4

SIMPLE EQUATION

CONTENTS

DEFINITION

A statement of equally which involves one or more variables is called an **equation**.

For example **:**

S.No.	Statements	Equations
(i)	A number x increased by 7 is 15	$x + 7 = 15$
(ii)	9 exceeds a number x by 3	$9 - x = 3$
(iii)	4 times a number x is 24	$4x = 24$
(iv)	A number y divided by 5 is 7	$\frac{y}{5} = 7$
(v)	The sum of the number x and twice the number y is 12	$x + 2y = 12$

Clearly, each one of the above statements is a statement of equality, containing one or more variables. Thus, each one of them is an equation.

Each of the equations through (i) to (iv) involves only one unknown (i.e. variable), while the equation (v) contains two unknown, namely, x and y.

LINEAR EQUATION

An equation in which the highest power of the variables involved is 1 is called a **linear equation**. Clearly, the sign of equality in an equation divides it into two sides, namely, the left-hand side and the right-hand side, written as LHS and RHS respectively.

SOLUTION OF AN EQUATION

A number which makes LHS = RHS when it is substituted for the variable in an equation is said to satisfy the equation and is called a **solution** or **root** of the equation.

Solving an equation is finding the roots of the equation.

SOLVING A LINEAR EQUATION BY THE TRIAL AND ERROR METHOD

In this method, we often make a guess of the root of the equations. We try several values of the variables and find the values of the LHS and the RHS in each case. When LHS = RHS for a particular value of the variable, we say that it is a root of the equation.

❖ EXAMPLES ❖

Ex.1 Find the solution of the equation $4x = 12$ by the trial-and-error method.

Sol. We try several values of x and find the values of the LHS and the RHS. We stop when for a particular value of x, LHS = RHS.

x	LHS	RHS
1	$4 \times 1 = 4$	12
2	$4 \times 2 = 8$	12
3	$4 \times 3 = 12$	12

$\therefore$ $x = 3$ is the solution of the given equation.

Ex.2 Solve the equation $3x - 5 = 7 - x$ by the trail and error method.

Sol. We try several values of x and find the values of the LHS and the RHS. We stop when for a particular value of x, LHS = RHS.

x	LHS	RHS
1	$3 \times 1 - 5 = -2$	$7 - 1 = 6$
2	$3 \times 2 - 5 = 1$	$7 - 2 = 5$
3	$3 \times 3 - 5 = 4$	$7 - 3 = 4$

$\therefore$ $x = 3$ is the solution of the given equation.

Ex.3 Solve the equation $\frac{1}{3}y + 5 = 8$ by the trial and error method.

Sol. We make a guess and try several values of y, and find the values of the LHS as well as the RHS in each case. We stop when for a particular value of y, LHS = RHS.

y	LHS	RHS
3	$\frac{1}{3} \times 3 + 5 = 6$	8
6	$\frac{1}{3} \times 6 + 5 = 7$	8
9	$\frac{1}{3} \times 9 + 5 = 8$	8

Thus, when y = 9, we have: LHS = RHS

$\therefore$ $y = 9$ is the solution of the given equation.

➢ SYSTEMATIC METHOD FOR SOLVING AN EQUATION

We have the following rules :

Rule (i) :

We can add the same number to both the sides of an equation.

Rule (ii) :

We can subtract the same number from both the sides of an equation.

Rule (iii) :

We can multiply both the sides of an equation by the same nonzero number.

Rule (iv) :

We can divide both the sides of an equation by the same nonzero number.

❖ EXAMPLES ❖

Ex.4 Solve the equation $x - 5 = 7$ and check the result.

Sol. $x - 5 = 7$

$\Rightarrow$ $x - 5 + 5 = 7 + 5$ [adding 5 to both sides]

$\Rightarrow$ $x = 12$

So, $x = 12$ is the solution of the given equation.

Check : Substituting $x = 12$ in the given equation, we get LHS = $12 - 5 = 7$ & RHS = 7.

