Laws of Motion and Friction

FORCE

A pull or push which changes or tends to change the state of rest or of uniform motion or direction of motion of any object is called force. Force is the interaction between the object and the source (providing the pull or push). It is a vector quantity.

(I) Effect of resultant force :

(1) may change only speed

(2) may change only direction of motion.

(3) may change both the speed and direction of motion.

(4) may change size and shape of a body

(II) Unit of force : Newton and $\frac{kg \cdot m}{s^2}$ (MKS System)

dyne and $\frac{g \cdot cm}{s^2}$ (CGS System)

1 Newton = 10^5 dyne

(III) Kilogram force (kgf)

The force with which earth attracts a 1kg body towards its centre is called kilogram force, thus

$$\text{kgf} = \frac{\text{Force in newton}}{g}$$

FREE BODY DIAGRAM

A free body diagram consists of a diagrammatic representations of a single body or a subsystem of bodies isolated from its surroundings showing all the forces acting on it.

Steps for F.B.D.

Step 1: Identify the object or system and isolate it from other objects clearly specify its boundary.

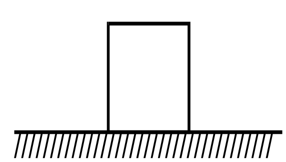
tep 2: First draw non-contact external force in the diagram. Generally it is weight.

Step 3: Draw contact forces which acts at the boundary of the object or system. Contact forces are normal, friction, tension and applied force.

In F.B.D, internal forces are not drawn, only external are drawn.

Solved Examples

Ex.1 A block of mass 'm' is kept on the ground as shown in figure.

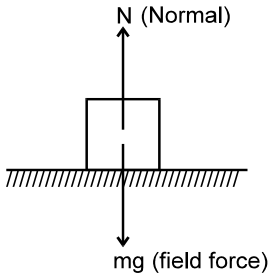


(i) Draw F.B.D. of block.

(ii) Are forces acting on block action–reaction pair.

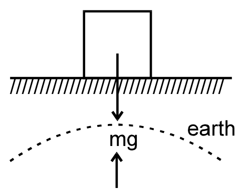
(iii) If answer is no, draw action reaction pair.

Sol. (i) F.B.D. of block

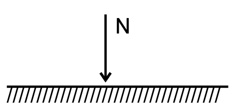


(ii) 'N' and mg are not action-reaction pair. Since pair act on different bodies, and they are of same nature.

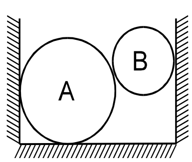
(iii) Pair of 'mg' of block acts on earth in opposite direction.



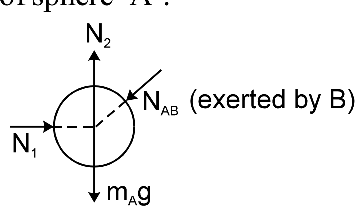
and pair of 'N' acts on surface as shown in figure.



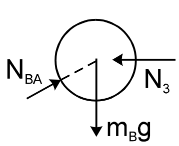
Ex.2 Two sphere A and B are placed between two vertical walls as shown in figure. Draw the free body diagrams of both the spheres.



Sol. F.B.D. of sphere 'A' :



F.B.D. of sphere 'B' : (exerted by A)

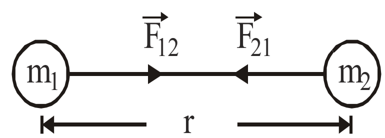


Note : Here N_{AB} and N_{BA} are the action–reaction pair (Newton's third law).

SOME PARTICULAR FORCES

(i) Gravitational force : The force of attraction between bodies by virtue of their masses is known as gravitational force.

Let two blocks of mass m_1 and m_2 are separated by a distance 'r'.



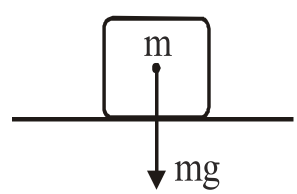
In the sense of magnitude, $F_{12} = F_{21} = F = \dfrac{Gm_1m_2}{r^2}$

Here, G = gravitational constant = 6.7×10^{-11} Nm^2/kg^2.

(ii) Weight of body (mg) : It is defined as the force by which earth attracts a body

towards its centre. If body is situated either on the surface of earth or near the

surface of earth, then gravitational acceleration is nearly constant and is equal to



g = 9.8 m/s^2. The force of gravity (weight) on a block of mass m is $w = mg$

acting centre of earth (shown in figure).

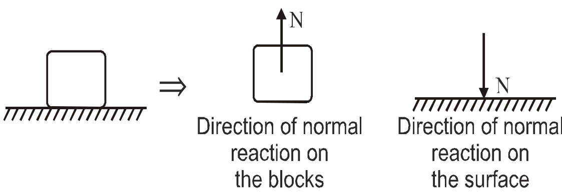
Caution : The weight of a body is not the mass of the body. Weight is the magnitude of force and is related to the mass by $w = mg$.

(iii) The normal reaction force : When a body presses against a surface, the surface (even a seemingly rigid surface) deforms and pushes on the body with a normal reaction force $\vec{N}$ that is perpendiuclar to the surface.

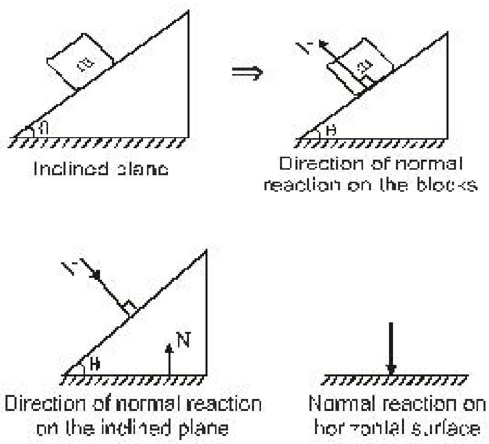
These forces are known as normal reaction forces.

Normal reaction forces in difference situations are shown below :

(a)

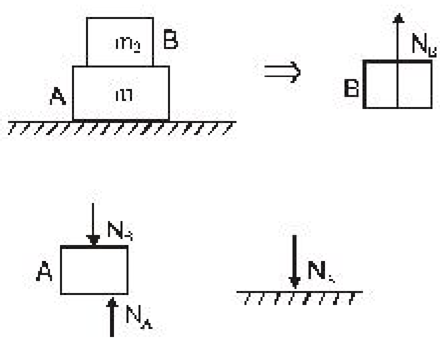


(b)



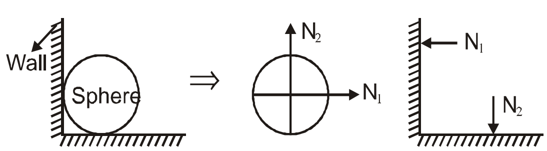
The number of normal reaction pairs is equal to number of contact surfaces.

(c)



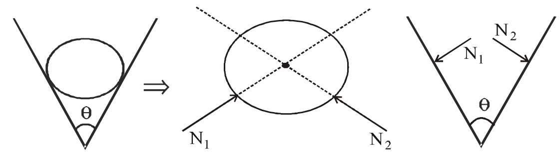
The normal reaction on upper block is in upward direction and normal reaction on lower block is in downward direction.

(d)

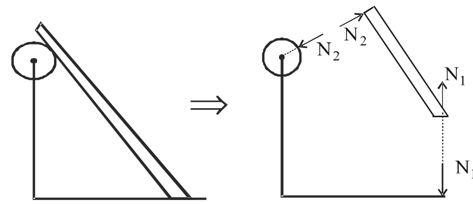


In case of spherical body normal reaction passes through centre of body.

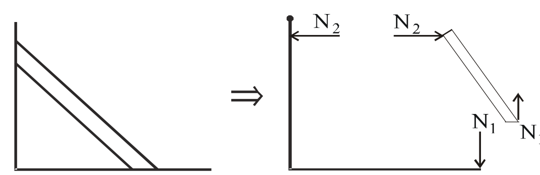
(e)



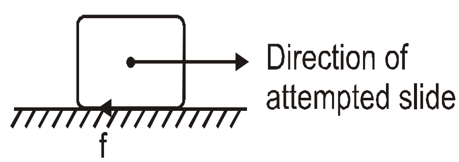
(f)



(g)

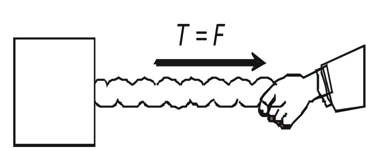


(IV) Friction : If we slide or attempt to slide over a surface, the moton is resisted by a bonding between the body and the surface. The resistance is considered to be single force $\vec{f}$, called the frictional force or simply friction. This force is directed along the surface, opposite the direction of the intended motion.



Some times to simplify a situation, friction is assumed to be negligible (the surface is frictionless).

(V) Tension:- The force exerted by the end of taut string, rope or chain against pulling (applied) force is called the tension. The direction of tension is so as to pull the body.



TENSION IN A STRING

It is an intermolecular force between the atoms of a string, which acts or reacts when the string is streched.

Important points about the tension in a string :

(1) Force of tension act on a body in the direction away from the point of contact or tied ends of the string.

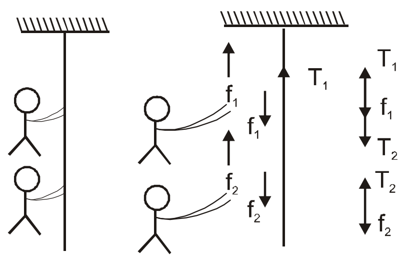
(2) If the string is extensible the acceleration of different masses connected through it will be different until the string can stretch.

(3) If String is massless and frictionless then tension throughout the string remains same.

(4) If the string is massless but not frictionless, at every contact tension changes.

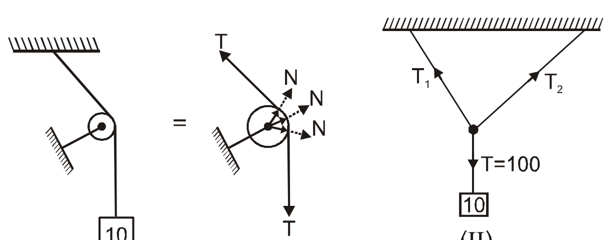
(5) If the string is not light, tension at each point will be different depending on the acceleration of the string.

(6) Any force which act on string and have component along string will change tension of string.



$$T_1 = f_1 + T_2$$
$$T_2 = f_2 = mg$$
$$T_1 = f_1 + f_2$$
$$= f_1 + mg$$

(7) Any force which is perpendicular to string will not change magnitude of tension force.



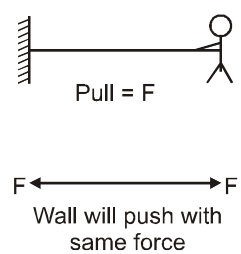
(I) (II)

In case I there will be no any component of normal force along string so tension will be same in string where as in case II there will be component of force along string so tension will be different.

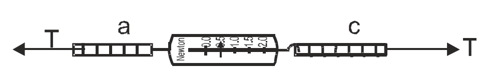
(8) If a force is directly applied on a string as say man is pulling a tied string from the other end with some force the tension will be equal to the applied force irrespective of the motion of the pulling agent, irrespective of whether the box will move or not, man will move or not.

(9) String is assumed to be massless unless stated, hence tension in it every where remains the same and equal to applied force. However, if a string has a mass, tension at different points will be different being maximum (= applied force) at the end through which force is applied and minimum at the other end connected to a body.

(10) In order to produce tension in a string two equal and opposite stretching forces must be applied. The tension thus produced is equal in magnitude to either applied force (i.e., T = F) and is directed inwards opposite to F. Here it must be noted that a string can never be compressed like a



(11) If string is cut so that element b is replaced by a spring scale (the rest of the string being undisturbed), the scale reads the tension T.



(12) Every string can bear a maximum tension, i.e. if the tension in a string is continuously increased it will break if the tension is increased beyond a certain limit. The maximum tension which a string can bear without breaking is called "breaking strength". It is finite for a string and depends on its material and dimensions.

(13) To calculate tension at a point cut the object from that point and apply newton's law

NEWTON'S LAWS OF MOTION

(i) First Law : "If no force acts on a body, then the body's velocity cannot change; that is the body cannot accelerate." In other words, if the body is at rest it stays at rest and if it is moving, it will continue to move with the same velocity (same magnitude and same direction) unless it experiences a net external force.

In simpler terms, we can say that the net force on Q body is zero, its acceleration is zero. That is, where $\Sigma F = 0$, then $a = 0$. From the first law. We conclude that an isolated body (a body that does not interact with its environment) is either at rest or moving with constant velocity.

(ii) Second Law : The change of motion is proportional to the magnitude of force applied and is made in the direction of the straight line in which that force is applied.

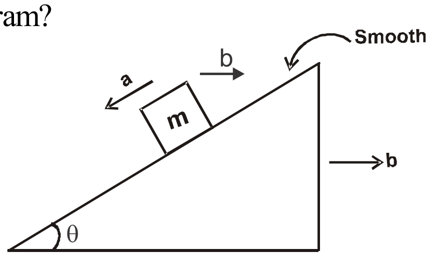
According to this law, the net force on a body is equal to the product of the body's mass and the acceleration of the body. In equation form

$$\vec{F}_{net} = M\vec{a} \qquad \vec{a} = \frac{\vec{F}_{net}}{M}$$

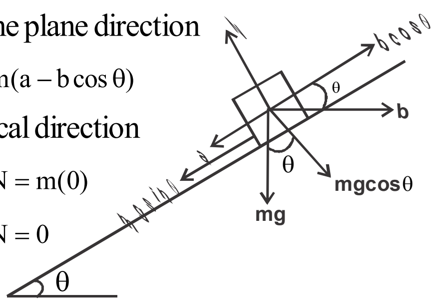
$$\sum F_x = Ma_x \qquad \sum F_y = Ma_y \qquad \sum F_z = Ma_z$$

Solved Examples

Ex.3 Apply Newton's second law on block along horizontal and vertical direction as shown in given diagram?



Sol. along incline plane direction



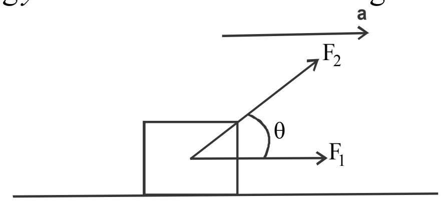
$mg\sin\theta = m(a - b\cos\theta)$

along vertical direction

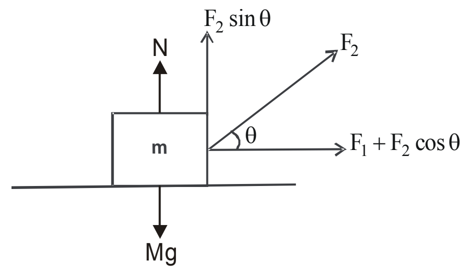
$mg\cos\theta - N = m(0)$

$mg\cos\theta - N = 0$

Ex.4 Apply Newton's second law on block along *x*-axis, along *y*-axis and *z*-axis as shown in given diagram ?



Sol. along x - axis



$F_1 + F_2\cos\theta = ma_x$

along y - axis

$F_2\sin\theta + N - mg = m(0)$

along z - axis

$F = 0$

(iii) Third Law : To every action there is always an equal and opposite reaction or the mutual actions of two bodies upon each other are always directed to contrary parts.

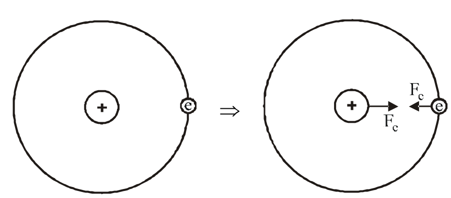
According to this law, when two bodies interact, the forces on the bodies from each other are always equal in magnitude and opposite in direction.

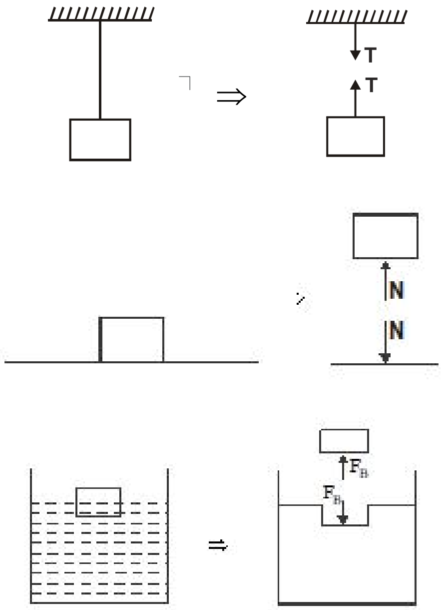
This is, $\vec{F}_{12} = \vec{F}_{21}$

This law, which is illustrated in fig., is equivalent to stating that forces always

occurs in pairs, or that a single isolated force cannot exist.

