysteresis losses in core.

(d) Flux leakage due to poor linking of magnetic flux.

* **Uses of transformer** :

(a) Step down and step up transformer are used in electrical power distribution.

(b) Audio frequency transformer are used in radiography, television, radio, telephone etc.

(c) Ratio frequency transformer are used in radio communication.

(d) Transformers are also used in impedance matching.

Solved Examples

Ex.26 If the number of turns in the primary and secondary coils of a transformer are 100 and 500 respectively, then the ratio of voltage will be -

(1) 1 : 1 (2) 1 : 2 (3) 5 : 1 (4) 1 : 5

Sol. $\frac{E_p}{E_s} = \frac{n_p}{n_s}$ $\therefore$ $\frac{E_p}{E_s} = \frac{100}{500} = \frac{1}{5}$

$\therefore$ Answer will be (4)

Ex.27 For an electric motor the voltage equation is -

(1) $E = E_b - I_aR_a$ (2) $E = E_b + I_aR_a$ (3) $E = \frac{E_b}{I_aR_a}$ (4) $E = \frac{I_aR_a}{E_b}$

Sol. Voltage equation for an electric motor is

$I_a = \frac{E - E_b}{R_a}$ or $E = E_b + I_aR_a$

$\therefore$ Answer will be (2)

Ex.28 10 ampere alternating current flows through the primary coil of transformer at 230 volt. If a voltage of 23000 volt is produced in the secondary coil and half of the power is lost in it, then the current in the secondary coil will be -

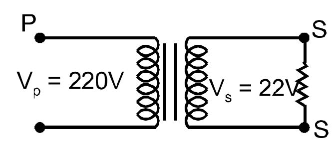
(1) 0.05A (2) 0.5A (3) 0.1 A (4) 1A

Sol. $\frac{1}{2} E_pI_p = E_sI_s$ $\therefore$ $I_s = \frac{E_pI_p}{2E_s}$

$= \frac{230 \times 10}{2 \times 23000} = \frac{1}{20} = 0.05$ A

$\therefore$ Answer will be (1)

Ex.29 The current in the primary coil of a transformer as shown in the figure will be -



(1) 0.0A (2) 1 A (3) 0.1A (4) 10^{-6}A

Sol. $V_s = I_sZ_s$ or $22 = I_s \times 200$

$\therefore I_s = 0.1$A

$\frac{V_s}{V_p} = \frac{I_p}{I_s}$ or $\frac{20}{220} = \frac{I_p}{0.1}$ or $I_p = 0.01$ A

Hence the correct answer will be (1)

Ex.30 The resistance of armature of a D.C motor is 20Ω. When it is a connected to a 220 V D.C. supply then a current of 1.5A flows in it. The value of back e.m.f will be -

(1) 190V (2) 90V (3) 280V (4) 360V

Sol. $I = \frac{E - e}{R}$

$1.5 = \frac{220 - e}{20}$ or $20 \times 1.5 = 220 - e$

or $e = 190$

Hence the correct answer will be (1)