LINEAR INEQUALITIES

1. Linear inequations in one variable are $x \geq a$, $x \leq a$, $x > a$, $x < a$, $y \geq a$, $y \leq a$, $y < a$, $y > a$. System of inequation in one variable is the combination of inequations.

2. To draw the graphical solution of $x \geq a$, $x \leq a$, $x > a$, $x < a$:

we keep the following points in consideration.

(i) First we draw graph of $x = a$ which is a line. For this equation x is always a and

x can take any value

x	a	a	a
y	0	1	2

etc.

and its graph is a line parallel to y-axis.

(ii) We draw a line if $x \geq a$, $x \leq a$ and a dotted line if $x > a$, $x < a$.

(iii) Take any point in the plane, not on the line and put in inequation $x \geq a$, $x \leq a$, $x > a$ or $x < a$. If it is a true statement then we shade the portion containing the point as the solution set and if it is a false statement we shade the portion not containgin the point as solution set.

3. To draw the graphical solution of $y \geq a$, $y \leq a$, $y > a$ or $y < a$:

(i) We consider the equation $y = a$ and draw its graph. For the equation $y = a$, is always a, can take any value.

x	0	1	2
y	a	a	a

etc.

Its graph is a line parallel to x-axis and the follow parts (ii), (iii) of point 2

4. To find the graphical solution of system of inequations in one variable we proceed as point 2 and point 3 and the *common shaded* portion represents the solution set.

5. Linear inequations in two variables are

(i) $ax + by + c \geq 0$ (ii) $ax+by + c \leq 0$

(iii) $ax + by + c > 0$ (iv) $ax+by +c < 0$.

Their combination is known as a system of linear inequations in two variables.

6. To find the graphical solution of linear inequation in two variables we follow as under :

(i) Consider the equation $ax + by + c = 0$ and find the values of x and y which satisfy the equation. Find at least three such ordered pairs (x, y) and plot these on a graph. Graph is a straight line. We draw a line for in equation

$ax + by + c \geq 0$ or $ax + by + c \leq 0$ and a dotted line for the inequation $ax + by + c > 0$ or $ax + by + c < 0$.

(ii) Take any point in a plane not lying on the line (generally we take (0, 0) if it does not lie on a point) and put in the given inequation. If it is a true statement we shade the portion containing that point and if it is a false statement we shade the portion not containing the point under consideration.

7. To find the graphical solution of system of linear inequations, we draw the graphical solution of inequations in the system and the common shaded portion represents the solution of system of inequations.

8. $|x-a| \geq b \Rightarrow x \geq a + b$ or $x \leq a - b$.

9. $|x-a| \leq b \Rightarrow a - b \leq x \leq a + b$.

SOLVED PROBLEMS

Ex.1 Solve $\frac{2x-3}{4} + 8 \geq 2 + \frac{4x}{3}$
Represent the solution set on number line.

Sol. The given inequation is

$$\frac{2x-3}{4} + 8 \geq 2 + \frac{4x}{3}$$

$\Rightarrow$ $3(2x-3)+96 \geq 24+16x$
(Multiplying both sides by 12)

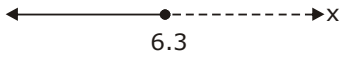
$\Rightarrow$ $6x-9+96 \geq 24+16x \quad \Rightarrow 6x-16x \geq 24+9-96$

$\Rightarrow$ $\frac{-10x}{10} \leq \frac{-63}{10} \quad \Rightarrow \quad x \leq 6.3$

$\therefore$ Solution set is the set of all numbers which are less than or equal to 6.3.

$\therefore$ $S = (-\infty, 6.3]$

Solution set on number line is represented as



Ex.2 Solve the following inequations: $\frac{x-3}{x-5} > 0$

Sol. The given inequation is

$$\frac{x-3}{x-5} > 0 \qquad ....(1)$$

Since $\frac{x-3}{x-5}$ is + ve.

$\therefore$ either $x-3>0, x-5>0$

or $x-3<0, x-5<0$

i.e., either $x>3, x>5$ or $x<3, x<5$

i.e., either $x> 5$ or $x<3$

[$\because$ $x>5 \Rightarrow x>3$ and $x<3 \Rightarrow x<5$]

Hence ths solution set is $(-\infty, 3), \cup (5, \infty)$, which on the number line as below:

Ex.3 Solve $\left|\frac{3x-4}{2}\right| \leq \frac{5}{12}$.

