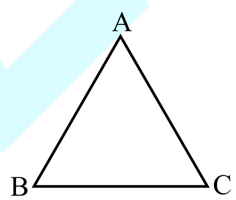
TRIANGLES AND ITS PROPERTIES

6

CONTENTS

➢ TRIANGLE

A geometrical figure formed by joining three non-collinear points by three line segments is called a triangle.



The triangle ABC has :

Sides : $\overline{AB}$, $\overline{BC}$, $\overline{CA}$

Vertices : A, B and C.

Angles : ∠BAC or ∠CAB, ∠ABC or ∠CBA and ∠ACB or ∠BCA.

A triangle is denoted by the symbol 'Δ'.

The three sides and three angles taken together are called six elements or six parts of a triangle.

❖ EXAMPLES ❖

Ex.1 Do three collinear points A, B and C form a triangle ?

Sol. No, three collinear points form a line.

Ex.2 For the triangle ΔLMN, name

(a) the side opposite to ∠M.

(b) the angle opposite to side LM.

(c) the vertex opposite to side NL.

(d) the side opposite to vertex N.

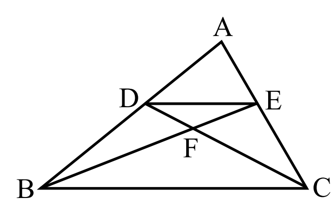
Sol. (a) The side opposite to ∠M is LN.

(b) The angle opposite to side LM is ∠N.

(c) The vertex opposite to side NL is M.

(d) The side opposite to vertex N is LM.

Ex.3 How many different triangles are in figure ? Name each of them.



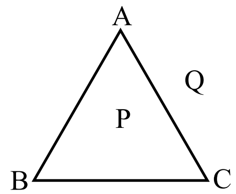
Sol. ΔABC,ΔADE,ΔABE,ΔADC,ΔBFC,ΔBFD, ΔBDE,ΔCEF,ΔCED,ΔDEF,ΔBCD,ΔBEC.

So, there are 12 different triangles in the given figure.

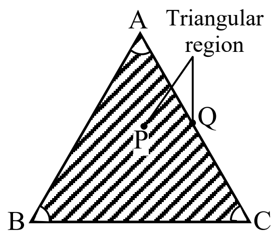
➤ INTERIOR AND EXTERIOR OF A TRIANGLE

Interior of a triangle is the region of the plane enclosed by ΔABC.

Here, point P is in the interior of ΔABC.

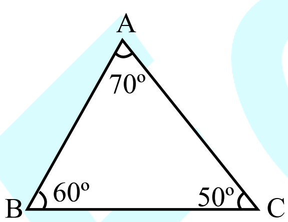
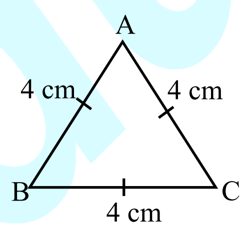
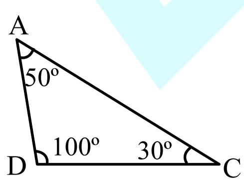
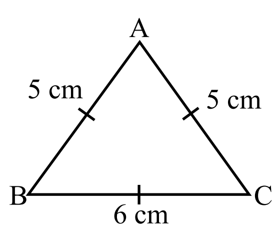
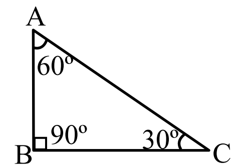
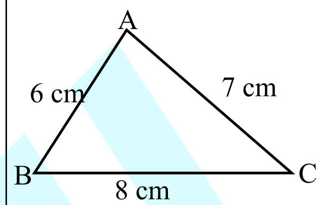


Exterior of a triangle is the region of the plane which lies beyond or not enclosed by the boundary of ΔABC. In figure, Q is the point which is in the exterior of the ΔABC.



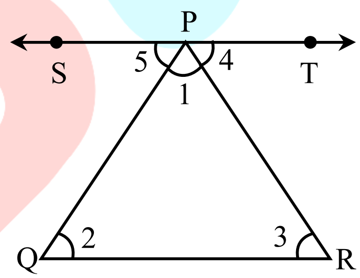
Interior of ΔABC (as shown by the shaded region P in figure) together with the points on the boundary of ΔABC (as shown by point Q) is known as the triangular region ABC.

➤ TYPES OF TRIANGLE

Based on angles	Based on sides
1. Acute angled triangle : A triangle whose all angles are acute i.e., less than 90º. 	**1. Equilateral triangle :** A triangle with all sides equal to one another. 
2. Obtuse angled triangle : A triangle whose one angle is obtuse i.e., greater than 90º.  A triangle cannot have more than one obtuse angle.	**2. Isosceles triangle :** A triangle with any two sides equal to each other. 
3. Right angled triangle : A triangle whose one angle is of measure 90º also the other two angles are acute angles whose sum is 90º.  The side of opposite to the right angle is called the hypotenuse, and other two side are called legs of the right triangle.	**3. Scalene triangle :** A triangle in which all sides are unequal. 

➤ ANGLE SUM PROPERTY OF A TRIANGLE

The sum of the angles of a triangle is 180º or two right angles.



Given : A triangle PQR.

To prove : $\angle 1 + \angle 2 + \angle 3 = 180^\circ$

i.e., sum of all angles of a triangle is 180º.

Construction : Through P, draw a line ST parallel to QR.

Proof : As ST || QR and transversal PQ cuts them.

$\therefore \quad \angle 2 = \angle 5$ (alternate angles) …(1)

Again ST || QR and transversal PR cuts them.

$\therefore \quad \angle 3 = \angle 4$ (alternate angles) …(2)

Adding (1) and (2), we get

$\angle 2 + \angle 3 = \angle 5 + \angle 4$ …(3)

Now adding ∠1 on both sides to equation (3), we get

$\angle 1 + \angle 2 + \angle 3 = \angle 1 + \angle 5 + \angle 4$

$\Rightarrow \quad \angle 1 + \angle 2 + \angle 3 = 180^\circ$

(as $\angle 1 + \angle 5 + \angle 4 = 180^\circ$)

Note :

(i). Each angle of an equilateral triangle measures 60º

(ii) The angles opposite to equal sides of an isosceles triangle are equal.

(iii) A scalene triangle has all angles unequal.

(iv) A triangle cannot have more than one right angle

(v) A triangle cannot have more than one obtuse angle.

(vi) In a right triangle, the sum of two acute angles is 90º.

(vii) The sum of the lengths of the sides of a triangle is called perimeter of triangle.

