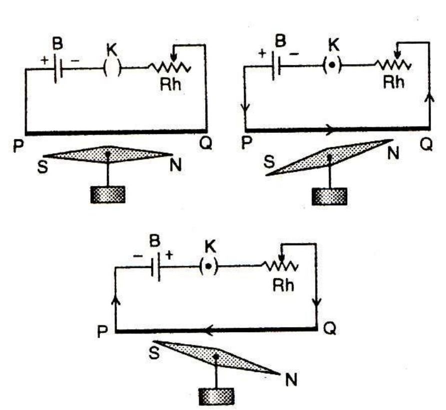
Moving Charges and Magnetism

4

OERSTED EXPERIMENT AND OBSERVATIONS

(i) Oerseted performed an experiment in 1819 whose arrangement is shown in the following figure. Following observations were noted from this experiment.

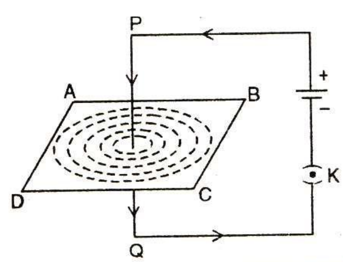


(a) When no current is passed through the wire AB, the magnetic needle remains undefelcted.

(b) When current is passed through the wire AB, the magnetic needle gets deflected in a particular direction and the deflection increases as the current increases.

(c) When the current flowing in the wire is reversed, the magnitude needle gets deflected in the opposite direction and its deflection increases as the current increases.

(ii) Oersted concluded from this experiment that on passing a current through the conducting wire, a magnetic field is produced around this wire. As a result the magnetic needle is deflected. This phenomenon is called magnetic effect of current.



(iii) From another experiment, it is found that the magnetic lines of force due to the current flowing in the wire are in the form of concentric circles around the conducting wire.

MAGNETIC FIELD ($\vec{B}$)

(Magnitude, Direction and unit)

(i) When a charged particle q moving with a velocity $\vec{v}$ passes through a magnetic field $\vec{B}$ (when electric field is not present), a force acts on it. It is called magnetic force. The force acting on a charged paritcle due to its motion is given by $\vec{F} = q(\vec{v} \times \vec{B})$

Its magnitude, $|\vec{F}| = qvB \sin\theta$

where θ is the angle between $\vec{B}$ and $\vec{v}$.

(ii) It is found from this relation that the direction of $\vec{F}$ is perpendicualr to the plane of $\vec{B}$ and $\vec{v}$. If v = 0 or B = 0 or $\vec{v}$ is parallel to $\vec{B}$, then $\vec{F}$ = 0.

(iii) If keeping the magnitude of v constant its direction is continuously changed, then the magnitude of $\vec{F}$ changes but the direction of $\vec{F}$ always remains perpendicular to $\vec{v}$. That direction of $\vec{v}$ for which the magnitude $\vec{F}$ is zero or no force acts on the moving charge, is the direction of vector $\vec{B}$.

(iv) If the test charge q moves with a velocity v perpendicular to the magnetic field, then the magnitude of F will be maximum, i.e., $F_{max} = qvB$

The magintude of B can be defined from $B = \frac{F_{max}}{qv}$

If q = 1C and v = 1 m/s, then $B = F_{max}$

Thus the magnetic force acting on a charge of 1C moving normal to the magnetic field with a velocity of 1 m/s is equal to the magnitude of the intensity of magnetic field.

(v) If a charged particle passes through such region where both electric and magnetic fields are present, then the resultant force acting on the particle is the vector sum of electric and magnetic forces,

i.e. $$\vec{F} = \vec{F}_E + \vec{F}_B$$

$$= q\vec{E} + q(\vec{v} \times \vec{B}) = q(\vec{E} + \vec{v} \times \vec{B})$$

This force is called Lorentz force. It is a microscopic force.

(vi) Unit of $\vec{B}$ in M.K.S. system is weber/meter^2 or tesla or newton/ampere-meter. It is defined as

If a charge of 1 coulomb is moving in a direction perpendicular to the direction of magnetic field with a velocity of 1 m/s and one newton force acts on it, then the magnitude of magnetic intensity of field is 1 weber/meter^2.

In C.G.S. system the unit of $\vec{B}$ is maxwell/ cm^2 or gauss. The relation of these units is

1 tesla $= 10^4$ maxwell/cm^2

$= 10^4$ gauss

Magnetizing Field

(i) The magnetic induction B depends upon the external current which produces field as well as the magnetization of the medium. In absence of medium, that is in vacuum where effect of medium is zero, it the magnetic induction is B_0, then $\frac{B_0}{\mu_0}$ represents a vector which depends upon the external currents only. This vecot is called magnetizing field

$$\vec{H} = \frac{\vec{B}_0}{\mu_0}$$

(ii) For solenoid, the magnetising field is given by

$$H = \frac{\mu_0 ni}{\mu_0} = ni$$

where n is the number of turns per unit length of solenoid and i is the current flowing in the solenoid.

(iii) Units : In C.G.S. system-oersted

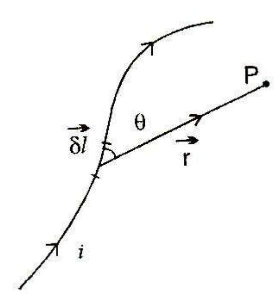
In M.K.S. system - ampere/metre

1 ampere/metre = $4\pi \times 10^{-3}$ oersted.

BIOT AND SAVARTS LAW

(i) The intensity of magnetic field due to current carrying conductor is determined with the help of this law.

(ii) This law is obtained on the basis of experiments.



(iii) The intensity of magnetic field (δB) at a certain point due to current carrying element of length ($\delta \ell$) of the conductor depends upon the following

(a) δB is directly proportional to the current flowing through the element, i.e. $\delta B \propto i$.

(b) δB is directly proportional to the length of the element of the conductor, i.e., $\delta B \propto \delta \ell$.

(c) δB is inversely proportional to the square of the distance r of the observation point P from the element, i.e., $\delta B \propto 1/r^2$

(d) dB is directly proportional to the sine of the angle θ between the position vector $\vec{r}$ of the observation point P with respect to the element carrying element is given by

(in rationalized M.K.S. system)

$$\delta B = \frac{\mu_0}{4\pi}\frac{i\delta\ell \sin\theta}{r^2}$$

where μ_0 is magnetic permeability of vacuum and its value is $4\pi \times 10^{-7} = 12.57 \times 10^{-7}$ weber/amperemeter or henry/meter (H/m).

(v) In vector representation

$$\vec{\delta B} = \frac{\mu_0}{4\pi}.\frac{i\vec{\delta\ell} \times \vec{r}}{r^3}$$

$$= \frac{\mu_0}{4\pi}.\frac{i\vec{\delta\ell} \times \hat{r}}{r^2}$$

The direction of the element $\vec{\delta\ell}$ is taken in the direction of flow of current.

(vi) The direction of magnetic field is always perpendiular to the plane formed by the vectors $\vec{\delta\ell}$ and $\vec{r}$

(vii) The intenstiy of magnetic field due to entire length of the conductor will be

$$B = \int \delta B = \int \frac{\mu_0}{4\pi}.\frac{i\delta\ell \sin\theta}{r^2}$$

or $$B = \sum \delta B = \sum \frac{\mu_0}{4\pi}.\frac{i\delta\ell \sin\theta}{r^2}$$

(viii) If $\theta = 0$, that is the observation point is on the current carrying element,

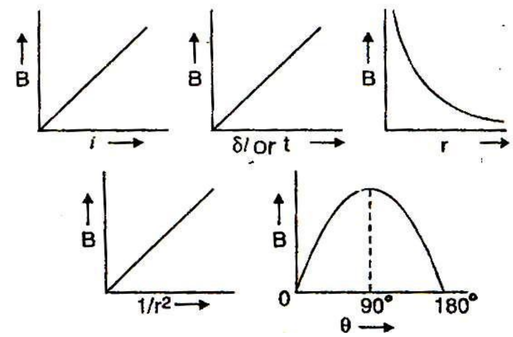
$\therefore \quad \delta B = 0 \quad$ or $\quad B = 0$ (minimum)

(ix) If the observation point is on the line perpendicular to the current carrying element, then

$\theta = 90^o$ and $\sin\theta = 1$

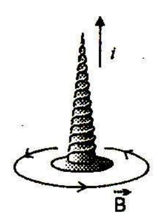
$$\therefore \quad \delta B = \frac{\mu_0}{4\pi}\frac{i\delta\ell}{r^2} \text{ (maximum)}$$

(x) The graph between B and i, B and $\delta\ell$ or ℓ, B and r, B and $1/r^2$ and B and θ are as follows.

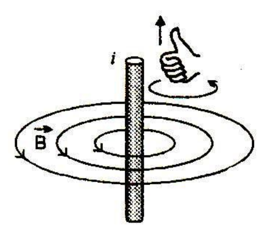


(xi) **Direction of magnetic field :** The direction of magnetic field is determined with the help of the following simple laws :

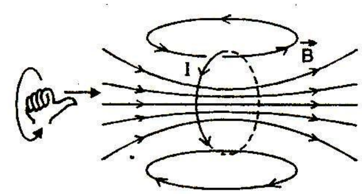
(a) Maxwell's cork screw rule : According to this law if a right handed cork screw is rotated in such a way that it moves forward in the direction of current in the conductor, then the direction of the rotation of the screw will show the direction of lines of foce. (fig.-a)



(fig.-a)



(fig.-b)



(fig.-c)

(b) Right hand palm rule : According to this rule if a current carrying conductor is held in the right hand such that the thumb of the hand represents the direction of current flow, then the direction of folding fingers will represent the direction of magnetic lines of force. (fig.-b)

(c) Right hand palm rule for circular currents : According to this rule if the direction of current in circular conducting coil is in the direction of folding fingers of right hand, then the direction of magnetic field will be in the direction of stretched thumb. (fig.-c)

(xii) The magnetic lines of force due to a current carrying element are in the form of concentric circles with their centres at the element.

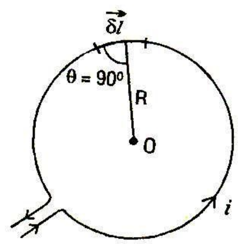
APPLICATION OF BIOT AND SAVARTS LAW

Magnetic field due to a circular coil carrying current

(i) If a circular coil of radius R is made from a wire of length L, then the number of turns in the coil will be

$$n = \frac{L}{2\pi R}$$

(ii) If the current flowing in the coil is i amperes, then from Biot-Savart's law the magnetic field at the centre of current carrying coil is



The direction of magnetic field is along the axis of the coil. If the current flowing in the coil is anticlockwise, then the direction of B_0 will be upward along the axis of the coil. If the current is clockwise, then the direction of B_0 will be inward along the axis of the coil.