Sol. We have

$$\left|\frac{3x-4}{2}\right| \leq \frac{5}{12} \quad \therefore \quad \frac{5}{12} \leq \frac{3x-4}{2} \leq \frac{5}{12}$$

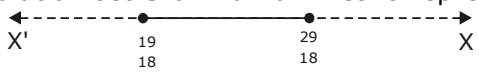
$\Rightarrow$ $-5 \leq 6(3x-4) \leq 5$

$\Rightarrow$ $-5+24 \leq 18x \leq 5+24$

$\Rightarrow$ $19 \leq 18x \leq 29$

$\Rightarrow$ $\frac{19}{18} \leq x \leq \frac{29}{18} \quad \therefore S = \left[\frac{19}{18}, \frac{29}{18}\right]$

$\therefore$ Solution set S on numbr lines is represented as



Ex.4 Solve the following system of inequations:
$2x - 7 > 5 - x$, $11 - 5x \leq 1$

Sol. The given inequations are :

$2x - 7 > 5 - x$ ...(1)

and $11 - 5x \leq 1$ ...(2)

From (1), $2x+x>7+5 \Rightarrow 3x > 12$

$\Rightarrow$ $x > 4$

Thus the solution set of (1) is $(4, \infty)$

From (2), $11 - 5x \leq 11$

$\Rightarrow$ $-5x \leq 1 - 11$

$\Rightarrow$ $-5x \leq -10 \Rightarrow x \geq 2$

Thus the solution set of (2) is $[2, \infty)$

Clearly the intersection of these solution sets is the sets $(4, \infty)$

Hence the solution set of the given system of inequations is $(4, \infty)$

Ex.5 Solve the following system of inequations
$\frac{5x}{4} + \frac{3x}{8} > \frac{39}{8}$ and $\frac{2x-1}{12} - \frac{x-11}{3} < \frac{3x+1}{4}$

Sol. The given inequations are :

$$\frac{5x}{4} + \frac{3x}{8} > \frac{39}{8} \qquad ...(1)$$

and $\frac{2x-1}{12} - \frac{x-11}{3} < \frac{3x+1}{4}$ ...(2)

From (1) $\frac{10x+3x}{8} > \frac{39}{8} \Rightarrow 13x > 39$

$\Rightarrow$ $x > 3$

Thus the solution set of (1) is $(3, \infty)$

From (2),

$\frac{(2x-1)-4(x-11)}{12} < \frac{3x+1}{4} \Rightarrow \frac{-2x+43}{12} < \frac{3x+1}{4}$

$\Rightarrow$ $-2x + 43 < 3(3x+1)$
[Multiplying both sides by 12]

$\Rightarrow$ $-2x + 43 < 9x + 3$

$\Rightarrow$ $-11x < -40 \quad \Rightarrow \quad x > \frac{40}{11}$

Thus the solution set of (2) is $\left(\frac{40}{11}, \infty\right)$

Clearly, the intersection of these solution sets is the set $\left(\frac{40}{11}, \infty\right)$

Hence, the solution set of the given system of inequations is $\left(\frac{40}{11}, \infty\right)$

Ex.6 Find the region enclosed by the following inequations :
$x+y-2 \leq 0$, $2x+y-3 \leq 0$, $x \geq 0$, $y \geq 0$.
Also find the ordered pairs of the vertices of the regions.

Sol. We have $x + y - 2 \leq 0$

$2x + y - 3 \le 0$ and $x \ge 0$, $y \ge 0$

The correspondin equation of (1) is

$x + y - 2 = 0$

$\Rightarrow$ $y = 2 - x$

To draw the graph :

Table :

x	....	1	2	....
Y	....	1	0	....

Plot the points (1, 1), (2, 0) join them, we get the graph of (5)

This straight line meet y-axis at (0, 2)

The corresponding equation of (2) is

$2x + y - 3 = 0 \Rightarrow y = 3 - 2x$.

To draw the graph :

Table :

x	....	0	1	....
Y	....	3	1	....

Plot the points (0, 3) (1, 1) and join them, we get the graph of (6)

This st. line meets x-axis at $\left(\frac{3}{2},0\right)$

Put (0, 0) in $x + y - 2 \le 0$, we get

$0 + 0 - 2 \le 0 \Rightarrow -2 \le 0$, true

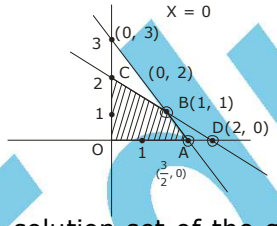
So we shade the portion containing (0, 0)

Put (0,0) in $2x+y-3 \le 0$, we get $-3 \le 0$, true

So we shade the portion containing (0,0)

Also $x \ge 0$ is the potion on and right of y-axis and $y \ge 0$ is the portion on and above x-axis.