❖ EXAMPLES ❖

Ex.4 Classify the triangles as Scalene, isosceles or equilateral, if their sides are :

(i) 2 cm, 3 cm, 2 cm (ii) 2 cm, 2 cm, 2 cm

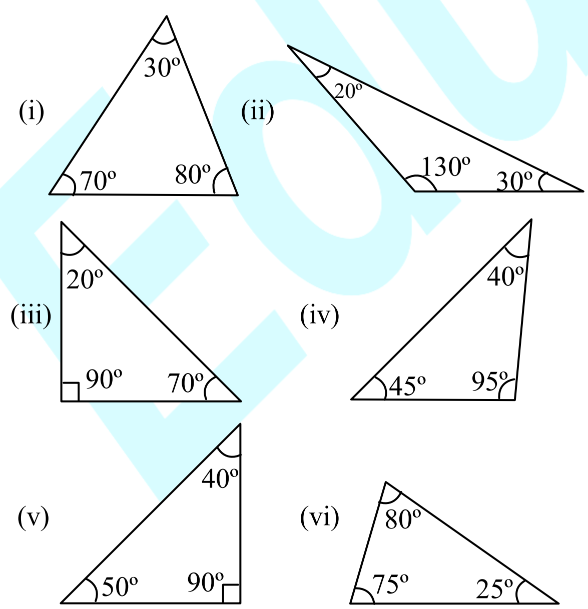
(iii) 3 cm, 6 cm, 4 cm

Sol. (i) As two sides are equal, so this is an isosceles triangle.

(ii) As all sides are equal, so this is an equilateral triangle.

(iii) As all sides are unequal, so this is a scalene triangle.

Ex. 5 Classify the following triangles according to their angles :



Sol. (i) As all the angles of this triangle are acute, so this is an acute triangle.

(ii) As one of the angles (130°) is obtuse, so this is an obtuse triangle.

(iii) As one of the angles is a right angle (90°), so this is a right triangle.

(iv) As one of the angles is obtuse (95°), so this is an obtuse triangle.

(v) As one of the angles is a right angle (90°), so this is a right triangle.

(vi) As all the angles are acute, so this is an acute triangle.

Ex. 6 Classify the triangles as acute, obtuse or right, whose angles are :

(i) 50°, 40°, 90° (ii) 120°, 30°, 30°

(iii) 70°, 60°, 50°

Sol. (i) As one of the angles is a right angle, so this is a right triangle.

(ii) As one of the angles is an obtuse angle, so this is an obtuse triangle.

(iii) As all the angles are acute, so this is an acute triangle.

Ex.7 Classify the triangles according to their given sides as scalene, isosceles or equilateral :

(a) 3.5 cm, 4 cm, 4 cm (b) 6 cm, 7 cm, 9 cm

(c) 6.2 cm, 6.2 cm, 6.2 cm

Sol. (a) As two sides are equal so it is an isosceles triangle.

(b) As all the sides are different so it is an scalene triangle.

(c) As all the sides are equal so it is equilateral triangle.

Ex.8 Classify the triangles as acute, obtuse or right if angles are :

(a) 60º, 30º, 90º

(b) 120º, 40º, 20º

(c) 60º, 60º, 60º

Sol. (a) As one angle of 90º so, it is a right triangle.

(b) As one angle (120º) is greater than 90º i.e., obtuse, so it is an obtuse triangle.

(c) As each angle is of 60º, so it is an equilateral triangle.

Ex.9 Two angles of a triangle are of measures 70º and 30º. Find the measure of the third angle.

Sol. Let PQR be a triangle such that ∠P = 70º, ∠Q = 30º. Then, we have to find the measure of third angle R.

As ∠P + ∠Q + ∠R = 180º

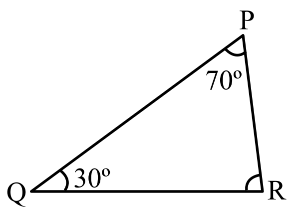
(angle sum property of triangle)

70º + 30º + ∠R = 180º

100º + ∠R = 180º

∠R = 180º – 100º

⇒ ∠R = 80º



Ex.10 One of the angles of a triangle has measure 70º and the other two angles are equal. Find these two angles.

Sol. Let PQR be a triangle such that :

∠P = 70º and ∠Q = ∠R = x (let)

As ∠P + ∠Q + ∠R = 180º

(angle sum property of Δ)

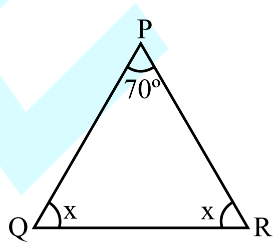
70º + x + x =180º

2x = 180º – 70º

2x = 110º

$$x = \frac{110^\circ}{2}$$

x = 55º



So, measure of each of remaining two angles is 55º.

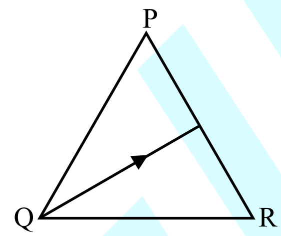
Ex.11 Write the

(i) side opposite to the vertex Q of ΔPQR

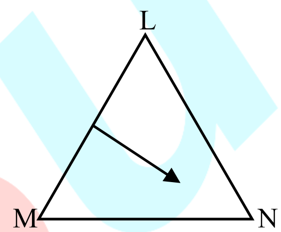
(ii) angle opposite to the side LM of ΔLMN

(iii) vertex opposite to the side RT of ΔRST.

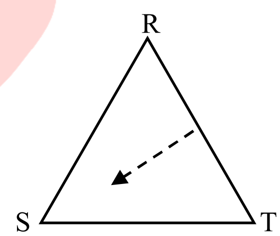
Sol. (i) The side opposite to vertex Q is PR.



(ii) Angle opposite to side LM is ∠N.



(iii) Vertex opposite to the side RT of ΔRST is S.



Ex.12 In each of the following, the measures of three angles are given. State in which case the angles can possibly be those of a triangle :

(i) 53°, 73°, 83° (ii) 59°, 12°, 109°

(iii) 45°, 45°, 90° (iv) 30°, 120°, 30°

Sol. (i) 53° + 73° + 83° = 209° > 180°

Therefore, not possible

(ii) 59° + 12° + 109° = 180°

Therefore, possible

(iii) 45° + 45° + 90° = 180°

Therefore, possible

(iv) 30° + 120° + 30° = 180°

Therefore, possible

Ex.13 The three angles of a triangle are equal to one another. What is the measure of each angle ?

Sol. Let each angle be of measure x in degrees. Then, by angle sum property

$x + x + x = 180°$

$\Rightarrow 3x = 180°$

$\Rightarrow x = 60°$

So, the measure of each angle is 60°.

Ex.14 The angles of a triangle are in the ratio 2 : 3 : 4. Find the angles.

Sol. Given ratio between the angles of a triangle = 2 : 3 : 4.

Let the angles be 2x, 3x and 4x

Since the sum of angles of a Δ is 180°

$\therefore 2x + 3x + 4x = 180°$

$\Rightarrow 9x = 180°$

$\Rightarrow x = \frac{180°}{9} = 20°$

Hence the angles are 2x, 3x and 4x

i.e., $2 \times 20°, 3 \times 20°, 4 \times 20°$

$\Rightarrow$ 40°, 60° and 80°.