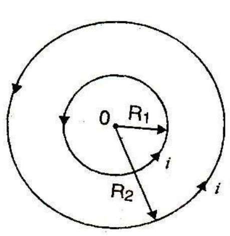
If a coil of one turn is made from a wire of length L and another coil of n turns is made from a wire of same length and same amount of current flows through both coils, then the relation between the magnetic fields produced at their centres will be

$$B_{0n} = n^2 B_{01}$$

or $$\frac{B_{0n}}{B_{01}} = n^2$$

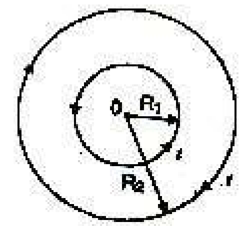
(iii) If same amount of current is passed through two concentric and coplanar circular coils of radii R_1 and R_2 and number of turns n_1 and n_2 respectively, then the intensity of magnetic field at the centre will be

(a) when current flows in the same direction,



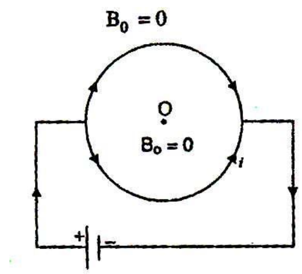
$$\vec{B}_0 = \vec{B}_{01} + \vec{B}_{02} \quad \text{or} \quad B_0 = \frac{\mu_0 n_1 i}{2R_1} + \frac{\mu_0 n_2 i}{2R_2}$$

(b) when current flows in opposite directions,

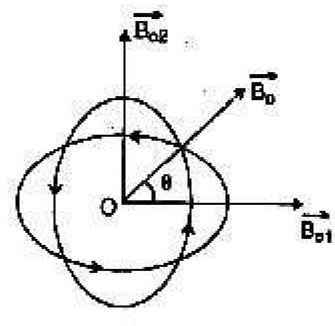


$$B_0' = \frac{\mu_0 n_1 i}{2R_1} - \frac{\mu_0 n_2 i}{2R_2}$$

(iv) If any two points of a circular coil are connected to a battery and then current is passed through the coil. The magnetic field at the centre of the coil will be zero, i.e., $B_0 = 0$



(v) The resultant magnetic field at the centre of two perpendicular concentric circular current carrying two perpendicular concentric circular current carrying coils is



$$B_0 = \sqrt{B^2_{01} + B^2_{02}}$$

If B_0 makes an angle θ with B_{01}, then

$$\tan\theta = \frac{B_{02}}{B_{01}}$$

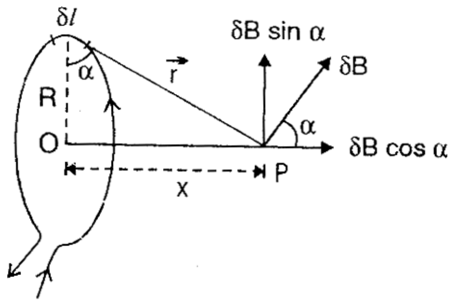
or $$\theta = \tan^{-1}\left(\frac{B_{02}}{B_{01}}\right)$$

If the two coils are identical, then

$$B_0 = \sqrt{2}\left(\frac{\mu_0 n i}{2R}\right)$$

and $$\theta = 45°$$

(vi) The magnetic field at an axial point situated at a distance x from the centre of a current carrying circular coil:



In this position the observation point P is at the same distance r from each element of the coil where $r = \sqrt{R^2 + x^2}$ and the angle θ between the current element $\vec{\delta\ell}$ and the position vector $\vec{r}$ is 90°. Hence $\sin\theta = 1$.

According to Biot-Savart's law the magnetic induction due to a current element is

$$\delta B = \frac{\mu_0}{4\pi}\frac{I\delta l}{r^2}$$

The components $\delta B\cos\alpha$ of magnetic induction $\vec{\delta B}$ due to different elements along the axis are in the same direction but the perpendicular components $\delta B\sin\alpha$ are symmetrically in different directions around the axis. Thus magnetic induction due to entire coil at the observation point, $\vec{B} = \Sigma\vec{\delta B} = \Sigma\delta B\cos\alpha$ along the axis and $\Sigma\delta B\sin\alpha = 0$ normal to the axis.

Thus $$B = \sum \frac{\mu_0}{4\pi}.\frac{I\delta l\cos\alpha}{r^2}$$

$$= \frac{\mu_0 niR^2}{2(R^2 + x^2)^{3/2}} \text{ along the axis}$$

or $$B = \frac{\mu_0 niR^2}{2R^3(1 + x^2/R^2)^{3/2}}$$

$$= \frac{\mu_0 ni}{2R}\left(1 + \frac{x^2}{R^2}\right)^{-3/2} = B_0\left(1 + \frac{x^2}{R^2}\right)^{-3/2}$$

If x >> R, then

$$B = \frac{\mu_0 niR^2}{2x^3} \quad \therefore \quad B \propto \frac{1}{x^3}$$

(a) B depends upon the distance x. B decreases as x increases and B = 0 at x = ∞

(b) At a distance ± R from the centre of the coil

$$B = \frac{\mu_0 ni}{(4\sqrt{2})R} \quad \therefore \quad \frac{B}{B_0} = \frac{1}{2\sqrt{2}}$$

(c) when x = ± 0.766 R, then $B = \frac{B_0}{2}$.

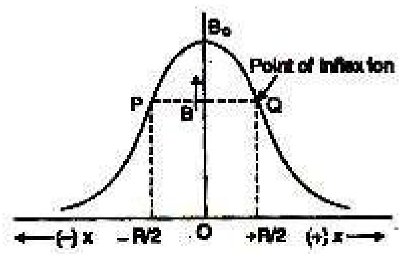
(d) B is not uniform near the coil. The rate of change of magnetic field with distance is different at different points.

At the centre of the coil, i.e. at x = 0

$$B_0 = \frac{\mu_0 ni}{2R}$$

Thus the magnetic field at the centre of the coil is maximum.

(e) The graph between B and x is found to be of the following type.



(f) The points of inflexion are those points on the curve where curvature becomes zero and the direction of curvature changes sign. The rate of change of magnetic field becomes constant at these points, i.e.,

$$\frac{dB}{dx} = \text{constant}$$

or $$\frac{d^2B}{dx^2} = 0.$$

(g) In the (B – x) graph of current carrying coil two points of inflexion are found. They are at a distance x = ± R/2 from the centre. The distance between the two points of inflexion is equal to the radius of the coil.

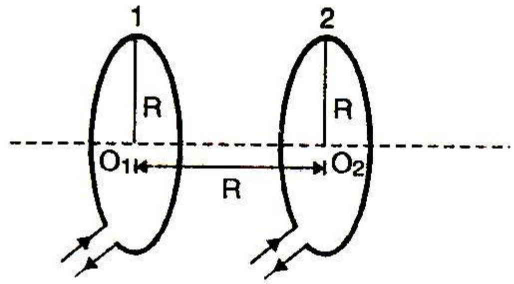
Helmholtz Coils

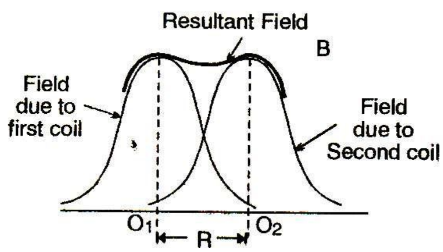
(i) If two identical coils placed coaxially parallel to each other in such a way that the distance between them is equal to the radius of the coil and same amount of current is passed through them in the same direction, then in the middle region of the coil (near x = ± R/2), on increasing the distance from the first coil field intensity will decrease but on decreasing the distance from the second coil the field intensity will increase by the same amount. Thus in this middle region the resultant magnetic field will be uniform. The two coils arranged in this way are called Helmholtz coils.

(ii) These coils are used to produce uniform magnetic field.

(iii) The uniform magnetic field produced in the middle region is

$$B = 2\left[\frac{\mu_0 n i R^2}{2(R^2 + R^2/4)^{3/2}}\right] = \frac{8\mu_0 ni}{5\sqrt{5}R} = 0.716\frac{\mu_0 ni}{R}$$



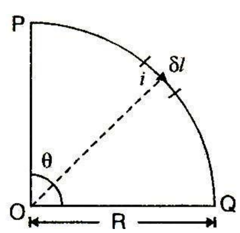


Magnetic Field due to a Current Carrying Arc

(i) If an arc of radius R subtends an angle θ at its centre, then

the lenght of the arc = $R\theta$

The magnetic field due to an element of the arc at the point O (from Biot-Savart's law)



$$\delta B_0 = \frac{\mu_0}{4\pi}\frac{i\delta l \sin\theta}{R^2}$$

but $\theta = 90^\circ$ $\therefore$ $\sin 90^\circ = 1$

Thus $\delta B_0 = \frac{\mu_0}{4\pi}\frac{i\delta l}{R^2}$

The magnetic field due to entire arc at the point O

$$B_0 = \Sigma \delta B_0$$

$$= \sum \frac{\mu_0}{4\pi}\frac{i\delta l}{R^2} = \frac{\mu_0}{4\pi}\frac{i}{R^2}\Sigma\delta l$$

but $\Sigma\,\delta l = R\theta$ where θ is in radians

$$\therefore \quad B_0 = \frac{\mu_0}{4\pi}\frac{i}{R^2}R\theta = \frac{\mu_0 i\theta}{4\pi R}$$

Thus $B_0 \propto i$, $B_0 \propto \theta$ and $B_0 \propto \frac{1}{R}$

(a) If length of the are is one fourth of the coil, then

$\theta = \pi/2$

$$\therefore \quad B_0 = \frac{\mu_0 i}{4\pi R}\left(\frac{\pi}{2}\right) = \frac{\mu_0 i}{8R}$$

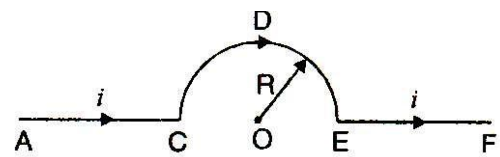
(b) If length of the arc is half of the coil, then

$\theta = \pi$

$$\therefore \quad B_0 = \frac{\mu_0 i}{4\pi R}\pi = \frac{\mu_0 i}{4R}$$

(ii) Magnetic field due to current carrying conductor shown in the following figure

The magnetic field at O



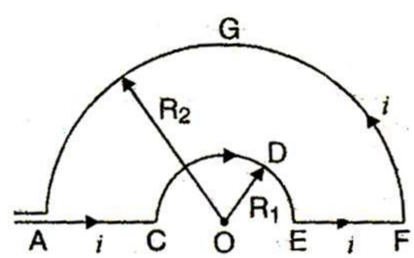
$$B_0 = B_{AC} + B_{CDE} + B_{EF}$$

$$= 0 + \left(-\frac{\mu_0 i\pi}{4\pi R}\right) + 0$$

$$= -\frac{\mu_0 i}{4R}$$

(Inward normal to the plane of the paper)

(iii) Mangetic field at the centre O of a current carrying conductor shown in the following figure



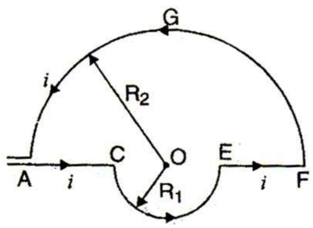
$$B_0 = B_{AC} + B_{CDE} + B_{EF} + B_{FGA}$$

$$= 0 + \left(-\frac{\mu_0 i}{4R_1}\right) + 0 + \left(\frac{\mu_0 i}{4R_2}\right)$$

$$= -\frac{\mu_0 i}{4}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = -\frac{\mu_0 i}{4}\left(\frac{R_2 - R_1}{R_1 R_2}\right)$$

Negative sign shown that the magnetic field is inward normal to the palne of paper.

(iv) The magnetic field at the centre O



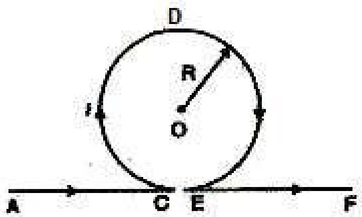
$$B_0 = B_{AC} + B_{CDE} + B_{EF} + B_{FGA}$$

$$B_0 = 0 + \frac{\mu_0 i}{4R_1} + 0 + \frac{\mu_0 i}{4R_2}$$

$$\text{or} \quad B_0 = \frac{\mu_0 i}{4}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{\mu_0 i}{4}\left(\frac{R_1 + R_2}{R_1 R_2}\right)$$

Upward normal to the plane of paper.

