PERMUTATION AND COMBINATION

1. FACTORIAL NOTATION

The continuous product of first n natural numbers is called **factorial** and it can be represented by notation $\underline{|n}$ or n! .

So $n! = 1.2.3 .... (n-1).n$

or $n! = n(n-1)(n-2) ....3.2.1$

$= n\{(n-1)(n-2) ....3.2.1.\}$

$\therefore\ n! = n(n-1)! = n(n-1)(n-2)!$

$= n(n-1)(n-2)(n-3)!$

$$n(n-1) ....(n-r+1) = \frac{n!}{(n-r)!}$$

Some useful results :

$0! = 1 \quad 4! = 24 \quad 8! = 40320 \quad 1! = 1$

$5! = 120 \quad 9! = 362880$

$2! = 2 \quad 6! = 720 \quad 10! = 3628800$

$3! = 6 \quad 7! = 5040$

$-n! =$ Meaningless

2. FUNDAMENTAL PRINCIPLES OF OPERATION

When one or more operations can be accomplished by number of ways then there are two principles to find the total number of ways to accomplish one, two, or all of the operations without counting them as follows :

Fundamental Principle of Multiplication

Let there are two parts A and B of an operation and if these two parts can be performed in m and n different number of ways respectively, then that operation can be completed in m × n ways.

Fundamental Principle of addition :

If there are two operations such that they can be done independently in m and n ways respectively, then any one of these two operations can be done by (m + n) number of ways.

3. COMBINATIONS

The different groups or selections of a given number of things by taking some or all at a time without paying any regard to their order, are called their **combinations.**

For example, if three things a, b and c are given then ab, bc and ac are three different groups, because ab and ba will give only one group, similarly bc and cb will give one group and ac and ca will give another group. Thus taking two things out of three different things a, b and c, the following three groups can be formed :

ab, bc, ca

The number of combinations of n **<u>different</u>** things taken r at a time is denoted by

${}^nC_r$ or $C(n, r)$

$${}^nC_r = \frac{n!}{r!(n-r)!}$$

$$\text{So } {}^nC_r = \frac{n(n-1)(n-2).....(n-r+1)}{r!}$$

Particular cases : ${}^nC_r = \dfrac{{}^np_r}{r!}$

${}^nC_n = 1$

${}^nC_0 = 1$

Some Important Results :

* ${}^nC_r = {}^nC_{n-r}$

* ${}^nC_x = {}^nC_y \Rightarrow x + y = n$

* ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

* ${}^nC_r = \dfrac{n}{r}.{}^{n-1}C_{r-1}$

* ${}^nC_r = \dfrac{1}{r}(n-r+1)\,{}^nC_{r-1}$

* ${}^nC_1 = {}^nC_{n-1} = n$

* Greatest value of ${}^nC_r$

$= {}^nC_{n/2}$, when n is even

$= {}^nC_{(n-1)/2}$ or ${}^nC_{(n+1)/2}$, when n is odd

4. PERMUTATIONS

An arrangement of some given things taking some or all of them, is called a **permutation** of these things.

For Example, three different things a, b and c are given, then different arrangements which can be made by taking two things from the three given things are

ab, ac, bc, ba, ca, cb

Therefore, the number of permutations will be 6.

The number of permutations of n **different** things taken r at a time is ^nP_r, where

$$^nP_r = \frac{n!}{(n-r)!}$$

$$= n\,(n-1)\,(n-2)\,.....\,(n-r+1)$$

The number of permutations of n dissimilar things taken all at a time = ^nP_n = n!

Explanation : Arrangement of n things at r places may be done as follows **:**

First we make arrangement for first place which may be done in n ways. After this, we make arrangement for second place which may be done in (n–1) ways. Continuing this process, we shall find the arrangements for third, fourth....r^{th} places may be done in (n–2), (n–3) ....(n–r +1) ways respectively.

Hence total number of permutations of n things taking r at a time

$$= n\,(n-1)\,(n-2)\,.......(n-r+1)$$

$$.^nP_1 = n$$

Note : $.^nP_r = n.\ ^{n-1}P_{r-1}$

$.^nP_r = (n-r+1)\ .^nP_{r-1}$

$.^nP_n = {}^nP_{n-1}$

Difference between Permutation and Combination :

(a) In combinations the order of things has no role to play, while in permutations the order of things plays an important role.

(b) In combinations the different groups are formed by taking some or all given things, while in case of permutations all possible arrangements of things are made in each group. Thus the number of permutations is always greater than the number of combinations.

Note: In general, we use the methods of finding combinations in case of forming groups, teams or committees, while the methods of finding permutations are used in forming numbers with the help of digits, words with the help of letters, distribution of prizes etc.

