WORK, POWER & ENERGY

1. WORK :

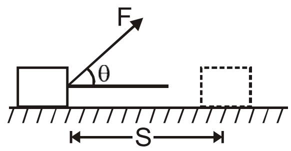
Work is said to be done by a force when the force produces a displacement in the body on which it acts in any direction except perpendicular to the direction of the force.

1.1 Work done by constant force

Consider an object undergoes a displacement S along a straight line while acted on a force F that makes an angle θ with S as shown.

The work done W by the agent is the product of the component of force in the direction of displacement and the magnitude of displacement.

i.e., $W = FS\cos\theta$ ...(1)



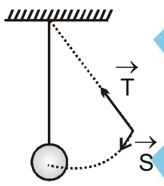
Work done is a scalar quantity and its S.I. unit is N-m or joule (J). We can also write work done as a scalar product of force and displacement.

$W = \vec{F}.\vec{S}$ ...(2)

where S is the displacement of the point of application of the force

From this definition, we conclude the following points

(A) work done by a force is zero if displacement is perpendicular to the force ($\theta = 90°$)



Example.

The tension in the string of a simple pendulum is always perpendicular to displacement. (Figure)

So, work done by the tension is zero.

(B) if the angle between force and displacement is acute ($\theta < 90°$), we say that the work done by the force is positive.

Example :

When a load is lifted, the lifting force and the displacement act in the same direction. So, work done by the lifting force is positive.

Example :

When a spring is stretched, both the stretching force and the displacement act in the same direction. So work done by the stetching force is positive.

(C) If the angle between force and displacement is obtuse ($\theta > 90°$), we say that the work done by the force is negative.

Example :

When a body is lifted, the work done by the gravitational force is negative. This is because the gravitational force acts vertically downwards while the displacement is in the vertically upwards direction.

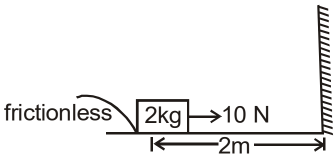
Important points about work :

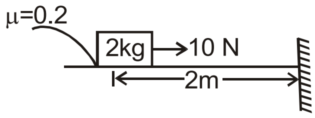
1. Work is said to be done by a force when its point of application moves by some distance.Force does no work if point of application of force does not move (S = 0)

Example :

A person carrying a load on his head and standing at a given place does no work.

2. Work is defined for an interval or displacement. There is no term like instantaneous work similar to instantaneous velocity.





Work done by 10 N force in both the cases are same = 20 N

3. For a particular displacement, work done by a force is independent of type of motion i.e. whether it moves with constant velocity, constant acceleration or retardation etc.
4. If a body is in dynamic equilibrium under the action of certain forces, then total work done on the body is zero but work done by individual forces may not be zero.
5. When several forces act, work done by a force for a particular displacement is independent of other forces.
6. A force is independent of reference frame. Its displacement depends on frame so work done by a force is frame dependent therefore work done by a force can be different in different reference frame.

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2. UNITS OF WORK :

In cgs system, the unit of work is erg.

One erg of work is said to be done when a force of one dyne displaces a body through one centimetre in its own direction.

$\therefore$ 1 erg = 1 dyne × 1 cm = 1 g cm s^{-2} × 1 cm = 1 g cm^2 s^{-2}

Note : Another name for joule is newton metre.

Relation between joule and erg

1 joule = 1 newton × 1 metre

1 joule = 10^5 dyne × 10^2 cm = 10^7 dyne cm

1 joule = 10^7 erg

1 erg = 10^{-7} joule

Dimensions of Work :

[Work] = [Force] [Distance] = $[MLT^{-2}]$ [L] = $[ML^2T^{-2}]$

Work has one dimension in mass, two dimensions in length and '–2' dimensions in time, On the basis of dimensional formula, the unit of work is kg m^2 s^{-2}.

Note that 1 kg m^2 s^{-2} = (1 kg m s^{-2}) m = 1 N m = 1 J.

3. WORK DONE BY MULTIPLE FORCES :

If several forces act on a particle, then we can replace $\vec{F}$ in equation W = $\vec{F} \cdot \vec{S}$ by the net force $\sum \vec{F}$ where

$$\sum \vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ldots\ldots$$

$$\therefore \quad W = \left[\sum \vec{F}\right] \cdot \vec{S} \qquad \ldots(i)$$

This gives the work done by the net force during a displacement $\vec{S}$ of the particle.

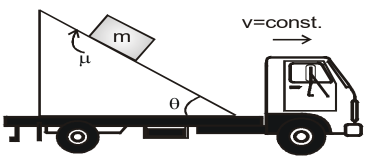
We can rewrite equation (i) as :

$$W = \vec{F}_1 . \vec{S} + \vec{F}_2 . \vec{S} + \vec{F}_3 . \vec{S} + \ldots\ldots$$

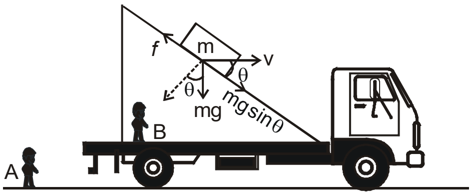
or $W = W_1 + W_2 + W_3 + \ldots\ldots\ldots$

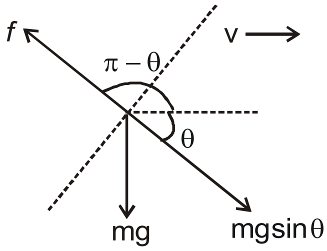
So, the work done on the particle is the sum of the individual work done by all the forces acting on the particle.