Ex.15 In ΔABC, if ∠A = 2∠B and ∠C = 3∠B, then find all the angles of ΔABC.

Sol. In ΔABC

$\angle A + \angle B + \angle C = 180°$

$\Rightarrow 2\angle B + \angle B + 3\angle B = 180°$

$\Rightarrow 6\angle B = 180°$

$\Rightarrow \angle B = \frac{180°}{6} = 30°$

$\Rightarrow \angle B = 30°$

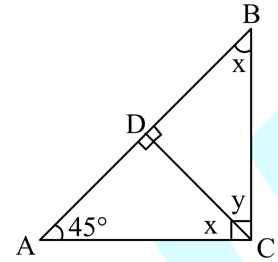
Now, $\angle A = 2\angle B = 2 \times 30° = 60°$ and

$\angle C = 3\angle B = 3 \times 30° = 90°$

Hence, ∠A = 60°, ∠B = 30° and ∠C = 90°.

Ex.16 In the Fig., CD ⊥ AB. Also, ∠A = 45°. Find ∠ADC, ∠CDB, ∠ABC, ∠DCB and ∠DCA.

Sol.



Since CD ⊥ AB

$\therefore \angle ADC = \angle CDB = 90°$

Now in ΔADC, we have

$\angle ADC + \angle DAC + \angle DCA = 180°$

(angle sum property of triangle)

$90° + 45° + z = 180°$

$\Rightarrow z = 180° - 135° = 45°$

$\therefore \angle y = 90° - 45° \Rightarrow \angle y = 45°$

In ΔACB

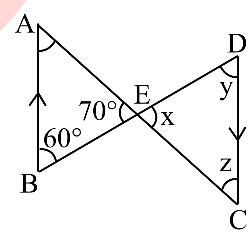
$\angle A + 90° + \angle x = 180°$ $45° + 90° + \angle x = 180°$

$135° + \angle x = 180°$ $\angle x = 180° - 135° = 45°$

Hence, x = 45°, y = 45° and z = 45° .

Ex.17 In the fig. AB || DC. Find the values of x, y and z.

Sol.



∠DEC = ∠AEB [vertically opposite angles]

$\Rightarrow x = 70°$ and ∠ABE = ∠EDC

[∵ AB || DC, ∴ alternate angles are equal]

$\Rightarrow y = 60°$

Now in ΔDEC, we have

$x + y + z = 180°$

[sum of interior angles of a Δ is 180°]

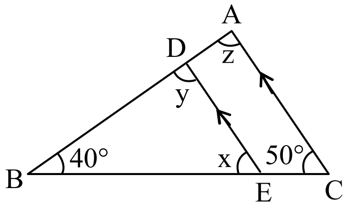
$\Rightarrow 70° + 60° + z = 180°$

$\Rightarrow z = 180° - 30° \Rightarrow z = 50°$

Hence, x = 70°, y = 60° and z = 50° respectively.

Ex.18 In the Fig., DE ∥ AC. If ∠B = 40° and ∠C = 50°, then find x, y and z.

Sol.



In ΔABC,

∠A + ∠B + ∠C = 180°

[In a Δ sum of all interior angles is 180°]

⇒ z + 40° + 50° = 180°

⇒ z = 90°

Now in ΔBDE, we have

y = z = 90°

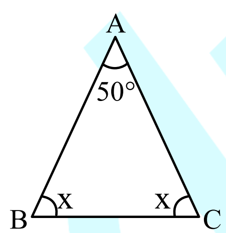
[∵ AC ∥ DE ∴ Corresponding angles are equal]

and x = ∠ACB = 50°

Hence, x = 50°, y = 90° and z = 90°.

Ex.19 One angle of a ΔABC is 50° and the other two angles are of same measure as in Fig. Find the measure of each angle.

Sol.



Let ∠A = 50° and ∠B = ∠C = x

We know that in a Δ, sum of angles is 180°.

∴ ∠A + ∠B + ∠C = 180°

⇒ 50° + x + x = 180°

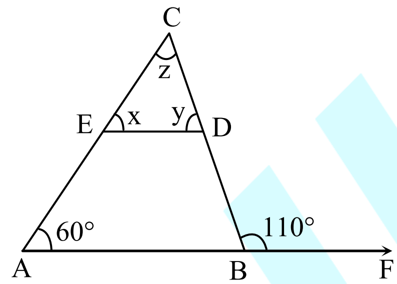
⇒ 2x = 180° – 50° = 130°

⇒ $x = \frac{130°}{2} = 65°$

Hence, the measure of equal angles is 65° each.

Ex.20 In (Fig.) ΔABC, DE ∥ AB, find the values of x, y and z.

Sol.



Since DE ∥ AB, therefore,

∠CED = ∠CAB [Corresponding angles]

⇒ x = 60° ...... (i)

and ∠CDE = ∠DBA ...... (ii)

[Corresponding angles]

But ∠DBA + ∠DBF = 180° [linear pair]

⇒ ∠DBA + 110° = 180°

⇒ ∠DBA = 180° – 110° = 70°

Substituting ∠DBA = 70° in (ii), we get

∠CDE = 70°

⇒ y = 70°

Now in ΔDBE, we have

x + y + z = 180°

[sum of the interior angles of a triangle is 180°]

⇒ 60° + 70° + z = 180°

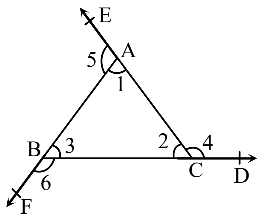
⇒ 130° + z = 180°

⇒ z = 180° – 130° = 50°

Hence, x = 60°, y = 70° and z = 50° respectively.

Ex.21 Show that sum of exterior angles of a triangle is 360°.

Sol. Let the triangle is ABC as shown in Fig.



Interior angles are marked with numbers 1, 2 and 3 while exterior angles are marked with 4, 5 and 6.

Since $\angle 2 + \angle 4 = 180°$ [Linear pair] ....(i)

$\angle 3 + \angle 6 = 180°$ [Linear pair] ......(ii)

$\angle 5 + \angle 1 = 180°$ [Linear pair] ...... (iii)

Adding (i), (ii) and (iii) on both the sides, we get

$\angle 2 + \angle 4 + \angle 3 + \angle 6 + \angle 5 + \angle 1 = 540°$

$\Rightarrow \angle 4 + \angle 5 + \angle 6 + (\angle 1 + \angle 2 + \angle 3) = 540°$

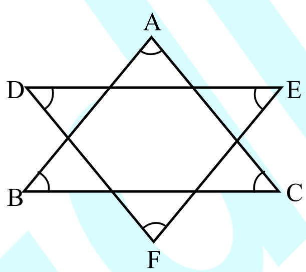
$\Rightarrow \angle 4 + \angle 5 + \angle 6 + 180° = 540°$

[$\because$ $\angle 1$, $\angle 2$ and $\angle 3$ are the interior angles of the ΔABC ($\therefore$ sum will be 180°)]

$\Rightarrow \angle 4 + \angle 5 + \angle 6 = 540° - 180° = 360°$.