Permutations in which all things are not different :

The number of permutations of n things taken **all at a time** when p of them are alike and of one kind, q of them are alike and of second kind, r of them are alike and of third kind and all remaining being different is $\frac{n!}{p!\,q!\,r!}$

Explanation : If all n things were different then total number of permutations will be n!, but if p things are similar, then these p things can be mutually arranged only in one way, Whereas in the case when they were different, their mutual arrangements could be p!. Similarly when q things and r things were different, their mutual arrangements could be q! and r! respectively. Thus the total number of permutations is obtained by dividing n! by p! . q! .r!.

Note : Above formula is applicable only when all n things are taken at a time.

Permutations in which things may be repeated :

The number of permutations of n **different things** taken r at a time when each thing can be used once, twice, ...upto r times in any permutation is $\mathbf{n^r}$.

In particular, in above case when n things are taken at a time then total number of permutation is $\mathbf{n^n}$.

Permutation of numbers when given digits include zero :

If the given digits include 0, then two or more digit numbers formed with these digits cannot have 0 on the extreme left. In such cases we find the number of permutations in the following two ways.

(a) (The number of digits which may be used at the extreme left) x (The number of ways in which the remaining places may be filled up)

(b) If given digits be n (including 0) then total number of m- digit numbers formed with them

$$= {}^nP_m - {}^{n-1}P_{m-1}.$$

because $^{n-1}P_{m-1}$ is the number of such numbers which contain 0 at extreme left.

SOLVED PROBLEMS

Ex.1 If $\frac{1}{9!}+\frac{1}{10!}=\frac{x}{11!}$, then find the value of x

Sol. $\frac{1}{9!}+\frac{1}{10!}=\frac{x}{11!} \Rightarrow \frac{1}{9!}+\frac{1}{10.9!}=\frac{x}{11.10.9!}$

$\Rightarrow \frac{1}{9!}\left[1+\frac{1}{10}\right]=\left(\frac{x}{11.10}\right)\cdot\frac{1}{9!} \Rightarrow 1+\frac{1}{10}=\frac{x}{11.10}$

$\Rightarrow \frac{11}{10}=\frac{x}{11.10} \Rightarrow x = 11 . 11 = 121$

Ex.2 Find the number of different words (meaningful or meaningless) can be formed by taking four different letters from English alphabets

Sol. The first letter of four letter word can be chosen by 26 ways, second by 25 ways, third by 24 ways and fourth by 23 ways. So number of four letter words

$= 26 \times 25 \times 24 \times 23 = 358800$

Ex.3 If ${}^{56}P_{r+6} : {}^{54}P_{r+3} = 30800 : 1$ then find the value of **r**

Sol. $\frac{{}^{56}P_{r+6}}{{}^{54}P_{r+3}}=\frac{30800}{1}$

$\Rightarrow \frac{56!}{(56-r-6)!} = \frac{(30800)\,x54!}{(54-r-3)!}$

$\Rightarrow 56 \times 55 \times (51-r) = 30800$

$\Rightarrow (51-r) = \frac{30800}{56\,x\,55} = 10$

$\Rightarrow r = 51 - 10 = 41$

Ex.4 Find the number of ways in which 2 vacancies can be filled up by 13 candidates

Sol. The no. of ways to fill up 2 vacancies by 13 candidates is-

${}^{13}P_2 = 13 \times 12 = 156$

Ex.5 Find how many different words beginning with A and ending with L can be formed by using the letters of the word' ANILMANGAL'?

Sol. After fixing the letters A and L in the first and last places, the total number of available places are 8 and the letters are also 8. Out of these 8 letters there are 2 groups of alike letters.

Therefore no. of words

$= \frac{8!}{2!2!} = 10080$

Ex.6 Find how many numbers can be formed between 20000 and 30000 by using digits 2, 3, 5, 6, 9 when digits may be repeated?

Sol. First digit between 20000 and 30000 will be 2 which can be chosen by one way. Every number will be of five digits and all the digits can be anything from the given five digit except first digit. So each digit of the remaining four digits can be chosen in 5 0ways

$\therefore$ required numbers

$= 1 \times 5 \times 5 \times 5 \times 5 = 625$

Ex.7 Find the number of three letters words can be formed from the letters of word 'SACHIN' when I do not come in any word

Sol. There are 6 letters in the given word. Then the number of three letters words from the remaining 5 letters after removing I is -

$= {}^5P_3 = 5 \times 4 \times 3 = 60$

Ex.8 Find the number of numbers lying between 100 and 1000 which can be formed with the digits 0, 1, 2, 3, 4, 5, 6

Sol. Required numbers will have 3 digits so their total number $= {}^7P_3 - {}^6P_2 = 180$

Ex.9 Find how many numbers between 1000 and 4000 (including 4000) can be formed with the digits 0,1,2,3,4 if each digit can be repeated any number of times?