Thus the required region is as shown shaded in the fig.



Hence the solution set of the given system of inequation is the quadrilateral OABC whose vertices are

O(0, 0), A $\left(\frac{3}{2},0\right)$, B (1, 1), C(0, 2).

Ex.7 Solve the following system of inequations graphically.

$2x+y \le 24$, $x+y \le 11$, $2x+5y \le 40$, $x \ge 0$, $y \ge 0$

Sol. The given inequations are

$2x + y \le 24$ ...(i)

$x + y \le 11$ ...(ii)

$2x + 5y \le 40$ ...(iii)

$x \ge 0$, $y \ge 0$ ...(iv)

Consider a set of rectangular cartesian axes OXY in the plane.

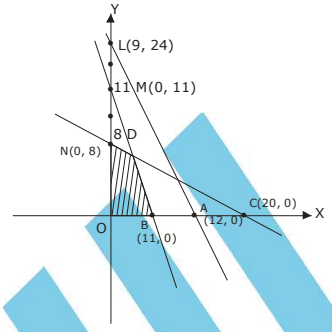
It is clear that any point which satisfies $x \ge 0$, $y \ge 0$ lies in the first quadrant

Let us draw the graph of $2x + y = 24$

For $x = 0$, $y = 24$

For $y = 0$, $2x = 24$ i.e., $x = 12$

$\therefore$ line $2x + y = 24$ meets OX in A (12, 0) and OY in L (0, 24).



Again we draw the graph of $x + y = 11$

For $x = 0$, $y = 11$

For $y = 0$, $x = 11$

$\therefore$ line $x + y = 11$ meets OX in B (11, 0) and OY in M (0, 11).

Now, we draw the graph of $2x + 5y = 40$

For $x = 0$, $5y = 40$ i.e., $y = 8$

For $y = 0$, $2x = 40$ i.e., $x = 20$

$\therefore$ line $2x + 5y = 40$ meets OX in C (20 , 0) and OY on N (0, 8)

Put (0, 0) in (i), (ii) and (iii) we get $0 \le 24$, $0 \le 11$, $0 \le 40$ respectively which are all true.

So in all the three cases we shade the portin containing (0, 0).

Since feasible region is the region which satisfies all the given constraints.

$\therefore$ OBDN is the feasible region, which represents the solution set of the given system of inequations.

Ex.8 A manufacturer has 600 litres of a 12% solution of acid. How many litres of a 30% acid solution must be added to it so that acid content in the resulting mixture will be more than 15% but less than 18% ?

Sol. Let number of litres of 30% acid solution be x.

From given condition

30% of x+12% of 600>15% of (x+600)

and 30% of x+12% of 600<18% of (x+600)

$\therefore$ $\frac{30}{100}x + \frac{12}{100} \times 600 > \frac{15}{100}(x + 600)$

and $\frac{30}{100}x + \frac{12}{100} \times 600 < \frac{15}{100}(x + 600)$

$\therefore$ $30x + 7200 > 15x + 9000$

and $30x + 7200 < 18x + 10800$

$\therefore$ $15x > 1800$ and $12x < 3600$

$\therefore$ $x > 120$ and $x < 300$

$\therefore$ $120 < x < 300$

$\therefore$ number of lines of the 30% solution of acid will have to be more than 120 but less than 300.

EXERCISE - I UNSOLVED PROBLEMS

Q.1 Draw the graphical solution of each of the following inequations :
(i) $x>3$ (ii) $x \le 1$ (iii) $x \ge -2$
(iv) $x < 5$ (v) $x > -2$ (vi) $x < -3$

Q.2 Draw the graphical solutions of each of the following inequations :
(i) $y \ge 0$ (ii) $y > 3$ (iii) $y < -2$ (iv) $y \le 1$

Q.3 Draw the graphical solutions of each of the following inequations :
(i) $|x| > 2$ (ii) $|x| \le 3$ (iii) $|y| \ge 1$
(iv) $|y| < 3$ (v) $|3x-2| \le \frac{1}{2}$
(vi) $|x+\frac{1}{4}| > \frac{7}{4}$ (vii) $|x+1| \ge 3$.

Q.4 Draw the graphical solutions of each of the following inequastions :
(i) $|x-1|>2$ (ii) $|y+3| \le 5$ (iii) $|x-2| \ge 3$
(iv) $|y-1|<4$ (v) $|3x-2| \le \frac{1}{2}$
(vi) $|x+\frac{1}{4}| > \frac{7}{4}$ (vii) $|x+1| \ge 3$.