Ex.22 Observe the Fig. and find $\angle A + \angle B + \angle C + \angle D + \angle E + \angle F$.

Sol.



We know that sum of interior angles of a triangle is 180°.

$\therefore$ In ΔABC, we have

$\angle A + \angle B + \angle C = 180°$ .....(i)

Similarly, in ΔDEF, we have

$\angle D + \angle E + \angle F = 180°$ .....(ii)

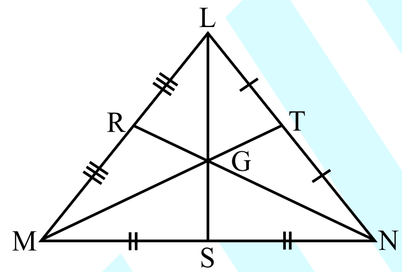
Adding (i) and (ii), we have

$\angle A + \angle B + \angle C + \angle D + \angle E + \angle F = 360°$.

➢ MEDIAN OF A TRIANGLE

A line segment that joins a vertex of a triangle to the mid-point of the opposite side is called a median of the triangle.

For example, consider ΔLMN. Let S be the mid-point of MN, then LS is the line segment joining vertex L to the mid point of its opposite side.



The line segment LS is said to be the median of ΔLMN.

Similarly, RN and MT are also medians of ΔLMN.

Note :

(i) A triangle has three medians.

(ii) All the three medians meet at one point G (called centroid of the triangle) i.e., all medians of any triangle are concurrent.

(iii) The centroid of the triangle always lies inside of triangle.

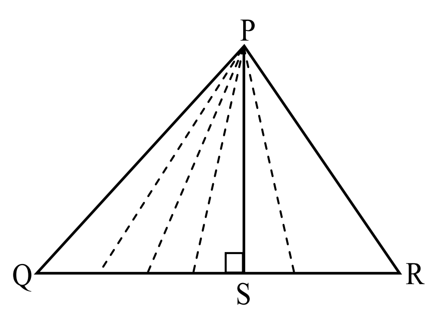
(iv) The centroid of a triangle divides each one of the medians in the ratio 2 : 1.

(v) The medians of an equilateral triangle are equal in length.

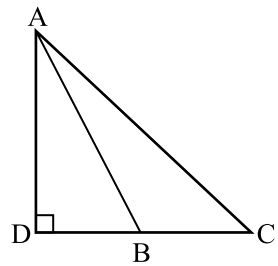
➢ ALTITUDE OF A TRIANGLE

An altitude of a triangle is the line segment drawn from a vertex of a triangle, perpendicular to the line containing the opposite side.

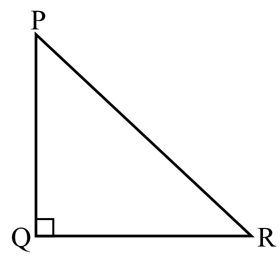
(i) PS is an altitude on side QR in figure.



(ii) AD is an altitude, with D the foot of perpendicular lying on BC in figure.



(iii) The side PQ, itself is an altitude to base QR of right angled ΔPQR in figure.

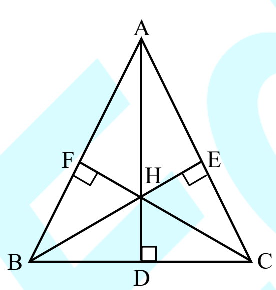


Note :

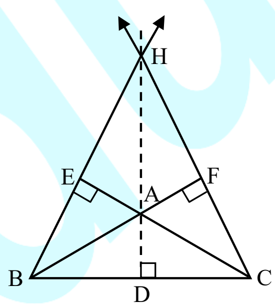
(i) A triangle has three altitudes.

(ii) All the three altitudes meet at a point H (called orthocentre of triangle) i.e., all altitudes of any triangle are concurrent.

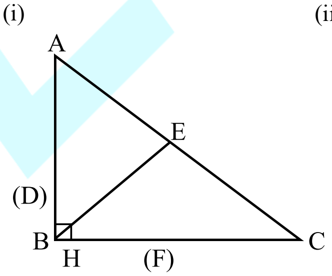
(iii) Orthocentre of the triangle may lie inside the triangle [Figure (i)],, outside the triangle [Figure (ii)] and on the triangle [Figure (iii)].



(i)



(ii)



(iii)

◈ Orthocentre

The point of concurrence of the altitudes of a triangle is called the orthocentre of the triangle.

Notes :

1. Since the altitudes of a triangle are concurrent, therefore to locate the orthocentre of a triangle, it is sufficient to draw its two altitudes.

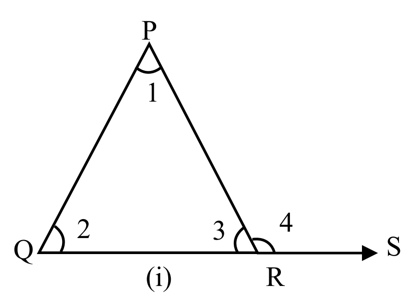
2. Although altitude of a triangle is a line segment, but in the statement of their concurrence property, the term altitude means a line containing the altitude (line segment).

Properties of Altitudes	**Properties of Orthocentre**
1. The altitudes of an equilateral triangle are equal.	**1.** The orthocentre of an acute-angled triangle lies in the interior of the triangle.
2. The altitude bisects the base of an equilateral triangle.	**2.** The orthocentre of a right-angled triangle is the vertex containing the right angle.
3. The altitudes drawn on equal sides of an isosceles triangle are equal.	**3.** The orthocentre of an obtuse-angled triangle lies in the exterior of the triangle.

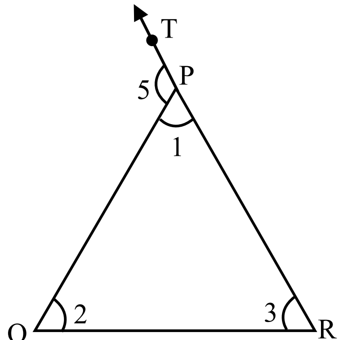
➤ EXTERIOR ANGLE OF A TRIANGLE

If a side of a triangle is produced, the exterior angle so formed is equal to the sum of two interior opposite angles.

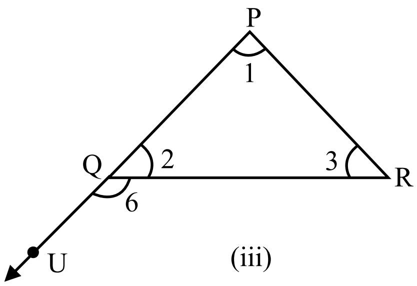
Let ΔPQR be a triangle such that its side QR is produced to form ray QS. Then ∠PRS(∠4) is the exterior angle of ΔPQR at R in [Figure (i)] and angle ∠1 and ∠2 are its two interior opposite angles i.e., ∠4 = ∠1 + ∠2.