Sol. Required number will have 4 digits and their thousand digit will be 1 or 2 or 3 or 4. The number of such numbers will be 124, 125, 125 and 1 respectively.

$\therefore$ Total numbers

$= 124 + 125 + 125 + 1 = 375$

Ex.10 Find the number of ways in which 7 girls can be stand in a circle so that they do not have the same neighbour in any two arrangements?

Sol. Seven girls can keep stand in a circle by $\frac{(7-1)!}{2!}$ number of ways, because there is no difference in anticlockwise and clockwise order of their standing in a circle.

$\therefore \frac{(7-1)!}{2!} = 360$

Ex.11 Find $^{47}C_4 + \sum_{r=1}^{5} {}^{52-r}C_3$

Sol. The given expression can be written as

$\sum_{r=1}^{5} {}^{52-r}C_3 + {}^{47}C_4$

$= {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3 + {}^{47}C_4$

[We know that $^{n+1}C_{r+1}$ $^{n}C_r$ + $^{n}C_{r+1}$]

$= {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{48}C_4$

$= {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{49}C_4$

$= {}^{51}C_3 + {}^{50}C_3 + {}^{50}C_4$

$= {}^{51}C_3 + {}^{51}C_4 = {}^{52}C_4$

Ex.12 A candidate is required to answer 6 out of 10 questions which are divided into two groups each containing 5 questions and he is not permitted to attempt more than 4 from each group. Find the number of ways in which he can make up his choice

Sol. Let there be two groups A and B each containing 5 questions. Questions to be attempted is 6, but not more than 4 from any group. The candidate can select the questions in following ways:

(i) 4 from group A and 2 from group B.

(ii) 3 from group A and 3 from group B.

(iii) 2 from group A and 4 from group B

The number of selections in the above cases are $^5C_4 \times {}^5C_2$, $^5C_3 \times {}^5C_3$, $^5C_2 \times {}^5C_4$ respectively.

$\therefore$ Number of ways of selecting 6 questions

$= {}^5C_4 \times {}^5C_2 + {}^5C_3 \times {}^5C_3 + {}^5C_2 \times {}^5C_4$

$= 50 + 100 + 50 = 200$

Ex.13 Find in how many ways can a committee consisting of one or more members be formed out of 12 members of the Municipal Corporation

Sol. Required number of ways

$= {}^{12}C_1 + {}^{12}C_2 + {}^{12}C_3 + \ldots.. + {}^{12}C_{12} = 2^{12} - 1$

$= 4096 - 1 = 4095$

Ex.14 Out of 10 white, 9 black and 7 red balls, find the number of ways in which selection of one or more balls can be made.

Sol. The required number of ways are

$(10 + 1)\ (9 + 1)\ (7 + 1) - 1$

$= 11 \times 10 \times 8 - 1 = 879$

Ex.15 Find the number of words which can be formed taking 4 different letters out of the letters of the word 'ASSASSINATION',

Sol. Total No. of selections of 4 different letters

$= {}^6C_4$

$\therefore$ Total no. of different words

$= {}^6C_4 \cdot 4!$

Ex.16 There are four balls of different colours and four boxes of colours same as those of the balls. Find the number of ways in which the balls, one each box, could be placed such that a ball does not go to box of its own colour

Sol. Number of derangements are

$= 4! \left\{\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!}\right\} = 12 - 4 + 1 = 9$

{Since number of derangements in such a problems is given by n!

$\left\{1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \ldots (-1)^n \frac{1}{n!}\right\}$

Ex.17 Find the number of 4 digit numbers divisible by 5 which can be formed by using the digits 0, 2, 3, 4, 5

Sol. For a number to be divisible by 5, unit place should be occupied by 0 or 5-

(i) If unit place is 0 then remaining 3 places can be filled by $^4P_3$ ways = 24

(ii) If unit place is 5 then no. of ways

$= 4! - 3! = 18$

$\therefore$ Total number of ways

$= 24 + 18 = 42$

Ex.18 Find the sum of all 5 digit numbers which can be formed using digits 1, 2, 3, 4, 5

Sol. Using formula (Here n = 5)

Sum = (1+ 2 + 3 + 4 + 5) 4! (11111)

= 15 × 24 × 11111 = 3999960

Ex.19 Find the number of diagonals in an octagon

Sol. Here n = 8 (given)