(v) The magnetic field at the centre O



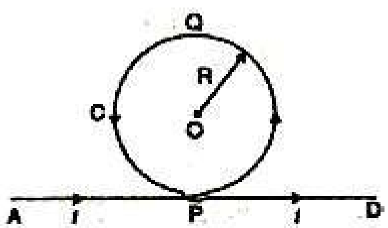
$$B_0 = B_{CDE} + B_{AF}$$

$$= -\frac{\mu_0 i}{2R} + \frac{\mu_0 i}{2\pi R} \qquad = -\frac{\mu_0 i}{2R}\left(1 - \frac{1}{\pi}\right)$$

Inward normal to the plane of paper.

(vi) The magnetic field at the centre O

At point P the wires do not touch each other

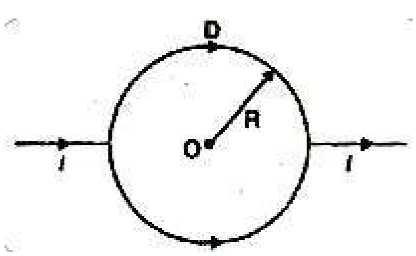


$$B_0 = B_{AD} + B_{PQC}$$

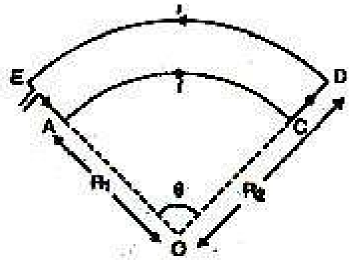
$$= \frac{\mu_0 i}{2\pi R} + \frac{\mu_0 i}{2R} \qquad = \frac{\mu_0 i}{2R}\left(\frac{1}{\pi} + 1\right)$$

Upward normal to the plane of paper.

(vii) The magnetic field at the centre O will be zero.



(viii) The magnetic field at the O

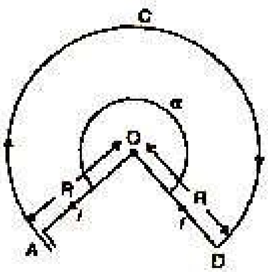


$$B_0 = B_{AC} + B_{CD} + B_{DE} + B_{EA}$$

$$= -\frac{\mu_0 i\theta}{4\pi R_1} + 0 + \frac{\mu_0 i\theta}{4\pi R_2} + 0 \qquad = -\frac{\mu_0 i\theta}{4\pi}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Negative sign shows that the magnetic field will be inward normal to the plane of paper.

(ix) The magnetic field at the centre O

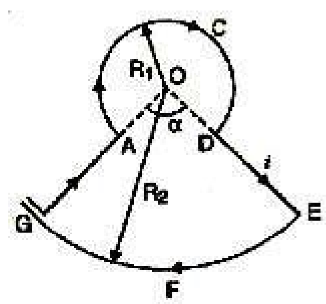


$$B_0 = B_{OA} + B_{ACD} + B_{DO}$$

$$= 0 - \frac{\mu_0 i\alpha}{4\pi R} + 0 \qquad = -\frac{\mu_0 i\alpha}{4\pi R}$$

Inward normal to the plane of paper.

(x) The magnetic field at the centre O



$$B_0 = B_{ACD} + B_{DE} + B_{EFG} + B_{GA}$$

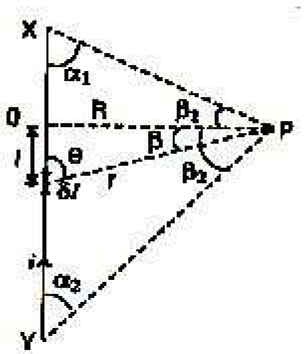
$$= -\frac{\mu_0 i(2\pi - \alpha)}{4\pi R_1} + 0 + (-)\frac{\mu_0 i\alpha}{4\pi R_2} + 0$$

$$= -\frac{\mu_0 i}{4\pi}\left[\frac{2\pi - \alpha}{R_1} + \frac{\alpha}{R_2}\right]$$

Inward normal to the plane of paper.

Magnetic Field due to straight current carrying wire of finite length

(i) If a current i is flowing through a conductor XY, then the magnetic field $\vec{B}$ due to it at a point P can be determined using Biot-Savart's law. The normal distance of point P from the conductor is R.



If the lines joining the ends X and Y of the conductor make angles β_1 and β_2 respectively with the perpendicular drawn on the conductor from the point P, then resultant magnetic induction at P due to current carrying conductor

$$B_P = \frac{\mu_0 i}{4\pi R}(\sin\beta_1 + \sin\beta_2)$$

If the lines joining the ends X and Y of the conductor with the point P make angles α_1 and α_2 respectively with the conductor, then

$$B_P = \frac{\mu_0 I}{4\pi R}(\cos\alpha_1 + \cos\alpha_2)$$

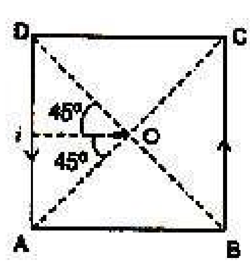
(ii) If the current carrying conductor is of infinite length, then

$$\beta_1 = \beta_2 = \frac{\pi}{2}$$

$$B = \frac{\mu_0 i}{2\pi R}\,\text{wb/m}^2$$

(iii) The magnetic field at the centre of current carrying square coil of side a :

If a current i is passed through a square coil ABCD of side a, then the magnetic field due to straight current carrying conductor AD of finite length



$$= \frac{\mu_o i}{4\pi(a/2)}(\sin 45^\circ + \sin 45^\circ) \qquad = \frac{\mu_0 i}{\sqrt{2}\pi a}$$

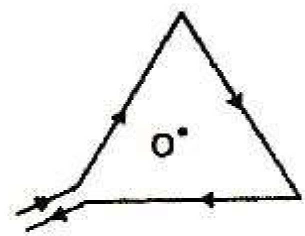
Due to symmetry the magnetic field due to each conductor will be $\frac{\mu_0 i}{\sqrt{2}\pi a}$. The direction of magnetic field in this case will be $\frac{\mu_0 i}{\sqrt{2}\pi a}$. The direction of magnetic field in this case will be upward perpendicular to the plane of the coil.

$\therefore$ The magnetic field due to all the four sides of the square

$$B_0 = 4\frac{\mu_0 i}{\sqrt{2}\pi a} \qquad = 2\sqrt{2}\frac{\mu_0 i}{\pi a}\text{tesla}$$

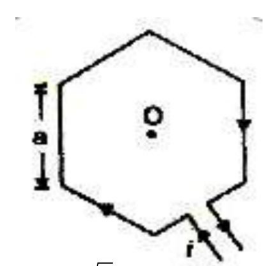
The direction of magnetic field will be normal to the plane of the coil.

(iv) If a current i ampere is passed through the arms of an equilateral triangle of side a, then the magnetic field at its centre will be



$$B_0 = \frac{9\mu_0 i}{2\pi a}$$

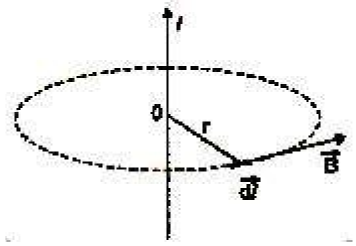
(v) If a current i ampere is passed in a hexagonal coil of side a. The magnetic field at the centre of the coil will be



$$B_0 = \frac{\mu_0 i\sqrt{3}}{\pi a}$$

Magnetic Field due to Infinitely Long Current carrying Wire

(i) Let current i be flowing in a long and straight wire. Wire is placed in the plane of paper. Magnetic field is determined at a distance r from this wire. The length of wire being very large relative to distance r, the wire can be assumed to be of infinite length. The magnetic lines of force due to current carrying wire will be in the form of concentric circles.



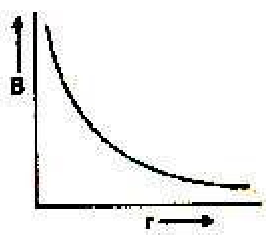
(ii) If the magnetic induction at a distance r from the wire is B, then the circulation on the circular closed path of radius r.

$$\oint \vec{B}, d\vec{\ell} = B(2\pi r) = \mu_0 i$$

$\therefore$ $$B = \frac{\mu_0 i}{2\pi r}$$

or $$B = \frac{\mu_0}{4\pi}\left(\frac{2i}{r}\right) \quad = K\left(\frac{2i}{r}\right)$$

(iii) The magnetic field $\vec{B}$ due to long current carrying wire at a point near it (a) is proportional to the current i and (b) is inversely proportiional to the distance r. This result can also be obtained from Bio-Savart's law.



(iv) The curve between B and r is or following type

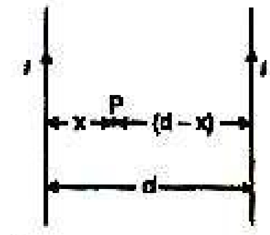
(v) The direction of magnetic field is perpendicular to the plane made by the wire and the position vector of the point.

Magnetic filed due to two parallel and straight current carrying wires

In the following figure, two long current carrying parallel wires are shown. The magnitude of current flowing in both wires is same. The distance between the wires is d. The magnetic field due to these wires at a point P will be.

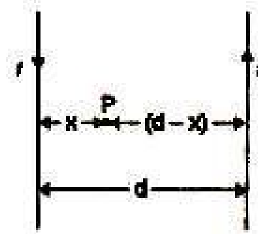
(I) If the point P is in between the wires and

(a) currents in both wires are in the same direction



$$\vec{B}_P = \vec{B}_1 - \vec{B}_2 \quad = \frac{\mu_0 i}{2\pi}\left(\frac{1}{x} - \frac{1}{d-x}\right)$$

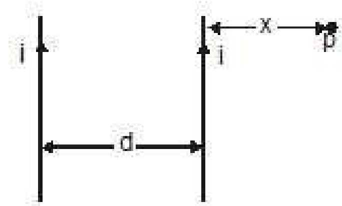
(b) currents in the two wires are in opposite directions



$$\vec{B}_P = \vec{B}_1 + \vec{B}_2 \quad = \frac{\mu_0 i}{2\pi}\left(\frac{1}{x} + \frac{1}{d-x}\right)$$

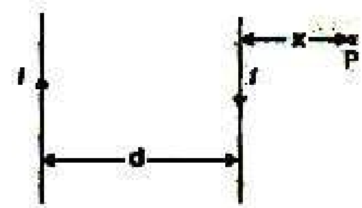
(ii) If the point P is outside the wire and

(a) currents in both wires are in the same direction



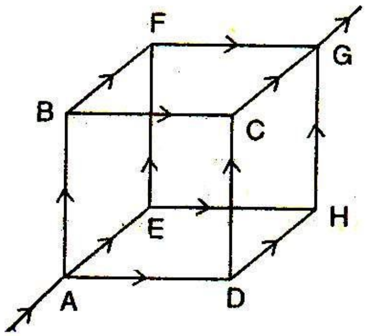
$$\vec{B}_P = \vec{B}_1 + \vec{B}_2 \quad = \frac{\mu_0 i}{2\pi}\left(\frac{1}{x} + \frac{1}{d+x}\right)$$

(b) current in the two wires are in opposite directions

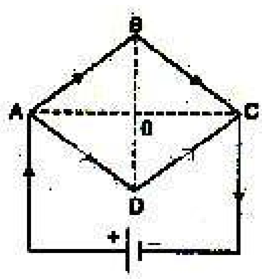


$$\vec{B}_P = \vec{B}_1 - \vec{B}_2 \quad = \frac{\mu_0 i}{2\pi}\left(\frac{1}{x} - \frac{1}{d+x}\right)$$

(vii) In a cubical structure made by wires if current is passed from one of its corner, then the magnetic field at its centre will be zero. Because AB and HG; AE and CG, AD and FG, BF and DH, FE and CD, EH and BC are 6 identical current carrying condutor pairs and each pair produces equal and opposite magnetic field at the centre. Thus the resultant field is zero.