Action- Reaction pairs :-





(i) Type of action- reaction will be same

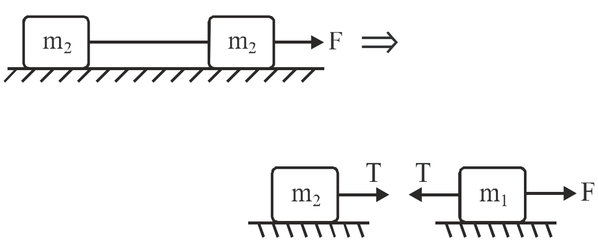
(ii) action- reaction pairs always acts in opposite direction and equal in magnitude

(iii) cause of action-reaction will be same

(iv) action -reaction force acts on different - different objects

FREE BODY DIAGRAM OF TENSION

(a)



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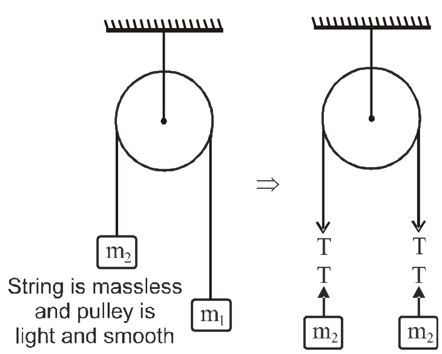

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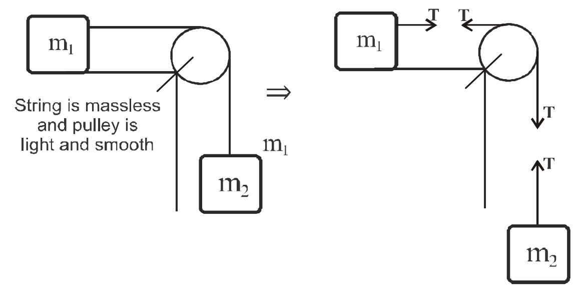

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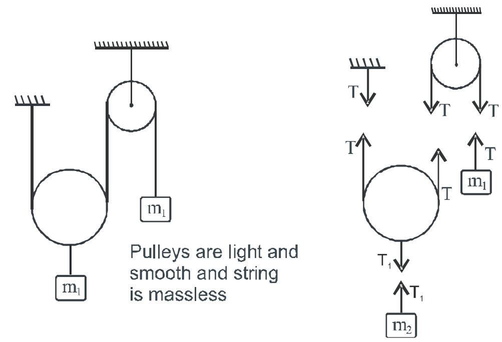

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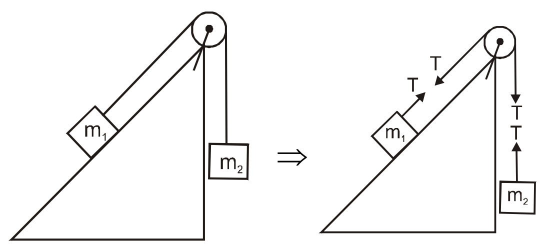

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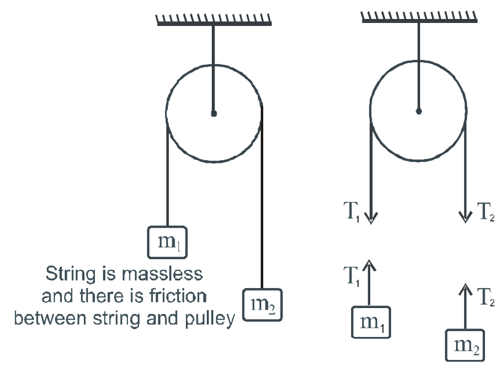

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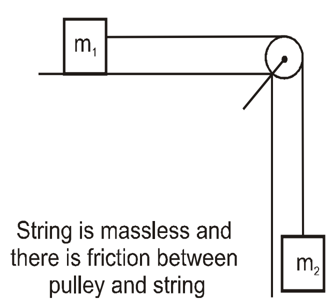

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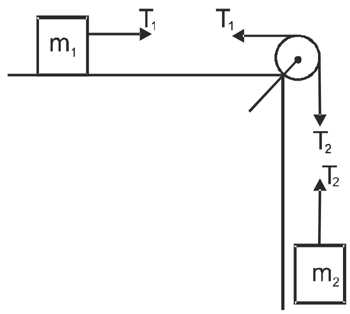

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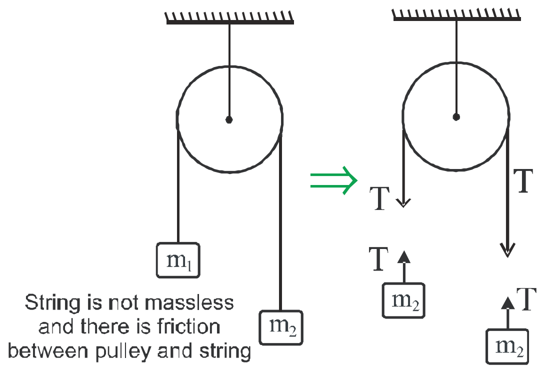
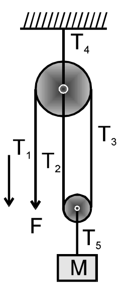


Solved Examples

Ex.5 A mass M is held in place by an applied force F and a pulley system as shown in figure. The pulleys are massless and frictionless.

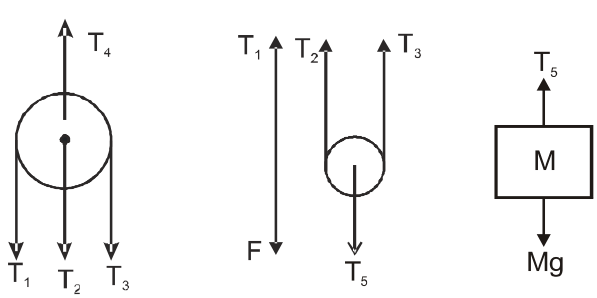


(a) Draw a free body diagram for each pulley

(b) Find the tension in each section of rope T_1, T_2, T_3, T_4 and T_5.

(c) Find the magnitude of F.

Sol.



$T_1 = F$ [Equilibrium of string]

$T_3 = T_1$ [String is massless and pulley is friction less so tension must be same on both sides of string]

⇒ $T_3 = F$

similarly $T_2 = F$

$T_5 = T_2 + T_3$ [Equilibrium of lower pulley]

⇒ $T_5 = 2F$

$T_5 = Mg$ [Equilibrium of block]

$F = T_2 = T_3 = \frac{Mg}{2}$ $T_4 = T_1 + T_2 + T_3$

[Equilibrium of upper pulley]

⇒ $T_4 = \frac{3Mg}{2}$

PROBLEMS SOLVING STEPS

Normally problem based on Newton's law can be solved in following steps :

(i) Concentrate your mind on the system (body) which is considered by you. The considered body may be a single particle, a block or a combination of two or more blocks, two blocks connected by a string. etc.

(ii) Show all forces acting on the system but it is restricted that the forces that the system exerts on other should not be shown here.

(iii) Thus, we draw, a free body diagram of the system and indicate the magnitude and directions of all the forces as discussed in step 2.

(iv) Choose a co-ordinate system including mutually perpendicular 'x' and 'y' axes in the plane of the forces.

Write the components of all the forcs acting along the acceleration (assumed as x-axis) and perpendicular to acceleration (assumed as y-axis). That is

$\Sigma F_x = ma$ ...(i)

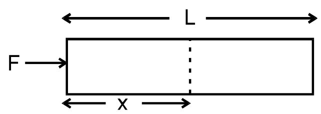
and for y-axis, $\Sigma F_y = 0$ ...(ii)

Note : Above Eqs (i) and (ii) hold in case of coplanar forces.

If the forces are collinear, the Eq. (ii) is not needed.

Solved Examples

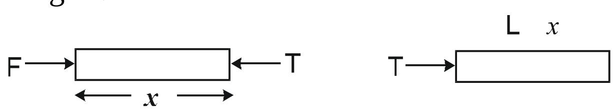
Ex.6 A horizontal force is applied on a uniform rod of length L kept on a frictionless surface. Find the tension in rod at a distance 'x' from the end where force is applied.



Sol. Considering rod as a system, we find acceleration of rod

$$a = \frac{F}{M}$$

now draw F.B.D. of rod having length 'x' as shown in figure.



Using Newton's second law

$$F - T = \left(\frac{M}{L}\right)x.a \quad \Rightarrow \quad T = F - \frac{M}{L}x.\frac{F}{M}$$

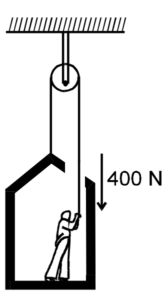
$$\Rightarrow \quad T = F\left(1 - \frac{x}{L}\right).$$

Ex.7 A 60 kg painter stands on a 15 kg platform. A rope attached to the platform and passing over an overhead pulley allows the painter to raise himself along with the platform.

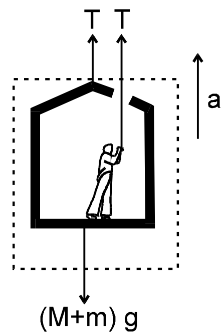
(i) To get started, he pulls the rope down with a force of 400 N. Find the acceleration of the platform as well as that of the painter.

(ii) What force must he exert on the rope so as to attain an upward speed of 1 m/s in 1s?

(iii) What force should he apply now to maintain the constant speed of 1 m/s?



Sol. The free body diagram of the painter and the platform as a system can be drawn as shown in the figure. Note that the tension in the string is equal to the force by which he pulls the rope.



(i) Applying Newton's Second Law

$2T - (M + m)g = (M + m)a$ or

$$a = \frac{2T - (M+m)g}{M+m}$$

Here M = 60 kg; m = 15 kg; T = 400 N

$g = 10\ m/s^2$

$$a = \frac{2(400) - (60+15)(10)}{60+15} = 0.67\ m/s^2$$

(ii) To attain a speed of 1 m/s in one second, the acceleration a must be $1\ m/s^2$.

Thus, the applied force is

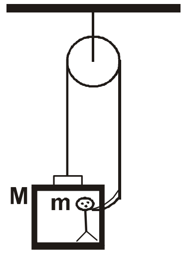
$$F = \frac{1}{2}(M+m)(g+a) = \frac{1}{2}(60+15)(10+1) = \mathbf{412.5\ N}$$

(iii) When the painter and the platform move (upward) together with a constant speed, it is in a state of dynamic equilibrium.

Thus, $2F - (M + m)\,g = 0$ or

$$F = \frac{(M+m)g}{2} = \frac{(60+15)(10)}{2} = \mathbf{375\ N}$$

Ex.8 A man of mass m stands on a crate of mass M. He pulls on a light rope passing over a smooth light pulley. The other end of the rope is attached to the crate. For the system to be is equilibrium, the force exerted by the man on the rope will be:



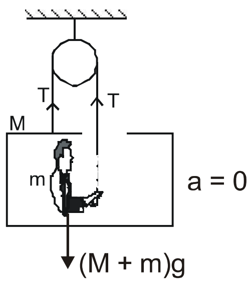
(A) mg (B) Mg

(C) $\frac{1}{2}(M+m)g$ (D) $(m+M)g$

Sol. $2T = (M + m)g$

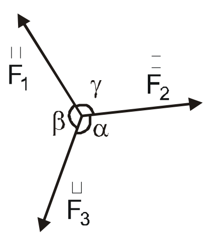
$T = \dfrac{(M+m)g}{2}$

Ans. (C)



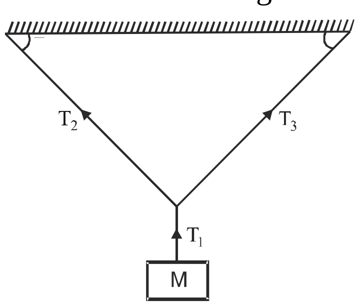
LAMI'S THEOREM : FOR CONCURRENT FORCES

$$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$$



Solved Examples

Ex.9 Find out tension in each string.



Sol.

From Lami's the orem

$$\frac{T_1}{\sin\{180-(\alpha+\beta)\}} = \frac{T_2}{\sin(90+\beta)} = \frac{T_3}{\sin(90+\alpha)}$$

$\because$ $T_1 = mg$

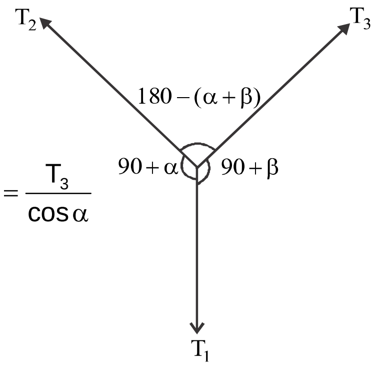
So,

$$\frac{mg}{\cos(\alpha+\beta)} = \frac{T_2}{\cos\beta} = \frac{T_3}{\cos\alpha}$$

on solving

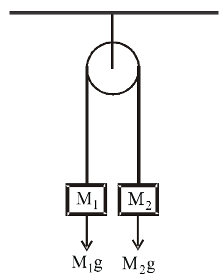
$$T_2 = \frac{mg\cos\beta}{\cos(\alpha+\beta)}$$

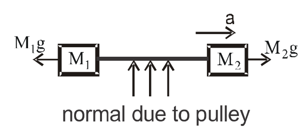
$$T_3 = \frac{mg\cos\alpha}{\cos(\alpha+\beta)}$$



MOTION OF CONNECTED BODIES

(i) Unequal masses ($m_1 > m_2$) suspended from a pulley :





$$M_2g - M_1g = (M_2 + M_1)a$$

$$a = \frac{m_2g - m_1g}{m_1 + m_2}$$

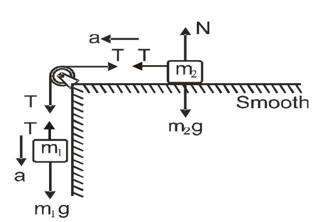
(ii) Bodies accelerated on a horizontal surface by a falling body :

$T = m_2a$ ...(i)

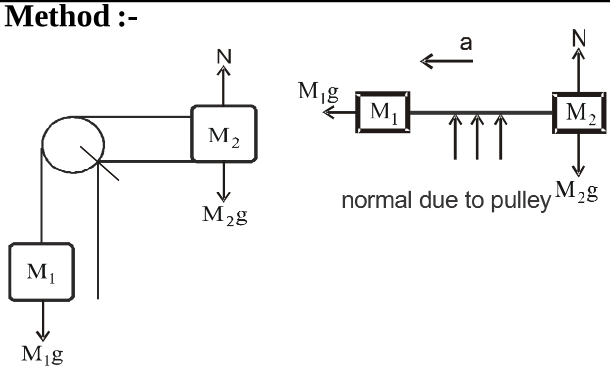
$m_1g - T = m_1a$ ...(ii)

$$\text{Acceleration} = a = \left(\frac{m_1}{m_1+m_2}\right)g$$

$$\text{Tension} = T = \left(\frac{m_1m_2}{m_1+m_2}\right)g$$



Alt. Method :-



$$m_1 g = (m_1 + m_2)a$$

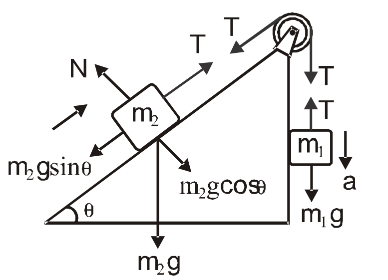
$$a = \frac{m_1 + m_2}{m_1 g}$$

(iii) Motion on a smooth inclined plane :

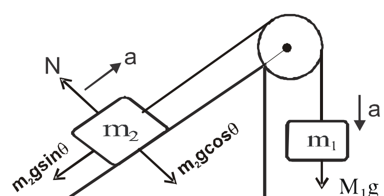
$m_1 g - T = m_1 a$ ...(i)

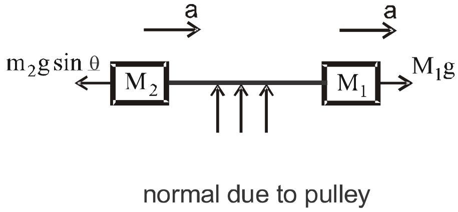
$T - m_2 g \sin\theta = m_2 a$ ...(ii)

$$\therefore \; a = \left(\frac{m_1 - m_2 \sin\theta}{m_1 + m_2}\right) g \text{ and } T = \frac{m_1 m_2 (1 + \sin\theta) g}{(m_1 + m_2)}$$



Alt. Method :





$$M_1 g - M_2 g \sin\theta = (M_1 + M_2)a$$

$$a = \frac{m_1 g - m_2 g \sin\theta}{m_1 + m_2}$$

CONSTRAINED MOTION

(i). string constrained

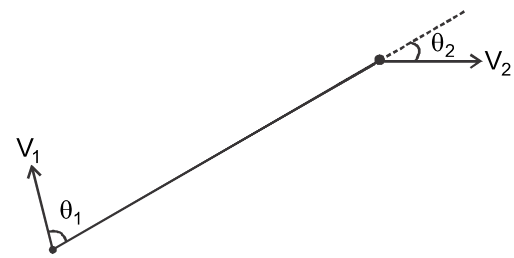
a. Length of string remains constant.

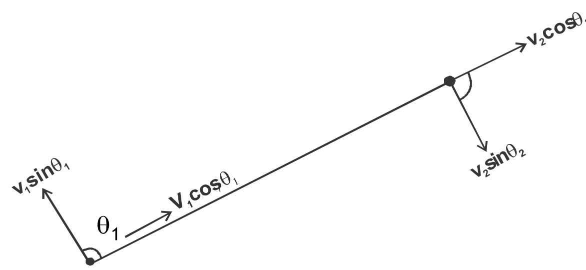
b. We assume string never gets slacked.

c. We assume string never breaks.

Constrained due to constant length :

Case (I):- When string is taken as straight

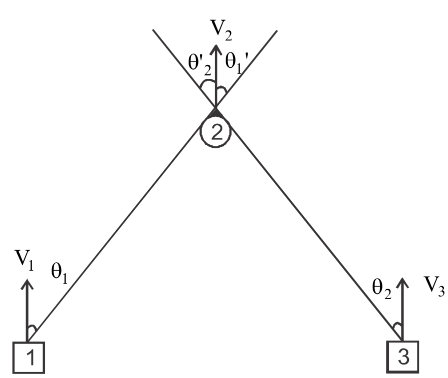


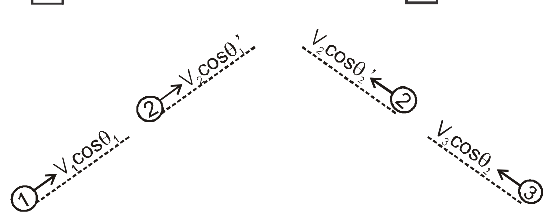


When the two particles are connected with the help of an object such that whose length remains constant then their component of velocity along the length will be equal in magnitude. i.e.

Vapp = 0 Vsep = 0

Case (II):- When string has one or more turns





$\therefore$ Length of strting remains unchanged.

So Length of string = (Distance b/w 1 & 2) + (Distance b/w 2 & 3)

Differentiate on both sides of eq^n :-

$L' = D'_{12} + D'_{23}$

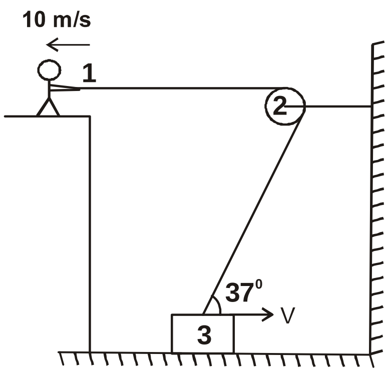
0=(Velocity of separationb/w 1 & 2) +

(Velocity of separation b/w 2&3)

$0 = (V_2\cos\theta_1' - V_1\cos\theta_1) + (V_2\cos\theta_2' - V_3\cos\theta_2)$

Solved Examples

Ex.10 Find velocity of block



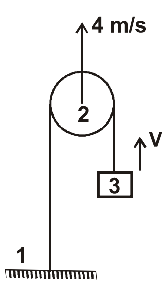
Sol. 0 = (velocity of separation between 1 & 2) + (Velocity of separation between 2 & 3)

$$0 = 10 + (0 - v\cos 37^0)$$

$$v\cos 37^0 = 10$$

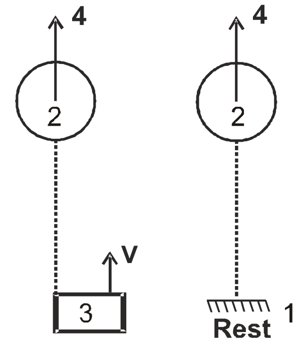
$$v = \frac{50}{4}\,m/s)$$

Ex. 11



Find Velocity of given block.