The number of diagonals are given by

$= \frac{n(n-3)}{2}$

$\Rightarrow \frac{8(8-3)}{2} = \frac{8.5}{2} = 20$

Ex.20 Out of 10 given points 6 are in a straight line. Find the number of the triangles formed by joining any three of them

Sol. A triangle can be formed by joining three points, so there will be ${}^{10}C_3$ triangles joining any three out of 10 points. But 6 of these 10 points are collinear so these 6 points will give no triangle. Hence the required number of triangles

$= {}^{10}C_3 - {}^{6}C_3$

= 120 – 20 = 100

Ex.21 Find the number of ways in which 5 biscuits can be distributed among two children

Sol. Each biscuit can be distributed in 2 ways. Therefore number of ways of distributing the biscuits = 2^5 = 32

Now number of ways in which either of the two children does not get any biscuit = 2.

∴ Required number of ways of distribution

= 32 – 2 = 30

Ex.22 Find how many five-letter words containing 3 vowels and 2 consonants can be formed using the letters of the word "EQUATION' so that the two consonants occur together?

Sol. There are 5 vowels and 3 consonants in the word 'EQUATION'. Three vowels out of 5 and 2 consonants out of 3 can be chosen in ${}^5C_3 \times {}^3C_2$ ways. So, there are ${}^5C_3 \times {}^3C_2$ groups each containing two consonants and three vowels. Now, each group contains 5 letters which are to be arranged in such a way that 2 consonants occur together. Considering 2 consonants as one letter, we have 4 letters which can be arranged in 4! ways. But two consonants can be put together in 2! ways. Therefore, 5 letters in each group can be arranged in 4! × 2! ways.

Hence, the required number of words

$= ({}^5C_3 \times {}^3C_2) \times 4! \times 2! = 1440$

Ex.23 Find how many numbers can be formed with the digits 0,1,2, 3,4,5 which are greater than 3000?

Sol. The numbers greater than 3000 may contain 4,5 or 36 digits. Now the 6 digit numbers can be formed by using all 6 given digits which will be ${}^6P_6$. These numbers include the numbers which starts with 0. Such type of numbers are ${}^5P_5$.

Therefore 6 digit numbers greater than 3000

$= {}^6P_6 - {}^5P_5 = 600$

Similarly 5 digit numbers greater than 3000

$= {}^6P_5 - {}^5P_4 = 600$

Now 4 digit numbers which are greater than 3000 should begin with the digit 3,4 or 5. If the first place of the number is occupied by the digit 3, 4 or 5, then the remaining three places of the number can be filled in ${}^5P_3$ = 60 ways.

Therefore numbers of 4 digits greater than 3000 = 3 x 60 = 180

Hence required numbers

= 600 + 600 + 180 = 1380

EXERCISE

Q.1 Compute:

(i) $\frac{9!}{(5!)\times(3!)}$ (ii) $\frac{32!}{29!}$ (iii) $\frac{(12!)-(10!)}{9!}$

Q.2 Find LCM [3!, 6!, 8!].

Q.3 Write the following products in factorial notation:

(i) $5 \cdot 6 \cdot 7 \cdot 8 \cdot 9 \cdot 10 \cdot 11 \cdot 12$

(ii) $3 \cdot 6 \cdot 9 \cdot 12 \cdot 15$

Q.4 Prove that :

$(n!) \cdot (n + 2) = [(n!) + (n + 1)!]$.

Q.5 Which of the following are true ?

(i) $(2 + 3)! = 2! + 3!$

(ii) $(2 \times 3)! = (2!) \times (3!)$

Q.6 Evaluate: $\frac{n!}{(r!)\times(n-r)!}$, when n = 15 and r = 12.

Q.7 Find n, if :

(i) $(n + 2)! = 60 \times (n - 1)!$

(ii) $(n + 2)! = 2550\ (n!)$

Q.8 Prove that:

(i) $\frac{n!}{r!} = n(n-1)(n-2)....(r+1)$

(ii) $(n - r + 1) \cdot \frac{n!}{(n-r+1)!} = \frac{n!}{(n-r)!}$

Q.9 If $\frac{n!}{2(n-2)!}$ and $\frac{n!}{4!(n-4)!}$ are in the ration 2 : 1. find n.

Q.10 There are 10 buses running between Delhi and Agra. In how many can a man go from Delhi to Agra and return by a different bus ?

Q.11 There are three routes: air, rail and road for going from Madras to Hyderabad. But from Hyderabad to Vikarabad, there are two routes, rail and road. How many kinds of routes are there from Madras to Vikarabad via Hyderabad

Q.12 In a text book on mathematics there are 3 exercise A, B, C consisting of 12, 18 and 9 questions respectively. In how many ways can three questions be selected choosing one from each exercise ?