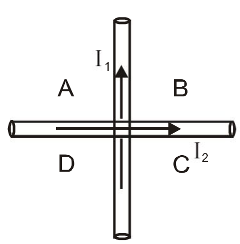


(viii) If a square ABCD is made by joining four identical wires and current is passed from a battery, then the magnetic field will be zero at the centre of the square.



Solved Examples

Ex.1 As shown in the diagram two mutually perpendicular wires are placed very close to each other, but are not touching each other. The regions where the intensity of magnetic field is zero is (are)



(1) A only (2) B, D
(3) A, B (4) B only

Sol. In region B, field due to I_1 is into the page and field due to I_2 is out of page. So at some points we expect, these fields to cancel and add up to give zero. Similarly in region D, field due to I_1 is out of page, while field due to I_2 is into the page. Thus, again, some where fields A and C, fields due to I_1 and I_2 are in the same directions and can not give zero on addition. Thus the correct answer is (2) B and D.

Ex.2 A straight horizontal stretch of copper wire carries a current i = 30 A. The linear mass density of the wire is 45 g/m. What is the magnitude of the magnetic field needed to "float" the wire, that is to be balance its weight ?

(1) 147 G (2) 441 G
(3) 14.7 G (4) zero G

Sol. For L length of wire, to balance

$F_{magnetic} = mg$

$ILB = mg$

Therefore $B = mg/IL = (m/L)\, g/I$

$= \dfrac{45 \times 10^{-3} \times 9.8}{30} = 1.47 \times 10^{-2}$ tesla

The correct answer is therefore (1) 147 Gauss

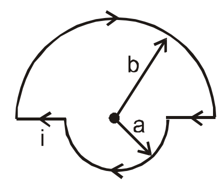
Ex.3 You are given a closed circuit with radii a and b carrying current i.

The magnetic dipole moment of the circuit is

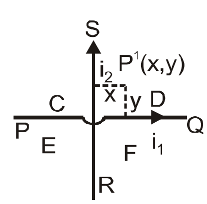
Sol. m = current × area

$= i\left(\frac{1}{2}\pi a^2 + \frac{1}{2}\pi b^2\right)$

$= \frac{1}{2} i\pi(a^2 + b^2)$



Ex.4 If i_1 and i_2 currents are flowing in two wires mutually perpendicular to each other and placed very near, then find the equation of locus of the points of zero magnetic field.



Sol. $B_{pQ} = \left(\frac{\mu_0 i_1}{2\pi y}\right) \odot$

$B_{Rs} = \left(\frac{\mu_0 i_2}{2\pi x}\right) \otimes$

For zero magnetic field $(B_{PQ}) \odot = (B_{RS}) \otimes$

or $\frac{\mu_0 i_1}{2\pi y} = \frac{\mu_0 i_2}{2\pi x}$ or $y = \left(\frac{i_1}{i_2}\right) x$

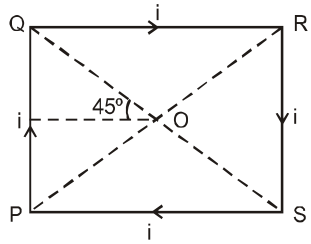
This locus is straight line.

Ex.5 A current i is flowing in a square coil of side a, find the value of magnetic field at centre.

Sol. PQRS is current carrying square of side a due to which magnetic field at its centre O

$B = 4 \times B_{PQ}$

$$= 4 \times \frac{\mu_0 i}{4\pi r}[\sin\theta_1 + \sin\theta_2]$$



here $\theta_1 = \theta_2 = 45^o$

and $r = \frac{a}{2}$

$$B = 4 \times \frac{\mu_0 i}{4\pi(a/2)}[\sin 45^o + \sin 45^o]$$

$$B = \frac{2\mu_0 i}{\pi a}\left(\frac{2}{\sqrt{2}}\right) \quad \text{or} \quad B = \frac{2\sqrt{2}\,\mu_0 i}{\pi a}$$

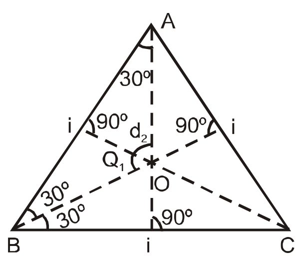
Ex.6 In an equilateral triangle of side 4.5×10^{-2} m, 1 ampere current is flowing. Find magnitude of B at its centroid.

Sol. Due to AB value of magnetic induction B_1 at O

$$B_1 = \frac{\mu_0 i}{4\pi a}(\sin\phi_1 + \sin\phi_2)$$

here i = 1 A, $\phi_1 = \phi_2 = 60^o$

by figure $\frac{a}{\ell/2} = \tan 30$



or $a = \frac{\ell}{2} \times \frac{1}{\sqrt{3}}$

$$= \frac{4.5 \times 10^{-2}}{2\sqrt{3}}$$

At the centre of triangle O, $B = 3B_1$ and this will be upward perpendicular.

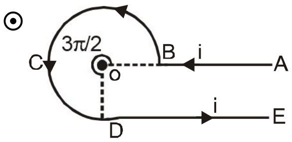
$$B = 3\frac{4\pi \times 10^{-7} \times 1}{4\pi \times \frac{4.5 \times 10^{-2}}{2\sqrt{3}}} \times (\sin 60^o + \sin 60^o)$$

$$= \frac{3 \times 10^{-7} \times 2\sqrt{3}}{4.5 \times 10^{-2}}\left(\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}\right) \quad = 4 \times 10^{-5}\ \text{Wb/m}^2$$

Ex.7 In the body shown in diagram i current is flowing. Radius of circular part is r and linear parts are very long. Find magnitude of magnetic field at O.

Sol. $\vec{B}_0 = \vec{B}_{AB} + \vec{B}_{BCD} + \vec{B}_{DE}$

$$= 0 + \frac{3}{4}\left\{\frac{\mu_0 i}{2r}\right\} \odot \quad + \frac{1}{2}\left\{\frac{\mu_0 i}{2\pi r}\right\} \odot$$

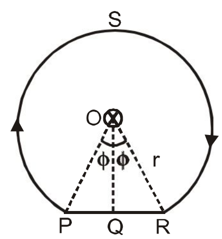


$$= \frac{\mu_0 i}{4\pi r}\left\{\frac{3}{2}\pi + 1\right\} \odot$$

Ex.8 Find the magnitude field at O due to curved and linear portions of conducting wire as shown in figure.

Sol. $\vec{B}_0 = \vec{B}_{PSR} + \vec{B}_{RQP}$

$$B_{PSR} = \frac{\mu_0 i}{4\pi}\left[\frac{2\pi - 2\phi}{r}\right]$$



$$= \frac{\mu_0 i}{2\pi r}[\pi - \phi]$$

$$B_{RQP} = \frac{\mu_0 i}{4\pi}\frac{2\sin\phi}{OQ}$$

$$= \frac{\mu_0 i}{4\pi}\frac{2\tan\phi}{r} \quad [\because OQ = r\cos\phi]$$

so $\quad B = \frac{\mu_0 i}{2\pi r}[\pi - \phi] + \frac{\mu_0 i}{2\pi r}\tan\phi = \frac{\mu_0 i}{2\pi r}(\pi - \phi + \tan\phi)$

Ex.9 There are two concentric coils having same number of turns and radius 10 cm and 30 cm respectively. A current is made to flow in them (i) in same direction, (ii) in opposite direction. In both the cases the ratio of resultant magnetic fields at the centre of the coils will be :

Sol. (i) When the current flows in same direction

$$|\vec{B}_0| = |\vec{B}_0 + \vec{B}_{02}| = \frac{\mu_0 ni}{2R_1} + \frac{\mu_0 ni}{2R_2}$$

$$= \frac{\mu_0 ni}{2}\left(\frac{1}{0.1} + \frac{1}{0.3}\right)$$

$$= \frac{\mu_0 ni}{2} \times \frac{40}{3}$$

(ii) When the current flows in opposite direction

$$\vec{B}_0' = \vec{B}_{01} - \vec{B}_{02}$$

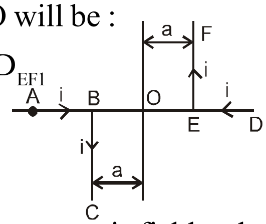
$$B_0' = \frac{\mu_0 ni}{2R_1} - \frac{\mu_0 ni}{2R_2} = \frac{\mu_0 ni}{2}\left[\frac{1}{0.1} - \frac{1}{0.3}\right]$$

$$= \frac{\mu_0 ni}{2} \times \frac{20}{3}$$

$$\therefore \quad \frac{B_0}{B_0'} = \frac{2}{1} = 2:1$$

$\therefore$ (2) is correct answer.

Ex.10 Two wire ABC and DEF are arranged as shown in figure. Current i is flowing through them. value of magnetic field at point O will be :

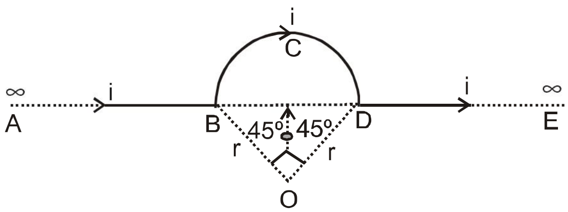


Sol. $B_0 = B_{AB} + B_{BC} + B_{DE} + D_{EF1}$

$$= 0 + \frac{\mu_0 i}{4\pi a} + 0 + \frac{\mu_0 i}{4\pi a} = \frac{\mu_0 i}{2\pi a}$$

Ex.11 In the given figure, find the magnetic field at the centre O :

Sol. $B_0 = B_{AB} + B_{BCD} + B_{DE}$



$$= \frac{\mu_0 i}{4\pi x} + \frac{1}{4}\frac{\mu_0 i}{2r} + \frac{\mu_0 i}{4\pi x} \qquad = 2\frac{\mu_0 i}{4\pi x} + \frac{\mu_0}{4}\cdot\left(\frac{\pi i}{2r}\right)$$

$$= \frac{\mu_0}{4\pi}\cdot\frac{2i\sqrt{2}}{r} + \frac{\mu_0}{4\pi}\left(\frac{\pi i}{2r}\right) \qquad = \frac{\mu_0}{4\pi}\left(\frac{2i}{r}\right)\left[\sqrt{2} + \frac{\pi}{4}\right]$$

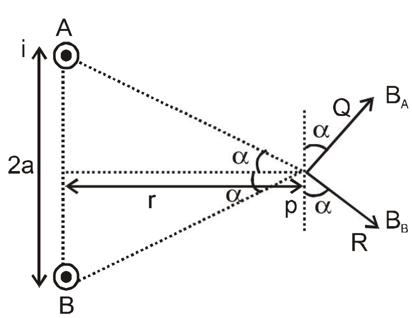
Ex.12 A current of 2 ampere is made to flow through a coil which has only one turn. The magnetic field produced at the centre is $4\pi \times 10^{-6}$ Wb/m^2, then radius of the coil will be :

Sol. $B_0 = \frac{\mu_0 ni}{2R}$ (but n = 1)

$B_0 = \frac{\mu_0 i}{2R}$ or $R = \frac{\mu_0 i}{2B_0}$

or $R = \frac{4\pi \times 10^{-7} \times 2}{2 \times 4\pi \times 10^{-6}} = 0.1\text{m}$

Ex.13 Two parallel wires situated at a distance 2a are there in which current i is flowing in opposite direction as shown in the figure. Value of magnetic field at point P which is situated at a distance r from both the wires will be :



Sol. Magnetic field at P due to wire A is

$$B_A = \frac{\mu_0 i}{2\pi r},$$

in the direction PQ and due to B,

$$B_B = \frac{\mu_0 i}{2\pi r},$$

in the direction PR.