Ex.1 A block of mass m is placed on an inclined plane which is moving with constant velocity v in horizontal direction as shown in figure. Then find out work done by the friction in time t if the block is at rest with respect to the incline plane.



Sol. F.B.D of block with respect to ground.





Block is at rest with respect to wedge

$\Rightarrow$ f = mg sin θ

In time t the displacement of block with respect to ground d = vt

Work done by friction for man A

W_f = (component of friction force along displacement) × displacement

W_f = mgsinθ.vt cos(180°–θ)

W_f = – mg vt cosθ sin θ

W_f for man B = 0 (displacement is zero with respect to man B)

4. WORK DONE BY A VARIABLE FORCE :

When F as a function of x, y, z

When the magnitude and direction of a force vary in three dimensions, it can be expressed as a function of the position. For a variable force work is calculated for infinitely small displacement and for this displacement force is assumed to be constant.

$$dW = \vec{F}.d\vec{s}$$

The total work done will be sum of infinitely small work

$$W_{A\to B} = \int_A^B \vec{F}.d\vec{s} = \int_A^B (\vec{F}\cos\theta)d\vec{s}$$

It terms of rectangular components,

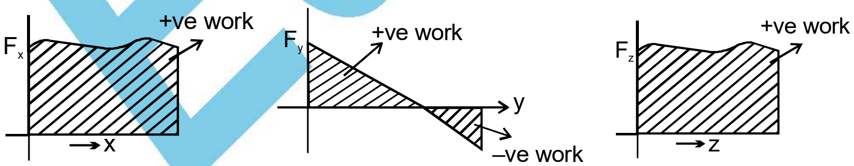
$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$$

$$d\vec{s} = dx\,\hat{i} + dy\,\hat{j} + dz\hat{k}$$

$$W_{A\to B} = \int_{x_A}^{x_B} F_x dx + \int_{y_A}^{y_B} F_y dy + \int_{z_A}^{z_B} F_z dz$$

5. AREA UNDER FORCE DISPLACEMENT CURVE :

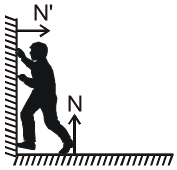
Graphically area under the force-displacement is the work done



The work done can be positive or negative as per the area above the x-axis or below the x-axis respectively.

6. INTERNAL WORK :

Suppose that a man sets himself in motion backward by pushing against a wall. The forces acting on the man are his weight 'W' the upward force N exerted by the ground and the horizontal force N′ exerted by the wall. The works of 'W' and of N are zero because they are perpendicular to the motion. The force N' is the unbalanced horizontal force that imparts to the system a horizontal acceleration. The work of N', however, is zero because there is no motion of its point of application. We are therefore confronted with a curious situation in which a force is responsible for acceleration, but its work, being zero, is not equal to the increase in kinetic energy of the system.



The new feature in this situation is that the man is a composite system with several parts that can move in relation to each other and thus can do work on each other, even in the absence of any interaction with externally applied forces. Such work is called internal work. Although internal forces play no role in acceleration of the composite system, their points of application can move so that work is done; thus the man's kinetic energy can change even though the external forces do no work.

"Basic concept of work lies in following lines

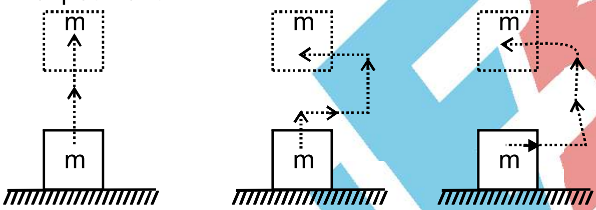
Draw the force at proper point where it acts that give proper importance of the point of application of force.

Think independently for displacement of point of application of force, Instead of relation the displacement of applicant point with force relate it with the observer or reference frame in which work is calculated.

$$W = (\text{Force vector}) \times \begin{pmatrix} \text{displacement vector of point of} \\ \text{application of force as seen by} \\ \text{observer} \end{pmatrix}$$

7. CONSERVATIVE FORCE :

A force is said to be conservative if work done by or against the force in moving a body depends only on the initial and final positions of the body and does not depend on the nature of path followed between the initial and final positions.



(a) (b) (c)

Consider a body of mass m being raised to a height h vertically upwards as shown in above figure. The work done is mgh. Suppose we take the body along the path as in (b). The work done during horizontal motion is zero. Adding up the works done in the two vertical parts of the paths, we get the result mgh once again. Any arbitrary path like the one shown in (c) can be broken into elementary horizontal and vertical portions. Work done along the horizontal path is zero. The work done along the vertical parts add up to mgh. Thus we conclude that the work done in raising a body against gravity is independent of the path taken. It only depends upon the intial and final positions of the body. We conclude from this discussion that the force of gravity is a conservative force.

Examples of Conservative forces.

(i) Gravitational force, not only due to Earth due in its general form as given by the universal law of gravitation, is a conservative force.
(ii) Elastic force in a stretched or compressed spring is a conservative force.
(iii) Electrostatic force between two electric charges is a conservative force.
(iv) Magnetic force between two magnetic poles is a conservative force.

Forces acting along the line joining the centres of two bodies are called central forces. Gravitational force and Electrosatic forces are two important examples of central forces. Central forces are conservative forces.

Properties of Conservative forces

- **Work done by or against a conservative force depends only on the initial and final position of the body.**
- Work done by or against a conservative force does not depend upon the nature of the path between initial and final position of the body.
- Work done by or against a conservative force in a round trip is zero.

If a body moves under the action of a force that does no total work during any round trip, then the

force is conservative; otherwise it is non-conservative.
The concept of potential energy exists only in the case of conservative forces.