Q.13 In a school, there are four sections of 40 students each, in XI standard. In how many ways can a set of 4 student representatives be chosen, one from each section ?

Q.14 In how many ways can a vowel, a consonant and a digit chosen out of the 26 letters of the English alphabet and the 10 digits ?

Q.15 In a class there are 27 boys and 14 girls. The teacher wants to select 1 boy and 1 girl to represent the class in a function. In how many ways can the teacher make this selection ?

Q.16 Given A = {2, 3, 5} and B = {0, 1}. Find the number of different ordered pairs in which the first entry is an element of A and the second is an element of B.

Q.17 How many arithmetic progressions with 10 terms are there whose first terms is in the set {1 , 2, 3} and whose common difference is in the set {2, 3, 4}?

Q.18 There are 6 items in column A and 6 items in column B. A student is asked to match each item in column A with an item in column B. How many possible (correct or incorrect) answers are there to this question ?

Q.19 There are 6 multiple choice questions in an examination. How many sequences of answers are possible, if the first three questions have 4 choice each and the next three have 5 each?

Q.20 A mint prepares metallic calendars specifying months, dates and days in the form of monthly sheets (one plate for each month). How many types of February calendars should it prepare to serve for all possibilities in the future years?

Q.21 In a city, telephone numbers consist of 6 digits and none of them begins with 0. How many telephone numbers could be possible in that city?

Q.22 In how many ways can 6 letters be posted in 5 letters boxes available in the locality?

Q.23 How many 3–digits number of distinct digits can be formed from the digits 2, 3, 7, 8, 9?

Q.24 Find the number of all even 2-digit numbers, not having 0 at the unit's place.

Q.25 How many 3-digit numbers are there, with distinct digits, with each digit odd ?

Q.26 How many 3-digits numbers are three with no digit repeated ?

Q.27 How many 4-digit numbers are there with distinct digits?

Q.28 (i) How many 2-digit numbers are there ?
(ii) How many 3-digit numbers are there ?
(iii) How many 4-digit numbers are there ?

Q.29 There are 6 books on physics and 5 books on chemistry in a bookshop. In how many ways can a student purchase either a book on physics or a book on chemistry

Q.30 From among the 36 teachers in a school, one principal and one vice-principal are to be appointed. In how many ways can this be done

Q.31 Evaluate:
(i) ${}^{10}P_4$ (ii) ${}^{62}P_3$ (iii) ${}^{9}P_9$

Q.32 Prove that ${}^9P_3 + 3\,{}^9P_2 = {}^{10}P_3$.

Q.33 (i) If ${}^{2n}P_3 = 100 \times {}^nP_2$, find n.
(ii) If $16\,{}^nP_3 = 13\,{}^{n+1}P_3$, find n.
(iii) If ${}^nP_5 = 20 \times {}^nP_3$, find n.

Q.34 (i) If ${}^5P_r = 2 \times {}^6P_{r-1}$, find r.
(ii) If ${}^{20}P_r = 13 \times {}^{20}P_{r-1}$, find r.

Q.35 (i) If ${}^nP_4 : {}^nP_5 = 1 : 2$, find n.
(ii) If ${}^{n-1}P_3 : {}^{n+1}P_3 = 5 : 12$, find n.
(iii) If ${}^{2n-1}P_n : {}^{2n+1}P_{n-1} = 22 : 7$, find n.
(iv) If ${}^{2n+1}P_{n-1} : {}^{2n-1}P_n = 3 : 5$, find n.

Q.36 If ${}^{11}P_r = {}^{12}P_{r-1}$, find r.

Q.37 If ${}^{15}P_{r-1} : {}^{16}P_{r-2} = 3 : 4$, find r.

Q.38 Prove that ${}^nP_n = 2\,{}^nP_{n-2}$.

Q.39 Find the number of permutations of 7 objects taken 3 at a time.

Q.40 In how many ways can 5 persons occupy 3 vacant seats ?

Q.41 In how many ways can 10 people line up at a ticket window of a cinema hall?

Q.42 In how many ways can 5 children stand in a queue?

Q.43 In how many ways can 6 women draw water from 6 wells, if no well remains unused ?

Q.44 In how many ways can 4 different books, one each in chemistry, physics, biology and mathematics be arranged in a shelf?

Q.45 Raju wants to arrange 3 economics, 2 history and 4 language books on a shelf. If the books on the same subject are different. Determine the number of all possible arrangements.

Q.46 Seven students are contesting the election for the presidentship of the students' union. In how many ways can their names be listed on the ballot papers

Q.47 Find the number of words formed (may be meaningless) by using all the letters of the word 'EQUATION', using each letter exactly once.