(i)



(ii)



(iii)

In Figure (ii), ∠5 is exterior angle at point P and ∠2 and ∠3 are its two interior opposite angle i.e., ∠5 = ∠2 + ∠3.

In Figure (iii), ∠6 is the exterior angle at point Q and ∠1 and ∠3 are its two interior opposite angle i.e., ∠6 = ∠1 + ∠3

Note :

(i) In a triangle an exterior angle is greater than each of the interior opposite angles.

(ii) An exterior angle and the interior adjacent angle form a linear pair.

(iii) An exterior angle of a triangle is equal to the sum of its interior opposite angles.

Therefore, we conclude that in an equilateral triangle, altitudes and medians are the same.

❖ EXAMPLES ❖

Ex.23 How many altitudes can a triangle have ?

Sol. A triangle can have three altitudes.

Ex.24 Fill in the blanks :

(i) A triangle has _______ medians.

(ii) The medians of a triangle are_______

(iii) The point where all the medians meet is said to be the ________ of the triangle.

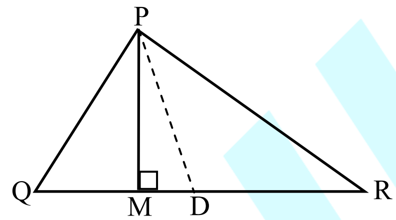
Sol. (i) three (ii) concurrent (iii) centroid.

Ex.25 In ΔPQR, D is the mid point of $\overline{QR}$.

(i) PM is _______ (ii) PD is _______

(iii) Is QM = MR ?

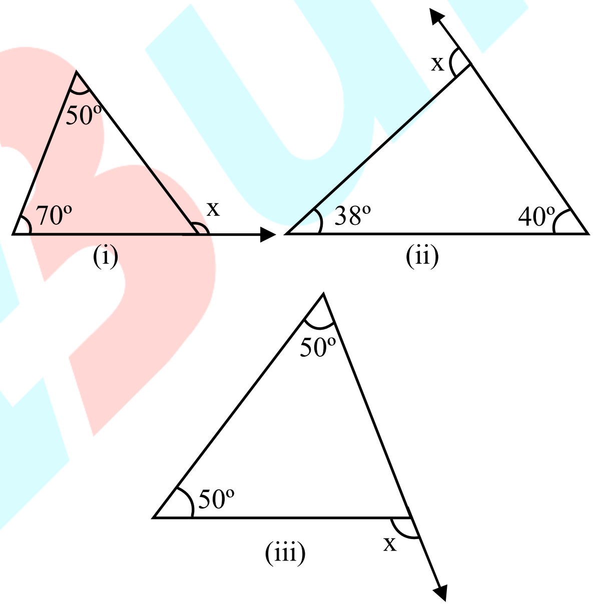
Sol.



(i) PM is altitude. (ii) PD is median.

(iii) No, QM ≠ MR.

Ex.26 Find the value of x in the following diagrams.



Sol. (i) ∠x = 50º + 70º

(∵ exterior angle is equal to sum of its opposite interior angles)

So, ∠x = 120º

(ii) ∠x = 38º + 40º

(∵ exterior angle is equal to sum of its opposite interior angles)

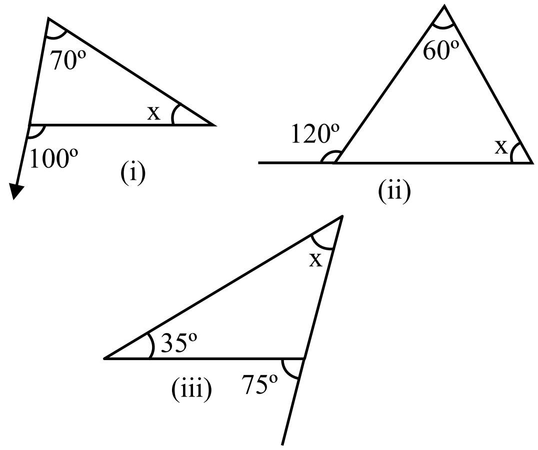
So, ∠x = 78º

(iii) ∠ x = 50º + 50º

(∵ exterior angle is equal to sum of its opposite interior angles)

So, ∠x = 100º.

Ex.27 Find the value of unknown interior angle x in the following figures :



Sol. (i) 100° = 70° + x

(∵ exterior angle is equal to sum of its opposite interior angles)

100° – 70° = x

30° = x

So, x = 30°

(ii) 120° = 60° + x

(∵ exterior angle is equal to sum of its opposite interior angles)

120° – 60° = x

60 ° = x

So, x = 60°

(iii) 75° = 35° + x

75° – 35° = x

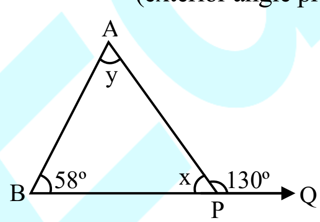
40° = x

So, x = 40°

Ex.28 In the given figure find the values of x and y.

Sol. ∠APQ = ∠BAP + ∠ABP

(exterior angle property of Δ)



130° = y + 58°

130° – 58° = y

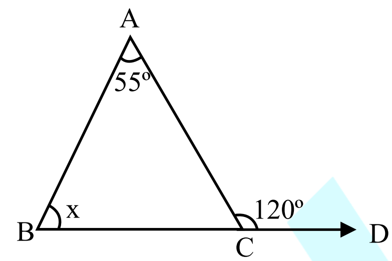
So, y = 72°

Now, x + 130° = 180° (By linear pair)

x = 180° – 130°

So, x = 50°

Ex.29 In the figure, find x.



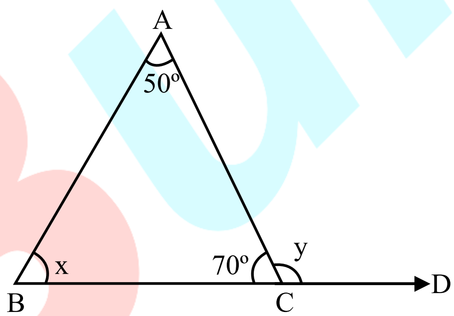
Sol. We know that

exterior angle of the triangle = sum of its two interior opposite angles

∴ 55° + x = 120°

⇒ x = 120° – 55° = 65°

Ex.30 In figure, find the values of x and y using exterior angle property.



Sol. Since 70° + y = 180° (Linear pair)

⇒ y = 180° – 70°

= 110°

We know that

exterior angle of the triangle = sum of its two interior opposite angles

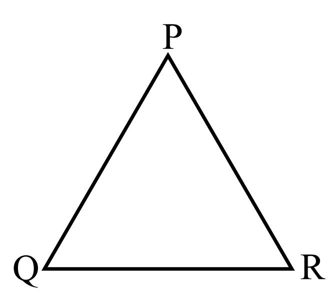
⇒ y = x + 50°

⇒ 110° = x + 50°

⇒ x = 60°

➢ TRIANGLE INEQUALITY

The sum of any two sides of a triangle is greater than the third side. PQ + QR > PR or PR + QR > PQ or PQ + PR > QR



❖ EXAMPLES ❖

Ex.31 Is it possible to have triangle with the following sides ?