Sol.

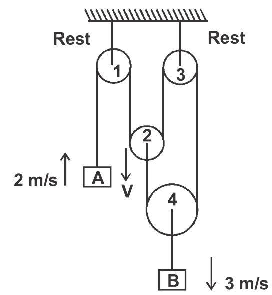


0 = (velocity of separation between 1 & 2) + (Velocity of separation between 2 & 3)

= (+ 4) + (4 –V)

V = 8m/s

Ex.12 Find velocity of pully 2



ℓ = (Distanceb/w a & 1)+(Distanceb/w1&2)+(Distanceb/w 2 &3)+ (Distance b/w 3&4)+(Distanceb/w4&2)

Differentiating on both side.

$$\frac{d\ell}{dt} = \frac{d}{dt}(\text{Distance b/w})\frac{d\ell}{dt} = \frac{d}{dt} + (\text{Ditance b/w A\&1})$$

$$+\frac{d}{dt}(\text{Ditanceb/w1\&2}) + \frac{d}{dt}(\text{Ditance b/w2 \&3})$$

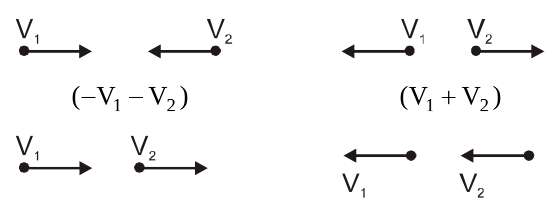
$$+\frac{d}{dt}(\text{Distance b/w 3 \& 4}) + \frac{d}{dt}(\text{Distance b/w 4 \& 2})$$

0 = (0 – 2) + (0 + v) + (0 + v) + (0 + 3) + (3 – v)

v = 4 m/s

0 $\rightarrow$ upward.

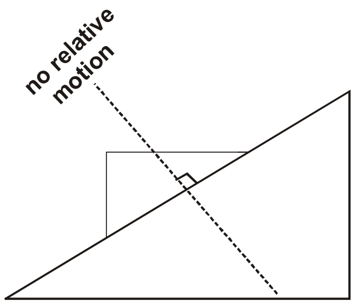
Note : Sign convention regarding velocity.



$(-V_1 - V_2)$ $(V_1 + V_2)$

$(-V_1 + V_2)$ $(V_1 - V_2)$

(ii) Wedge Constraint :



Assumptions :

(a) When two objects remains in contact

(b) Shape and size of object does not change

Results :

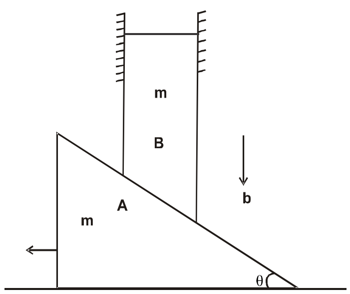
(a) Relative motion b/w the wedges can only occur along the common surface.

(b) There will be no relative motion perpendicular to the common surface.

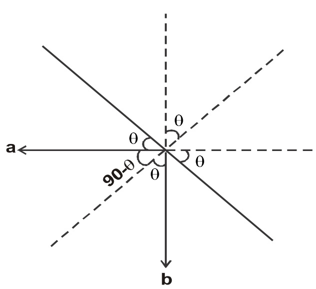
(c) Both the wedges will have same velocity and same acceleration along the line perpen dicular to the common surface.

Solved Examples

Ex.13 Find out reletion between acceleration of both blocks.



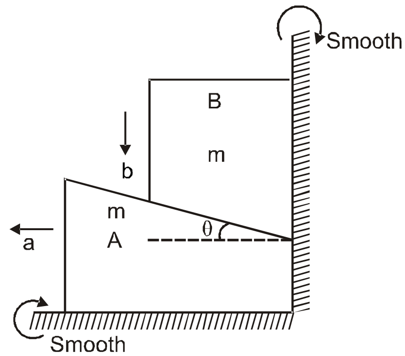
Sol.



accelerations of both blocks along normal will be same

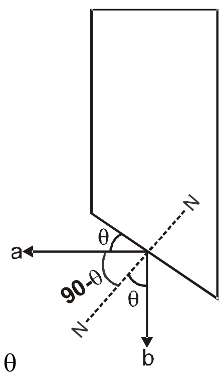
$\therefore\ a\sin\theta = b\cos\theta$

Ex.14



Find the relation between acceleration of both the blocks.

Sol.



$a\sin\theta = b\cos\theta$

NEWTON'S LAW FOR A SYSTEM

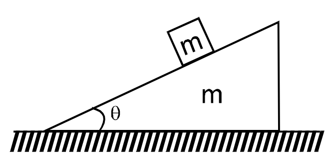
$\vec{F}_{ext} = m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3 + ...$

$\vec{F}_{ext} =$ Net external force on the system.

m_1, m_2, m_3 are the masses of the objects of the system and $\vec{a}_1, \vec{a}_2, \vec{a}_3$ are the acceleration of the objects respectively.

Solved Examples

Ex.15 The block of mass m slides on a wedge of mass 'm' which is free to move on the horizontal ground. Find the accelerations of wedge and block. (All surfaces are smooth).

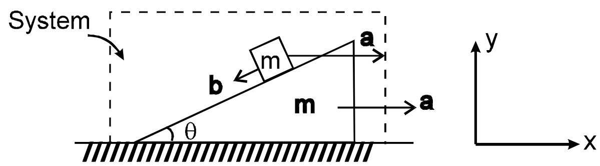


Sol. Let.

$a \Rightarrow$ acceleration of wedge

$b \Rightarrow$ acceleration of block with respect to wedge

Taking block and wedge as a system and applying Newton's law in the horizontal direction



$F_x = m_1\ \vec{a}_{1x} + m_2\ \vec{a}_{2x} = 0$

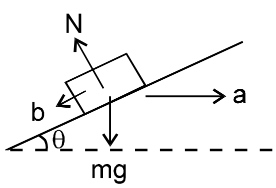
$0 = ma + m(a - b\cos\theta)$ --------(i)

here 'a ' and 'b' are two unknowns , so for making second equation, we draw F.B.D. of block.

F.B.D of block.

using Newton's second law along inclined plane

$mg\sin\theta = m\,(b - a\cos\theta)$ --------(ii)



Now solving equations (1) and (2) we will get

$$a = \frac{mg\sin\theta\cos\theta}{m(1+\sin^2\theta)} = \frac{g\sin\theta\cos\theta}{(1+\sin^2\theta)} \text{ and } b = \frac{2g\sin\theta}{(1+\sin^2\theta)}$$

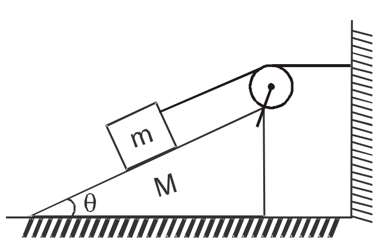
So in vector form :

$$\vec{a}_{\text{wedge}} = a\hat{i} = \left(\frac{g\sin\theta\cos\theta}{1+\sin^2\theta}\right)\hat{i}$$

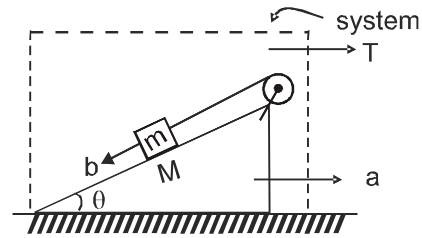
$$\vec{a}_{\text{block}} = (a - b\cos\theta)\,\hat{i} - b\sin\theta\,\hat{j}$$

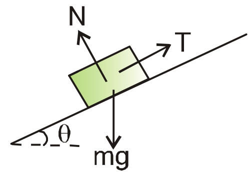
$$\vec{a}_{\text{block}} = -\frac{g\sin\theta\cos\theta}{(1+\sin^2\theta)}\hat{i} - \frac{2g\sin^2\theta}{(1+\sin^2\theta)}\hat{j}\,.$$

Ex.16 For the arrangement shown in figure when the system is released, find the acceleration of wedge. Pulley and string are ideal and friction is absent.



Sol. Considering block and wedge as a system and using Newton' s law for the system along x– direction





$T = Ma + m\,(a - b\cos\theta)$ --------(i)

F.B.D of m

along the inclined plane

$mg\sin\theta - T = m\,(b - a\cos\theta)$ -------(ii)

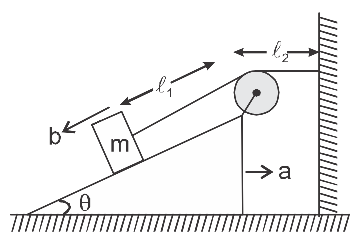
usi ng string constraint equation.

$\ell_1 + \ell_2 = \text{constant}$

$\ell_1'' + \ell_2'' = 0$

$(0 - a) + b = 0$

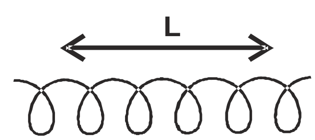
$a = b$



Solving above equations (i) & (ii) we get

$$a = \frac{mg\sin\theta}{M + 2m(1-\cos\theta)}$$

SPRING : HELICAL SPRING



Natural Length of a spring :- Length of a spring in relaxed state. when force by spring is zero or tension in spring is zero i.e. F= 0 & T = 0

Spring force :- It is the force applied by the spring on the objects. conneted to the both ends.

→ Extended spring pulls and compressed spring pushes.

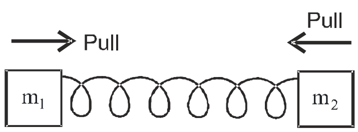
→ Spring always tries to attain its natural length.

→ Spring can push and pull

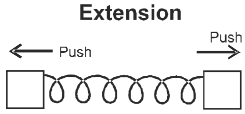
→ If one end is at push then another end will be also push and vise versa

→ Tension is a mass less spring will be same at each point.

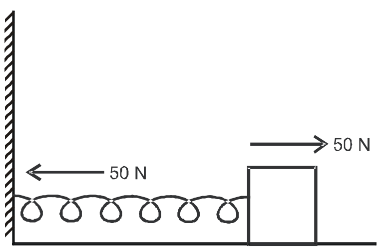
→ Force apply by the spring on the objects connected at the both ends will be equal of magnitude. and opposite in direction.



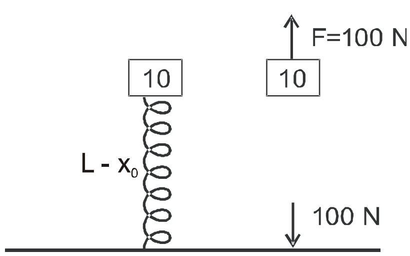
Extension



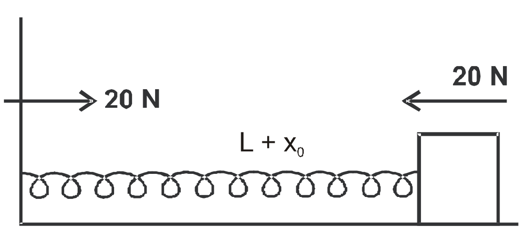
Compression



Compression



Compression



Extension

Force applied by the spring is directly proportional to change in length.

$F \propto \Delta\ell$

$F = K\Delta\ell$

where $\Delta\ell = \ell - L$

$\Delta\ell$ = Current length - natural length

If $\ell > L$ then there will be pull

and If $\ell < L$ then there will be push

$F = k(\ell - L)$

If $F > 0$ then pull and If $F < 0$ then push.

Spring Constant : It is the force required to change in length of spring by one mt. (1unit)

$$K = \frac{F}{\Delta\ell} = N/m \text{ (unit)}$$

→ spring constant is independent from its extension & compression.

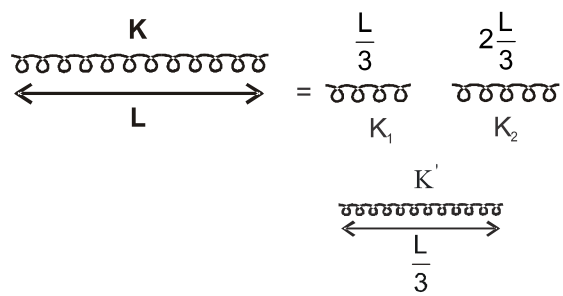
→ When ever a spring is cut its natural length decreases & more force is required for same extension so spring constant increases

$$K \propto \frac{1}{\text{Natural length}}$$

$$k_1L_1 = k_2L_2 = ...$$

Solved Examples

Ex.17 Spring is cut as shown in figure. find K_1 & K_2



Sol. $K \propto \frac{1}{L}$

$$K_1L_1 = K_2L_2 = KL$$

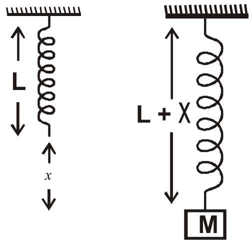
$$K_1\frac{L}{3} = K_2\frac{2L}{3} = KL$$

$$K_1 = 3K$$

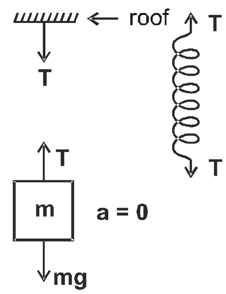
$$K_2 = \frac{3K}{2}$$

Ex.18 A block having mass M is suspended from a given spring. Find out tension & extension in spring

Sol.



F.B.D.....



Where $\Delta\ell = (L + x) - L$

$\Delta\ell = x$

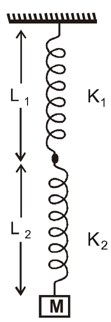
mg - T = 0

T = mg

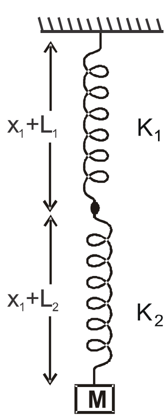
$kx = mg$

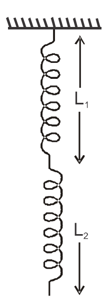
$x = \dfrac{mg}{K}$

Ex.19 Two springs having natural length L_1 and L_2 respectively are joined in series combination. Mass M is suspended from this combination. Find out tension and extension in both spring ?



Sol.





F.B.D...

where $mg - T_2 = 0$

$T_2 - T_1 = 0$

$T_2 = T_1$

$T_2 = Mg$

$T_1 = T_2 = mg$

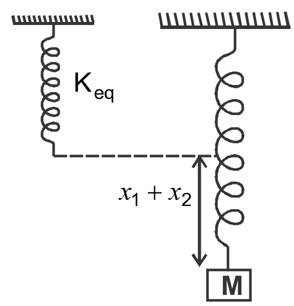
$K_1 x_1 = K_2 x_2 = mg$

$x_1 = \dfrac{mg}{K_1}$

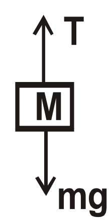
$x_2 = \dfrac{mg}{K_2}$

(a) Series combination of spring :- In series combination of massless spring tension will be same in each spring.

$$\frac{1}{K_{eq}} = \frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3} + \ldots\ldots..$$



F.B.D....



$K_{eq}(x_1 + x_2) = mg$

$x_1 + x_2 = \dfrac{mg}{K_{eq}}$

$\dfrac{mg}{K_1} + \dfrac{mg}{K_2} = \dfrac{mg}{K_{eq}}$

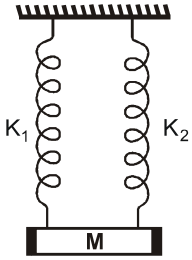
$\dfrac{1}{K_1} + \dfrac{1}{K_2} = \dfrac{1}{K_{eq}}$

$$\frac{1}{K_{eq}} = \frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3} + \ldots\ldots\ldots$$

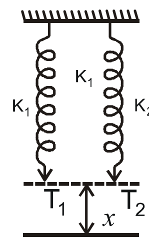
(b) Parallel combination of spring :- In parallel combination Extention will be same

$K_{eq} = K_1 + K_2 + K_3 + ...$

Ex. 20 Two springs having spring constant K_1 and K_2 respectively. Mass M is suspended from this combination. Find out tension and extenion in both spring



Sol. $T_1 = K_1 x$

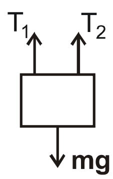


$T_2 = K_2 x$

$T_1 + T_2 = mg$

$K_1 x + K_2 x = mg$

$x = \dfrac{mg}{K_1 + K_2}$



$T_1 = \dfrac{K_1 mg}{K_1 + K_2}$

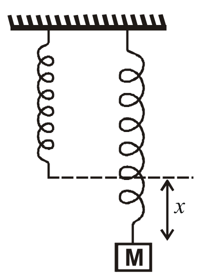
$T_2 = \dfrac{K_2 mg}{K_1 + K_2}$

Ex.21 Find out K_{eq} in above question

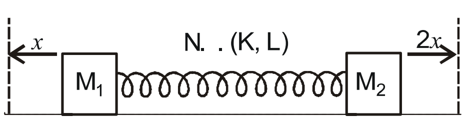
Sol. $K_{eq} x = mg$

$K_{eq} x = K_1 x + K_2 x$

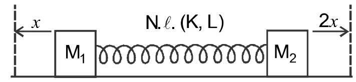
$K_{eq} = K_1 + K_2$

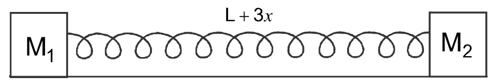


Ex.22



Sol.





$\Delta\ell = (L + 3x) - L$

$= 3x$

$T = K(3x)$

$a_1 = \dfrac{T}{m_1} = \dfrac{3Kx}{m_1}$ $\qquad a_2 = \dfrac{3Kx}{m_2}$

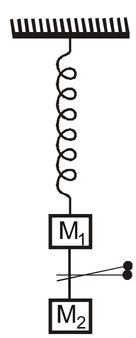
Note: **(1)** When ever a spring is connected between two masses the length of the spring cannot change suddenly. because to change the length masses must move and for the displacment of masses time interval is required

(2) Whenever a spring is out one end of the spring becomes free (massless) & spring attains natural length. and tension becomes zero instantaneusly.

(3) Tension in a string can change suddenly because tension is independent from length of this string and to change tension no displacement is required.