Q.48 Find the numbers of different 4-letter words (may be meaningless) that can be formed for the letters of the word 'NUMBERS'.

Q.49 Ten students are participating in a race. In how many ways can the first three prizes be won ?

Q.50 There are 3 different rings to be worn in 4 fingers with at most one in each finger. In how many ways can this be done?

Q.51 Four toys shaped as the letters B, V, R, S, one of each, were purchased from a plasticware shop. How many ordered pairs of letters, to be used as initials, can be formed from them?

Q.52 If there are 6 periods in each working day of a school, in how many ways can one arrange 5 subjects such that each subject is allowed at least one period ?

Q.53 In how many ways can 6 boys and 5 girls be arranged for a group photograph if the girls are to sit on chairs in a row and the boys are to stands in a row behind them?

Q.54 How many words can be formed from the letters of the word 'SUNDAY'?

Q.55 How many words beginning with C and ending with Y can be formed by using the letters of the word 'COURTESY' ?

Q.56 Find the number of permutations of the letters of the word 'ENGLISH'. How many of these begin with E and end with I?

Q.57 In how many ways can the letters of the word 'HEXAGON' be permuted ? In how many words will the vowel be together ?

Q.58 How many words can be formed out of the letters of the word 'ORIENTAL' so that the vowels always occupy the odd places?

Q.59 In how many words can the letters of the word 'FAILURE' be arranged so that the consonants may occupy only odd positions?

Q.60 How many permutations can be formed by the letters of the word 'VOWELS', when
(i) there is no restriction on letters;
(ii)each word begins with E;
(iii) each word begin with O and ends with L;
(iv) all vowels come together;
(v) all consonant come together?

Q.61 In how many arrangements of the word 'GOLDEN' will the vowels never occur together?

Q.62 In how many ways can 6 books on economics, 5 on history and 3 on Hindi be arranged on a shelf so that the books on each subject are always together?

Q.63 Find the number of ways in which 5 ladies and 5 gentlemen may be seated in a row so that no two ladies are together.

Q.64 Find the number of ways in which m boys and n girls may be arranged in a row so that no two of the girls are together, it being given that $m > n$.

Q.65 In how many ways can 5 children be arranged in a line such that
(i) two of them, Ram and Shyam, are always together;
(ii) two of them, Ram and Shyam, are never together ?

Q.66 When a group photograph is taken, all the seven teachers should be in the first row and all the twenty students should be in the second row. If the two corners of the second row are reserved for the two tallest students, interchangeable only between them, and if the middle seat of the front row is reserved for the principal, how many arrangements are possible?

Q.67 Find a formula for the number of permutations of n different things taken r at a time such that two specified things occur together.

Q.68 How many numbers divisible by 5 and lying between 3000 and 4000 can be formed by using the digits 3, 4, 5, 6, 7, 8, when no digit is repeated in any such number ?

Q.69 In an examination 10 candidates have to appear. 4 candidates are to appear in mathematics and the rest in different subjects, In how many ways can they be seated in a row, if candidates appearing in mathematics are not to sit together?

Q.70 A child has plastic toys bearing the digits 2, 2 and 5. How many three digits numbers can it make, using them?

Q.71 There are five round stickers, 3 of them are red the other 2 green. It is desired to make a design by passing them in a row. How many such designs are possible?

Q.72 There are three blue balls, four red balls and five green balls. In how many ways can they be arranged in a row ?

Q.73 How many arrangements can be made out of the letters of the word :

(i)	INDIA	(ii)	ENGINEERING
(iii)	COMMERCE	(iv)	CHANDIGARH
(v)	INTERMEDIATE	(vi)	EXAMINATION

Q.74 How many permutations can be made out of the word 'SERIES' ?

Q.75 How many numbers can be formed with the help of the digits 5, 4, 3, 5, 3?

Q.76 How many 7-digit numbers can be formed using the digits 1, 2, 0, 2, 4, 2 and 4?

Q.77 How many numbers can be formed with the digits 1, 2, 3, 4, 3, 2, 1 so that the odd digits always occupy the odd place ?

Q.78 How many different signals can be made from 4 red, 2 white and 3 green flags by arranging all of them vertically on a flagstaff?

Q.79 How many 3-digits numbers can be formed by using the digits 0, 1, 3, 5, 7 while each digit may be repeated any number of times

Q.80 In how many ways can 4 letters be posted in 3 letter boxes ?

Q.81 In how many ways can 5 prize be distributed among 4 students; when each student may receive any number of prizes?

Q.82 How many natural numbers less than 1000 can be formed from the digits 0, 1, 2, 3, 4, 5 when a digit may be repeated any number of times?