(i) 2 cm, 3 cm, 5 cm

(ii) 3 cm, 6 cm, 7 cm

(iii) 6 cm, 3 cm, 2 cm

Sol. (i) No

As $2 + 3 \ngtr 5$

(as the sum of two sides (2 cm, 3 cm) is 5 cm which is not greater than the third side)

(ii) 3 cm, 6 cm, 7 cm

As $3 + 6 = 9 > 7$

$6 + 7 = 13 > 3$

$7 + 3 = 10 > 6$

So, these are the possible sides of the triangle.

(iii) 6 cm, 3 cm, 2 cm

As $6 + 3 = 9 > 2$

$3 + 2 = 5 \ngtr 6$

$6 + 2 = 8 > 3$

As $3 + 2 = 5 \ngtr 6$

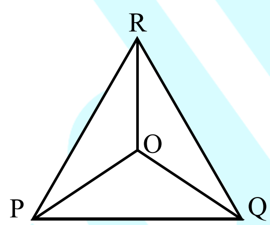
So, these are not the possible sides of triangle.

Ex.32 Take any point O in the interior of triangle PQR. Is

(i) OP + OQ > PQ ? (ii) OQ + OR > QR ?

(iii) OR + OP > RP ?

Sol.



(i) OP + OQ > PQ is true.

($\because$ in ΔPOQ the sum of two sides is greater than the third side.)

(ii) OQ + OR > QR is true.

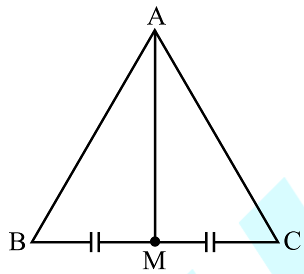
($\because$ in ΔROQ the sum of two sides is greater than the third side.)

(iii) OR + OP > RP is true.

($\because$ in ΔPOR the sum of two sides is greater than the third side)

Ex.33 AM is a median of triangle ABC.

Is AB +BC + CA > 2AM ?



Sol. In ΔABM,

AB + BM > AM …(1)

($\because$ in triangle the sum of any two sides is greater than the third side)

Also in ΔAMC

AC + MC > AM …(2)

Adding (1) and (2), we get

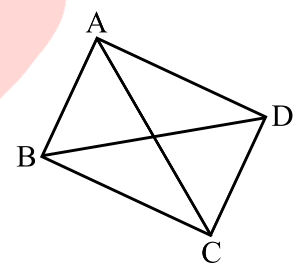
AB + BM + AC + MC > AM + AM

$\Rightarrow$ AB + AC + (BM + MC) > 2 AM

$\Rightarrow$ AB + AC + BC > 2 AM ($\because$ BM + MC = BC)

Ex.34 ABCD is a quadrilateral.

Is AB + BC + CD + DA > AC + BD ?



Sol. In ΔABC

AB + BC > AC …(1)

($\because$ sum of two sides is greater than the third side)

Now, in ΔADC

AD + DC > AC …(2)

($\because$ sum of two sides is greater than the third side)

In ΔABD, AB + AD > BD …(3)

In ΔBCD, BC + CD > BD …(4)

Adding (1), (2), (3) and (4), we get

2(AB + BC + CD + DA) > 2 (AC + BD)

$\Rightarrow$ AB + BC + CD + DA > AC + BD.

Ex.35 The lengths of two sides of a triangle are 6 cm and 10 cm. Between which two numbers can length of third side fall ?

Sol. We know that the sum of two sides of a triangle is always greater than the third side.

∴ The third side has to be less than the sum of the two sides.

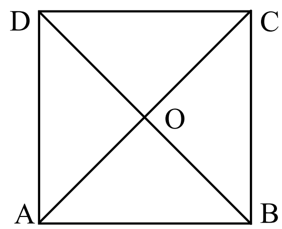
The third side is thus less than 6 + 10 = 16 cm. The side cannot be less than the difference of the two sides. Thus the side has to be more than 10 – 6 = 4 cm.

The length of third side could be any length greater than 4 cm and less than 16 cm.

Ex.36 ABCD is a quadrilateral.

Is AB + BC + CD + DA < 2 (AC + BD) ?

Sol. Let ABCD be a quadrilateral in which diagonals intersect at point O.



In ΔOAB,

OA + OB > AB ...(1)

(as the sum of any two sides is greater than the third side)

Similarly, in ΔOBC,

OB + OC > BC ...(2)

(as the sum of any two sides is greater than the third side)

In ΔDOC, OC + OD > DC ...(3)

In ΔAOD, OA + OD > AD ...(4)

Adding (1), (2), (3) and (4), we get

2(OA + OB+ OC + OD) > AB + BC + DC + AD

⇒ 2(OA + OC) + 2 (OB + OD)

> AB + BC + DC + AC

⇒ 2(AC + BD) > AB + BC + DC + AD

[∵ OA + OC = AC and OB + OD = BD]

or AB + BC + CD + DA < 2(AC + BD)

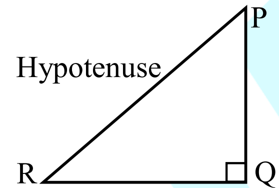
◈ **Rule for angles and sides of triangle :**

(i) The side opposite to the measure of the greatest angle is the greatest and vice-versa.

(ii) The side opposite to the measure of the smallest angle is the smallest and vice-versa.

➤ PYTHAGORAS THEOREM

In a right triangle, the square of the hypotenuse (The side opposite to right angle) is equal to the sum of the squares of its remaining two sides.



In ΔPQR, ∠Q = 90º, we have

$$PR^2 = PQ^2 + RQ^2$$

Note :

(i) In a right triangle, the hypotenuse opposite to right angle is the longest side.

(ii) Of all the line segments that can be drawn to a given line from a point outside it the perpendicular line segment is the shortest.

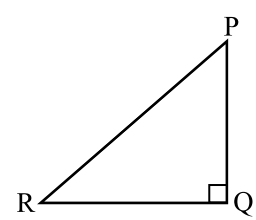
(iii) The two sides of a right triangle other than the hypotenuse are called its legs.

(iv) Three positive integers a, b, c in the same order are said to form a **Pythagoras triplet**, if $c^2 = a^2 + b^2$, for example, (3, 4, 5), (8, 15, 12) are Pythagoras triplets as $3^2 + 4^2 = 5^2$, $8^2 + 12^2 = 15^2$.

◈ **Converse of Pythagoras Theorem :**

If there is a triangle such that the sum of the squares of two of its sides is equal to the square of the third side, it must be a right-angled triangle.