- The work done by a conservative force is completely recoverable.

Complete recoverability is an important aspect of the work done by a conservative force.

Work done by conservative forces

I^st format : (When constant force is given)

8. NON-CONSERVATIVE FORCES :

A force is said to be non-conservative if work done by or against the force in moving a body depends upon the path between the initial and final positions.
The frictional forces are non-conservative forces. This is because the work done against friction depends on the length of the path along which a body is moved. It does not depend only on the initial and final positions. Note that the work done by fricitional force in a round trip is not zero.
The velocity-dependent forces such as air resistance, viscous force, magnetic force etc., are non conservative forces.

9. CONSERVATIVE FORCE AND POTENTIAL ENERGY :

$$F_s = -\frac{\partial U}{\partial s}$$

i.e. the projection of the force field , the vector F, at a given point in the direction of the displacement r equals the derivative of the potential energy U with respect to a given direction, taken with the opposite sign. The designation of a partial derivative $\partial/\partial s$ emphasizes the fact of deriving with respect to a definite direction.
So, having reversed the sign of the partial derivatives of the function U with respect to x, y, z, we obtain the projection F_x, F_y and F_z of the vector F on the unit vectors i, j and k. Hence, one can readily find the vector itself :

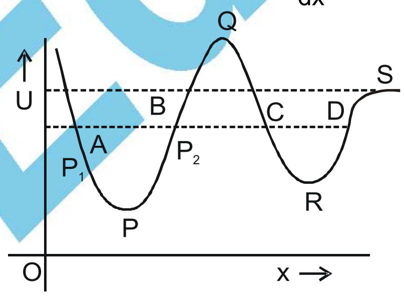
$$F = F_x i + F_y j + F_z k, \text{ or } F = -\left(\frac{\partial U}{\partial x} i + \frac{\partial U}{\partial y} j + \frac{\partial U}{\partial z} k\right)$$

The quantity in parentheses is referred to as the scalar gradient of the function U and is denoted by grad U or ∇U. We shall use the second, more convenient, designation where ∇ ("nabla") signifies the symbolic vector or operator

$$\nabla = i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}$$

Potential Energy curve :

- A graph plotted between the PE of a particle and its displacement from the centre of force field is called PE curve.
- Using graph, we can predict the rate of motion of a particle at various positions.
- Force on the particle is $F_{(x)} = -\frac{dU}{dx}$



Case : I On increasing x, if U increases, force is in (–) ve x direction i.e. attraction force.
Case : II On increasing x, if U decreases, force is in (+) ve x-direction i.e. repulsion force.

Different positions of a particle :

Position of equilibrium

If net force acting on a body is zero, it is said to be in equilibrium. For equilibrium $\frac{dU}{dx} = 0$. Points P, Q, R and S are the states of equilbrium positions.

Types of equilirbium :

- **Stable equilibrium :**

When a particle is displaced slightly from a position and a force acting on it brings it back to the initial position, it is said to be in stable equilibrium position.

Necessary conditions:– $\frac{dU}{dx} = 0, \text{and} \frac{d^2U}{dx^2} = +\text{ ve}$

In figure P and R point shows stable equilibrium point.

- **Unstable Equilibrium :**

When a particle is displaced slightly from a position and force acting on it tries to displace the particle further away from the equilibrium position, it is said to be in unstable equilibrium.

Condition : $\frac{dU}{dx} = 0$ potential energy is maximum i.e. $= \frac{d^2U}{dx^2} = -\text{ ve}$

Q point in figure shows unstable equilibrium point

- **Neutral equilibrium** :

In the neutral equilibrium potential energy is constant. When a particle is displaced from its position it does not experience any force acting on it and continues to be in equilibrium in the displaced position. This is said to be neutral equilibrium.

In figure S is the neutral point

Condition : $\frac{dU}{dx} = 0$, $\frac{d^2U}{dx^2} = 0$

10. WORK ENERGY THEOREM :

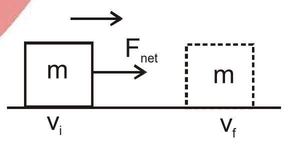
If the resultant or net force acting on a body is F_{net} then Newton's second law states that

$F_{net} = ma$ ...(1)

If the resultant force varies with x, the acceleration and speed also depend on x.

then $a = v\frac{dv}{dx}$ ...(2)

from eq. (1)



$F_{net} = mv\frac{dv}{dx} \Rightarrow F_{net}.dx = m\,v\,dv$

$\int F_{net}.dx = \int_{v_i}^{v_f} mvdv$

$W_{net} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

$W_{net} = k_f - k_i$

$W_{net} = \Delta K$ ...(3)

Work done by net force F_{net} in displacing a particle equals to the change in kinetic energy of the particle i.e.

we can write eq. (3) in following way

$(W.D)_c + (W.D)_{N.C} + (W.D)_{ext.} + (W.D)_{pseudo}$

$= \Delta K$ ...(4)

where $(W.D)_c$ = work done by conservative force

$(W.D)_{N.C}$ = work done by non conservative force.

$(W.D)_{ext}$ = work done by external force

$(W.D)_{pseudo}$ = work done by pseudo force.

we know that

$(W.D)_c = -\Delta U$

$\Rightarrow -\Delta U + (W.D)_{N.C} + (W.D)_{ext} + (W.D)_{pseudo} = \Delta K$

$\Rightarrow (W.D)_{N.C} + (W.D)_{ext.} + (W.D)_{pseudo} = (k_f + u_f) - (k_i + u_i)$

$\because$ k + u = Mechanical energy.