Q.5 Draw the graphical solution of each of the following system of inequation:
(i) $x \ge 3, y < -2$
(ii) $x < 3, y > -4, y < 2$
(iii) $|x| \le 1; |y| \ge 1$
(iv) $|x-3| < 2; |y+2| \le 5$
(v) $x > 3, y \le 5, x \le -2, y \ge 7$

Q.6 Draw the graph of each of the following inequations:
(i) $x+y < 6$ (ii) $3x-4y \ge 12$ (iii) $2x-4y \ge 1$
(iv) $y-x \le 0$ (v) $5x-2y < 10$ (vi) $x-2y \le -1$
(vii) $x \le 8 - 4y$

Q.7 Draw the graphical solution of each of the following system of inequation :
(i) $2x+y < 2, x-3y \ge 6$.
(ii) $3x+4y \le 12, 4x+3y \le 12, x \ge 0, y \ge 0$
(iii) $2x+y \ge 2, x-y \le 1, x+2y \le 8, x \ge 0, y \ge 0$
(iv) $|y-x| \le 3$
(v) $x+2y \ge 0, 2x + y \le 4, x \ge 0, y \le 2$
(vi) $6x+5y \le 150$,
$x+4y \le 80, x \le 15, x \ge 0, y \ge 0$

Q.8 Draw the graphical solution of each of the following:
(i) $x \ge 0, x \le 6, y \ge 0, y \le 7$.
(ii) $x \ge 0, y \ge 0, x \le y, x+y \ge 6$.
(iii) $x \le 5, y \le 3, x \ge y, x+2y \le 6$
(iv) $y \ge -4, y \le 4, x+2y \ge 2, x-y \ge 6$
(v) $x \ge 2, x \le 8, y \le x+3, 2x+y \le 10, y \ge 4$
(vi) $3x+2y \le 24, x + 2y \le 16, x + y \le 10$,
$x \ge 0, y \ge 0$.

Q.9 Draw the graphical solution of each of the following:
(i) $y < 0, 3x - y \ge 0$ (ii) $x \ge 0, x-y \le 0$
(iii) $x \ge 0, y \le 5, x - y \le 6$.
(iv) $x \ge 4, y \ge -3, x + y \le 6$
(v) $x + y > 6, 2x - y > 0$

Q.10 Solve: $11x < 101$ when (i)x is a natural number (ii) x is an integer.

Q.11 Solve of: $3x-4 \ge -2x + 6$ when (i) x is a natural number (ii) x is a real number.

Q.12 Solve for x: $\frac{1}{2}\left(\frac{3}{5}x+4\right) \ge \frac{1}{3}(x-6)$.

Q.13 Solve for x: $|x| < 4$.

Q.14 Solve: $|3x-7| \ge 2$.

Q.15 Solve for x: $\frac{x-5}{x+2} < 0$

Q.16 A recharger manufacturing company produces rechargers and its cost function for a week is $C(x) = \frac{1}{10}(4270 + 23x)$ and its revenue function is $R(x) = 3x$, where x in the number of rechargers produced and sold per week. How many rechargers mus be sold for the company to make a profit.

Q.17 To receive grade A in a subject, one must obtain an average of 90 or higher marks in five examinations. If a student's marks in the first four examinations are 91, 88, 93 and 95, find the minimum marks, a student must obtain to get grade A in the subject.

Q.18 Solve the following inequations and represent the solution on the number line.
(i) $8x-2 \ge 5x+1$ (ii) $\frac{4-3x}{5} < \frac{2x-5}{4}$ (iii) $\frac{x+3}{x+5} > 5$

Q.19 Solve the following inequations and represent the solution on the number line.
(i) $|3x-2| < \frac{1}{2}$ (ii) $\left|\frac{4x-5}{3}\right| \le \frac{5}{3}$ (iii) $\left|\frac{2(3-x)}{5}\right| < \frac{9}{5}$

Q.20 I.Q of a person is given by formula
$I.Q. = \frac{M.A.}{C.A.} \times 100$ where M.A. stands for mental age and C.A. stands for chronological age. If $75 \le I.Q. \le 135$ for a group of 9 year children. Find the range of their mental age.

ANSWER KEY

10. (i) 1,2,3, ..., 9 (ii) ....–4,–3,–2,–1,0,1,2,...9
11. (i) 2,3,4, ... (ii) $\{x \in R/x \ge 2\}$.
12. $\{x \in R/x \le 120\}$ **13.** $-4 < x < 4$.
14. $\{x \in R/x \le 5/3, x \ge 3\}$.
15. $\{x \in R/-2 < x < 5\}$
16. 610 **17.** 83