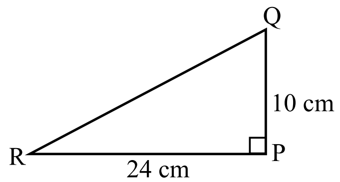
In ΔPQR if $PR^2 = PQ^2 + RQ^2$, then the triangle is right angled at Q.



❖ **EXAMPLES** ❖

Ex.37 PQR is a triangle, right angled at P. If PQ = 10 cm and PR = 24 cm, find QR.

Sol. In ΔRPQ using Pythagoras theorem,



$RQ^2 = PQ^2 + PR^2$

$RQ^2 = (10)^2 + (24)^2 = 100 + 576$

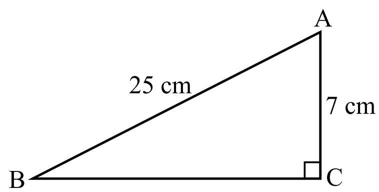
$RQ^2 = 676$

$RQ^2 = 26^2 \quad (\because 676 = 26 \times 26)$

$RQ = 26$ cm

Ex.38 ABC is a triangle, right angled at C. If AB = 25 cm and AC = 7 cm, find BC.

Sol. In ΔABC, using Pythagoras theorem,



$AB^2 = AC^2 + BC^2 \quad (25)^2 = (7)^2 + BC^2$

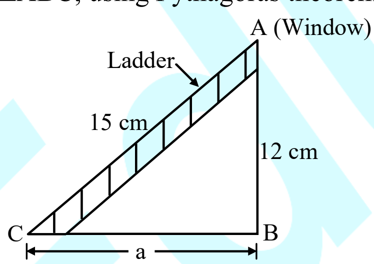
$625 = 49 + BC^2 \quad 625 - 49 = BC^2$

$576 = BC^2 \quad 24^2 = BC^2 \ (\because 24 \times 24 = 576)$

$\Rightarrow$ BC = 24 cm

Ex.39 A 15 m long ladder reached a window 12 m high from the ground on placing it against a wall at a distance 'a'. Find the distance of the foot of the ladder from the wall.

Sol. In ΔABC, using Pythagoras theorem, we get



$AC^2 = AB^2 + BC^2$

$15^2 = 12^2 + BC^2$

$225 = 144 + BC^2$

$225 - 144 = BC^2$

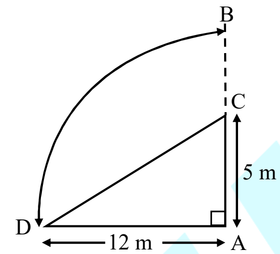
$81 = BC^2 \quad 9^2 = BC^2$

$\Rightarrow$ BC = 9 m i.e., a = 9 m $(\because BC = a)$

Ex.40 A tree has broken at a height of 5 m from the ground and its top touches the ground at a distance of 12 m from the base of the tree. Find the original height of tree.

Sol. Let AB be the tree and let C be the point at which it broke.

Then CB takes the position CD.



To find : Original height of tree i.e., AB

i.e., AC + BC

$\Rightarrow$ AC + CD $(\because BC = CD)$

In ΔACD, using Pythagoras theorem, we have

$CD^2 = AC^2 + AD^2 \quad CD^2 = (5)^2 + (12)^2$

$= 25 + 144 = 169$

$CD^2 = 13^2 \quad CD = 13$ m

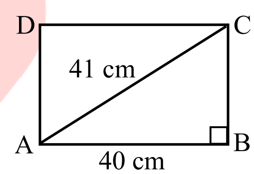
So, height of tree = AC + BC

= AC + CD $(\because BC = CD)$

= (5 + 13)m = 18 m

Hence, height of tree = 18 m

Ex.41 Find the perimeter of the rectangle whose length is 40 cm and a diagonal is 41 cm.



Sol. Let ABCD is a rectangle, in which length AB = 40 cm, and a diagonal AC = 41 cm.

In rectangle each angle is of 90º. So, ∠ABC = 90º

In ΔABC, using Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$(41)^2 = (40)^2 + BC^2$

$\Rightarrow \quad 1681 = 1600 + BC^2$

$\Rightarrow \quad 1681 - 1600 = BC^2$

$\Rightarrow \quad 81 = BC^2$

$\Rightarrow \quad 9^2 = BC^2$

$\Rightarrow$ BC = 9 cm

Hence, breadth of rectangle = 9 cm

Now, perimeter of rectangle

= 2 (length + breadth)

= 2 (40 + 9) cm = 2 × 49 cm

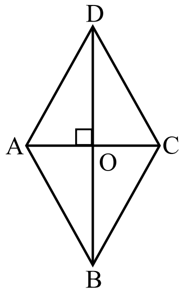
Hence, perimeter of rectangle = 98 cm

Ex.42 The diagonals of a rhombus measure 16 cm and 30 cm. Find its perimeter.

Sol. Let ABCD be a rhombus, in which diagonals

AC and BD are of lengths 16 cm and 30 cm respectively.

We know that in rhombus diagonals bisect each other at right angle i.e., AO = OC and OB = OD.



So, $\angle AOD = 90^\circ$

$$AO = \frac{AC}{2} = \frac{16}{2} = 8 \text{ cm}$$

$$DO = \frac{BD}{2} = \frac{30}{2} = 15 \text{ cm}$$

In ΔAOD, using Pythagoras theorem,

$AD^2 = AO^2 + DO^2$ $\quad AD^2 = (8)^2 + (15)^2$

$= 64 + 225$

$AD^2 = 289 \quad AD^2 = 17^2 \quad AD = 17$ cm

Perimeter of rhombus = 4 × side

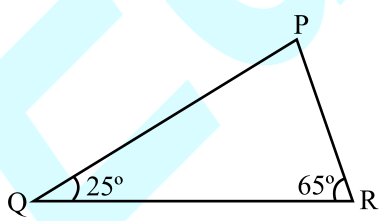
= 4 × AD = 4 × 17 cm

Hence, perimeter of rhombus = 68 cm

Ex.43 Angles Q and R of a ΔPQR are 25° and 65°. Which of the following is true :

(i) $PQ^2 + QR^2 = RP^2$ (ii) $PQ^2 + RP^2 = QR^2$

(iii) $RP^2 + QR^2 = PQ^2$?



Sol. In ΔPQR

$\angle P + \angle Q + \angle R = 180^\circ$

$\angle P + 25^\circ + 65^\circ = 180^\circ$

$\angle P + 90^\circ = 180^\circ$

$\angle P = 180^\circ - 90^\circ = 90^\circ$

$\Rightarrow$ ΔPQR is right triangle in which $\angle P = 90^\circ$

$\therefore$ By Pythagoras theorem,

$$QR^2 = PQ^2 + PR^2$$

Hence, (ii) is true.

Ex.44 Which of the following can be the sides of a right triangle ?