$\Rightarrow$ work done by forces (except conservative forces)

= change is mechanical energy.

If $(W.D)_{N.C} = (W.D)_{ext} = (W.D)_{pseudo} = 0$

$K_f + U_f = K_i + U_i$

Initial mechanical energy = final mechanical energy

This is called mechanical energy conservation law.

Questions Based on work Energy Theorem :

(A) When only one conservative force is acting

11. POWER

Power is defined as the time rate of doing work.

When the time taken to complete a given amount of work is important, we measure the power of the agent doing work.

The average power $(\bar{P} \text{ or } P_{av})$ delivered by an agent is given by

$$\bar{P} \text{ or } P_{av} = \frac{\Delta W}{\Delta t} = \frac{\text{Total work done}}{\text{Total time}}$$

where ΔW is the amount of work done in time Δt.

Power is the ratio of two scalars-work and time. So, power is a scalar quantity. If time taken to complete a given amount of work is more, then power is less.

- The instantaneous power is, $P = \frac{dW}{dt}$ where dW is the work done by a force $\vec{F}$ in a small time dt.

- $P = \frac{dW}{dt} = \vec{F}.\frac{d\vec{r}}{dt} = \vec{F}.\vec{v}$ where $\vec{v}$ is the velocity of the body.

By definition of dot product,

$P = Fv\cos\theta$

where θ is the smaller angle between $\vec{F}$ and $\vec{v}$

This P is called as instantaneous power if dt is very small.

11.1 Unit of Power :

A unit power is the power of an agent which does unit work in unit time.

The power of an agent is said to be one watt if it does one joule of work in one second.

1 watt = 1 joule/second = 10^7 erg/second

Also, $\quad 1 \text{ watt} = \frac{1 \text{ newton} \times 1 \text{ metre}}{1 \text{ second}} = 1 \text{ N ms}^{-1}$.

Dimensional formula of power

$$[\text{Power}] = \frac{[\text{Work}]}{[\text{Time}]} = \frac{[ML^2T^{-2}]}{[T]} = [ML^2T^{-3}]$$

SOLVED EXAMPLE

Ex.1 Find the angle between force $\mathbf{F}$ = $(3\hat{i}+4\hat{j}+5\hat{k})$ unit and displacement $\mathbf{d}$ = $(5\hat{i}+4\hat{j}+3\hat{k})$ unit. Also find the projection of $\mathbf{F}$ on $\mathbf{d}$.

Ans. $\mathbf{F.d} = F_x d_x + F_y d_y + F_z d_z$

$= 3\ (5) + 4\ (4) + (-3)\ (3)$

$= 16$ unit

Hence $\mathbf{F.d} = F\ d \cos\theta = 16$ unit

Now $\mathbf{F.F} = F^2 = F^2_x + F^2_y + F^2_z$

$= 9 + 16 + 25 = 50$ unit

and $\mathbf{d.d} = d^2 = d_x^{\ 2} + d_y^{\ 2} + d_z^{\ 2}$

$= 25 + 16 + 9 = 50$ unit

$$\therefore \quad \cos\theta = \frac{16}{\sqrt{50}\sqrt{50}} = \frac{16}{50} = 0.32$$

$\theta = \cos^{-1}\ 0.32$

Ex.2 It is well known that a raindrop falls under the influence of the downward gravitational force and the opposing resistive force. The latter isknown to be proportional to the speed of the drop but is otherwise undetermined. Consider a drop of mass1.00 g falling from a height 1.00 km. It hits the ground with a speed of 50.0 m s-1. (a) What is the work done by the gravitational force ? What is the work done by the unknown resistive force?

Ans. (a) The change in kinetic energy of the drop is

$$\Delta K = \frac{1}{2} m\upsilon^2 - 0$$

$$= \frac{1}{2} \times 10^{-3} \times 50 \times 50 = 1.25 \text{ J}$$

where we have assumed that the drop is initially at rest.

Assuming that g is a constant with a value 10 m/s^2, the work done by the gravitational force is, W_g = mgh

$= 10^{-3} \times 10 \times 10^3 = 10.0$ J

(b) From the work-energy theorem $\Delta K = W_g + W_r$ where W_r is the work done by the resistive force on the raindrop.

Thus $W_r = \Delta K - W_g = 1.25 - 10 = -8.75$ J is negative.

Ex.3 A cyclist comes to a skidding stop in 10 m. During this process, the force on the cycle due to the road is 200 N and is directly opposed to the motion. (a) How much work does the road do on the cycle ? (b) How much work does the cycle do on the road ?

Ans. Work done on the cycle by the road is the work done by the stopping (frictional) force on the cycle due to the road. (a) The stopping force and the displacement make an angle of 180º (θ rad) with each other. Thus, work done by the road, $W_r = Fd\ \cos\theta$

$= 200\ .\ 10\ .\ \cos\theta = -2000$ J

It is this negative work that brings the cycle to a halt in accordance with WE theorem. (b) From Newton's Third Law an equal and opposite force acts on the road due to the cycle. Its magnitude is 200 N. However, the road undergoes no displacement. Thus, work done by cycle on the road is zero. . .

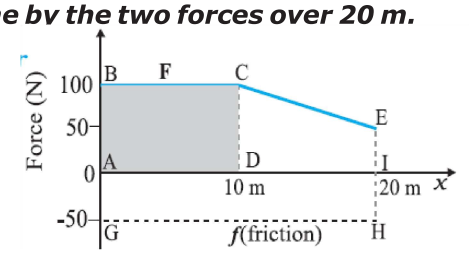
Ex.4 In a ballistics demonstration a police officer fires a bullet of mass 50.0 g with speed 200 m s^{-1} (see Table 6.2) on soft plywood of thickness 2.00 cm. The bullet emerges with only 10% of its initial kinetic energy. What is the emergent speed of the bullet ?

