Current Electricity

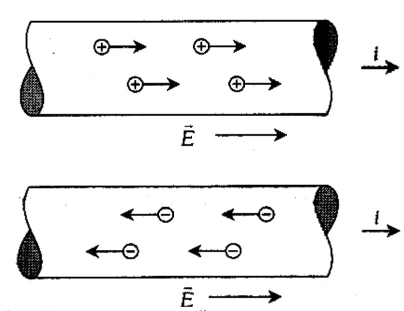
3

ELECTRIC CURRENT

* Flow of charge is called **electric current** and the rate of flow of charge in a circuit represents the magnitude of the electric current in that circuit. It is generally represented by I.
* If through a croos section, ΔQ charge 4passes in time Δt then $i_{av} = \frac{\Delta Q}{\Delta t}$ and instantaneous current

$$i = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}$$

* The conventional direction of current is taken to be the direction of flow of positive charge, i.e. field and is opposite to the direction of flow of negative charge as shown below.



Though conventionally a direction is associated with current (opposite to the motion of electron), it is not a vector. It is because the current can be added algebraically. Only scalar quantities can be added algebraically not the vector quantities.

* Electric current is a scalar quantity.
* S.I. unit of electric current is coulomb/second or ampere, and C.G.S. unit is stat ampere and ab ampere (biot)

$$1 \text{ ampere} = 3 \times 10^9 \text{ stat ampere} = \frac{1}{10} \text{ ab ampere}$$

Dimensions of electric current = $M^0L^0T^0A^1$

$$1A = \frac{1\,\text{coulomb}}{1\,\text{second}}$$

* If n is the number of free electrons passing through a point in t seconds, then the total charge passing through that point in t seconds is $q = ne$

and the current flowing through the conductor

$$I = \frac{q}{t} = \frac{ne}{t}$$

$\therefore$ For one ampere current $n = \frac{1}{1.6 \times 10^{-19}}$

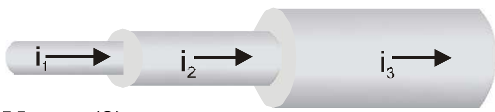
$= 6.25 \times 10^{18}$ electrons / sec

* The number of electrons flowing through a conductor in t seconds $n = \frac{I\,t}{e}$

Note: (1)

(a) Charge on a current carrying conductor : In conductor the current is caused by electron (free electron). The no. of electron (negative charge) and proton (positive charge) in a conductor is same. Hence the net charge in a current carrying conductor is zero.

(b) Current through a conductor of non-uniform cross-section : For a given conductor current does not change with change in cross-sectional area. In the following figure, $i_1 = i_2 = i_3$



Note: (2)

Current carriers : The charged particles whose flow in a definite direction consitiutes the electric current are called carriers. In different situation current carriers are different.

(i) Solids : In solid conductors like metals current carriers are free electrons.

(ii) Liquids : In liquids current carriers are positive and negative ions.

(iii) Gases : In gases current carriers are positive ions and free electrons.

(iv) Semi conductor : In semi conudctor current carriers are holes and free electrons.

Solved Examples

Ex.1 How many electrons flow per second through any cross-section of a wire, if it carries a current of one ampere?

Sol. $I = \frac{q}{t} = \frac{ne}{t}$

$\Rightarrow n = \frac{It}{e} = \frac{1 \times 1}{1.6 \times 10^{-19}} = 6.25 \times 10^{18}$

Ex.2 How many electrons pass through a heater wire in one minute, if current flowing is 8 ampere?

Sol. $n = \frac{It}{e} = \frac{8 \times 60}{1.6 \times 10^{-19}} = 3 \times 10^{21}$

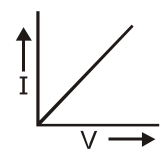
OHM'S LAW

* If the physical conditions of a conductor (such as temperature etc.,) remain same, then the potential difference across the ends of the conductor is directly proportional to the current flowing in it, that is

$V \propto I$ or $V = RI$

where R is a constant, called resistance of the conductor.

* Ohm's law is valid for all metal condcutors in which there are free electrons.
* Onm's law is not obeyed in thermionic tube, crystal, rectifier, semi-conductor, transistor, ionized gases etc.,
* The graph between V and I for a conductor is a straight line.



Solved Examples

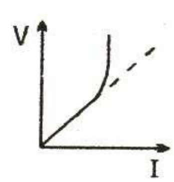
Ex.3 In a wire of length 4 m and diameter 6 mm, a current of 120 ampere is passed. The potential difference across the wire is found to be 18 volt. What is the resistance of the wire?