$\therefore$ when $x = 12$, we have : LHS = RHS

Ex.5 Solve the equation $8 + x = 3$ and check the result.

Sol. $8 + x = 3$

$\Rightarrow$ $8 + x - 8 = 3 - 8$ [subtracting 8 from both sides]

$\Rightarrow$ $x + 8 - 8 = 3 - 8$ [$\because$ $8 + x = x + 8$]

$\Rightarrow x = -5$

So, $x = -5$ is the solution of the given equation.

Check : Substituting $x = -5$ in the given equation, we get LHS = $8 - 5 = 3$ and RHS = 3

$\therefore$ When $x = -5$, we have : LHS = RHS

Ex.6 Solve the equation $8x = 24$ and check the result.

Sol. $8x = 24$

$\Rightarrow \frac{8x}{8} = \frac{24}{8}$ [dividing both sides by 8]

$\Rightarrow x = 3$

$\Rightarrow x = 3$ is the solution of the given equation.

Check : Substituting $x = 3$ in the given equation, we get

LHS $= 8 \times 3 = 24$ and RHS $= 24$

$\therefore$ when $x = 3$,

we have : LHS = RHS

Ex.7 Solve the equation $\frac{2}{3}x = 18$ and check the result.

Sol. $\frac{2}{3}x = 18 \Rightarrow \frac{2}{3}x \times \frac{3}{2} = 18 \times \frac{3}{2}$

[multiplying both sides by $\frac{3}{2}$]

$\Rightarrow \frac{2}{3} \times \frac{3}{2} \times x = 27 \Rightarrow x = 27$

$\therefore x = 27$ is the solution of the given equation.

Check : Substituting $x = 27$ in the given equation, we get

LHS $= \frac{2}{3} \times 27 = 18$ and RHS $= 18$

$\therefore$ when $x = 27$, we have : LHS = RHS

➢ TRANSPOSITION

You know that one can add or subtract a number from both sides of the equation. So, for the equation $x - 4 = 5$. We can write

$x - 4 + 4 = 5 + 4 \Rightarrow x = 5 + 4$

Similarly, for the equation $x + 5 = 3$, we can write

$x + 5 - 5 = 3 - 5 \Rightarrow x = 3 - 5$.

In both these cases you will notice that after this operation, the number appears on the other side of the equation, but with the opposite sign. So, you can straightway change the sign of a term and transfer it from one side of an equation to the other side. This is called **transposition.**

❖ EXAMPLES ❖

Ex.8 Solve : $3x + 5 = 13 - x$. Check the result.

Sol. $3x + 5 = 13 - x$

$\Rightarrow 3x + x = 13 - 5$

[transposition $-x$ to LHS and $+5$ to RHS]

$\Rightarrow 4x = 8$

$\Rightarrow \frac{4x}{4} = \frac{8}{4}$ [dividing both sides by 4]

$\Rightarrow x = 2$

$\therefore x = 2$ is the solution of the given equation

Check : Substituting $x = 2$ in the given equation, we get

LHS $= 3 \times 2 + 5 = 11$ and RHS $= 13 - 2 = 11$

$\therefore$ LHS = RHS, when $x = 2$.

Ex.9 Solve : $x - 7 = 5 + \frac{x}{2}$. Check the result.

Sol. $x - 7 = 5 + \frac{x}{2} \Rightarrow x - \frac{x}{2} = 5 + 7$

[transposing $\frac{x}{2}$ to LHS and -7 to RHS]

$\Rightarrow \frac{x}{2} = 12$

$\Rightarrow \frac{x}{2} \times 2 = 12 \times 2$ [multiplying both sides by 2]

$\Rightarrow x = 24$ is the solution of the given equation.

Check : Substituting $x = 24$ in the given equation, we get

LHS $= (24 - 7) = 17$

and RHS $= \left(5 + \frac{1}{2} \times 24\right) = 17$.