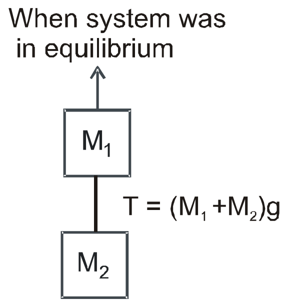
Solved Examples

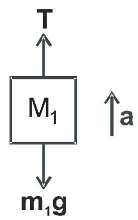
Ex.23 Find out acceleration of two block just after the string is cut ?



When system was in equilibrium

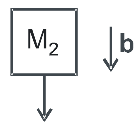
Sol.





$(m_1 + m_2)g - m_1g = m_1a$

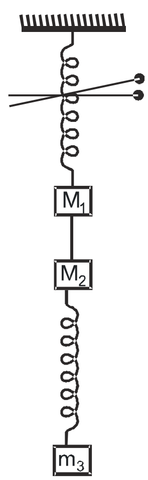
$a = \frac{m_2}{m_1}g$



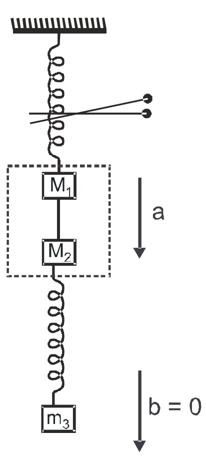
$m_2g = m_2b$

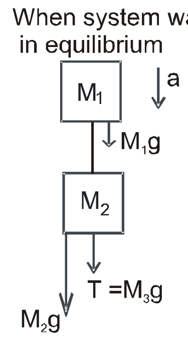
$b = g$

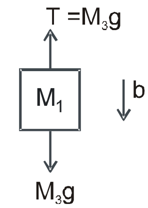
Ex.24 Find out acceleration of blocks shown in figure. Just after spring is cut.



Sol.







$M_3g - M_3g = M_3b$

$b = 0$

$(M_1 + M_2 + M_3)g = (M_1 + M_2)a$

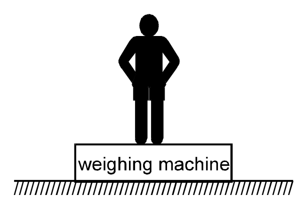
$$a = \left(\frac{M_1 + M_2 + M_3}{M_1 + M_2}\right)g$$

WEIGHING MACHINE

A weighing machine does not measure the weight but measures the force exerted by object on its upper surface.

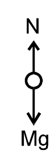
Solved Examples

Ex.25 A man of mass 60 Kg is standing on a weighing machine placed on ground. Calculate the reading of machine ($g = 10\ m/s^2$).

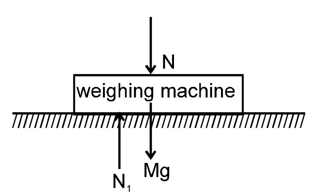


Sol. For calculating the reading of weighing machine, we draw F.B.D. of man and machine separately.

F.B.D. of man   F.B.D. of eighing machine



$N = Mg$

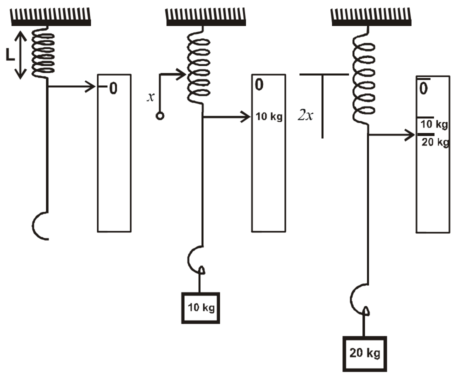


Here force exerted by object on upper surface is N

Reading of weighing machine

$N = Mg \quad = 60 \times 10 \qquad N = 600\ N.$

SPRING BALANCE

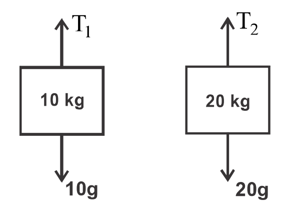


It does not measure the weight. It measures the force exerted by the object at the hook. Symbolically, it is represented as shown in figure.

A block of mass ‘m’ is suspended at hook.

When spring balance is in equilibrium, we draw the F.B.D. of mass m for calculating the reading of balance.

F.B.D. of ‘m’.



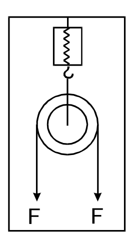
$10g - T = 0$ $\quad$ $20g - T = 0$

$T = 100\,N$ $\quad$ $T = 200\,N$

Magnitude of T gives the reading of spring balance.

Solved Examples

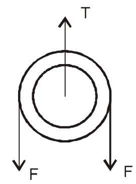
Ex.26 Find the reading of spring balance in the adjoining figure, pulley and strings are ideal.



Sol. **FBD of spring balance**

$R = T$ ....(i)

FBD of pulley



$T = 2\,F$ ....(ii)

$R = 2F$

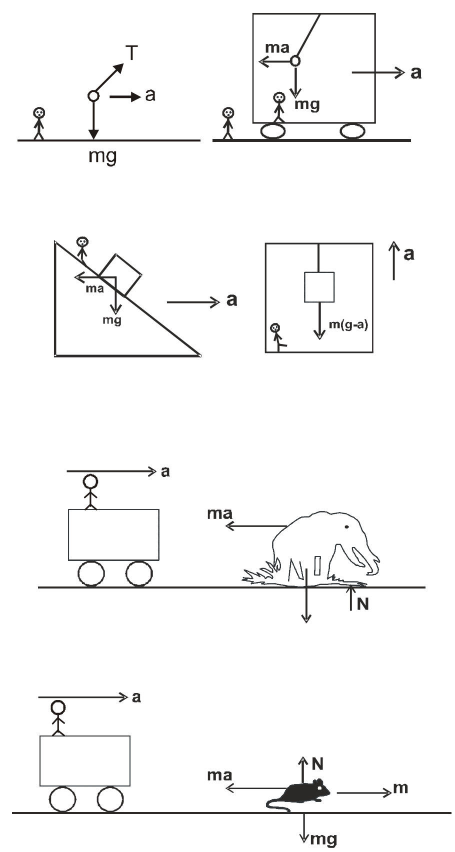
PSUEDO FORCE

(i) It is very clear that newton’s law can not be applied by every type of observer.

(ii) For an accelerated observer a block kept on smooth floor appear to be accelerater without any force.

(iii) Only observer at rest or moving with constant velocity can apply newton’s law.

(iv) Psuedo force makes us us capable to apply newton’s law according to accelerated observer.



F_{psuedo} on elephant = $M_{eliphant}\ a_{observer/\ ground}$

F_{psuedo} on rat = $M_{rat}\ a_{observer/\ ground}$

(v) If object is massless or observer is unaccelerated.i.e F_{pf} =0

(vi) Direction of psuedo force is opposite to the direction of observer with respect to ground.

There are two type of Reference frame

(a) Inertial Refrence frame

(b) Non Inertial reference frame

(a) <u>Inertial Reference frame</u> : A reference frame in which " Law of inertia " is applicable that means newton's Laws are applicable is called as inertial reference frame .

A frame moving with constant velocity or at rest B inestial Refrence frame.

A frame having zero acceleration.

Newton's Law for Inertial observer

$$\sum \vec{F}_{ext} = Ma_{object/graund}$$

Newton's Law for accelerated observer

$$\sum \vec{F}_{ext} + \vec{F}_{pf} = Ma_{object/observer}$$

$\vec{F}_{pf}$ = Psuedo force

Psuedo force is an additional Imaginary force that must be applied on an object to apply newton's law in accelerated reference frame in addition to the real force.

(vii) Psuedo force will not act for stationary or an accelerated observer. It should be applied only when observer is accelerated.

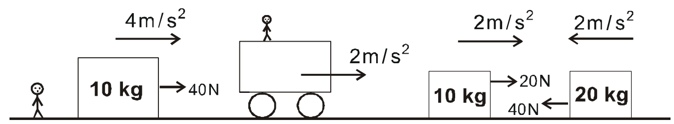
$$\vec{F}_{pf} = 0$$

when object is massless or when acceleration of observer is zero

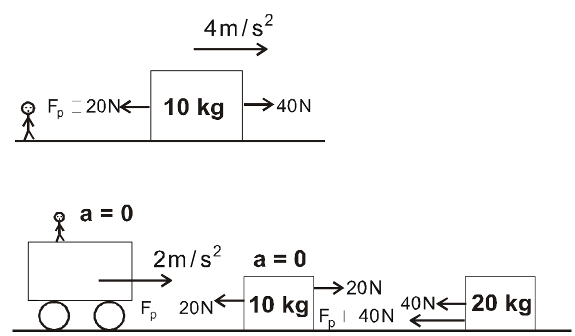
Solved Examples

Ex.27 Draw psuedo force on each object.

Force and acceleration with respect to ground



Sol. Froce and acceleration with respect to cart

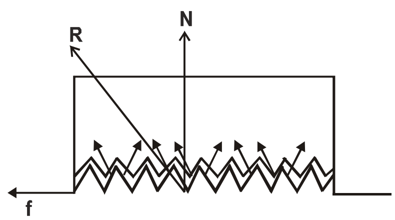


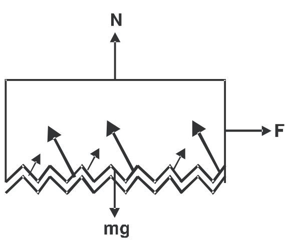
FRICTION

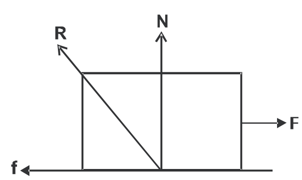
INTRODUCTION

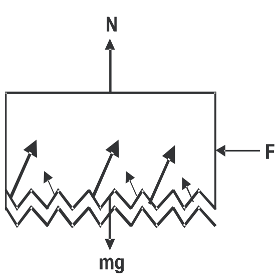
When two bodies are kept in contact, electromagnetic forces act between the charged particles (molecules) at the surfaces of the bodies. Thus, each body exerts a contact force on the other. The magnitudes of the contact forces acting on the two bodies are equal but their directions are opposite and therefore the contact forces obey Newton's third law.

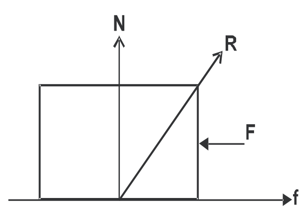
The direction of the contact force acting on a particular body is not necessarily perpendicular to the contact surface. We can resolve this contact force into two components, one perpendicular to the contact surface and the other parallel to it.











Friction - It is the component of resultant contact force parallel to the contact surface.

Normal - It is the component of resultant contact force perpendicular to the contact surface.

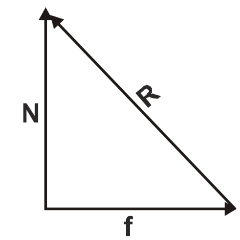
ANGLE OF FRICTION (ϕ)

The angle of friction (ϕ) may be defined as the angle between the normal reaction N and the resultant of friction force f and the normal reaction.

Thus $\tan\phi = \frac{f}{N}$

since $f = \mu N$. therefore

$\tan\phi = \mu$



Causes of friction :-

(i) Interlocking of projected parts of the object into the contact surface and projected parts of the surface into the object

(ii) Temporary bodies behaves as the molecules of the object and contact surface.

Key points :-

(i) Force of friction is practicaly independent of microscopic area of surface in contact and relative velocity between them. (if it is not high)

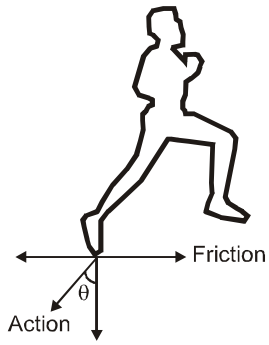
(ii) However, it depends on the nature of material of the surface in contact (force of adhesion) and their roughness and smoothness.

(iii) Normally with increase in smoothness friction decreases. But if the surface area are made too smooth by polishing and cleaning the bonding force of adhesion will increase and so the friction will increase resulting in 'Cold welding'

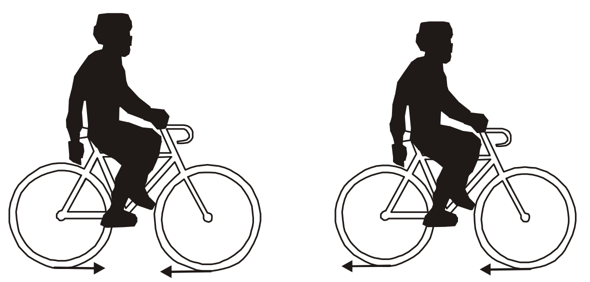
(iv) Friction is a non conservative force, i.e. work done against friction is path dependent.

(v) In its presence mechanical energy is not conserved. Thus friction reduces efficiency of a machine.

(vi) In moving, a person or vehicle pushes the ground backwards (action) and the rough surface of ground reacts and exerts a forward force due to friction which causes the motion. If there had been no friction there will be slipping and no motion.

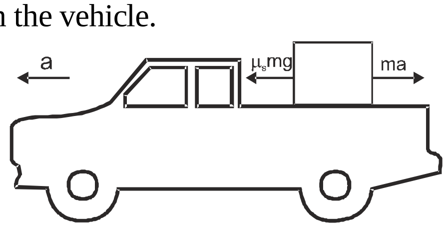


(vii) In cycling, the rear wheel moves by the force communicated to it by pedaling while front wheel moves by itself. So, when peddling a bicycle, the force exerted by rear wheel on ground makes force of friction act on it in the forward direction (like walking). Front wheel moving by itself experience force of friction in backward direction (like rolling of a ball). [However, if peddling is stopped both wheels move by themselves and so experience force of friction in backward direction.]



While pedaling Pedaling is stopped

(viii) If a body is placed in a vehicle which is accelerating, the force of friction is the cause of motion of the body along with the vehicle (i.e., the body will remain at rest in the accelerating vehicle until If there had been no friction between body and vehicle the body will not move along with the vehicle.



From these examples it is clear that without friction motion cannot be started, stopped or transferred from one body to the other.

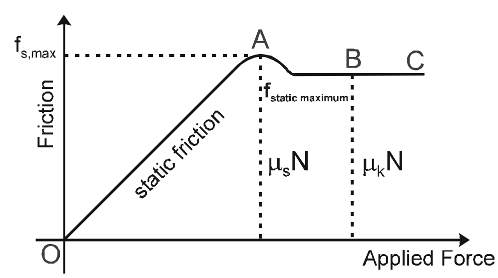
Graph Between Applied Force and Force of Friction :-

(1) Part OA of the curve represents static friction. Its value increases linearly with the applied force

(2) At point A the static friction is maximum. This represent limiting friction .

(3) Beyond A, the force of friction is seen to decrease slightly. The portion BC of the curve represents the kinetic friction .

(4) As the portion BC of the curve is parallel to x-axis therefore kinetic friction does not change with the applied force, it remains constant, whatever be the applied force



COEFFICIENT OF FRICTION

Ratio of maximum limiting friction force to normal is called as friction coefficient

$$\mu = \frac{f_{max}}{N}$$

Types of friction coefficient :-

(i) static friction coefficient (μ_s)

(ii) kinetic friction coefficient (μ_k)

Properties of friction coefficient :-

(i) It depends on the degree of smoothness of two contact surfaces.

(ii) It depends upon the material of two contact surfaces.

(iii) Friction coefficient is the combined property of two contact surfaces.

(iv) The value of friction coefficient can be from 0 to ∞.

(v) $\mu_s > \mu_k$ for a given pair of surfaces. If not mentioned then $\mu_s = \mu_k = \mu$ can be taken.

(vi) μ_s & μ_k are dimensionless quantities independent of shape and area of contact. It is a property of the two contact.

(viii) Following table gives a rough estimate of the values of coefficient of static friction between certain pairs of materials. The actual value depends on the degree of smoothness and other environmental factors.For example, wood may be prepared at various degrees of smoothness and the friction coefficient will vary.

Material	μ_s	Material	μ_s
Steel and steel	0.58	Copper and copper	1.60
Steel and brass	0.35	Teflon and teflon	0.04
Glass and glass	1.00	Rubber tyre on dry concrete road	1.0
Wood and wood	0.35		
Wood and metal	0.40	Rubber tyre on wet concrete road	0.7

TYPES OF FRICTION

Kinetic friction :- When there is a relative motion between two contact surfaces then the frictional force betveen them will be kinetic friction.

or

If the applied force is increased further and sets the body in motion, the friction opposing the motion is called dynamic or kinetic friction.

Direction of Kinetic friction on an object :–

It is opposite to the relative velocity of the object with respect to the other object in contact considered.

Note that its direction is not opposite to the force applied it is opposite to the relative motion of the body considered which is in contact with the other surface.

Magnitude of Kinetic Friction :-

The magnitude of the kinetic friction is proportional to the normal force acting between the two bodies. We can write

$f_k = \mu_k N$

where N is the normal force. **The proportionality constant μ_k is called the coefficient of kinetic friction and its value depends on the nature of the two surfaces in contact.**

Conditions for kinetic friction:-

(i) There must be contact and compression

(ii) There must be relative velocity w.r.t. to contact surface.

(iii) Contact surfaces must be rough

Duration of kinetic friction :- It will act upto when there is relative velocity and when relative velocity becomes zero then kinetic friction force stops. When there is slipping then there must be kinetic friction.

Key Points :

(i) When there is slipping then there will be kinetic friction.

(ii) Kinetic friction always acts to the opposite direction to relative velocity.

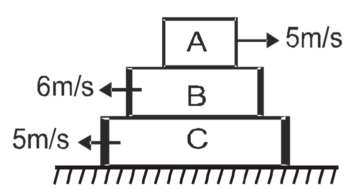
(iii) It tries to stop relative motion between two contact surfaces

(iv) Friction acts in the direction of the object when velocity is smaller .

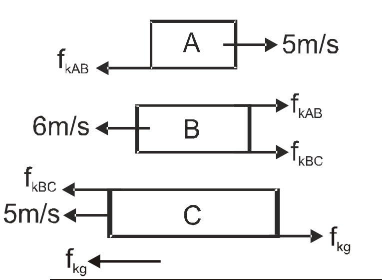
(v) It acts opposite to the direction of velocity when the velocity of the object is greater.

Solved Examples

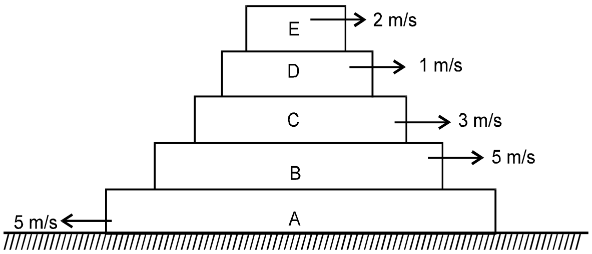
Ex:28 Draw the direction of friction force on each block.