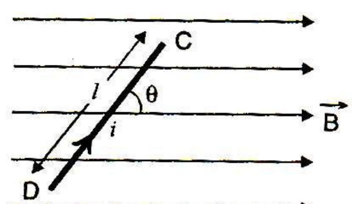
On resolving the magnetic field into their components, resultant of components parallel to AB will be zero whereas components perpendicular to AB will be added.

Resultant magnetic field

$$B = 2B\ \sin\alpha = \frac{2\mu_0 i}{2\pi r}\frac{a}{r} = \frac{\mu_0 ia}{\pi r^2}$$

FORCE ON A CURRENT CARRYING CONDUCTOR DUE TO MAGNETIC FIELD

(i) If a current i is passed through a conductor of length *l* and it is placed in a magnetic field B, then the force acting on it will be



$$F = iBl\sin\theta$$

or $\vec{F} = i(\vec{l} \times \vec{B})$

(ii) If $\theta = 0$, that is, $\vec{B} \parallel \vec{l}$, then

F = 0 (minimum)

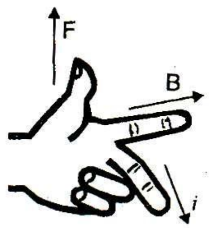
(iii) If $\theta = 90°$, that is, $\vec{B} \perp \vec{l}$, then

F = Bi*l* (maximum)

(iv) The direction of magnetic force is perpendicular to the plane of $\vec{l}$ and $\vec{B}$ according to right hand screw rule. Following two rules are used in determining the direction of the magnetic force.

(a) **Right hand palm rule :** If the right hand and the palm are stretched such that the thumb points in the direction of current and the stretched fingers in the direction of the magnetic field, then the force on the conductor will be perpendicular to the palm in the outward direction.

(b) **Fleming left hand rule :** If the thumb, fore finger and central finger of the left hand are stretched such that first finger points in the direction of magnetic field and the central finger in the direction of current, then the thumb will point in the direction of force acting on the conductor.



Solved Examples

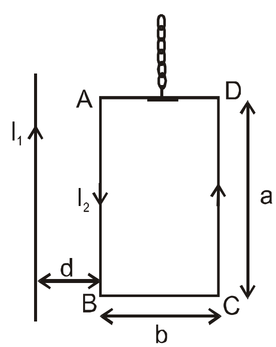
Ex.14 The distance between the wires of electric mains is 12 cm. These wires experience 4 mg wt. per unit length. The value of current flowing in each wire will be –

Sol. $\frac{F}{l} = \frac{\mu_0 i^2}{2\pi d}$

$$9.8 \times 4 \times 10^{-6} = \frac{4\pi \times 10^{-7} \times i^2}{2 \times \pi \times 12 \times 10^{-2}}$$

$$i = \sqrt{\frac{4 \times 10^{-6} \times 9.8 \times 0.12}{2 \times 10^{-7}}} = 4.85A$$

Ex.15 In the adjoining fig the force on current carrying loop ABCD due to a long current carrying (i_1) wire will be–



Sol. The (repulsive) force on the side AB of the loop, due to current i_1 is $F_1 = \frac{\mu_0}{2\pi}\frac{i_1 i_2 L}{R}$

given $\frac{\mu_0}{2\pi} = 2 \times 10^{-7}$ N/A^2, i_1 = 20A, i_2 = 16A, R = d = 4 cm = 0.04 m and L = a = 15 cm = 0.15 m & b = 6 cm

$$\therefore \; F_1 = (2 \times 10^{-7}) \times \frac{20 \times 16 \times 0.15}{0.04} = 2.40 \times 10^{-4} N.$$

This force will act away from the current-carrying wire. Similarly, the (attractive) force on the side CD of the loop, due to current i_1 is (now R = d + b = 10 cm = 0.10 m)

$$\therefore \; F_2 = (2 \times 10^{-7}) \times \frac{20 \times 16 \times 0.15}{0.10} = 0.96 \times 10^{-4} N$$

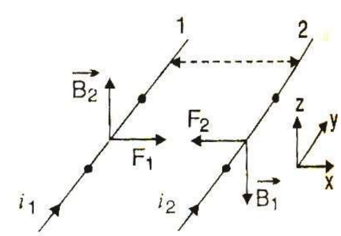
This force will act towards the current carrying wire. The force on the sides AD and BC of the loop will be equal and acting downward and upward respectively. Hence they will neutralize each other.

$\therefore$ net force on the loop = $F_1 - F_2$ = (2.40 – 0.96) $\times 10^{-4}$ = 1.44 $\times 10^{-4}$ N

acting away from the current - carrying wire.

MAGNETIC FORCE BETWEEN TWO CURRENT CARRYING PARALLEL CONDUCTORS

(i) In the figure two parallel conductors are shown. The distance between them is d. Let i_1 and i_2 be the currents flowing in these conductors. When current flowing in these conductors are in the same direction, they attract each other. When currents flowing in them are in opposite direction, they repel each other. The force of attraction or repulsion acting on unit length of each conductor will be



$$F = \frac{\mu_0 i_1 i_2}{2\pi d} N/m$$

(ii) Force acting on a conductor of length ℓ

$$F' = F\ell = \frac{\mu_0 i_1 i_2 \ell}{2\pi d} \text{ newton}$$

(iii) Attractive or repulsive force

$$|\vec{F}_{12}| = |\vec{F}_{21}|$$

and $\vec{F}_{12} = -\vec{F}_{21}$

i.e., they are equal in magnitude but opposite in directions, where $\vec{F}_{12}$ is the force on the conductor 1 due to conductor 2 and $\vec{F}_{21}$ is the force on second due to first conductor.

(iv) **Definition of Ampere :**

If $i_1 = i_2 = 1$ ampere and d = 1 metre, then

$$F = \frac{\mu_0}{2\pi} = \frac{4\pi\times10^{-7}}{2\pi}$$

$= 2 \times 10^{-7}$ N/m

If two conductors of infinite length and negligible cross-sectional area are placed parallel to each other at a distance 1m apart each carrying same current in the same direction produce a force of attraction of 2×10^{-7} N/m between them, then the current flowing in each wire will be 1 ampere.

From the above definition of ampere, the new unit of μ_0 is

$$2\times10^{-7}\frac{N}{m} = \frac{\mu_0}{2\pi}\frac{(1\text{ampere})^2}{(1\text{metre})}$$

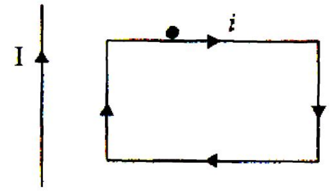
$\therefore \quad \mu_0 = 4\pi\times10^{-7} N/A^2$

Dimensions of $\mu_0 = \frac{MLT^{-2}}{A^2} = MLT^{-2}A^2$

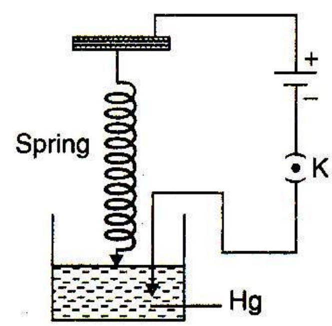
(v) Two parallel wires are placed one over the other. One wire is stationary and other is free. If free wire is above the stationary wire, then in order to stop the free wire falling currents will have to be passed in the two wires in opposite directions. The force of repulsion due to them will balance the weight of free wire. If free wire is below the stationary wire, then currents will have to be passed in the same direction. In this case force of attraction due to them will balance the weight of free wire.

(vi) Some weight is suspended from a spring. If current is passed through it, then there will be attraction between the turns due to flow of current in the turns in the same direction and spring will contract.

(vii) As shown in figure current i is flowing through a rectangular loop. It is placed near a long straight wire in such a way that the wire is parallel to one arm of the loop and is in the plane of the loop. If constant current is passed through the wire in a direction shown in the figure, then the loop will move towards the wire because there will be more attraction due to current flowing through nearby arm of the loop in the same direction.



(viii) **Current carrying spring :**



(a) If current is passed through a spring then it will contract because current will flow through all the turns in the same direction.

(b) In the figure shown, when key K is pressed, then the spring will contract. As a result curent will stop flowing and the spring will region its original position. Hence spring will again touch mercury and current will start flowing and again it will contract. In this ay the spring will oscillate up and down.

Solved Examples

Ex.16 Two parallel wires are situated at a distance 10 cm and if 1 ampere current is flowing through each wire. The calculate the force per unit length on any one of them.

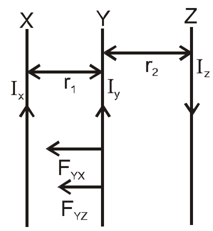
Sol. We know that

$$\frac{F}{\ell} = \frac{\mu_0 i_1 i_2}{2\pi r} = \left(\frac{4\pi\times10^{-7}\times10\times10}{2\pi\times10\times10^{-2}}\right)$$

$= 2 \times 10^{-4}$ N/m

Ex.17 As given in the figure X, Y and Z are three current carrying wires. What will be the direction of resultant magnetic force on wire Y ?

Sol. There will be attraction between X and Y and repulsion between X and Z. Both forces will act towards left



$\therefore$ Resultant force,

$$\vec{F}_{Total} = \vec{F}_{YX} + \vec{F}_{YZ}$$

$$\vec{F}_{Total} = \frac{\mu_0 I_x \times I_y}{2\pi r_1} + \frac{\mu_0 I_y \times I_z}{2\pi r_2}$$

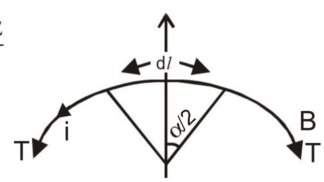
Ex.18 A flexible conducting loop of wire of length 0.5 meter is kept in a magnetic field perpendicular to the plane of loop of value 1 tesla. Show that when the current flows in the loop then it acquires circular shape. Also calculate the tension in the wire if 1.57 amperes current is flowing through it.

Sol. Force on each small element dl of loop placed in magnetic field will be $iBdl$ and it will be perpendicular to the element $d\ell$

$\therefore$ Loop will acquire circular shape At equilibrium,

$$2T \sin\frac{\alpha}{2} = iBdl$$

If α is small $\sin\frac{\alpha}{2} \simeq \frac{\alpha}{2}$



or $2T\frac{\alpha}{2} = iBdl$

or $T = \frac{Bidl}{\alpha}$ $\left\{\text{but } \frac{dl}{\alpha} = r\right\}$

$$= Bir = \frac{iBl}{2\pi}$$

$$= \frac{1.57 \times 1 \times 0.5}{2 \times 3.14} = 0.125\,\text{N}$$

FORCE BETWEEN TWO MOVING CHARGES

(i) Electric force acts between two stationary charges which is determined by Coulomb's law

$$F_e = K\frac{q_1q_2}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r^2}$$

(ii) If charge is not stationary, in that case magnetic field also acts in addition to the electric field.

(iii) Each charge q moving with velocity v produces a magnetic field whose intensity at a point having the position vector $\vec{r}$ will be

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q(\vec{v}\times\hat{r})}{r^2}$$

or $$B = \frac{\mu_0}{4\pi}\frac{qv\sin\theta}{r^2}$$

Position vector $\vec{r}$ is with respect to source charge.

(iv) If charges q_1 and q_2 are moving with velocities v_1 and v_2 respectively and at any instant the distance between them is r, then the force between them will be

$$F_m = \frac{\mu_0}{4\pi}\frac{q_1q_2v_1v_2}{r^2} \text{ newton}$$

(a) If both charges are of same nature and moving in the same direction, then the magnetic force acting between them will be attractive in nature.