$\therefore$ LHS = RHS, when $x = 24$.

Ex.10 Solve : $3(x + 3) - 2(x - 1) = 5(x - 5)$.

Check the result.

Sol. $3(x + 3) - 2(x - 1) = 5(x - 5)$

$\Rightarrow 3x + 9 - 2x + 2 = 5x - 25$

[removing parentheses]

$\Rightarrow x + 11 = 5x - 25$

$\Rightarrow x - 5x = -25 - 11$

[transposing $5x$ to LHS and 11 to RHS]

$\Rightarrow -4x = -36$

$\Rightarrow x = 9$ [dividing both sides by -4]

$\therefore x = 9$ is the solution of the given equation.

Check : Substituting x = 9 in the given equation, we get

LHS = 3(9 + 3) – 2 (9 – 1) = (3 × 12 – 2 × 8) = 36 – 16 = 20

RHS = 5(9 – 5) = 5 × 4 = 20

∴ LHS = RHS, when x = 9.

Ex.11 Solve : $\frac{x}{8}-\frac{1}{2}=\frac{x}{6}-2$. Check the result.

Sol. Multiplying each term by 24, the LCM of 8, 2 and 6, the given equation becomes :

3x – 12 = 4x – 48

⇒ 3x – 4x = – 48 + 12

[transposing 4x to LHS and –12 to RHS]

⇒ –x = –36

⇒ x = 36

∴ x = 36 is the solution of the given equation.

Check : Substituting x = 36 in the given equation, we get

$$\text{LHS}=\left(\frac{36}{8}-\frac{1}{2}\right)=\left(\frac{36-4}{8}\right)=\frac{32}{8}=4$$

$$\text{and RHS}=\left(\frac{36}{6}-2\right)=(6-2)=4$$

∴ LHS = RHS, when x = 36.

➢ APPLICATION OF EQUATIONS

If a problem on numbers is given, we shall first translate it in the form of an equation and then solve it.

❖ EXAMPLES ❖

Ex.12 If 5 is subtracted from three times a number, the result is 16. Find the number.

Sol. Let the required number be x. Then,

3x – 5 = 16

⇒ 3x = 16 + 5 [on transposing – 5 to RHS]

⇒ 3x = 21

⇒ x = 7 [dividing both sides by 3]

Hence, the required number is 7.

Ex.13 Find two numbers such that one of them exceeds the other by 9 and their sum is 81.

Sol. Let the smaller number be x.

Then, the other number = (x + 9)

∴ x + (x + 9) = 81 ⇒ 2x + 9 = 81

⇒ 2x = 81 – 9 [on transposing 9 to RHS]

⇒ 2x = 72

⇒ x = 36 [Dividing both sides by 2]

Hence, one number = 36, and the other number = (36 + 9) = 45.

Ex.14 The length of a rectangular field is twice its breadth. If the perimeter of the field is 228 meters. Find the dimension of the field.

Sol. Let the breadth of the field be x metres. Then, its length = 2x metres

∴ Perimeter of the field = 2 (length + breadth)

= 2(2x + x) metres = 6x metres.

So, 6x = 228 ⇒ $x=\frac{228}{6}$

[dividing both sides by 6]

⇒ x = 38

Hence, breadth of the field = 38 metres, and length of the field = (2 × 38) metres = 76 metres

Ex.15 Mona's father is thrice as old as Mona. After 12 years, his age will be twice that of his daughter. Find their present ages.

Sol. Let Mona's present age be x years.

Then, her father's present age = 3x years.

Mona's age after 12 years = (x + 12) years

Mona's father's age after 12 years = (3x + 12) years

∴ 3x + 12 = 2(x + 12)

⇒ 3x + 12 = 2x + 24

⇒ 3x – 2x = 24 – 12

[transposing 2x to LHS and 12 to RHS]

⇒ x = 12

∴ Mona's present age = 12 years

And, her father's present age

= (3 × 12) years = 36 years.