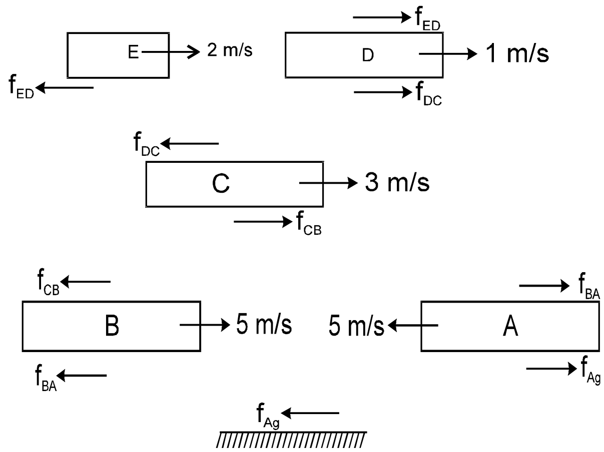
Sol.



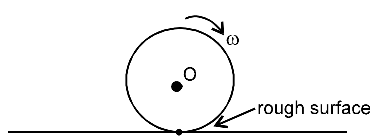
Ex.29 In the given diagram find the direction of friction forces on each block and the ground (Assume all surfaces are rough and all velocities are with respect to ground).



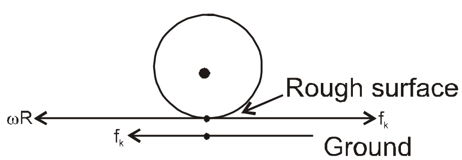
Sol.



Ex.30 The wheel shown in the diagram is fixed at 'O' and is in contact with a rough surface as shown. The wheel rotates with an angular velocity ω. What is the direction and nature of friction force on the wheel and on the ground.

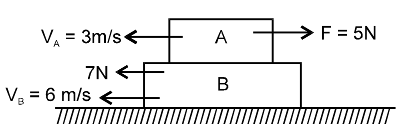


Sol.

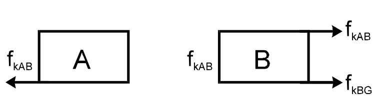


Kinetic friction is involved.

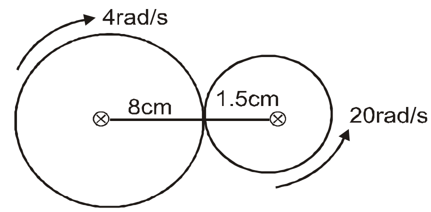
Ex.31 In the following figure, find the direction of friction on the blocks and ground .



Sol. Direction of friction is such that it opposes the relative velocity.



Ex.32 Draw the direction of friction force at contact point.



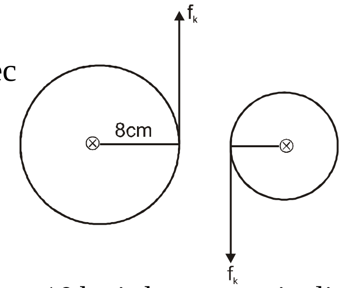
Sol. $\therefore v_1 = r_1\omega_1$

$= 8 \times 4 = 32$ rad/ sec

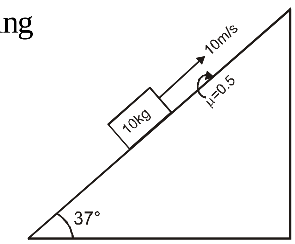
$v_2 = r_2\omega_2$

$= 1.5 \times 20$

$= 30$ rad /sec



Ex.33 A block having mass 10 kg is kept on an incline plane then find following



(i) Normal

(ii) friction

(iii) acceleration

(iv) distance travelled by block before it stops.

(v) velocity of block when its displacement becomes zero ?

Sol. (i) $N = 10g \cos 37°$

$N = 100 \times \frac{4}{5} = 80$

N = 80Nt.

(ii) Friction = μN

$= 0.5 \times 80 = 40N$

(iii) $a = g\sin\theta + 4$

$= 10 \times \frac{3}{5} + 4 = 10 m/s^2$

(iv) $V^2 = u^2 + 2as$

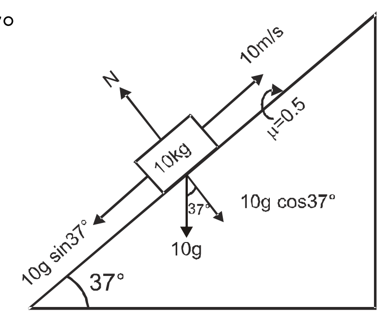
$0 = 100 - 2(10)s$

s = 5m

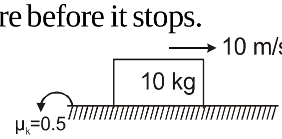
(v) $V^2 = u^2 + 2as$

$V^2 = 0 + 2 \times (+2) \times 5$

$V = 2\sqrt{5}$ m/s



Ex.34 Find out the distance travelled by the blocks shown in the figure before it stops.



Sol. $N - 10\,g = 0$

$N = 100\ N$

$f_k = \mu_k N$

$\mu = \mu_s = \mu_k$ when not mentioned

$f_K = 0.5 \times 100 = 50\ N$

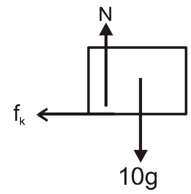
$F_K = ma$

$50 = 10\,a \Rightarrow a = 5$

$\therefore v^2 = u^2 + 2as$

$0^2 = 10^2 + 2(-5)(S)$

$\therefore S = 10\ m$



Ex.35 Find out the distance travelled by the block on an incline plane before it stops. Initial velocity of the block is 10 m/s and coefficient of friction between the block and incline is

Sol. $N = mg\cos 37°$

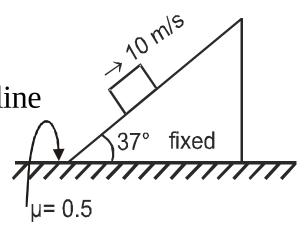
$\therefore mg\sin 37° + \mu N = ma$

$a = 10\ m/s^2$ down the incline

Now $\quad v^2 = u^2 + 2as$

$0 = 10^2 + 2(-10)\,S$

$\therefore S = 5\ m$



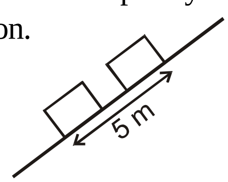
Ex.36 Find the time taken in the above example by the block to reach the initial position.

Sol. $a = g\sin 37° - \mu g\cos 37°$

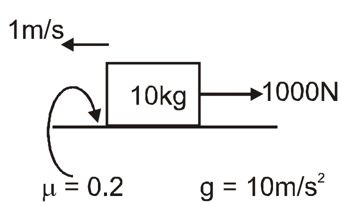
$\therefore a = 2\ m/s^2$ down the incline

$\therefore S = ut + \frac{1}{2}at^2 \quad \Rightarrow \quad S = \frac{1}{2} \times 2 \times t^2$

$\therefore t = \sqrt{5}$ sec.



Ex.37 Find out acceleration of block.

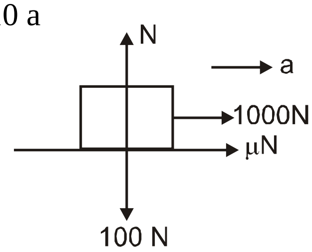


Sol. $1000 + 0.2 \times 100 = 10\,a$

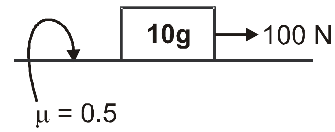
$1000 + 20 = 10\,a$

$a = \frac{1020}{10}$

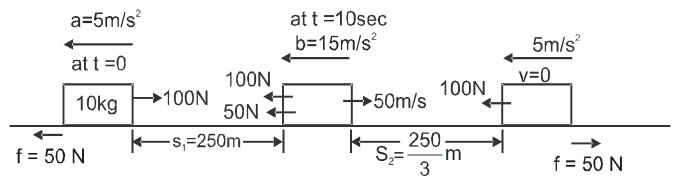
$= 102\ m/s^2$



Ex:38 After 10 sec. direction of applied force is reversed find out velocity of blcok when it again reaches initial point.



Sol.



$a = \frac{100-50}{10}$ $\quad b = \frac{100+50}{10}$ $\quad C = \frac{100-50}{10} = 5m/s^2$

$a = 5m/s^2 \qquad b = 15m/s^2$

Case : I After 10 sec.

$v = u + at$

$= 0 + 5\times 10 = 50$ m/s.

Distance travelled

$s = at + \frac{1}{2}at^2$

$= 0 + \frac{1}{2} \times 5 \times 100$

$s_1 = 250$ m

Case : I $v^2 = (50)^2 + 2(-15)s$

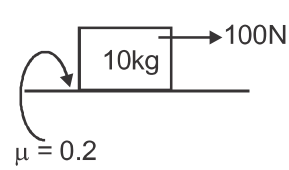
$s_2 = \frac{250}{3}$ m.

when block reaches initial point

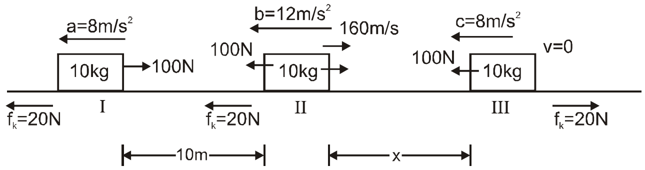
$v_f^2 = 0 + 2 \times 5\left(250 + \frac{250}{3}\right)$

$v_f = \frac{100}{\sqrt{3}}$

Ex:39 Find out the velocity of the block when it again reaches to the intial point. It is given that the applied force changes its direction & starts acting towards left side when the block is displaced 10m right.



Sol



Case : I $80 = 10a$

$a = 8m/s^2$

$v^2 = u^2 + 2as$

$v = \sqrt{2 \times 8 \times 10} = 4\sqrt{10} m/s$

Case : II $100 + 20 = 10b$

$b = 12m/s^2$

Case : III $100 - 20 = 10c$

$c = 8m/s^2$

$v^2 = u^2 + 2as$

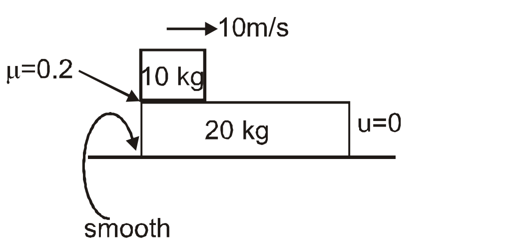
$0 = 160 - 2 \times 12 \times x$

$x = \dfrac{20}{3}$

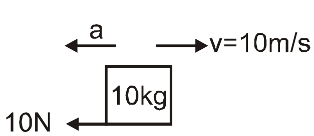
Now $v_f = 0 + 2 \times 8 \times \left(10 + \dfrac{20}{3}\right)$

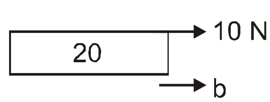
$= 16 \times \dfrac{50}{3}$ $\qquad v_f = 20 \times \sqrt{\dfrac{2}{3}} m/s$

Ex.40 Find out time after frictional force acting between the two block stops.



Sol





$10 = 10a$ $\qquad 10 = 20b$

$a = 1m/s^2$ $\qquad b = \dfrac{1}{2} m/s^2$

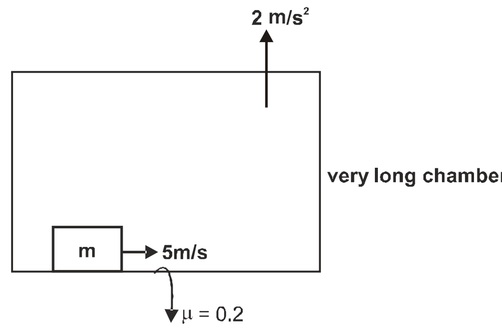
$V_A = 10 - 1 \times t = 10 - t$

$V_B = \dfrac{1}{2} t$

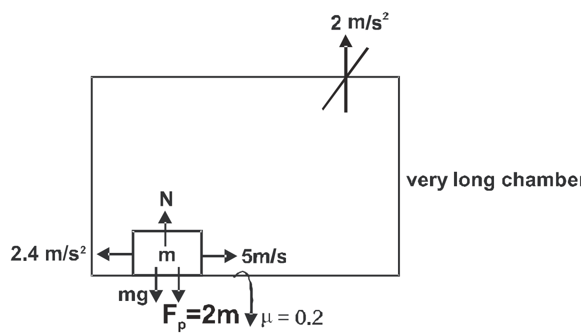
According to condction

$V_A = V_B \quad 10 - t = \dfrac{t}{2} \qquad t = \dfrac{20}{3}$ sec.

Ex.41 Calculate time after which the block will be at rest w.r.t. chamber



Sol. Where f_p = Psuedo force.



$N - mg = 2m$

$N = m(g + 2)$

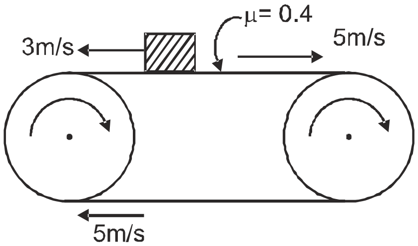
$f_k = (0.2m)(12)$

$a = \dfrac{f_k}{m} = \dfrac{0.2m \times 12}{m}$

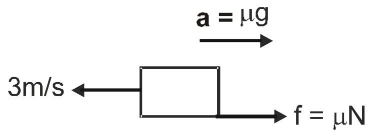
$\therefore = 2.4 m/s^{-2}$ using $v = u + at$

$0 = 5 + (-2.4) \times t \qquad t = \dfrac{25}{12} s$

Ex.42 Calculate time after which block will come to rest w.r.t belt.



Sol.



$5 = -3 + \mu gt$

$\frac{8}{\mu g} = \frac{8}{4} = 2s$

using

$v^2 = u^2 + 2as$

$25 = 9 + 2(4)s$

$s = 2m$

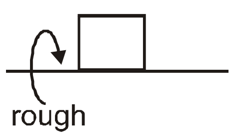
Static Friction : When there is tendency of relative motion between contact point of the object and the contact surface parallel to the surface but there is no relative motion then the frictional force acting between them is called as static friction forces.

Condition for static friction :-

(1) There must be contact and compression

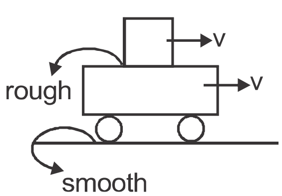
(2) There must be tendency of relative motion of contact points but there is no relative motion. Surface must be rough.

Ex.43



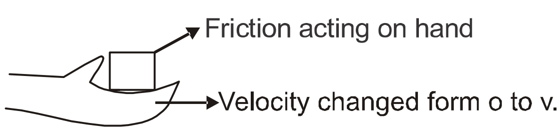
No tendency to move so no static friction even if the surface is rough.

eg :



No static friction, No kinetic friction.

eg.



Direction of Static Friction Force : Static frictional force always acts opposite to the tendency of relative motion of contact point parallel to the contact surface

Magnitude of Static friction force :-

Static frictional force is variable and self adjustable force, it can adjust its magnitude from zero to maximum limiting friction as per requirement. It has a maximum value called limiting friction

Magnitude of maximum limiting friction

$f_{s'max} \infty N$ $\quad f_{max} = \mu_s N$ $\quad 0 \le f \le \mu_s N$

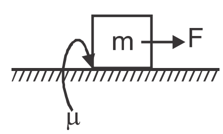
Key point :-

(i) Tendency of relative motion of contact point of the object w.r.t. the contact surface is decided by the reductant force acting parallel to the contact surface on the object in the reference frame of contact surface.

(ii) Static friction is not always greater than kinetic friction but $\mu_s > \mu_K$ (always) . and limiting friction > static & kinetic both (always).

(iii)Tendency of relative motion of non living body is decided by the body parallel to the contact force. Including pseudo force in the reference frame of the pseudo force.

Ex.44 A block is kept on rough surface it is pulled by external force then find out acceleration ?

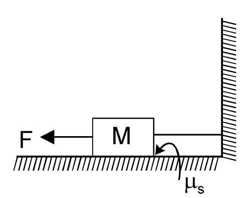


Sol. $F - \mu mg = ma$

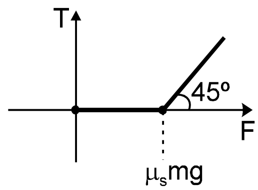
$a = \frac{F - \mu mg}{m}$ $\quad$ if $F > \mu mg$

$a = 0$ $\quad$ iF $\mu mg \ge F$.

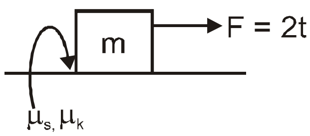
Ex.45 In the following figure force F is gradually increased from zero. Draw the graph between applied force F and tension T in the string. The coefficient of static friction between the block and the ground is μ_s.



Sol. As the external force F is gradually increased from zero it is compensated by the friction and the string bears no tension. When limiting friction is achieved by increasing force F to a value till $\mu_s$mg, the further increase in F is transferred to the string.



Ex.46 A block is kept on rough surface. It is pulled by a variable force then find



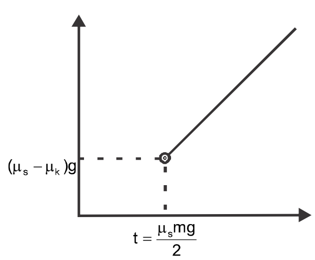
(a) Find the acceleration **(b)** Draw a vs t graph.