Ans. The initial kinetic energy of the bullet is $mv^2/2$ = 1000 J. It has a final kinetic energy of 0.1. 1000 = 100 J. If v_f is the emergent speed of the bullet,

$$\frac{1}{2} m\upsilon_f^2 = 100 \text{ J} \quad \upsilon_f = \sqrt{\frac{2 \times 100 \text{ J}}{0.05 \text{kg}}} = 63.2 \text{ ms}^{-1}$$

Ex.5 A woman pushes a trunk on a railway platform which has a rough surface. She applies a force of 100 N over a distance of 10 m. Thereafter, she gets progressively tired and her applied force reduces linearly with distance to 50 N. The total distance through which the trunk has been moved is 20 m. Plot the force applied by the woman and the frictional force, which is 50 N. Calculate the work done bv the two forces over 20 m.

Ans.



The plot of the applied force is shown in Fig. At x = 20 m, F = 50 N ($\neq 0$). We are given that the frictional force f is $|f| = 50$ N. It opposes motion and acts in a direction opposite to F. It is therefore, shown on the negative side of the force axis. The work done by the woman is $W_F \rightarrow$ area of the rectangle ABCD + area of the trapezium CEID

$$W_F = 100 \times 10 + \frac{1}{2}(100 + 50) \times 10$$

$= 1000 + 750 = 1750$ J

The work done by the frictional force is

$W_F \rightarrow$ area of the rectangle AGHI

$W_f = (-50) \times 20 = -1000$ J

The area on the negative side of the force axis has a negative sign.

Ex.6 A block of mass m = 1 kg, moving on a horizontal surface with speed v_i = 2 ms^{-1} enters a rough patch ranging from x = 0.10 m to x = 2.01 m. The retarding force F_r on the block in this range is inversely proportional to x over this range, $qF_1 = \frac{-k}{x}$ for 0.1 < x < 2.01 m = 0 for x < 0.1m and x > 2.01 m where k = 0.5 J. What is the final kinetic energy and speed v_f of the block as it crosses this patch ?

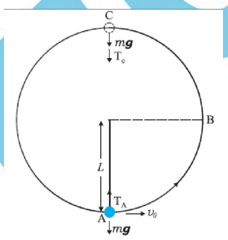
Ans. From Eq. $k_f - k_i = \int_{x_i}^{x_f} F\,dx$ $K_f = K_i + \int_{0.1}^{2.01} \frac{(-k)}{x} dx$

$$= \frac{1}{2} m\upsilon_t^2 - k\ln(x)|_{0.1}^{2.01} = \frac{1}{2} m\upsilon_t^2 - k\ln(2.01/0.1)$$

$= 2 - 0.5 \ln(20.1) = 2 - 1.5 = 0.5$ J

$\upsilon_f = \sqrt{2K_f/m} = 1$ ms^{-1}

Ex.7 A bob of mass m is suspended by a light string of length L . It is imparted a horizontal velocity v_o at the lowest point A such that it completes a semi-circular trajectory in the vertical plane with the string becoming slack only on reaching the topmost point, C. This is shown in Fig. 6.6. Obtain an expression for (i) v_o; (ii) the speeds at points B and C; (iii) the ratio of the kinetic energies (K_B/K_C) at B and C. Comment on the nature of the trajectory of the bob after it reaches the point C.



Ans. (i) There are two external forces on the bob : gravity and the tension (T) in the string. The latter does no work since the displacement of the bob is always normal to the string. The potential energy of the bob is thus associated with the gravitational force only. The total mechanical energy E of the system is conserved. We take the potential energy of the system to be zero at the lowest point A. Thus, at A :

$$E = \frac{1}{2} mv_0^2$$

$$T_A - mg = \frac{m\upsilon_0^2}{L} \quad \text{[Newton's Second Law]}$$

where T_A is the tension in the string at A. At the highest point C, the string slackens, as the tension in the string (T_C) becomes zero. Thus, at C

$$E = \frac{1}{2} m\upsilon_c^2 + 2mgL \qquad .....(1)$$

where v_C is the speed at C.

$$E = \frac{5}{2} mgL \qquad ....(2)$$

Equating this to the energy at A $\frac{5}{2} mgL = \frac{m}{2} \upsilon_0^2$

or $\upsilon_0 = \sqrt{5gL}$

(ii) It is clear from $\upsilon_c = \sqrt{gL}$

At B, the energy is

$$E = \frac{1}{2} m\upsilon_B^2 + mgL$$

Equating this to the energy at A and employing the result from (i), namely $v_0^2 = 5gL$

$$\frac{1}{2} m\upsilon_B^2 + mgL = \frac{1}{2} m\upsilon_0^2 = \frac{5}{2} m\,g\,L$$

$\therefore \upsilon_B = \sqrt{3gL}$

(iii) The ratio of the kinetic energies at B and C is :

$$\frac{K_B}{K_C} = \frac{\frac{1}{2} m\upsilon_B^2}{\frac{1}{2} m\upsilon_C^2} = \frac{3}{1}$$

At point C, the string becomes slack and the velocity of the bob is horizontal and to the left. If the connecting string is cut at this instant, the bob will execute a projectile motion with horizontal projection akin to a rock kicked horizontally from the edge of a cliff. Otherwise the bob will continue on its circular path and complete the revolution.