(b) If the two charges are moving in opposite directions or their nature are opposite, then the magnetic force acting between them will be repulsive in nature.

(v) If both charges are moving with the same velocity, then the magnetic force acting between them will be

$$F_m = \frac{\mu_0}{4\pi}\frac{q_1q_2v^2}{r^2} \text{ newton}$$

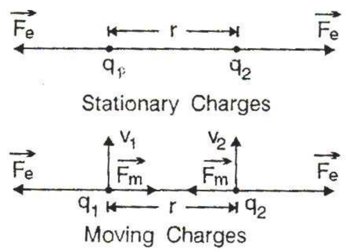
That is, the magnetic force acting between the charges depends upon their velocities.

(vi) If an observer moving with velocity v observes a charge moving with the same velocity v, then it will appear stationary. In that case electric force will appear to be acting between the charges and not the magnetic force.

(vii) If charges are stationary and they are seen by an observer moving with velocity *v*, then both electric and the magnetic force both will appear to be acting between them.

(viii) The existence of the magnetic force acting between two charges depends upon the motion of charges relative to the observer. Thus for the observers moving with different velocities the magnetic force acting between two charges will appear different.

(ix) The ratio of the magnetic force and the electric force acting between two moving charges is



$$\frac{F_m}{F_e} = \frac{\frac{\mu_0}{4\pi}\frac{q_1q_2v^2}{r^2}}{\frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r^2}} = \mu_0\epsilon_0 v^2$$

but $\mu_0\epsilon_0 = 1/c^2$ where c is velocity of light in vacuum.

$$\therefore \quad \frac{F_m}{F_e} = \left(\frac{v}{c}\right)^2$$

If v << c, then $F_m << F_e$.

(x) The force acting between the charges is represented in the following figure.

Solved Examples

Ex.19 A proton is moving in circular path perpendicular to some magnetic field of induction $\vec{B}$. If value of B is doubled and radius of circular path is kept constant then what will be the value of kinetic energy of the particle ?

Sol. Value of kinetic energy of proton

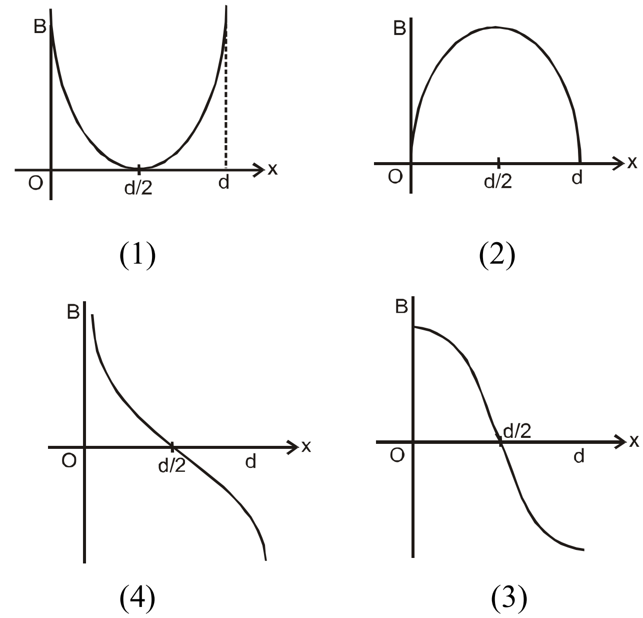
$$KE = \frac{1}{2}mv^2 = \frac{R^2q^2B^2}{2m}$$

Value of kinetic energy if value of B is doubled

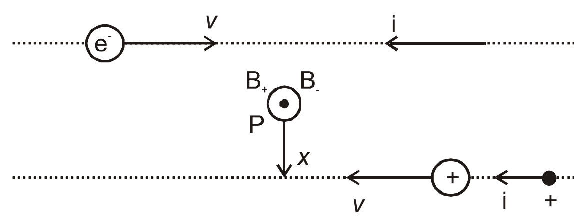
$$(KE)_{new} = \frac{R^2q^2(2B)^2}{2m} = \frac{4R^2q^2B^2}{2m}$$

Which is four times the initial kinetic energy.

Ex.20 Two beams of electrons and protons are equivalent to same value of current are at a distance d. Electron and proton are moving in opposite direction. There is a point P situated between these two beams at a distance x from one of the beams having magnetic flux density B. If curve is drawn between B and x, then the resultant curve will be :



Sol. According to the figure, at point P due to the motion of proton current i_+ will produce field B_+ whose direction will be inward ($B_+^{\otimes} = \mu_0 i/2\pi r$).



In the same way at point P, due to the motion of electrons a current i_- produces field B_-. Whose direction will be outward ($B_+^{\odot} = \mu_0 i/2\pi r$).

Ans. (3)

Ex.21 In hydrogen atom electron revolves around proton in an orbit of radius 0.5Å. If at the position of proton magnetic field of magnitude 14 tesla is produced, then what will be the value of angular frequency of motion of electron:

Sol. Magnitude of magnetic induction on proton will be

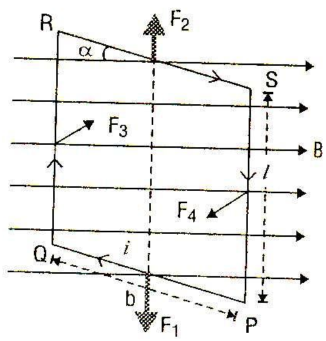
$$B_0 = \frac{\mu_0 ef}{2R}$$

$$\therefore \quad f = \frac{B_0 \times 2R}{\mu_0 e} = \frac{14 \times 2 \times 0.5 \times 10^{-10}}{12.57 \times 10^{-7} \times 1.6 \times 10^{-19}}$$

$$= \frac{14 \times 10^{-10}}{20.112 \times 10^{-26}} \quad \text{or} \quad = \frac{14}{2} \times 10^{15} = 7 \times 10^{15}$$

FORCE AND TORQUE ON A CURRENT CARRYING LOOP PLACED IN A UNIFORM MAGNETIC FIELD

(i) A rectangular conducting loop PQRS is placed in a uniform magnetic field. Its length is l and breadth is b. The rotational axis of the loop is perpendicular to the magnetic field B. When current i is passed through the loop, then the resultant force acting on the loop is zero but a torque acts on it.



(ii) If plane of the loop makes an angle α with the direction of magnetic field, then the torque acting on the loop

$$\tau = i\ell bB\cos\alpha$$

but ℓb = A = area of the rectangular loop

$\therefore$ $\tau = iBA\cos\alpha$

If the loop has N turns, then

$$\tau = NiAB\cos\alpha$$

(iii) If the vector area of the loop makes an angle θ with the direction of magnetic field, then $\theta = \frac{\pi}{2} - \alpha$

$\therefore$ Torque $\tau = iAB\sin\theta$

For N turns $\tau = NiAB\sin\theta$

(iv) NiA = M = magnetic moment of the loop

$\therefore$ $\tau = MB\sin\theta$

In vector form, $\vec{\tau} = \vec{M}\times\vec{B}$

(v) The torque acting on a current carrying loop placed in a magnetic field depends on its magnetic moment but not the shape of the loop.

(vi) For maximum torque $\theta = 90^\circ$ or $\alpha = 0$, i.e., the plane or the loop should be in the direction of magnetic field or the vector area of the loop should be normal to the magnetic field.

$$\tau_{max} = MB\sin 90^\circ = MB$$

(vii) For minimum torque $\theta = 0^\circ$ or $\alpha = 90^\circ$, i.e., the plane of the loop should be normal to the magnetic field or vector area should be in the direction of magnetic field.

$$\tau_{min} = MB\sin 0 = 0$$

(viii) The torque acting on a current - carrying loop placed in a magnetic field depends on the current. Moving coil galvanometer is based on this principle.

(ix) The work done in deflecting a coil from angle θ_1 to θ_2

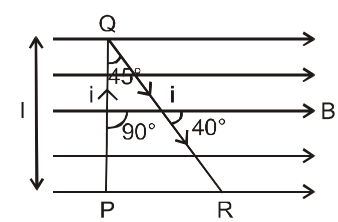
$$W = MB(\cos\theta_1 - \cos\theta_2)$$

The work done in deflecting a coil from an angle 0 to θ in a magnetic field

$$W = MB(1-\cos\theta).$$

Solved Examples

Ex.22 If i current is flowing in wire PQR. It is given the shape according to the figure and kept in a magnetic field B. If PQ = ℓ and $\angle PQR = 45^\circ$, then ratio of force on QR and PQ is :



Sol. $$\frac{F_{QR}}{F_{PQ}} = \frac{Bil\sin 90^\circ}{Bi(\ell\sqrt{2})\sin 45^\circ} = 1$$

Ex.23 The length of a solenoid is 0.4 m and the number turns in it is 500. A current of 3 amp. is flowing in it. In a small coil of radius 0.01 m and number of turns 10, a current of 0.4 amp. is flowing. The torque necessary to keep the axis of this coil perpendicular to the axis of solenoid will be –

Sol. $B_{solenoid} = \mu_0 n_s i_s$

$= \frac{\mu_0 N_s i_s}{L_s}$ or $\tau = B_s.iNA$

$= \frac{\mu_0 N_s i_s i N \pi r^2}{L_s}$

$\tau = \frac{4\pi \times 10^{-7} \times 500 \times 3 \times 0.4 \times 10 \times \pi \times (0.01)^2}{0.4}$

$= 5.92 \times 10^{-6}$ N-m.

Ex.24 The effective radius of a coil of 100 turns is 0.05 m and a current of 0.1 amp is flowing in it. The work required to turn this coil in an external magnetic field of 1.5 Tesla through 180° will be, if initially the plane of the coil is normal to the magnetic field.

Sol. W = 2MB

W = $2\pi i$ Na^{2}B

or W = 3.14 × 2 × 0.1 × 100 × $(0.05)^2$ × 1.5

= 0.236 Joule

Ex.25 A conducting wire of length l is turned in the form of a circular coil and a current i is passed through it. For torque due to magnetic field produced at its centre, to be maximum, the number of turns in the coil will be–

Sol. $\tau_{max} = MB$

or $\tau_{max} = ni\pi a^2 B$

Let number of turns in length ℓ is n

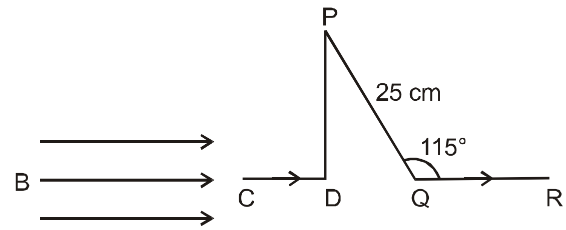
$\ell = n(2\pi a)$ or $a = \frac{\ell}{2\pi n}$

$\tau_{max} = \frac{ni\pi B\ell^2}{4\pi^2 n^2} = \frac{\ell^2 iB}{4\pi n_{min}}$

$\because \tau_{max} \propto \frac{1}{n_{min}}$

$n_{min} = 1$

Ex.26 The magnetic force on segment PQ, due to a current of 5 amp. flowing in it, if it is placed in a magnetic field of 0.25 Tesla, will be –



Sol. F = Bi$l$sin θ

= 0.25 × 5 × 0.25 sin 65°

= 0.3125 sin 65°

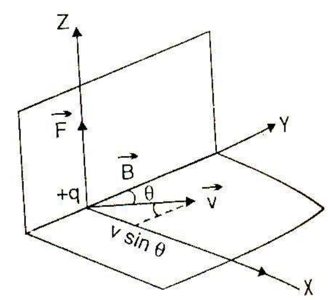
MOTION OF A CHARGED PARTICLE IN A MAGNETIC FIELD

(i) If a particle of charge q moves in a magnetic field $\vec{B}$ with a velocity $\vec{v}$, then the force acting on the particle will be

$\vec{F} = q(\vec{v} \times \vec{B})$

or $F = qvB \sin\theta$

where θ is the angle between $\vec{B}$ and $\vec{v}$.