Sol. (a) a = 0 if $2t < \mu_s mg$ $\quad t \le \dfrac{\mu_s mg}{2}$

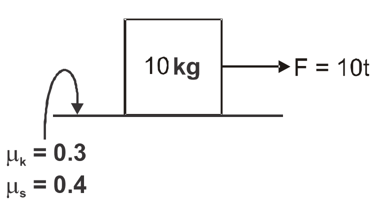
$a = \dfrac{2t - \mu_k mg}{m}$ if $2t > \mu_s mg$ $\quad t > \dfrac{\mu_s mg}{2}$

$a = \dfrac{2}{m}t - \mu_k mg.$

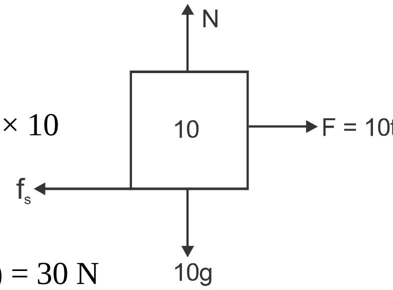
(b)



Ex.47 Find out acceleration of block and draw graph f vs t and a vs t



Sol. N = 10g



$f_{s,max} = \mu_s N$

$= 0.4 \times 10 \times 10$

$= 40$ N

$f_k = \mu_k N$

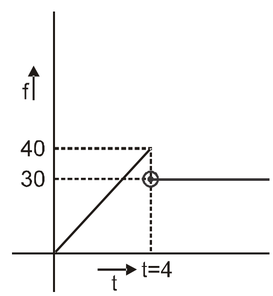
$= 0.3\,(10g) = 30$ N

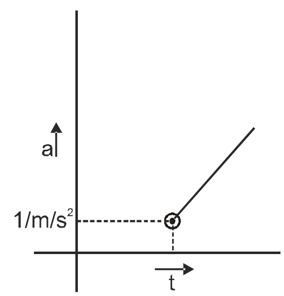
a = 0 $\qquad 0 \le t \le 4$

$a = \dfrac{10t - 30}{10}$

$= t - 3 \qquad t > 4$

Graph f vs t $\qquad$ Graph a vs t





Ex.48 A solid cube of mass 5 kg is placed on a rough horizontal surface, in xy-plane as shown. The friction coefficient between the surface and the cube is 0.4. An external force $\vec{F} = 6\hat{i} + 8\hat{j} + 20\hat{k}$ N is applied on the cube. (use g = 10 m/s^2)

(A) The block starts slipping over the surface

(B) The friction force on the cube by the surface is 10 N.

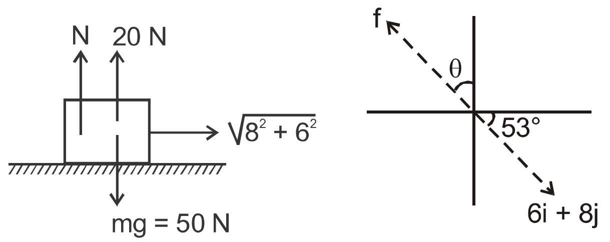
(C) The friction force acts in xy-plane at angle 127° with the positive x-axis in clockwise direction.

(D) The contact force exerted by the surface on the cube is $10\sqrt{10}$ N.

Sol. N = 50 – 20 = 30 N

Limiting friction force = μN = 12 N and applied force in horizontal direction is less than the limiting friction force, therefore the block will not slide.

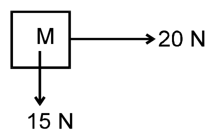
For equilibrium in horizontal direction, friction force must be equal to 10 N.



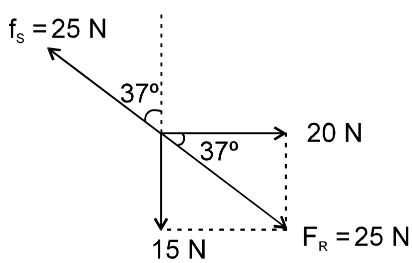
From the top view, it is clear that $\theta = 37°$ i.e. 127° from the x-axis that is the direction of the friction force. It is opposite to the applied force.

Contact force = $\sqrt{N^2+f^2} = 10\sqrt{10}$ N

Ex.49 In the following figure an object of mass M is kept on a rough table as seen from above. Forces are applied on it as shown. Find the direction of static friction if the object does not move.

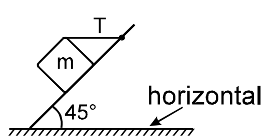


Sol. In the above problem we first draw the free body diagram of find the resultant force.



As the object doe not move this is not a case of limiting friction. The direction of static friction is opposite to the direction of the resultant force F_R as shown in figure by f_s. Its magnitude is equal to 25 N.

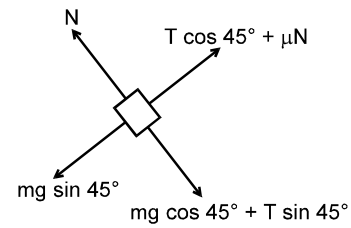
EX.50 A block of mass 15 kg is resting on a rough inclined plane as shown in figure. The block is tied up by a horizontal string which has a tension of 50 N. The coefficient of friction between the surfaces of contact is ($g = 10\ m/s^2$)



(A) 1/2 (B) 2/3

(C) 3/4 (D) 1/4

Sol. The free body diagram of the block is N is the normal reaction exerted by inclined plane on the block.



Applying Newton's second law to the block along and normal to the incline.

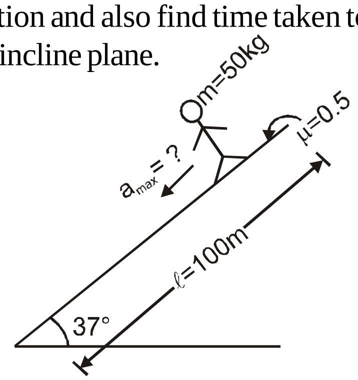
$mg \sin 45° = T\cos 45° + \mu N$ .... (1)

$N = mg\cos 45° + T \sin 45°$ .... (2)

Solving we get

$\mu = 1/2$

Ex.51 A man is moving down the incline plane. Find the maximum acceleration and also find time taken to reach the bottom of incline plane.



Sol. $f_{s,max} = \mu_s N$

$= 0.5\ (50g\cos 37°)$

$= 0.5 \times 50 \times 10 \times \frac{4}{5}$

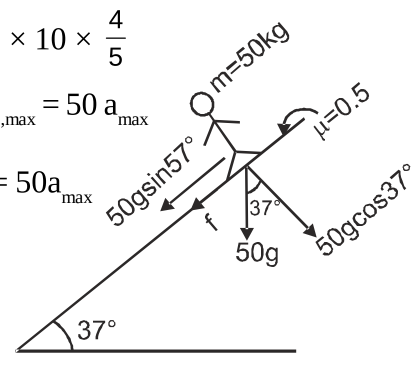
$50g\sin 37^0 + f_{s,max} = 50\ a_{max}$

$500 \times \frac{3}{5} + 200 = 50a_{max}$

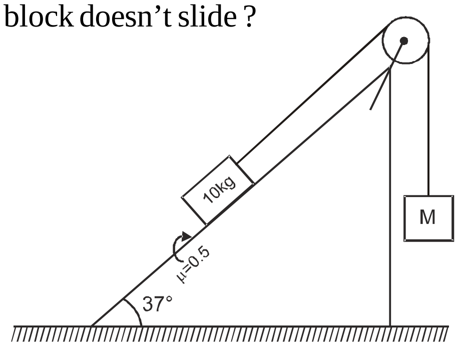
$6 + 4 = a_{max}$

$a_{max} = 10 m/s^2$

$100 = \frac{1}{2}10t^2$ $\quad t = 5\sqrt{2}$



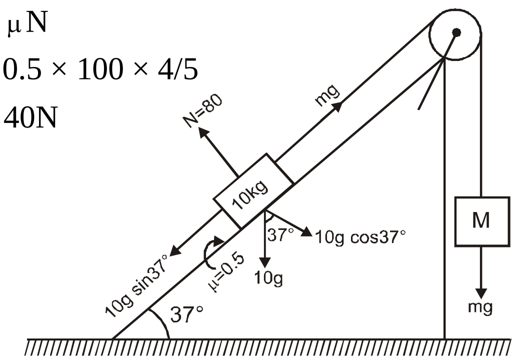
Ex.52 In the shown in figure what is the range of M so that 10kg block doesn't slide ?



Sol. $f_{s,max} = \mu N$

$f = 0.5 \times 100 \times 4/5$

$= 40N$



Case - I

Let the tendency of block is upward.

$Mg = 10g\sin 37^0 + f_{s,max}$

$10\ M = 100 \times \dfrac{3}{5} + 40$

$M = 10$ kg

Case - II

Let the tendency of block is downward.

$\therefore\ 10g\sin 37^0 = Mg + f_{s,max}$

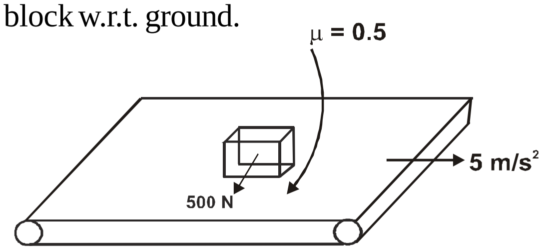
$100 \times \dfrac{3}{5} = 10\ M + 40$

$60 - 40 = 10\ M$

$20 = 10M \qquad\qquad M = 2$ kg

So the range of mass M is [2 , 10]

Ex.53 In the figure shown a block having mass m = 50 kg is kept on conveyor belt. Find out acceleration of block w.r.t. ground.



Sol. Where f_P = Psuedo force.

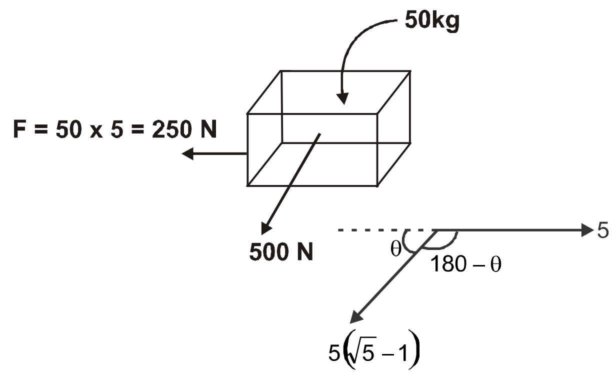
$$F_{net} = \sqrt{(500)^2 + (250)^2} = 250\sqrt{5}$$

$\Rightarrow\ = F_{net} - \mu N = 50\ a$

$$a = \frac{F_{net} - \mu N}{50} = \frac{250\sqrt{5} - 250}{50}$$

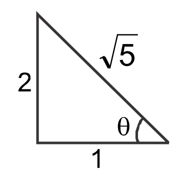
$a = 5\left(\sqrt{5} - 1\right)$ m/s^2

acceleration of block w.r.t ground.



$\tan\theta = \dfrac{500}{200}$

$= 2$



From above right angle triangle

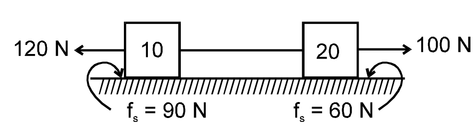
$$\cos\theta = \frac{1}{\sqrt{5}}$$

$$a_{b/g} = \sqrt{5^2 + 5^2\left(\sqrt{5}-1\right)^2 - 2\times 5\times 5\left(\sqrt{5}-1\right)\cos\theta}$$

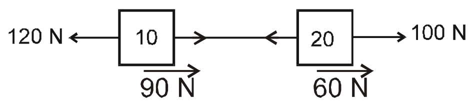
Where $a_{b/g}$ = acceleration of block with respect to ground.

$$= 5\sqrt{7 - 2\sqrt{5} - \frac{2}{\sqrt{5}}}\ \text{m/s}^2$$

Ex.54 Find the tension in the string in situation as shown in the figure below. Forces 120 N and 100 N start acting when the system is at rest and the maximum value of static friction on 10 kg is 90 N and that on 20 kg is 60N?

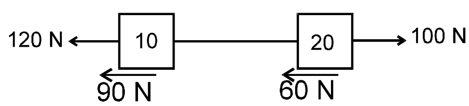


Sol. (i) Let us assume that system moves towards left then as it is clear from FBD, net force in horizontal direction is towards right. Therefore the assumption is not valid.



Above assumption is not possible as net force on system comes towards right. Hence system is not moving towards left.

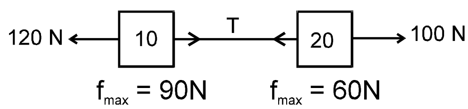
(ii) Similarly let us assume that system moves towards right.



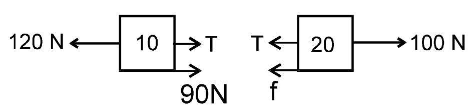
Above assumption is also not possible as net force on the system is towards left in this situation.

Hence assumption is again not valid.

Therefore it can be concluded that the system is stationary.



Assuming that the 10 kg block reaches limiting friction first then using FBD's.



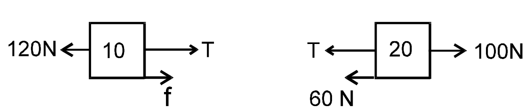
$120 = T + 90 \quad \Rightarrow \quad T = 30$ N

Also $\quad T + f = 100$

$\therefore\ 30 + f = 100 \Rightarrow$

f = 70 N which is not possible as the limiting value is 60 N for this surface of block.

$\therefore$ Our assumption is wrong and now taking the 20 kg surface to be limiting we have



$T + 60 = 100\text{ N} \Rightarrow \quad T = 40$ N

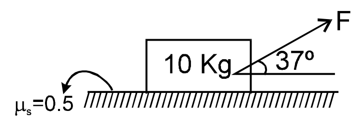
Also $\quad f + T = 120\text{ N} \quad \Rightarrow \quad f = 80$ N

This is acceptable as static friction at this surface should be less than 90 N.

Hence the tension in the string is

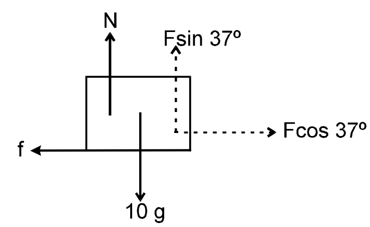
T = 40 N.

Ex.55 Force F is gradually increased from zero. Determine whether the block will first slide or lift up?



Sol. There are minimum magnitude of forces required both in horizontal and vertical direction either to slide on lift up the block. The block will first slide on lift up will depend upon which minimum magnitude of force is lesser.

For vertical direction to start lifting up



$F \sin 37^o + N - Mg \geq 0.$

N becomes zero just lifting condition.

$$F_{lift} \geq \frac{10g}{3/5}$$

$$\therefore \quad F_{lift} \geq \frac{500}{3}\text{N}$$

For horizontal direction to start sliding

$F \cos 37 \geq \mu_s N$

$F \cos 37^o > 0.5\ [\,10g - F \sin 37^o]$

$(\because\ N = 10\ g - F \sin 37^o)$

$$\text{Hence } F_{slide} > \frac{50}{\cos 37^o + 0.5 \sin 37^o}$$

$$F_{slide} > \frac{500}{11}\text{N} \qquad F_{lift} > \frac{500}{3}\text{N.}$$

$\Rightarrow F_{slide} < F_{lift}$

Therefore the block will begin to slide before lifting.

ANGLE OF REPOSE

The angle of repose (ϕ) is that minimum angle of inclination of the inclined plane at which a body placed at rest on the inclined plane is about to slide down in equilibrium condition.

$N = mg\cos\theta$

and $mg\sin\theta = f \le f_{max}$

if $\theta = \phi$ then $f = f_{max}$.

$\therefore\ mg\sin\phi = f_{max} = \mu N$

$\Rightarrow mg\sin\phi = \mu\, mg\cos\phi$

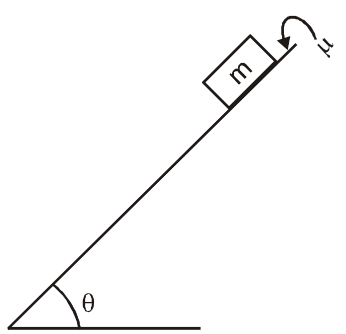
$\Rightarrow \tan\phi = \mu$

(a) When $\theta \le \phi$ (or $\tan^{-1}\mu$) the body is in equilibrium

(b) When the angle of inclination is more than the angle of friction ($\theta > \phi$) the block starts sliding down.

Solved Examples

Ex.56 Find out the angle at which block will slide ?



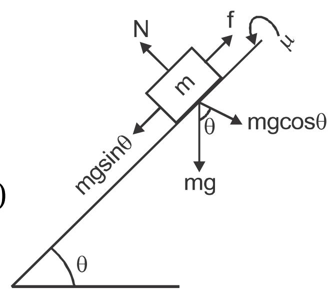
Sol. $mg\sin\theta > f_{s,max}$

$mg\sin\theta > \mu N$

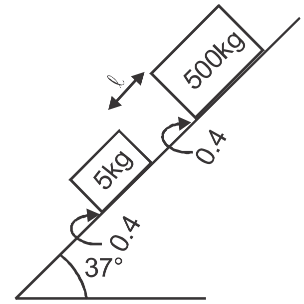
$mg\sin\theta > \mu mg\cos\theta$

$\tan\theta > \mu$ (for sliding)

$\tan\theta \le \mu$ (for no sliding)



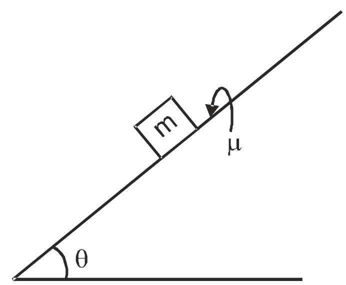
Note : Sliding does not depend upon mass it depends only on mass of the block so whatever mass be placed on the incline plane the displacement will be free from mass of the block.



Distance between both blocks never change.

Solved Examples

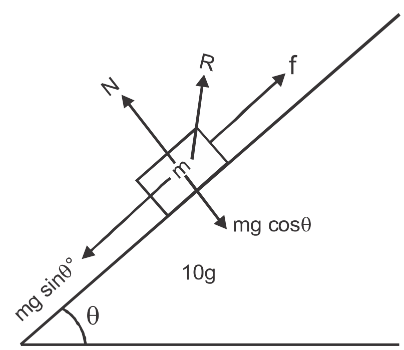
Ex.57



(i) In the figure shown a block having mass m is kept on the Incline plane then find friction, normal and resultant contact force in case of sliding and without sliding

(i) Draw graph fr vs θ, R vs θ and N vs θ

Sol.