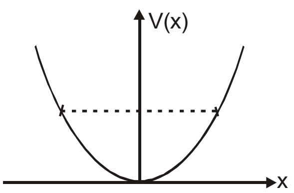
Exercise - I UNSOLVED PROBLEMS

Q.1 The sign of work done by a force on a body is important to understand. State carefully if the following quantities are positive or negative : **NCERT XI_6.1**
(a) work done by a man in lifting a bucket out of a well by means of a rope tied to the bucket.
(b) work done by gravitational force in the above case.
(c) work done by friction on a body sliding down an inclined plane.
(d) work done by an applied force on a body moving on a rough horizontal plane with uniform velocity,
(e) work done by the resistive force of air on a vibrating pendulum in bringing into rest.

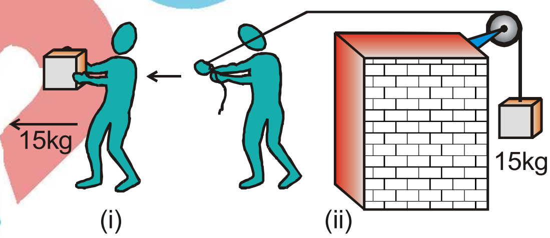
Q.2 A body of mass 2 kg initially at rest moves under the action of an applied horizontal force of 7 N on a table with coefficient of kinetic friction - 0.1. Compute the
(a) work done by the applied force in 10 s,
(b) work done by friction in 10 s,
(c) work done by the net force on the body in 10 s
(d) change in kinetic energy of the body in 10 s, and interpret your results.

Q.3 Given in figure are examples of some potential energy functions in one dimension. The total energy of the particle is indicated by a cross on the ordinate axis. In each case, specify the regions, if any, in which the particle cannot be found for the given energy. Also, indicate the minimum total energy the particle must have in each case. Think of simple physical contexts for which these potential energy shapes are relevant.

Q.4 The potential energy function for a particle executing linear simple harmonic motion is given by $V(x) = kx^2/2$, where k is the force constant of the oscillator. For $k = 0.5\ N\ m^{-1}$, the graphj of V(x) versus x is shown in figure. Show that a particle of total energy 1 J moving under this potential must 'turn back' when it reaches $x = \pm 2$ m.



Q.5 Answer the following :
(a) The casing of a rocket in flight burns up due to friction. At whose expense is the heat energy required for burning obtained ? The rocket or the atmosphere ?
(b) Comets move around the sun in highly elliptical orbits. The gravitational force on the comet due to the sun is not normal to the comet's velocity in general. Yet the work done by the gravitational force over every complete orbit of the comet is zero. Why ?
(c) An artificial satellite orbiting the earth in very thin atmosphere loses its energy gradually due to dissipation against atmospheric resistance, however small. Why then as it comes closer and closer to the earth ?
(d) In figure the man walks 2 m carrying a mass of 15 kg on his hands. In figure (ii) he walks the same distance pulling the rope behind him. The rope goes over a pulley, and a mass of 15 kg hangs at its other end. In which case is the work done greater ?



Q.6 Underline the correct alternative :
(a) When a conservative force does positive work on a body, the potential energy of the body increases/ decreases/remains unaltered.
(b) Work done by a body against friction always results in a losses of its kinetic/potential energy.
(c) The rate of change of total momentum of a many-particle system is proportional to the external force/ sum of the internal forces on the system.
(d) In an inelastic collision of two bodies, the quantities which do not change after the collision are the total kinetic energy/total linear momentum/total energy of the system of two bodies.

Q.7 State if each of the following statements is true or false. Give reasons for your answer.
(a) In an elastic collision of two bodies, the momentum and energy of each body is conserved.
(b) Total energy of a system is always conserved, no matter what internal and external forces on the body are present.

(c) Work done in the motion of a body over a closed loop is zero for every force in nature.
(d) In an inelastic collision, the final kinetic energy is always less than the initial kinetic energy of the system. **[NCERT pg. 141, 6.7]**

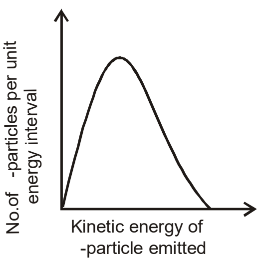
Q.8 Answer carefully, with reasons :
(a) In an elastic collision of two billiard balls, is the total kinetic energy conserved during the short time of collision of the balls (i.e. when they are in contact) ?
(b) Is the total linear momentum conserved during the short time of an elastic collision of two balls ?
(c) What are the answers to (a) and (b) for an inelastic collision ?
(d) If the potential energy of two billiard balls depends only on the separation distance between their centres, is the collision elastic or inelastic ? (Note, we are talking here of potential energy corresponding to the force during collision, not gravitational potential energy).

Q.9 Which of the following potential energy curves in Figure cannot possibly describe the elastic collision of two billiard balls ? Here r is the distance between centres of the balls.

Q.10 Consider the decay of a free neutron at rest :
$n \rightarrow p + e^{-1}$

Show that the two-body decay of this type must necessarily give an electron of fixed energy and, therefore, cannot account for the observed continuous energy distribution in the β-decay of a neutron or a nucleus (figure).