(ii) If $\theta = 0$, i.e., the charged particle moves in the direction of magnetic field, then

$\sin\theta = 0$

$\therefore F = qvB \sin 0 = 0$

Thus no force will act on the particle. In this case the particle will remain in motion without any deflection and the energy and momentum of the particle will remain constant.

(iii) If $\theta = 90°$, i.e. the charged particle moves in a direction normal to the magnetic field, then

(a) F = qvB sin 90 = qvB, this force will be maximum.

(b) Force will act in a direction perpendicular to the motion of particle.

(c) Particle will move in a circular path.

(d) The kinetic energy of the particle will remain constant.

(e) The magnitude of momentum of the particle will remain constant but its direction will change continuously.

(f) If R is the radius of the circular path of the particle, then

$$\frac{mv^2}{R} = qvB$$

$$\therefore \qquad R = \frac{mv}{qB} = \frac{p}{qB}$$

where p is the momentum of the particle.

(g) Kinetic energy of the particle –

$$E = \frac{1}{2}mv^2 = \frac{R^2q^2B^2}{2m}$$

(h) If E is the energy of the particle, then the radius of the circular path will be

$$R = \frac{\sqrt{2mE}}{qB}$$

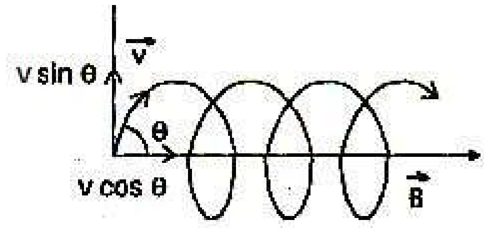
(i) The work done by the charged particle in completing a full circle will be zero.

(j) Period of the particle $T = \frac{2\pi m}{qB}$ and frequency

$$n = \frac{1}{T} = \frac{qB}{2\pi m}$$

Thus the period or the frequency does not depend upon the speed of the particle. As the speed of the particle increases, the radius of circular path increases in the same ratio so that the time taken in completing a full circle remains same. This frequency for electron is called cyclotron frequency.

(iv) When the velocity $\vec{v}$ of moving charge ($+q_0$) is not perpendicular to the magnetic field, i.e., $0 < \theta < 90°$, then the velocity component $v\cos\theta$ parallel to the magnetic field remains uneffected, as a result particle moves in the forward direction and due to velocity moves in the forward direction and due to velocity component $v\sin\theta$ normal to the magnetic field the particle moves in a circular path. Thus the particle has both types of motion simultaneously so the resultant motion is in the form of helix.



(v) Moving charged particle produces both electric and magnetic fields.

(vi) If a proton and α-particle having same kinetic energy move perpendicular to the magnetic field $\vec{B}$, then the ratio of the radii of their paths will be

$$R_p = \frac{\sqrt{2m_pE}}{q_pB} \quad \text{and} \quad R_\alpha = \frac{\sqrt{2m_\alpha E}}{q_\alpha B}$$

but $\qquad m_\alpha = 4m_p$ and $q_\alpha = 2q_p$

$$\therefore \qquad \frac{R_p}{R_\alpha} = \frac{\sqrt{2m_pE}}{q_pB} \times \frac{q_\alpha B}{\sqrt{2m_\alpha E}}$$

$$= \frac{\sqrt{2m_pE}}{q_pB} \times \frac{2q_pB}{\sqrt{2\times 4m_pE}} = \frac{2\sqrt{2}}{2\sqrt{2}} = \frac{1}{1}$$

$$\therefore \qquad R_p : R_\alpha :: 1 : 1$$

Solved Examples

Ex.27 A point charge of q coulomb is moving along a circular path of radius r meter at the rate of n rotations per second. Magnitude of magnetic induction at the centre of circle (in tesla) will be :

Sol. From any point on the circle charge flowing or circular current per second will be i = nq.

Magnetic induction produce at the centre due to circular current

$$B = \frac{\mu_0 i}{2r}$$

or $\qquad B = \frac{4\pi\times 10^{-7}}{2r}(nq)$

$$= \frac{2\pi nq}{r}\times 10^{-7}\,T$$

Ex.28 An electron is projected with a velocity of 10^5 m/s at right angles to a magnetic field of 0.019 G. Calculate the radius of the circular path described by the electron, if e = 1.6×10^{-19}C, m = 9.1×10^{-31} kg.

Sol. $\because$ v = 10^5 m/s; e = 1.6×10^{-19} C; m = 9.1×10^{-31} kg; B = 0.019 G = 0.019×10^{-4} T

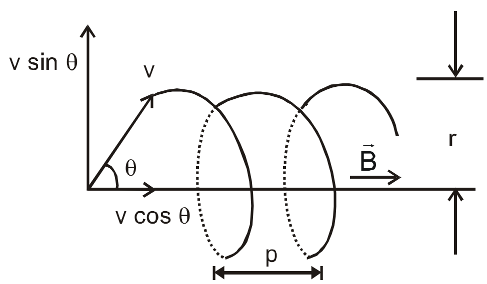
$$r = \frac{mv}{Be} = \frac{9.1\times10^{-31}\times10^5}{0.019\times10^{-4}\times1.6\times10^{-19}} = 0.299\,\text{m}$$

Ex.29 Electron moving at right angles to the uniform magnetic field completes a circular orbit in 10^{-6} s. Calculate the value of magnetic field.

Sol. Period of revolution $T = \frac{2\pi m}{Be}$ [$\because$ T = 10^{-6} s]

or $B = \frac{2\pi m}{eT} = \frac{2\pi\times9\times10^{-31}}{1.6\times10^{-19}\times10^{-6}} = 3.534\times10^{-5}\,\text{Tesla}$

Ex.30 A beam of protons with velocity 4×10^5 m/s enters a uniform magnetic field of 0.3 tesla at an angle of 60° to the magnetic field. Find the radius of the helical path taken by the proton beam. Also find the pitch of helix. Mass of proton = 1.67×10^{-27} kg.



Sol. $r = \frac{mv\sin\theta}{qB}$ ($\therefore$ component of velocity $\perp$ to field is v sin θ)

$$= \frac{(1.67\times10^{-27})(4\times10^5)\sqrt{3}/2}{(1.6\times10^{-19})0.3} = \frac{2}{\sqrt{3}}\times10^{-2}\text{m} = 1.2\text{cm}$$

Again, pitch $p = v\cos\theta\times T$

Where $T = \frac{2\pi r}{v\sin\theta}$

$$\therefore\ p = \frac{v\cos\theta\times2\pi r}{v\sin\theta} = \frac{\cos60^\circ\times2\pi\times(1.2\times10^{-2})}{\sin60^\circ}$$

= 4.35×10^{-2} m = 4.35 cm

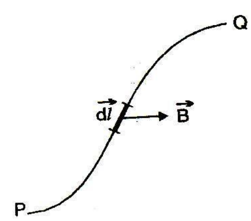
Ex.31 A test charge of 1.6×10^{-19} C is moving with velocity $\vec{v} = (2\hat{i} + 3\hat{j})$ m/s in a magnetic field $\vec{B} = (2\hat{i} + 3\hat{j})$ Wb/m^2. Find the force acting on the test charge.

Sol. e = 1.6×10^{-19} C; $\vec{v} = (2\hat{i} + 3\hat{j})$ m/s; $\vec{B} = (2\hat{i} + 3\hat{j})$ Wb m^{-2}

$\vec{F} = e(\vec{v}\times\vec{B})$ = 1.6×10^{-19} $[(2\hat{i}+3\hat{j})\times(2\hat{i}+3\hat{j})] = 0$

AMPERES LAW

(i) If an imaginary path is considered in a magnetic field, then the integral of the scalar product of the magnetic field $\vec{B}$ at the position of an element with the displacement $\vec{\delta\ell}$ along the path is called line integral. Thus the line integral of the magnetic field for the path PQ $= \int_P^Q \vec{B}.\vec{d\ell}$



(ii) If the path is closed, then the line integral along the claose path is called circulation.

$\therefore$ Circulation = line integral along the closed path

$$= \oint \vec{B}.\vec{d\ell}$$

(iii) The ratio of the magnetic induction due to current or moving charge to the permebaility μ of the medium, i.e. $\vec{B}/\mu$ is called magnetizing field H. The magnetizing field $\vec{H}$ depends upon the current flowing in the conductor and its geometry.

(iv) The circulation or the the integral of the magnetizing field along the closed path is called magnetomotive force.

Megnetomotive force $F_m = \oint \vec{H}.\vec{d\ell} = \frac{1}{\mu}\oint \vec{B}.\vec{d\ell} +$

Unit of magnetomotive force is ampere. This quantity is analogous to electromotive force which is equivalent to the circulation $\oint \vec{E}.\vec{d\ell}$, where E is the intensity of electric field. Unit of electromotive force is volt.

(v) According to Ampere's law, the integral of magnetic field along a closed path, i.e., circulation is equal to the product of μ_0 and the algebraic sum of currents crossing the area bound by the closed path, where m_0 is the permeability of the medium.

$$\text{Circulation} = \oint_C \vec{B}.\vec{d\ell} = \mu_0 \Sigma i$$

(vi) If no current passes through the area bound by the closed path, then the circulation is zero, i.e.

$$\oint_C \vec{B}.\vec{d\ell} = 0$$

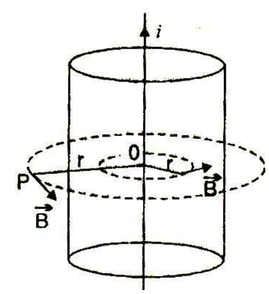
(vii) From Ampere's law the magnetic field $\vec{B}$ can be determined but it can be used only in those circumstance when the distribution of current is uniform and symmetrical, that is, current should be completely defined and amperian path should be symmetrical.

(viii) If magnetic field B is defined in terms of magnetising field H, i.e., $H = B/\mu_0$, then Ampere's law can be written in the following form $\oint \vec{H}.\vec{d\ell} = \Sigma i$

(ix) Magnetomotive force on the closed path is equivalent to the algebric sum of the electric currents crossing the area bound by the closed path.

(a) Magnetic Field due to a Long Cylindreical Current Carrying Conductor

(i) Current i is flowing symmetrically through a straigh long solid cylindrical conductor. Magnetic field is produced due to current flowing in the conductor around the conductor whose lines of force are concentric circles outside and inside the conductor.



(ii) Magnetic filed outside the conductor (r > R)

$$B_{ext} = \frac{\mu_0 i}{2\pi r}$$

Thus the magnetic field due to current carrying cylinder outside it is equivalent to the magnetic field due to thin current carrying wire.

(iii) Magnetic field at the surface of the cylinder (r = R)

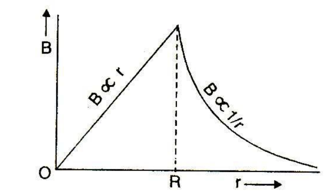
$$B_{sureface} = \frac{\mu_0 i}{2\pi R}$$

(iv) Magnetic field inside the conductor (r < R)

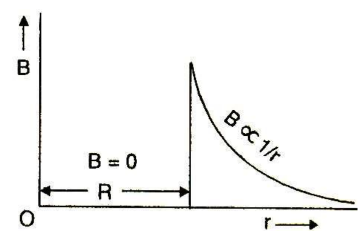
$$B_{in} = \frac{\mu_0 i}{2\pi R^2} r$$

Thus the magnetic field inside the cylindrical conductor is proportional to the distance r from the axis.