(i) Case - I :- When there is no sliding i.e. $\theta \le \tan^{-1}\mu$

$f = mg\sin\theta$

$N = mg\cos\theta$

$R = \sqrt{f^2 + N^2}$

$= \sqrt{(mg\sin\theta)^2 + (mg\cos\theta)^2}$

$= mg$

Case - II :- When there is sliding i.e.$\theta < \tan^{-1}\mu$

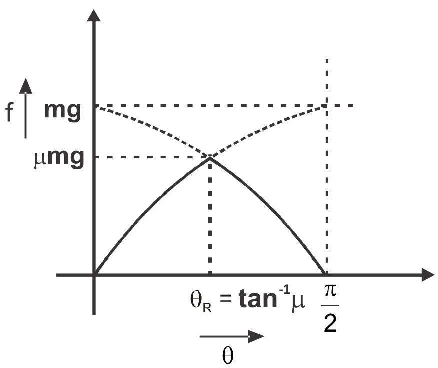
$$f = \mu mg\cos\theta$$

$$N = mg\cos\theta$$

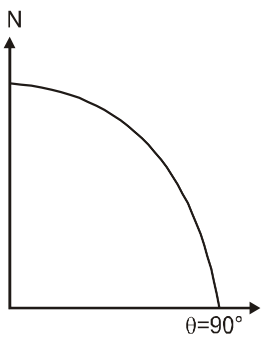
$$R = \sqrt{mg\cos\theta + \mu mg\cos\theta}$$

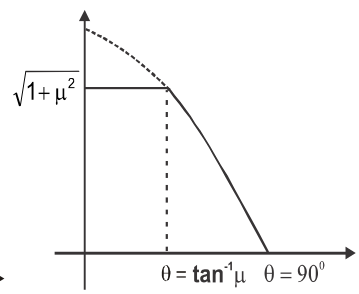
$$= mg\cos\theta\sqrt{1+\mu^2}$$

(ii) Graph f vs θ s



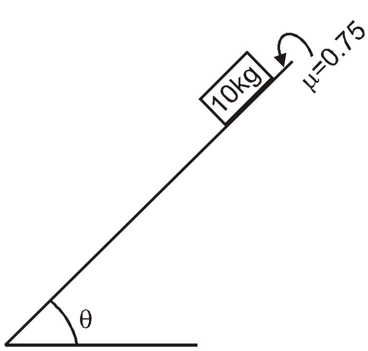
Graph N vs θ Graph R vs θ



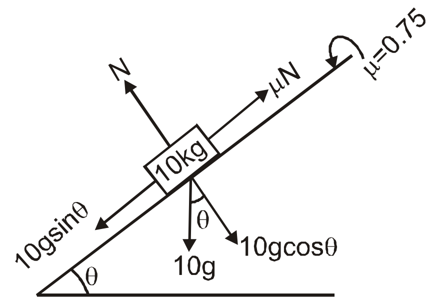


Ex.58(i) In the figure shown a block having mass 10kg is kept on the Incline plane then find friction, normal and resultant contact force in case of sliding and without sliding

(ii) Draw graph fr vs θ, R vs θ and N vs θ



Sol.



Case : I When slideing does not occur ($\theta \le 37°$)

$N = 10g\cos\theta$

$f_{s,max} = \mu s N$

$= (0.75)(10g\cos\theta)$

$= 75\cos\theta$

$\theta_r = \tan^{-1}(0.75)$

$= \tan^{-1}\left(\frac{3}{4}\right)$

where θ_r is angle of repose

$\theta_r = 37°$

$\because f = 10g\sin\theta$

$= 100\sin\theta$

$R = \sqrt{f^2 + N^2}$

When sliding occurs

$= \sqrt{(75\cos\theta)^2 + (100\cos\theta)^2}$

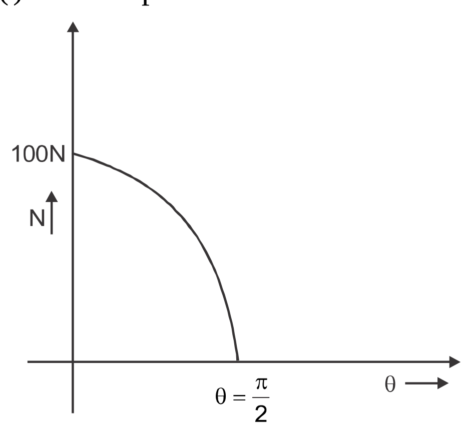
$R = \cos\theta\sqrt{(75)^2 + (100)^2}$

Case : II When sliding occurs ($\theta > 37°$)

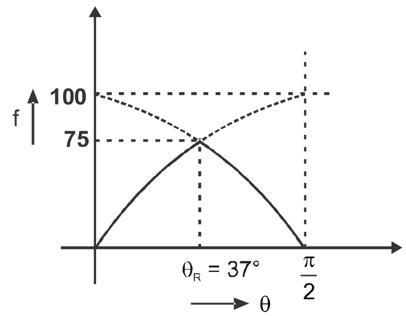
$f = 10g\cos\theta$

$= 100\cos\theta$

(i) Graph N vs θ



(ii) Graph f vs θ

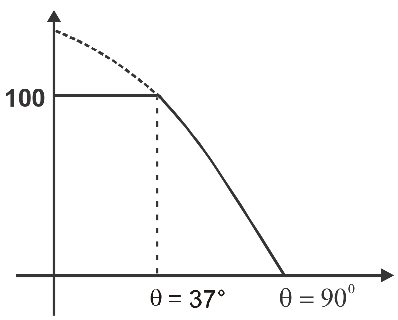


where θR = angle of relose.

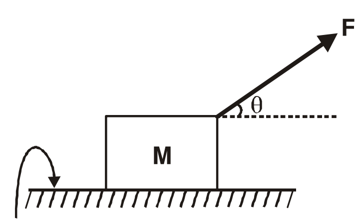
(ii) graph R vs θ

$$R = \sqrt{(100\cos\theta)^2 + (75\cos\theta)^2}$$

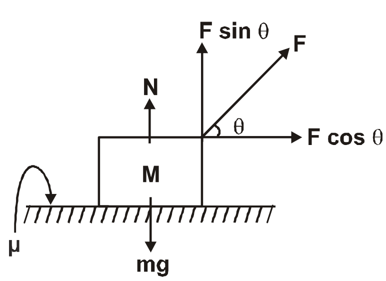
$$R = 125 \cos\theta$$

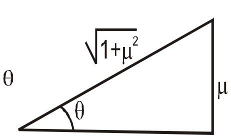


Ex.59 In the figure a force F is appliede on block at an angal θ. What is the minimum force required to slide block along horizontal direction ? At what angle this force should be applied.



Sol. N = mg – Fsinθ.





$$f = (mg - F\sin\theta)\mu$$

$$F\cos\theta - \mu(mg - F\sin\theta) \geq 0$$

$$F\cos\theta + \mu F\sin\theta \geq \mu mg$$

$$F \geq \frac{\mu mg}{\cos\theta + \mu\sin\theta}$$

$$F_{min} = \frac{\mu mg}{\cos\theta + \mu\sin\theta}$$

$$y = \cos\theta + \mu\sin\theta$$

$$\frac{dy}{d\theta} = \mu\cos\theta - \sin\theta = 0$$

$$\tan\theta = \mu$$

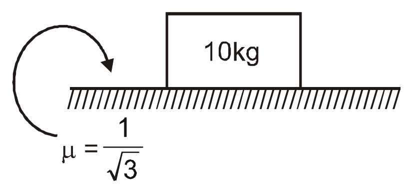
$$\theta = \tan^{-1}\mu$$

$$\sin\theta = \frac{\mu}{\sqrt{1+\mu^2}}$$

$$\cos\theta = \frac{1}{\sqrt{1+\mu^2}}$$

$$F_{min} = \frac{\mu mg}{\sqrt{1+\mu^2}}$$

Ex.60



(i) What is the minimum horizontal force required to slide the block.

(ii) What is the minimum force required to slide the bock. At what angle this must be applied.

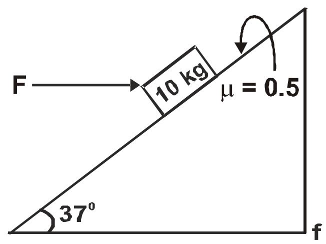
Sol (i) $\mu mg = \frac{1}{\sqrt{3}} \times 100 = \frac{100}{\sqrt{3}}$

(ii) $$F_{min} = \frac{\frac{100}{\sqrt{3}}}{\sqrt{1+\frac{1}{3}}} = \frac{\frac{100}{\sqrt{3}}}{\sqrt{\frac{4}{3}}} = \frac{100}{\sqrt{3}} \times \frac{\sqrt{3}}{2} = 50 \text{ N}$$

$$\theta = \tan^{-1}(\mu)$$

$$= \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30°$$

Ex.61 In the figure a block having mass 10 kg is kept on an incline plane. Find out minimum and maximum value of F at which block will slide.

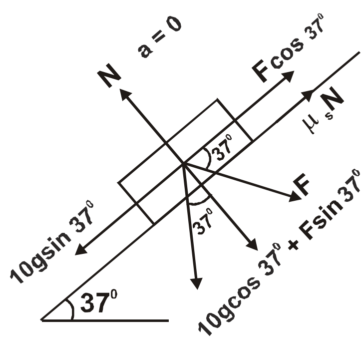


Sol. Case-I :- Condition of minimum force

(i) Friction and applied force will act in same direction

(ii) Acceleration of object will equal to the acceleration of incline plane on which it is kept.

(iii)Maximum limiting friction ($\mu_s N$) will act



$\theta > \tan^{-1} \mu$

$\tan 37^0 > 0.5$

$\frac{3}{4} > 0.5$

So it slides itself

$N = F\sin 37^0 + 10g \cos 37^0$

$f \cos 37^0 + \mu(F \sin 37^0 + 10g \cos 37^0)$

$= 10\, g \sin 37^0$

$$F = \frac{10g \sin 37^0 + \mu 10g\cos 37^0}{\cos 37^0 - \mu \sin 37^0}$$

$$F_{min} = \frac{200}{11} \text{ N}$$

Case-II :- Condition of maximum force

(i) Friction force and applied force will act in opposite direction

(ii) Acceleration of object will be equal to the surface on which it is kept.

(iii)Limiting friction force ($\mu_s N$) will act

$$F = \frac{10g \sin 37^0 + \mu 10g\cos 37^0}{\cos 37^0 - \mu \sin 37^0} = 200 \text{ N}$$

Ex.62 A bar of mass m is placed on a triangular block of mass M as shown in figure. The friction coefficient between the two surface is μ and ground is smooth. Find the minimum and maximum horizontal force F required to be applied on block so that bar will not slip on the inclined surface of block.

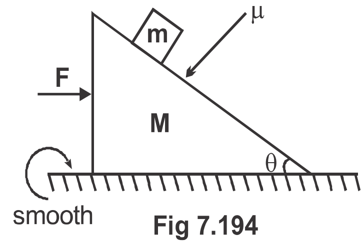


Fig 7.194

Sol. Here if both the masses are moving together, acceleration of the system will be $a = \frac{F}{M+m}$. If we observe the mass m relative to M, it experiences a pseudo force ma towards left. Along the incline it experiences two forces, mg sin θ is more than ma cos θ, it has a tendency of slipping downwards, so friction on it will act in upward direction. Here if block m is in equilibrium on inclined surface, we must have

$f = mg \sin\theta - ma\cos\theta \le \mu(mg\cos\theta + ma\sin\theta)$

or $a \le \frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta} g$

or $(M+m)a \le \frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}(M+m)g$

or $F_{max} \le \frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}(M+m)g$ ...(i)

If force is more than the value obtained in Eq. (i), ma cos θ will increase on m and the static friction on it will decrease. At a = g tan θ (when F =(M + m) g tan θ), we know that the force mg sin θ will be balanced by ma cos θ at this acceleration no friction will act on it. If applied force will increase beyond this valued, ma cos θ will exceed mg sin θ and friction starts acting in downward direction.

Here if block m is in equilibrium, we must have

$f = ma\cos\theta - mg\sin\theta \le m(mg\cos\theta + ma\sin\theta)$

$$\Rightarrow a \le \frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta} g$$

$$\Rightarrow (M+m)a \le \frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}(M+m)g$$

$$\Rightarrow F \le \frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}(M+m)g$$

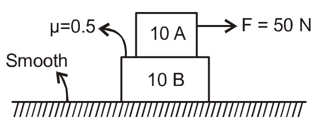
Hence $F_{min} \le \dfrac{\sin\theta - \mu\cos\theta}{\cos\theta + \mu\sin\theta}(M+m)g$

and $F_{max} \le \dfrac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}(M+m)g$

TWO & THREE BLCOK PROBLEMS

Solved Examples

Ex.63 Find the acceleration of the two blocks. The system is initially at rest and the friction coefficient are as shown in the figure?



Sol. <u>**Method of solving**</u>

Step 1 : Make force diagram.

Step 2 : Show static friction force by f because value of friction is not known.

Step 3 : Calculate separately for two cases.

<u>**Case 1 : Move together**</u>

Step 4 : Calculate acceleration.

Step 5 : Check value of friction for above case.

Step 6 : If required friction is less than available it means they will move together else move separately.

Step 7: (a) above acceleration will be common acceleration for both

<u>**Case 2 : Move separately**</u>

Step 7(b) If they move separately then kinetic friction is involved. whose value is μN.

Step 8 : Calculate acceleration for above case.

$f_{max} = \mu N$

$\therefore$ $f \le 50$ N (available friction)

<u>**Move together**</u>

<u>**Move separately**</u>

(i) $a = \dfrac{50}{10+10} = 2.5$ m/s^2

No need to calculate

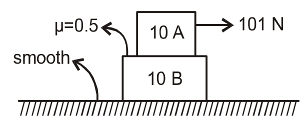
(ii) Check friction for B :

$f = 10 \times 2.5 = 25$

25 N is required which is less than available friction hence they will move together.

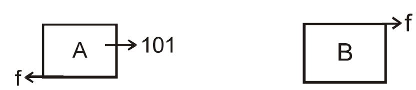
and $a_A = a_B = 2.5$ m/s^2

Ex.64 Find the acceleration of the two blocks. The system is initially at rest and the friction coefficient are as shown in the figure?



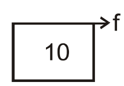
Sol. $f_{max} = 50$ N

$\therefore$ $f \le 50$ N



(i) If they move together $a = \dfrac{101}{20} = 5.05$ m/s^2

(ii) Check friction on B



$f = 10 \times 5.05 = 50.5$ (required)

$50.5 > 50$ (therefore required > available)

Hence they will <u>not</u> move together.

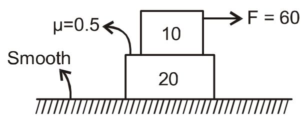
(iii) Hence they move separately so kinetic friction is involved.

$\therefore$ for $a_A = \dfrac{101-50}{10} = 5.1$ m/s^2

$\Rightarrow a_B = \dfrac{50}{10} = 5$ m/s^2

Also $a_A > a_B$ as force is applied on A.

Ex.65 Find the acceleration of the two blocks. The system is initially at rest and the friction coefficient are as shown in the figure?



Sol. **Move Together** | **Move Separately**

$a = \frac{60}{30} = 2\ m/s^2$ | No need to calculate.

Check friction on 20 kg.

$f = 20 \times 2$

$f = 40$ (which is required)

$40 < 50$ (therefore required < available)

$\therefore$ will move together.

Ex.66 In above example find maximum F for which two blocks will move together.

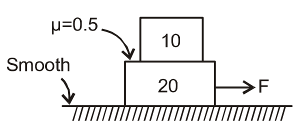
Sol. Observing the critical situation where friction becomes limiting.

$\therefore F - f_{max} = 10\ a$ .... (1)

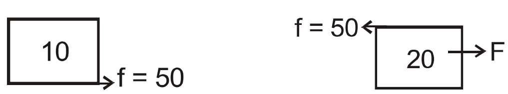
$f_{max} = 20\ a$ .... (2)

$\therefore F = 75\ N$

Ex.67 Initially the system is at rest. find out minimum value of F for which sliding starts between the two blocks.



Sol. At just sliding condition limiting friction is acting.



$F - 50 = 20\ a$ .... (1)

$f = 10\ a$ .... (2)

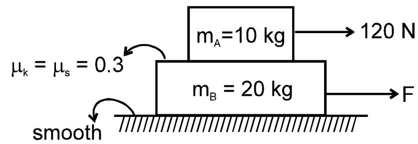
$50 = 10\ a$

$\therefore a = 5\ m/s^2$

hence $F = 50 + 20 \times 5 = 150\ N$

$\therefore F_{min} = 150\ N$

Ex.68 In the figure given below force F applied horizontally on lower block, is gradually increased from zero. Discuss the direction and nature of friction force and the accelerations of the block for different values of F (Take $g = 10\ m/s^2$).



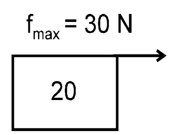
Sol. In the above situation we see that the maximum possible value of friction between the blocks is $\mu_s m_A g = 0.3 \times 10 \times 10 = 30\ N$.

Case : I When F = O.

Considering that there is no slipping between the blocks the acceleration of system will be

$$a = \frac{120}{20+10} = 4\ m/s^2$$

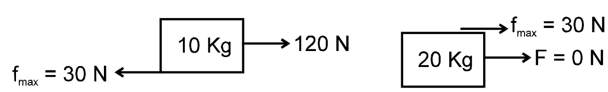
But the maximum acceleration of B can be obtained by the following force diagram.



$a_B = \frac{30}{20} = 1.5\ m/s^2$ ($\because$ only friction force by block A is responsible for producing acceleration in block B)

Because $4 > 1.5\ m/s^2$ we can conclude that the blocks do not move together.

Now drawing the F.B.D. of each block, for finding out individual accelerations.

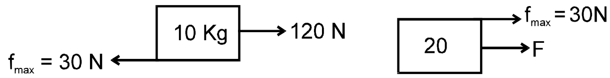


$$a_A = \frac{120-30}{10} = 9\ m/s^2 \text{ towards right}$$

$$a_B = \frac{30}{20} = 1.5\ m/s^2 \text{ towards right.}$$

Case : II F is increased from zero till the two blocks just start moving together.

As the two blocks move together the friction is static in nature and its value is limiting. FBD in this case will be



$$a_A = \frac{120 - 30}{10} = 9 \text{ m/s}^2 \quad \Rightarrow \quad a_B = \frac{F + 30}{20} = a_A$$

$$\Rightarrow \frac{F + 30}{20} = 9$$

$\therefore$ F = 150 N

Hence when 0 < F < 150 N the blocks do not move together and the friction is kinetic. As F increases acceleration of block B increases from 1.5 m/s^2.

At F = 150 N limiting static friction start acting and the two blocks start moving together.

Case : III When F is increased above 150 N.

In this scenario the static friction adjusts itself so as to keep the blocks moving together. The value of static friction starts reducing but the direction still remains same. This happens continuously till the value of friction becomes zero. In this case the FBD is as follows

$$a_A = a_B = \frac{120 - f}{10} = \frac{F + f}{20}$$

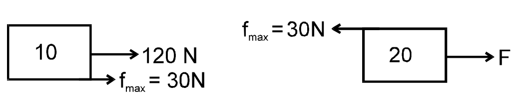
$\therefore$ when friction force f gets reduced to zero the above accelerations become

$$a_A = \frac{120}{10} = 12 \text{ m/s}^2 \quad \Rightarrow \quad a_B = \frac{F}{20} = a_A = 12 \text{ m/s}^2$$

$\therefore$ F = 240 N

Hence when 150 $\le$ F $\le$ 240 N the static friction force continuously decreases from maximum to zero at F = 240 N. The accelerations of the blocks increase from 9 m/s^2 to 12 m/s^2 during the change of force F.