[Note : The simple result of this exercise was one among the several arguments advanced by W. Pauli to predict the existence of at third particle in the decay products of β-decay. This particle is known as neutrino. We now know that it is a particle of intrinsic spin ½ (like e^{-1}, p or n), but is neutral, and either massless or having an extremely small mass (compared to electron's mass) and which interacts very weakly with matter. The correct decay process of neutron is : $n \rightarrow p + e^{-} + v$]



Q.11 A body is initially at rest. It undergoes on - dimensional motion with constant acceleration. The power delivered to it at time t is proportional to
(i) $t^{1/2}$ (ii) t (iii) $t^{3/2}$ (iv) t^2

Q.12 A body is moving uni-directionally under the influence of a source of constant power. Its displacement in time t is proportional to :
(i) $t^{1?2}$ (ii) t (iii) $t^{3/2}$ (iv) t^2

Q.13 A body constrained to move along the z-axis of a coordinate system is subject to a constant force F given by F= $-\hat{i}+2\hat{j}+3\hat{k}$ N, where $\hat{i}, \hat{j}, \hat{k}$ are unit vectors along the x-, y- and z-axis of the system respectively. What is the work done by this force in moving the body a distance of 4 m along the z-axis ?

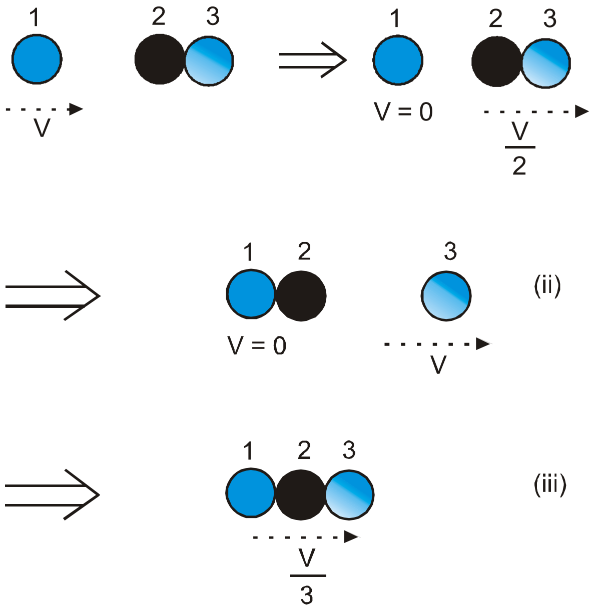
Q.14 An electron and a proton are detected in a cosmic ray experiment, the first with kinetic energy 10 keV, and the second with 100 keV. Which is faster, the electron or the proton ? Obtain the ratio of their speeds. (Electron mass = 9.11×10^{-31} kg, proton mass = 1.67×10^{-27} kg, 1 eV = 1.60×10^{-19} J).

Q.15 A rain drop of radius 2 mm falls from a height of 500 m above the ground. It falls with decreasing acceleration (due to viscous resistance of the air) until at half its original height, it attains its maximum (terminal) speed, and moves with uniform speed height, it attains its maximum (terminal) speed, and moves with uniform speed thereafter. What is the work done by the gravitational force on the drop in the first and second half of its journey ? What is the work done by the resistive force in the entire journey if its speed on reaching the ground is 10 m s^{-1} ?

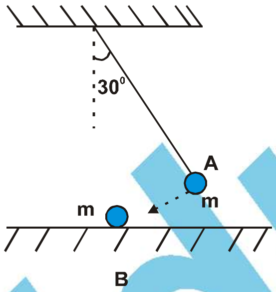
Q.16 A molecule in a gas container hits a horizontal wall with speed 200 m s^{-1} and angle 30° with the normal, and rebounds with the same speed. Is momentum conserved in the collision ? Is the collision elastic or inelastic ?

Q.17 A pump on the ground floor of a building can pump up water to fill a tank of volume 30 m^3 in 15 min. If the tank is 40 m above the ground, and the efficiency of the pump is 30%, how much electric power is consumed by the pump ?

Q.18 Two identical ball bearings in contact with each of the and resting on a frictionless table are hit head-on by another ball bearing of the same mass moving initially with a speed V. If the collision is elastic, which of the following figure) is a possible result after collision ?



Q.19 The bob A of a pendulum released from 30° to the vertical hits another bob B of the same mass at rest on a stable as shown in figure. How high does the bob A rise after the collision ? Neglect the size of the bobs and assume the collision to be elastic.



Q.20 The bob of a pendulum is released from a horizontal position A as shown in figure. If the length of the pendulum is 1.5 m, what is the speed with which the bob arrives at the lowermost point B, given that it dissipated 5% of its initial energy against air resistance ?

Q.21 A trolley of mass 300 kg carrying a sandbag of 25 kg is moving uniformly with a speed of 27 km/h on a frictionless track. After a while, sand starts leaking out of a hole on the trolley's floor at the rate of 0.05 kg s^{-1}. What is the speed of the trolley after the entire sand bag is empty ?

Q.22 A particle of mass 0.5 kg travels in a straight line with velocity $v = ax^{3/2}$ where $a = 5\ m^{-1/2}\ s^{-1}$. What is the work done by the net force during its displacement from x = 0 x = 2m ?

Q.23 The blades of a windmill sweep out a circle of area A. **NCERT XI_6.23**
(a) If the wind flows at a velocity v perpendicular to the circle, what is the mass of the air passing through it in time t ?
(b) What is the kinetic energy of the air ?
(c) Assume that the windmill converts 25% of the wind's energy into electrical energy, and that A = 30 m^2, v = 36 km/h and the density of air is 1.2 kgm^{-2}. What is the electrical power produced ?