(v) In the figure the dependence of magnetic field due to cylindrical conductor with the distance r from the axis is shown.



(vi) The magnetic field inside a current current pipe (hollow cylinder) : If cylindrical condcutor is hollow, i.e., it is in the form of pipe then the current flows through the surface only. In this case the current crossing an area πr^2 of a circular path inside the hollow part is zero. Thus from Ampere's law



$$\oint \vec{B}.\vec{d\ell} = B(2\pi r) = 0$$

$$B = 0$$

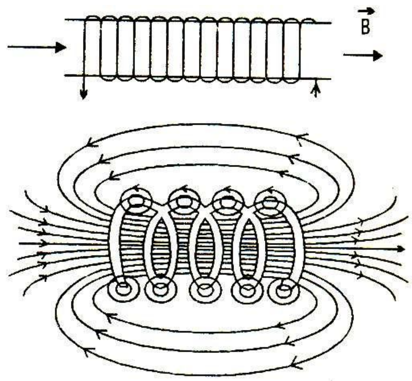
That is, the magnetic field inside the current carrying pipe is zero. For this case the graph between B and the distance r is given below.

(b) Megnetic Field Inside a Solenoid

(i) Solenoid is a long cylindrical tube on which electically insulated wire is wound symmetrically along its length.

(ii) The diameter of a solenoid is small in comparsion of its length and the plane of each turn of wire of an ideal solenoid can be considered to be normal to its axis.

(iii) When current is passed through the soleniod, the magnetic behaviour at the points near each turn of the wire is similar to the straight current carrrying wire and magnetic lines of force are in the form of concentric circles, as shown in figure.



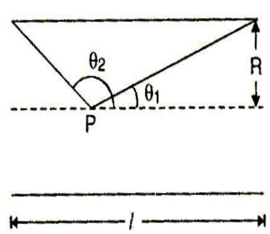
(iv) The magnetic field $\vec{B}$ at any point of the solenoid is equal to the vector sum of the magnetic fields produced by the different turns.

(v) The magnetic field inside a solenoid is almost uniform and along the axis and the magnetic field outside the solenoid at large distance is negligible in comparison to the magnetic field inside the solenoid thus the resultant magnetic field outside the solenoid at large distances can be considered to be zero.

(vi) **A solenoid of finite length :** The magnetic field at a point P inside a current carrying solenoid of finite length

$$B_0 = \frac{\mu_0 ni}{2}(\cos\theta_1 - \cos\theta_2)$$

If L is the length of the solenoid and R its radius, then the magnetic field at its centre



$$B_{oc} = \frac{\mu_0 niL}{\sqrt{4R^2 + L^2}}$$

where n is number of turns per unit length of solenoid.

(vii) **A Solenoid of infinite length :**

Magnetic field inside a solenoid

$B_0 = \mu_0 ni$

where n is number of turns per unit length of solenoid.

If N is number of turns in a solenoid of length L, then

$$n = \frac{N}{L}$$

$\therefore \quad B_0 = \mu_0 (N/L)\ i$

(viii) If a magnetic material of permeability μ is placed inside the solenoid, then

$$B = \mu ni$$

or $$B = \mu(N/L)\ i$$

(ix) The magnetic field at one end of the solenoid is half of the magnetic field at its centre i.e.,

$$B_{end} = \frac{B_0}{2} = \frac{\mu_0 ni}{2} = \frac{\mu_0 Ni}{2L}$$

(x) The magnetising field inside the solenoid is

$$H = ni = \frac{B_0}{\mu_0} \text{ ampere turn / metre}$$

$$H = \left(\frac{N}{L}\right) i$$

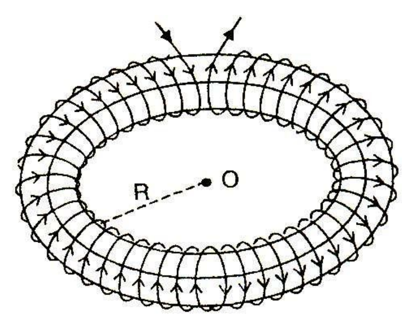
and at its end $H_{end} = \frac{ni}{2} = \frac{Ni}{2L}$

(xii) B is uniform over the cross-sectional area of the solenoid and it does not depend upon the length or diameter of the solenoid.

(c) Magnetic Field in a Toroid

(i) Toroid is like an endless cylindrical solenoid, i.e. if a long solenoid is bent round in the form of a closed ring, then it becomes a toroid.

(ii) Electrically insulated wire is wound uniformly over the toroid as shown in the firgure.



(iii) The thickness of toroid is kept small in comparison to its radius and the number of turns is kept very large.

(iv) When a current i is passed through the toroid, each turn of the toroid produces a magnetic field along the axis at its centre. Due to uniform distribution of turns this magnetic filed has same magnitude at their centres. Thus the magnetic lines of force inside the toroid are circular.

(v) The magnetic field inside a toroid at all points is same but outside the toroid it is zero.

(vi) If total number of turns in a toroid is N and R is its radius, then number of turns per unit length of the toroid will be

$$n = \frac{N}{2\pi R}$$

(vii) The magnetic field due to toroid is determined by Ampere's law.

(viii) The magnetic field due to toroid is

$$B_0 = \mu_0 ni$$

or $$B_0 = \mu_0\left(\frac{N}{2\pi R}\right)i$$

(ix) If a substance of permeability μ is placed inside the toroid, then

$$B = \mu ni$$

If μ_r is relative magnetic permeability of the substance, then

$$B = \mu_r\mu_0\ ni$$

MAGNETIC BEHAVIOUR OF A CURRENT CARRYING COIL AND ITS MAGNETIC MOMENT

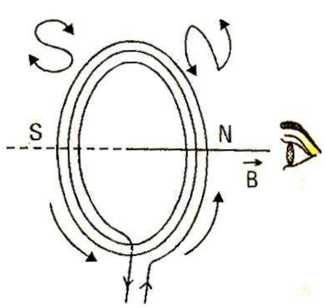
(i) If current is passed in a small coil (loop), then the magnetic field due to it at a point on its axis is equivalent to the magnetic field due to a small magnet or magnetic dipole at a point on its axis.

(ii) A current carrying small coil behaves like a small magnet.

(iii) If the current in the coil is observed anticlockwise, then that plane of the coil behaves like the north pole.

(iv) If the current in the coil is observed clockwise, then that plane of the coil behaves like the south pole.

(v) Current carrying coil is equivalent to a magnetic strip.



(vi) Magnetic moment of a current carrying coil is equal to the product of the current flowing in the coil and its effective area i.e.,

$$M = IA$$

(vii) For a coil of one turn

$$M = IA = I\pi R^2$$

where R is the radius of a coil and I is the current flowing in the coil.

For a coil of N turns

$$M = NAI = N\pi R^2 I$$

(viii) Unit of magnetic moment

$$\text{ampere} - m^2$$

(ix) The magnetic field B_0 at the centre of a current carrying circular coil and its magnetic moment M have the following relation. $B_0 = \frac{\mu_0 M}{2\pi R^3}$

(x) If the magnetic moment of a current carrying small coil is M, then the magnetic field at a distance x(x >> R) on its axis from the centre will be

$$B_P = \frac{\mu_0}{4\pi}\frac{2M}{x^3} = \frac{\mu_0 M}{2\pi x^3}$$

CURRENT AND MAGNETIC FIELD DUE TO CIRCULAR MOTION OF A CHARGE

(i) According to the theory of atomic structure every atom is made of electrons, protons and neutrons. protons and neutrons are in the nucleus of each atom and electrons are assumed to be moving in different orbits around the nucleus.

(ii) An electron and a proton present in the atom constitute an electric dipole at every moment but the direction of this dipole changes continuously and hence at any time the average dipole moment is zero. As a result static electric field is not observed.

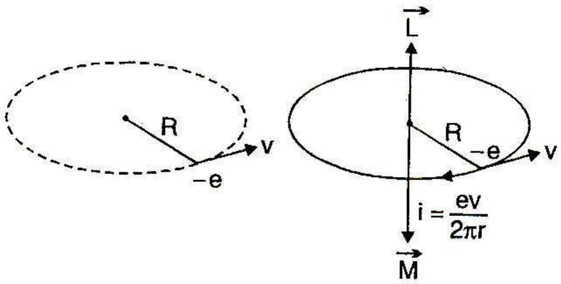
(iii) Moving charge produces magnetic field and the average value of this field in the atom is not zero.

(iv) In an atom an electron moving in a circular path around the nucleus. Due to this motion current appears to be flowing in the electronic orbit and the orbit behaves like a current carrying coil. If e is the electron charge, R is the radius of the orbit and f is the frequency of motion of electron in the orbit, then

(a) current in the orbit = charge × frequency = ef

If T is the period, then $f = \frac{1}{T}$

$\therefore \quad i = \frac{e}{T}$



(b) Magnetic field at the nucleus (centre)

$$B_0 = \frac{\mu_0 i}{2R} = \frac{\mu_0 ef}{2R} = \frac{\mu_0 e}{2RT}$$

(c) If the angular velocity of the electron is ω, then

$$\omega = 2\pi f \text{ and } f = \frac{\omega}{2\pi}$$

$$\therefore \quad i = ef = \frac{e\omega}{2\pi}$$

$$\therefore \quad B_0 = \frac{\mu_0 i}{2R} = \frac{\mu_0 e\omega}{4\pi R}$$

(d) If the linear velocity of the electron is v, then

$$v = R\omega = R(2\pi f)$$

or $$f = \left(\frac{v}{2\pi R}\right)$$

$$\therefore \quad i = ef = \frac{ev}{2\pi R}$$

$$\therefore \quad B_0 = \frac{\mu_0 i}{2R} = \frac{\mu_0 ev}{4\pi R^2}$$

(v) Magnetic moment due to motion of electron in an orbit

$$M = iA = ef\pi R^2 = \frac{e\pi R^2}{T}$$

or $$M = \frac{e\omega\pi R^2}{2\pi} = \frac{e\omega R^2}{2}$$

or $$M = \frac{ev\pi R^2}{2\pi R} = \frac{evR}{2}$$

If the angular momentum of the electron is L, then

$$L = mvR = m\omega R^2$$

Writing M in terms of L

$$M = \frac{em\omega R^2}{2m} = \frac{emvR}{2m} = \frac{eL}{2m}$$

According to Bohr's second postulate

$$mvR = n\frac{h}{2\pi}$$

In ground state n = 1

$$L = \frac{h}{2\pi}$$

$$\therefore \quad M = \frac{eh}{4\pi m}$$

(vi) If a charge q (or a charged ring of charge q) is moving in a circular path of radius R with a frequency f or angular velocity ω, then

(a) current due to moving charge

$$i = qf = q\omega/2\pi$$

(b) magnetic field at the centre of ring

$$B_0 = \frac{\mu_0 i}{2R} = \frac{\mu_0 qf}{2R}$$

or $$B_0 = \frac{\mu_0 q\omega}{4\pi R}$$

(c) magnetic moment

$$M = i(\pi R^2)$$

$$= qf\pi R^2 = \frac{1}{2}q\omega R^2$$

(vii) If a charge q is distributed uniformly over the surface of plastic disc of radius R and it is rotated about its axis with an angular velocity ω, then

(a) the magnetic field produced at its centre will be

$$B_0 = \frac{\mu_0 q\omega}{2\pi R}$$

(b) the magnetic moment of the disc will be

$$M = \frac{q\omega R^2}{4}$$