Case : IV When F is increased again from 240 N the direction of friction force on the block reverses but it is still static. F can be increased till this reversed static friction reaches its limiting value. FBD at this juncture will be



The blocks move together therefore.

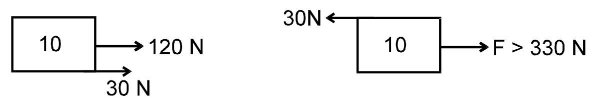
$$a_A = \frac{120 + 30}{10} = 15 \text{ m/s}^2 \quad \Rightarrow \quad a_B = \frac{F - 30}{20} = a_A$$

= 15 m/s^2

$$\therefore \frac{F - 30}{20} = 15 \text{ m/s}^2$$

Hence F = 330 N.

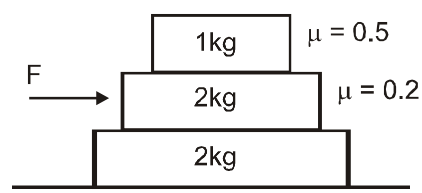
Case : V When F is increased beyond 330 N. In this case the limiting friction is achieved and slipping takes place between the blocks (kinetic friction is involved).



$\therefore$ a_A = 15 m/s^2 which is constant

$a_B = \frac{F - 30}{20}$ m/s^2 where F > 330 N.

Ex.69 In the situation shown in figure, there is no friction between 2kg and ground.



(a) For what maximum value of force F can all three blocks move together ?

(b) Find the value of force F at which sliding starts at other rough surfaces.

(c) Find acceleration of all blocks, nature and value of friction force for following value of force F

(i) 10N (ii) 18N (iii) 25N

Sol. Between 1kg and 2kg : $f_{\ell 1} = f_{k1} = 0.5 \times 1 \times g = 5N$

Between 2kg and 2kg : $f_{\ell 2} = f_{k2} = 0.2 \times (1+2)g = 6N$

Maximum possible acceleration of 1kg be

$\mu g = 0.5\, g = 5 \text{ ms}^{-2}$

Maximum possible acceleration of lower 2kg :

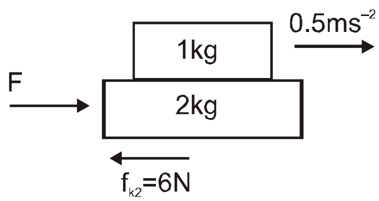
$$a = \frac{f\ell_2}{2} = \frac{6}{2} = 3\text{ms}^{-2}$$

For the entire system to move together, we have take lesser acceleration.

(a) Maximum force F that can be applied for the entire system to move together :

$F_{max} = m_{total}a \quad \Rightarrow \quad F_{max} = (1+2+2)\times 3 = 15N$

(b) Sliding will start between 2kg and 2kg if F> 15N for sliding to start between 1kg and 2kg : This will occur when acceleration of combined 1kg and 2kg will exceed $5ms^{-2}$

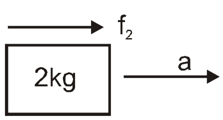


$F - f_{k2} = (1+2)\times 5 \Rightarrow F = 21$ N

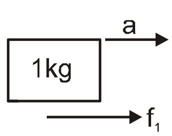
(c) (i) F = 10 < 15 N, so entire system will move together with common acceleration.

$$a = \frac{10}{1+2+2} = 2ms^{-2}$$

Friction between 2kg and k2kg : $f_2 = 2a\times 2\times 2 = 4N$



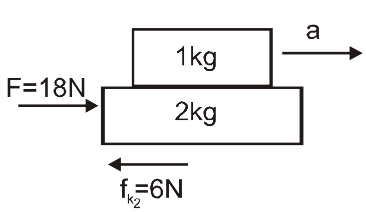
Friction between 1kg and 2kg : $f_1 = 1\times a = 1\times 2 = 2N$



(ii) F =18N, here 15N <F <21 N, so sliding will start between 2kg and 2kg and not between 1kg and 2kg.

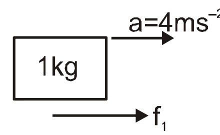
Acceleration of lower 2kg $a = \frac{f_{k_2}}{m} = 6/2 = 3ms^{-2}$

Acceleration of 1kg and 2kg : $a = \frac{18-6}{1+2} = 4ms^{-2}$



Friction between 2kg and ground is $f_{k_2} = 6N$

Friction between 1kg and 2kg : $f_1 = 1\times 4 = 4N$



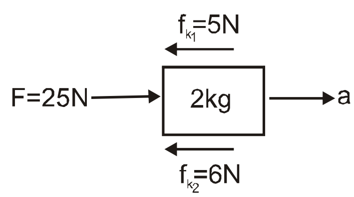
(iii) F =25N > 21 N, so sliding occurs at all the contact surface. Friction between 1kg and 2kg is $f_{k_1} = 5N$.

Friction between 2kg and 2kg is $f_{k_2} = 6N$

Accelerations of 1kg and lower 2kg are their maximum possible acceleration.

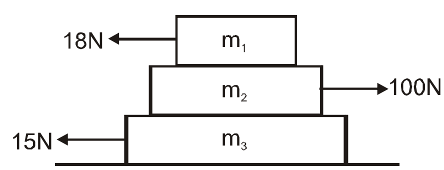
Acceleration of upper 2kg : $a = \frac{25-5-6}{2} = 7ms^{-2}$

$f_{k_1} = 5N$



Ex.70 Consider three blocks placed over the other as shown in figure. Let us now pull the blocks with the forces of magnitudes 18N, 100N and 15N. Take $m_1 = m_2 = m_3 = 10$kg. If the coefficients of static and kinetic friction between all contact surfaces are $\mu_s = 0.3$ and $\mu_k = 0.2$, respectively,

find the



(a) Acceleration of the blocks.

(b) Friction at each surface.

Sol. Let us first calculate limiting and kinetic friction force

Between m_1 and m_2 : $f_{\ell 1} = 0.3 \times 10g = 30$ N

$f_{k_1} = 0.2 \times 10g = 20$ N

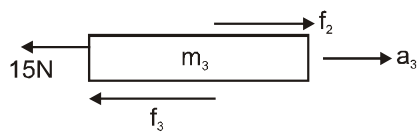
Between m_2 and m_3 : $f_{\ell 2} = 0.3 \times 20g = 60$N

$f_{k2} = 0.2 \times 20g = 40$N

Between m_3 and ground : $f_{\ell 3} = 0.3 \times 30g = 90$ N

$f_{k3} = 0.2 \times 30g = 60$ N

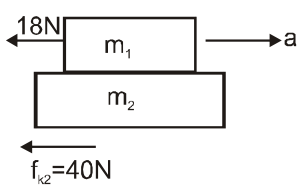
Consider FBD of m_3



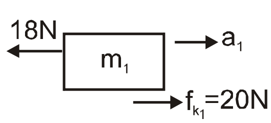
Maximum value of f_2 is $f_{\ell 2}$ =60N, which is less than the maximum value of 15 + f_3

which is 15 + 90=105N. Hence, m_3 will not accelerate, so $a_3 = 0$

Now assume that m_1 and m_2 move together with common acceleration a.

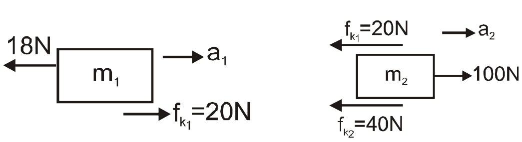


Let us calculate how much friction is required between m_1 and m_2 for common acceleration a. For this consider FBD of m_1.



$f_1 - 18 = 10a \Rightarrow f_1 - 18 = 10 \times 2.1 \Rightarrow f_1 = 39$N

This is not possible because maximum value of f_1 is $f\ell_1 = 30$N. Hence slipping occure between m_1 and m_2



$$a_1 = \frac{f_{k_1} - 18}{10} = \frac{20 - 18}{10} = 0.2\text{ms}^{-2}$$

$$a_2 = \frac{100 - f_{k_1} - f_{k_2}}{10} = \frac{100 - 20 - 40}{10} = 4\text{ms}^{-2}$$

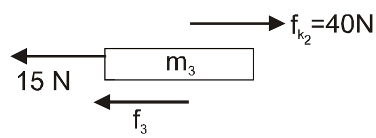
(a) So acceleration of m_1 is $a_1 = 0.2\text{ms}^{-2}$

Acceleration of m_2 is $a_2 = 40\text{ms}^{-2}$

Acceleration of m3 is $a_3 = 0$

(b) Since slipping occurs between m_1 and m_2 friction between m_1 and m_2 is f_{k_1} =20N. Similarly, slipping occurs between m_2 and m_3.friction between m_2 and m_3 is f_{k_2} = 40N

To find friction between m_3 and ground :



$f_{k_2} = 15 + f_3 \Rightarrow 40 = 15 + f_3 \Rightarrow f_3 = 25$ N

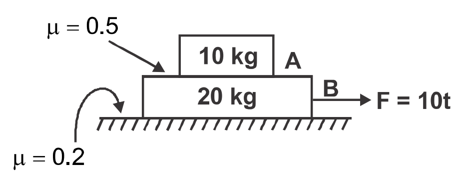
JUST SLIDING

(i) maximum static friction will act in this condition.

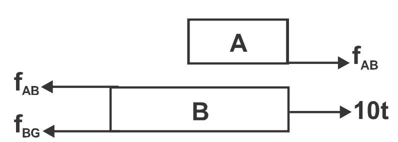
(ii) acceleration of two contact surfaces will be equal where just sliding is taken.

Solved Examples

Ex.71 Find out friction force and acceleration for just sliding



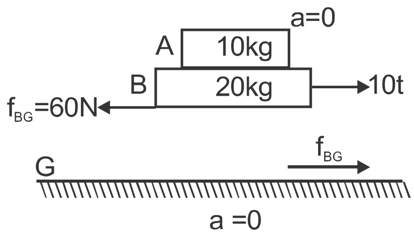
Sol. F.B.D...



f_{AB}, max = 0.5 × 10 × 10 = 50 N

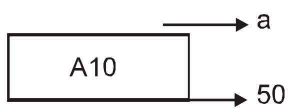
f_{BG}, max = 0.2 × 10 × 10 = 60 N

Case : I Let just sliding occurs between B and G and both block will move together.

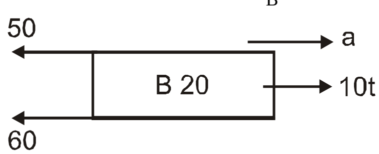


Friction between A and B $F_{AB} = 10\times0 = 0$

Let just sliding occurs between A and B

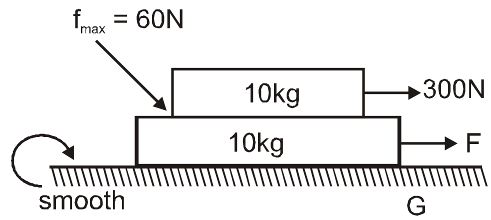


$50 = 10a$	$a_B = \frac{t}{2} - 5.5$	
$a = 5m/s^2$	$a_B = 0$	$0 \le t \le 6$

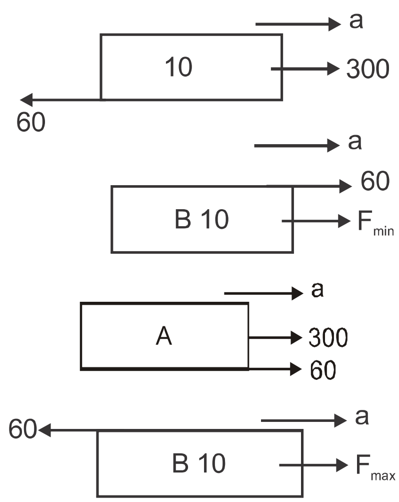


	$\frac{t}{3} - 2$	$6 \le t \le 21$
$10t - 50 - 60 = 20a$	$\frac{t}{2} - 5.5$	$21 < t$
$10t = 210$	$a_A = 0$	$0 \le t \le 21$
$t = 21sec.$		
In case of just sliding	$\frac{t}{3} - 2$	$6 \le t \le 21$
$10t - 60 = 30a$	5	$t > 21$
$\frac{t}{3} - 2 = a$	$f_G = 10t$	$0 \le t \le 6$
$10t - 60 - 50 = 20a$	60	$6 \le T$
$10t - 110 = 20a$	$F_{AB} = 0$	$0 \le t \le 6$
	$\frac{10t}{3} - 20$	$6 \le t \le 21$
	50	$t \le 21$

Ex.72 Find out the minimum and maximum force. At must be applied on B so that two block moves together.



Sol.



$300 - 60 = 10a$

$24m/s^2 = a$

$60 + F_{min} = 10a$

$F_{min} = 10a - 60$

$F_{min} = 240 - 60$

$F_{min} = 180N$

$300 + 60 = 10a$

$a = 36m/s^2$

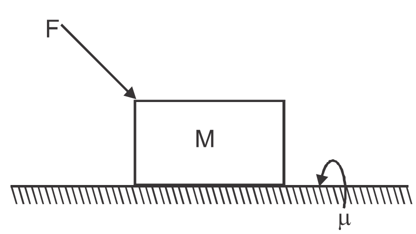
$f_{max} - 60 = 10a$

$f_{max} = 60 + 10a$

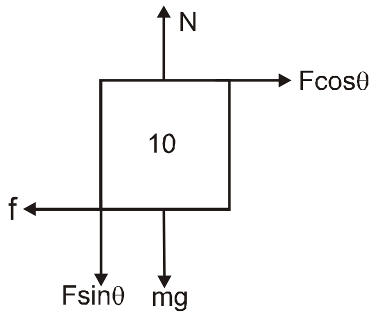
$= 60 + 360$

$F_{max} = 420N.$

Ex.73 Find out θ_{min} so that block does not slide whatever be the value of force.



Sol $N = F\sin\theta + mg$



$F\cos\theta \le f$

$F\cos\theta \le \mu(F\sin\theta + mg)$

$F\cos\theta \le \mu(\mu F\sin\theta + mg)$

$F(\cos\theta - \mu\sin\theta) \le \mu mg)$

$\cos\theta - \mu\sin\theta \le \frac{\mu mg}{F}$

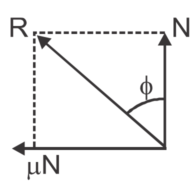
$\because$ the value F may be ∞ i.e. $F = \infty$

$\cos\theta - \mu\sin\theta \le 0$

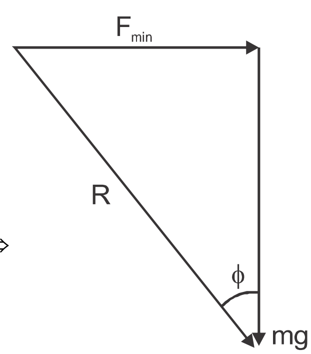
$\frac{1}{\mu} \le \tan\theta$

$\theta \ge \tan^{-1}\frac{1}{\mu}$

$\theta_{min} = \tan^{-1}\frac{1}{\mu}$



$\Rightarrow$

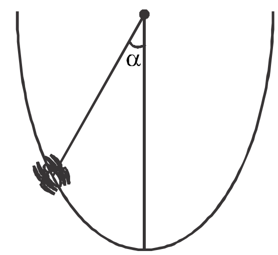


$\sin\phi = \frac{F_{min}}{mg}$ $\qquad F_{min} = mg\sin\phi$

$F_{min} = mg\frac{\mu}{\sqrt{1+\mu^2}}$

MOTION OF AN INSECT IN THE ROUGH BOWL

Ex.74 An insect crawls up a hemispherical surface very slowly. The coefficient of friction between the insect and the surface is 1/3. If the line joining the centre of the hemispherical surface to the insect makes an angle α with the vertical, the maximum possible value of α is given by



(A) $\cot\alpha = 3$

(B) $\tan\alpha = 3$

(C) $\sec\alpha = 3$

(D) $\operatorname{cosec}\alpha = 3$

Sol. a. The two forces acting on the insect are mg and N. Let us resolve mg into two components:

$mg\cos\alpha$ balances N

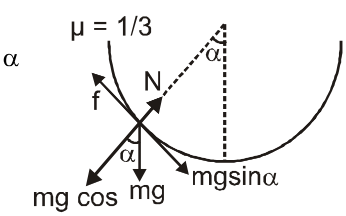
$mg\sin\alpha$ is balanced by the frictional force.

$N = mg\cos\alpha$

$f = mg\sin\alpha$

But

$f = \mu N = \mu mg\cos\alpha$



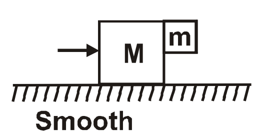
$\Rightarrow \cot\alpha = \frac{1}{\mu}$

$\Rightarrow \cot\alpha = 3$

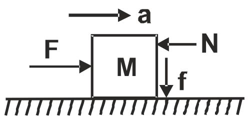
STICKING OF A BLOCK WITH ACCELERATED CART

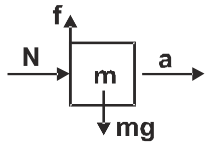
Solved Examples

Ex.75 The figure shows two blocks m and M which are pushed together on a smooth horizontal surface. If the coefficient of friction between the blocks is m, then determine the minimum value of the horizontal force F required to hold the blocks together.



Sol. Acceleration of system $a = \frac{F}{M+m}$





From free-body diagram of m:

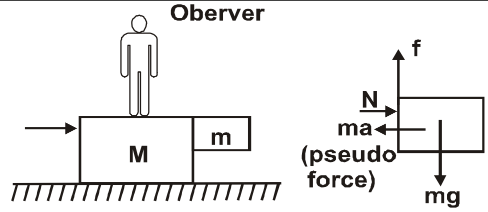
$N = ma = m\left(\frac{F}{M+m}\right)$ (i)

m is not sliding, so $f = mg$ (ii)

For no sliding case $f \le \mu N$

$\Rightarrow mg \le \mu \frac{mF}{(M+m)} \Rightarrow \quad F \ge \mu \frac{(M+m)g}{\mu}$

using the method of pseudo force :



Observing m in frame of M :

In the frame of M the block m is not sliding.

$N = ma \qquad f = mg$

If f is of static nature, $f \le \mu N$

$mg \le \mu ma \quad \Rightarrow mg \le \frac{\mu mF}{M+m} \qquad F \ge \frac{(M+m)g}{\mu}$