Q.24 A person trying to loose weight (dieter) lifts a 10 kg mass 0.5 m, 1000 times. Assume that the potential energy lost each time she lowers the mass is dissipated. (a) How much work does she do against the gravitational force ? (b) Fat supplies 3.8×10^7 J of energy per kilogram which is converted to mechanical energy with a 20% efficiency rate. How much fat will the dieter use up ? **NCERT XI_6.24**

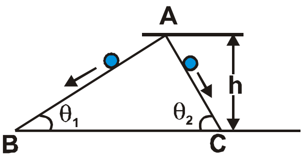
Q.25 A large family uses 8 kW of power. (A) Direct solar energy is incident on the horizontal surface at an average rate of 200 W per square meter. If 20% of this energy can be converted to useful electrical energy, how large an area is needed to supply 8 kW ? (b) Compare this area to that of the roof of a typical house.

Q.26 The heart rate (number of heart beats per minute) scales as 1/L where L is the characteristic length of the mammal. (a) Can you explain this ? (b) The scale factor of a human relative to a rhesus monkey is about 2.5. What is the monkey's heart rate ?

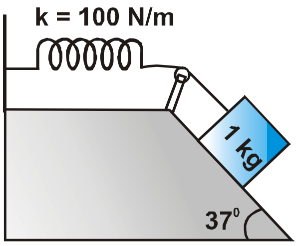
Q.27 In a nerve impulse about 10^5 neurons are fired. Estimate the energy used.

Q.28 A bullet of mass 0.012 kg and horizontal speed 70 ms^{-1} strikes a block of wood of mass 0.4 kg and instantly comes to rest with respect to the block. The block is suspended from the ceiling by means of thin wires. Calculate the height to which the block rises. Also, estimate the amount of heat produced in the block.

Q.29 Two inclined frictionless tracks, one gradual and he other steep meet at A from where two stones are allowed to slide down from rest, one on each track (Figure). Will the stones reach the bottom at the same time ? Will they reach there with the same speed ? Explain. Given $\theta_1 = 30°$, $\theta_2 = 60°$, and h = 10 m, what are the speeds and times taken by the two stones ?



Q.30 A 1 kg block situated on a rough incline is connected to a spring of spring constant 100 N m^{-1} as a shown in figure. The block is released from rest with the spring in the unstretched position. The block moves 10 cm down the incline before coming to rest. Find the coefficient of friction between the block and the incline. Assume that the spring has negligible mass and the pulley is frictionless.



Q.31 The potential energy of a 2 kg particle free to move along the x-axis is given by $U(x) = \left(\frac{x}{b}\right)^4 - 5\left(\frac{x}{b}\right)^2$ J, where b = 1 m. Plot this potential, identifying the extremum points. Identify the regions where particle may be found and its maximum speed given that the total mechanical energy is (1) 36 J ; (ii) 4 J.

NCERT XI_6.31

Q.32 A blot of mass 0.3 kg falls from the ceiling of an elevator moving down with an uniform speed of 7 ms^{-1}. It hits the floor of the elevator (length of the elevator = 3m) and does not rebound. What is the heat produced by the impact ? Would your answer be different if the elevator were stationary ?

Q.33 A trolley of mass 200 kg moves with a uniform speed of 35 km/h on a frictionless track. A child of mass 20 kg runs on the trolley from one end to the other (10 m away) with a speed of 4 ms^{-1} relative to the trolley in a direction opposite to the trolley's motion, and jumps out of the trolley. What is the final speed of the trolley ? How much has the trolley moved from the time the child begins to run ? **NCERT XI_6.33**

Q.34 Estimate the amount of energy released in the nuclear fusion reaction ${}^2_1H + {}^2_1H \rightarrow {}^3_1He + n$ (${}^2_1H$: deuterium, ${}^3_2He$: isotope of helium, given that $M({}^3_2He) = 3.0160\,\mu$, $M_n = 1.0087$ u and $M({}^2_1H) = 2.0141$ u where 1 u = 1.661 × 10^{-27} kg. Express your answer in units of MeV.

Q.35 When slow neutrons are incident on a target containing ${}^{235}_{92}U$, a possible (fission) reaction is ${}^{235}_{92}U + n \rightarrow {}^{141}_{36}Kr + 3n$

Estimate the amount of energy released using the following data :

$M({}^{235}U)$ = 235.04 u, $M({}^{141}Ba)$ = 140.91 u, $M({}^{92}Kr)$ = 91. 926 u, M_n = 1.0087 u. The energy released in a fusion reaction (problem 6.34) is much less than the energy release in a fission reaction of the heavy nucleus. Why then is a hydrogen bomb (based on nuclear fusion) far more powerful than the atom bomb (based on nuclear fission).

Q.36 The nucleus ${}^{57}Fe$ emits a γ-ray of energy 14.4 keV. If the mass of the nucleus is 56.935 u, calculate the recoil energy of the nucleus. You will notice that the recoil energy of a nucleus is inversely proportional to its mass. Now, if a nucleus such as ${}^{57}Fe$ is embedded in a lattice, the recoil momentum due to γ-emission is shared, roughly speaking. By the entire crystal. Consequently, the recoil energy is very greatly reduced nad the entire energy of the decay is converted (without any loss due to recoil) into the energy of the γ-ray. This the principle of recoilless emission employed in Mossbauer spectroscopy invented in 1958 and now an important tool for high resolution nuclear spectroscopic studies.

Q.37 Discuss how the work-energy theorem would stand modified if we consider motion in an accelerated frame of reference. The following two problems are calculator / computer based

Q.38 A 2 kg particle starts at the origin and moves along the positive x-axis. The net force acting on it measured at intervals of 1 m is 27.9, 28.3, 30.9, 34.0 34.5,46.9, 48.2, on the particle in this interval ?

Q.39 A 0.5 kg particle moves along the x-axis from x = 5 m to x = 17.2 m under the influence of a force F(x) $F = \frac{200}{2x + x^3}$ N Estimate (with at least 5% accuracy) the total work done by this force during this displacement.