Q.83 A boy has 6 pockets. In how many ways can he put 5 marbles in his pocket?

Q.84 In how many ways can 6 persons be arranges in (i) a line, (ii) a circle ?

Q.85 There are 5 men and 5 ladies to dine at a round table. In how many ways can they seat themselves so that no two no two ladies are together ?

Q.86 In how many ways can 11 members of committee sit at a round table so that the secretary and the joint secretary are always the neighbours of the president?

Q.87 In how many ways can 8 persons be seated at a round table so that all shall not have the same neighbours in any two arrangements ?

Q.88 In how many different ways can 20 different pearls be arranged to form a necklace ?

Q.89 In how many different ways can a garland of 16 different flowers be made?

Q.90 Evaluate:
(i) ${}^{15}C_3$ (ii) ${}^{12}C_9$ (iii) ${}^{50}C_{47}$
(iv) ${}^{71}C_{71}$ (v) ${}^{n+1}C_n$ (vi) $\sum_{r=1}^{6} {}^{5}C_r$

Q.91 Verify that ${}^9C_4 + {}^8C_3 = {}^9C_4$.

Q.92 (i) If ${}^nC_7 = {}^nC_5$, find n.
(ii) If ${}^nC_{14} = {}^nC_{16}$, find ${}^nC_{28}$.
(iii) If ${}^nC_{16} = {}^nC_{14}$, find ${}^nC_{27}$.

Q.93 (i) If ${}^{20}C_r = {}^{20}C_{r+6}$, find r.
(ii) If ${}^{18}C_r = {}^{18}C_{r+2}$, find ${}^rC_5$.

Q.94 If ${}^nP_r = 1680$ and ${}^nC_r = 70$, find n and r.

Q.95 If ${}^{2n}C_3 : {}^nC_3$ 11 : 1, find n.

Q.96 If ${}^{15}C_r : {}^{15}C_{r-1} = 11 : 5$, find r.

Q.97 If ${}^nC_{r-1} = {}^nC_{3r}$, find r.

Q.98 If ${}^{n+1}C_{r+1} : {}^nC_r = 11 : 6$ and ${}^nC_r : {}^{n-1}C_{r-1} = 6 : 3$, find n and r.

Q.99 Let r and n be positive integers such that $1 \le r \le n$. Prove that :
$$\frac{{}^nC_r}{{}^{n-1}C_{r-1}} = \frac{n}{r}$$

Q.100 From a class of 32 students, 4 are to be chosen for a competition. In how many ways can this be done?

Q.101 In how many ways can 5 sportsmen be selected from a group of 10?

Q.102 In how many ways can a student choose 5 courses out of 9 courses if two courses are compulsory for every student?

Q.103 If there are 12 persons in a party and if every two of them shakes hands with each other, how many handshakes happen in the party?

Q.104 In how many ways can a cricket team be selected from 17 players in which 5 players can bowl? Each cricket team must include 2 bowlers?

Q.105 A question paper has two parts, part 'A' and part 'B' each containing 10 questions. If the students has to choose 8 from part 'A' and 5 from part 'B', in how many ways can he choose the questions ?

Q.106 A sports team of 11 students is to be constituted, choosing at least 5 from Class XI and at least 5 from Class XII. If there are 20 students in each of these classes, in how many ways can the team be constituted ?

Q.107 From a class of 25 students, 10 are to be chosen for an excursion party. There are 3 students who decide that either all of them will join or none of them will join. In how many ways can the ten be chosen ?

Q.108 Out of 6 teachers and 8 students, a, committee of 11 is to be formed. In how many ways can his be done, if the committee contains

(i) exactly 4 teachers,

(ii) at least 4 teachers

Q.109 A cricket team of 11 players is to be selected from 16 players including 5 bowlers and 2 wicketkeepers. In how many ways can a team be selected so as to consist of exactly 3 bowlers and 1 wicketkeeper ?

Q.110 Find the numbers of diagonals of

(i) a hexagon

(ii) a polygon of 16 sides.

Q.111 (i) How many straight line can be obtained by joining 12 points, points, 5 of which are collinear ?

(ii) How many triangles can be obtained by joining 12 point, five of which, are collinear ?

Q.112 How many (i) lines (ii) triangles can be drawn through n points on a circle ?

Q.113 A polygon has 44 diagonals. Find the number of its sides.

Q.114 From a class of 14 boys and 10 girls, 10 students are to be chosen for a competition, at least including 4 boys and 4 girls. The 2 girls who won the prizes last year should be included. In how many ways can the selection be made ?

Q.115 In a village, there are 87 families of which 52 families have at most 2 children. In a rural development programme, 20 families are to be chosen for assistance, of which at least 18 families must have at most 2 children. In how many ways can the choice be made ?