(i) 2.5 cm, 6.5 cm, 6 cm

(ii) 2 cm, 2 cm, 5 cm

(iii) 1.5 cm, 2 cm, 2.5 cm ?

Sol. As we know that in a right angled triangle, the square of longest (hypotenuse) is equal to sum of squares of other two sides.

(i) Let a = 2.5, b = 6.5, c = 6

$a^2 + c^2 = [(2.5)^2 + (6)^2]$ cm^2

$= (6.25 + 36)$ cm^2

$a^2 + c^2 = 42.25$ cm^2

Now $b^2 = (6.5)^2 = 6.5 \times 6.5 = 42.25$ cm^2

$\Rightarrow$ $a^2 + c^2 = b^2$

$\Rightarrow$ 2.5 cm, 6.5 cm, 6 cm are the sides of the right angled triangle.

(ii) Let a = 2, b = 2, c = 5

$a^2 + b^2 = (2)^2 + (2)^2 = 4 + 4$

$a^2 + b^2 = 8$

Now, $c^2 = (5)^2 = 25$

$\Rightarrow$ $a^2 + b^2 \neq c^2$ $\quad (\because 8 \neq 25)$

$\Rightarrow$ 2 cm, 2 cm and 5 cm are not the sides of the triangle.

(iii) Let a = 1.5 cm, b = 2 cm, c = 2.5 cm

$a^2 + b^2 = (1.5)^2 + (2)^2$

$= 2.25 + 4 = 6.25$

$c^2 = (2.5)^2$

$= 6.25$

$\Rightarrow$ $a^2 + b^2 = c^2$

Hence, 1.5 cm, 2 cm and 2.5 cm are sides of the right angled triangle.

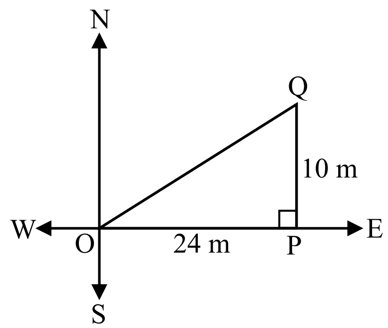
Ex.45 A man goes 24 m due east and then 10 m due north. How far is he away from his initial position ?

Sol. Let O be the initial position of the man. Let he cover OP = 24 m due east and then PQ = 10 m due north.

Finally, he reaches at point Q.

Join OQ which we have to find.

Now, in right ΔOPQ using Pythagoras theorem



$$OQ^2 = OP^2 + PQ^2$$

$$= (24)^2 + (10)^2$$

$$= 576 + 100 = 676$$

$$OQ^2 = 26^2$$

$$OQ = 26 \text{ m}$$

Hence, the man is at a distance of 26 m from his initial position.

Ex.46 A ladder 13 m long reaches a window which is 5 m above the ground, on one side of street. Keeping its foot at the same point, the ladder is turned to the other side of the street to reach a window at a height of 12 m. Find the width of the street.

Sol. Let AB be the street and C be foot of the ladder. Let D and E be the windows at the heights of 5 m and 12 m respectively from the ground. Then, CD and CE are the two position of the ladder. In ΔCDA, using Pythagoras theorem, we have

$$AC^2 + AD^2 = DC^2$$

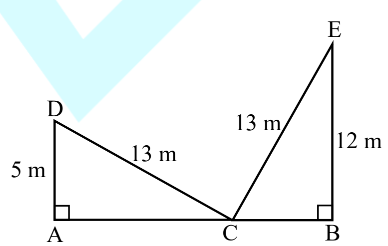
$$AC^2 = DC^2 - AD^2$$

$$= 13^2 - 5^2$$

$$= 169 - 25 = 144$$

$$AC^2 = 12^2$$

$$\Rightarrow \quad AC = 12 \text{ m}$$



Now, in ΔBEC, using Pythagoras theorem,

$$CE^2 = BE^2 + BC^2$$

$$(13)^2 = (12)^2 + BC^2$$

$$169 - 144 = BC^2$$

$$25 = BC^2$$

$$5^2 = BC^2 \Rightarrow BC = 5 \text{ m.}$$

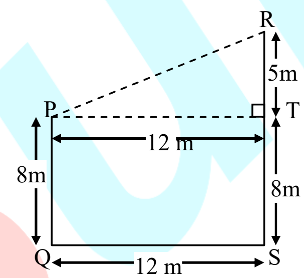
Hence, width of the street

= AB = AC + BC

= 12 m + 5 m = 17 m

Ex.47 Two poles of 8 m and 13 m stand upright on a plane ground. If the distance between their feet is 12 m, find the distance between their tops.

Sol. Let PQ and RS be the given poles such that PQ = 8 m, RS = 13 m and QS = 12 m.



Join PR (the distance between the tops of the poles which we have to find.)

From P, draw PT ⊥ RS.

$\therefore$ RT = RS – TS (TS = PQ = 8 m)

= (13 – 8) m

RT = 5 m

PT = QS = 12 m

In ΔPRT, using Pythagoras theorem,

$$PR^2 = PT^2 + RT^2$$

$$PR^2 = (12)^2 + (5)^2$$

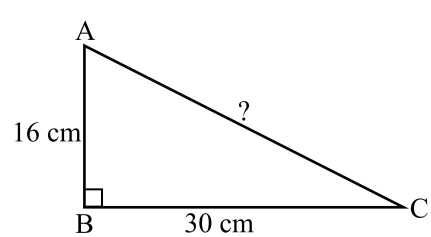
$$= 144 + 25 = 169$$

$$PR^2 = 13^2$$

$$\Rightarrow \quad PR = 13 \text{ m.}$$

Hence, the distance between the tops of the poles is 13 m.

Ex.48 Find the length of hypotenuse of the right-angled triangle given in figure.



Sol. In the figure, AC is the hypotenuse (the side opposite to right-angle).

From Pythagoras Theorem,

$$AC^2 = AB^2 + BC^2$$

$\Rightarrow AC \times AC = AB \times AB + BC \times BC$

$\Rightarrow AC \times AC = 16 \times 16 + 30 \times 30$

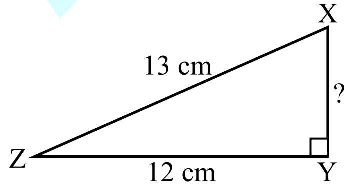
$= 256 + 900 = 1156$

$= 34 \times 34$

On comparing both sides, we get

AC = 34 cm.

Ex.49 Find the length of XY in the right-angled triangle.



Sol. In this Δ, XZ is the hypotenuse (because XZ lies opposite to the right-angle Y).

Therefore, using Pythagoras theorem, we have

$$XZ^2 = XY^2 + YZ^2$$

$\Rightarrow (13)^2 = XY^2 + (12)^2$

$\Rightarrow XY^2 = 13^2 - 12^2 = 169 - 144 = 25$

$\Rightarrow (XY) \times (XY) = 25 = 5 \times 5$

$\Rightarrow$ XY = 5 cm.