Q.116 A committee of 5 is to be selected from among 6 boys and 5 girls. Determine the number of ways of selecting the committee if it is to consist of at least 1 boy and 1 girl.

Q.117 A bag contains 4 red, 3 white and 2 blue marbles. 3 marbles are drawn at random. Determine the numbers of ways of selecting at least 1 white marble in this selection.

Q.118 A boy has 3 library tickets and books of his interest in the library. Of these 8, he does not want to borrow Chemistry part II unless Chemistry part I is also borrowed.

In how many ways can he choose the three books to be borrowed ?

Q.119 In how many ways can 5 white balls and 4 black balls be arranged in a row so that no two black balls are together ?

Q.120 How many different words, each containing 2 vowels and 3 consonants, can be formed with 5 vowels and 17 consonants?

Q.121 A man has 6 friends. In how many ways can he invite one or more to a party ?

Q.122 There are 5 equations in a question paper. In how many ways can a boy solve one more questions ?

ANSWER KEY

1. (i) 504 (ii) 29760 (iii) 1310 **2.** 40320 **3.** (i) $\frac{12!}{4!}$ (ii) $3^5 \cdot 5!$ **5.** None is true

6. 455 **7.** (i) 3 (ii) 49 **9.** n = 5 **10.** 90 **11.** 6 **12.** 1944

13. 2560000 **14.** 1050 **15.** 378 **16.** 6 **17.** 9 **18.** 720 **19.** 8000

20. 14 **21.** 900000 **22.** $5^6 = 15625$ **23.** 60 **24.** 36 **25.** 60 **26.** 648

27. 4536 **28.** (i) 90 (ii) 900 (iii) 9000 **29.** 11 **30.** 1260 **31.** (i) 5040 (ii) 226920 (iii) 362880 **33.** (i) 13 (ii) 15 (iii) 8 **34.** (i) 3 (ii) 8 **35.** (i) 6 (ii) 8 (iii) 10 (iv) 4 **36.** 9 **37.** 14 **39.** 210 **40.** 60 **41.** 10 ! **42.** 120 **43.** 720

44. 24 **45.** 9! **46.** 5040 **47.** 40320 **48.** 840 **49.** 720 **50.** 24

51. 12 **52.** 3600 **53.** 86400 **54.** 720 **55.** 720 **56.** 5040, 120

57. 5040, 720 **58.** 576 **59.** 576 **60.** (i) 720 (ii) 120 (iii) 24 (iv) 240 (v) 144

61. 480 **62.** 3110400 **63.** 86400 **64.** $\frac{m! \times (m+1)!}{(m+1-n)!}$ **65.** (i) 48 (ii) 72

66. $(18!) \times (6!) \times 2$ **67.** $2\,{}^{n-1}P_r$ **68.** 12 **69.** $6! \times {}^7P_4 = 604800$ **70.** 3 **71.** 10

72. 27720 **73.** (i) 60 (ii) 277200 (iii) 5040 (iv) 907200 (v) 19958400 (vi) 4989600

74. (i) 180, (ii) 12 **75.** 30 **76.** 360 **77.** 18 **78.** 1260 **79.** 100 **80.** 81

81. $4^5 = 1024$ **82.** 215 **83.** 6^5 **84.** (i) 720 (ii) 120 **85.** 2880 **86.** 80640

87. 2520 **88.** $\frac{1}{2} \times (19\,!)$ **89.** $\frac{1}{2} \times (15\,!)$ **90.** (i) 455 (ii) 220 (iii) 19600 (iv) 1 (v) n + 1 (vi) 31

92. (i) 12 (ii) 435 (iii) 4060 **93.** (i) 7, (ii) 56 **94.** n = 8, r = 4 **95.** 6

96. 8 **97.** $\frac{1}{4}(n+1)$ **98.** n = 10, r = 5 **100.** 35960 **101.** 252 **102.** 35 **103.** 66

104. 2200 **105.** 11340 **106.** $2 \times {}^{20}C_5 \times {}^{20}C_6$ **107.** 817190

108. (i) 120 (ii) 344 **109.** 722 **110.** (i) 9, (ii) 104 **111.** (i) 57, (ii) 210

112. (i) (1/2) n(n – 1) (ii) (1/6) n(n – 1)(n – 2) **113.** 11 **114.** 266266

115. ${}^{52}C_{18} \times {}^{35}C_2 + {}^{52}C_{19} \times {}^{35}C_1 + {}^{52}C_{20}$ **116.** 455 **117.** 64 **118.** 41 **119.** 20 **120.** 816000

121. 63 **122.** 31