Sol. $R = \frac{V}{I} = \frac{18}{120} = 0.15$ ohm

NON-OHMIC DEVICE - FAILURE OF OHM'S LAW

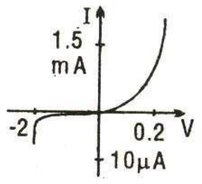
There are many electronic devices for which Ohm's law V = IR i.e., linear relation between current and voltage is not valid. For example,

* Current may vary nonlinearly with potential difference as in figure.



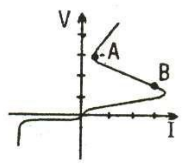
(a)

* The variation of current may depend upon sign of the potential difference applied [as vacuum diode or pn junction diode.



(b)

* Current may decrease on increasing the potential difference (negative resistance device as in Thyristor, Figure shown, portion AB)



(c)

Solved Examples

Ex.4 For a hypothetical electronic device, the potential difference V (in volts) is related to the current I (in mA) by $V = 3.5\ I^2$. Find

(i) the static resistance when current is 2.0 mA,

(ii) value of current for which the static or d.c. resistance is equal to 14 ohm.

Sol. (i) $R = \frac{V(\text{volt})}{I(\text{amp})} = 1000 \times \frac{V(\text{volt})}{I(\text{mA})}$

$= 1000 \times 3.5 I(\text{mA})$

$= 1000 \times 3.5 \times 2 = 7\ K\Omega$

(ii) $14 = 1000 \times \frac{V(\text{volt})}{I(\text{mA})}$

$= 1000 \times (3.5) I(\text{mA})$

or $I(\text{mA}) = \frac{14}{3.5 \times 1000} = 4 \times 10^{-3}\ (\text{mA})$

or $I = 4\mu A$

DRIFT VELOCITY OF FREE ELECTRONS, RELAXATION TIME,ELECTRIC CURRENT AND RESISTANCE

* Metals contain avery large number of free electrons and they behave like the gas molecules. These electrons move at random with a velocity about 10^5 m/s in the vacant space between the stationary positive ions and collide with these ions. The average time taken between two successive collisions is called relaxation time. It is represented by τ. Its value is of the order of 10^{-14} s.

* When a potential difference is applied between the ends of the conductor wire, it provides a small velocity to the electrons towards the higher potential end along the length of the wire which is superposed on the random motion of the electrons. This constant small velocity is called drift velocity. The drift velocity is generally represented by v_d and its value is found to be of the order of 10^{-4} m/s.

* If the potential difference between the ends of a wire of length ℓ is V, then the intensity of electric field in the wire is

when m is the mass of the electron.

The drift velocity is the velocity acquired by the electron in a direction opposite to applied electric field due to the acceleration of electron during the relaxation time τ. Hence,

drift velocity = acceleration x time

$$v_d = a\tau$$

$$= \frac{eE}{m}\tau = \frac{eV}{m\ell}\tau$$

* If area of cross–section of wire is A, the number of free electrons in its unit volume is n and the drift velocity of the electrons is v_d, then the electric current flowing through the wire will be $I = neAv_d$

or $$I = neA\left(\frac{eE}{m}\right)\tau = \frac{ne^2AE}{m}\tau$$

or $$I = neA\left(\frac{eV}{m\ell}\right)\tau = \left(\frac{ne^2\tau}{m}\right)\left(\frac{A}{\ell}\right)V$$

NOTE :

(i) The direction of drift velocity for electron in a metal is opposite to that of applied electric field (i.e. current density $\vec{J}$)

(ii) $v_d \propto E$ i.e., greater the electric field, larger will be the drift velocity.

(iii) When a steady current flows through a conductor of non-uniform cross-section drift velocity varies inversely with area of cross-sectional $\left(v_d \propto \frac{1}{A}\right)$

(iv) If diameter of a conductor is doubled, then drift velocity of electrons inside it will not change.

* If the resistance of the wire is R, then

$$R = \frac{V}{I} = \frac{m\ell}{ne^2\tau A} = \left(\frac{m}{ne^2\tau}\right)\left(\frac{\ell}{A}\right)$$

If the mean free path of the electron is λ and rms speed of electrons is v_{rms}, then

$$\tau = \frac{\lambda}{v_{rms}}$$

$$\therefore \quad R = \frac{m\ell v_{rms}}{ne^2\lambda A}$$

Resistivity of conductor

$$\rho = \frac{RA}{\ell} = \frac{m}{ne^2\tau}$$

and conductivity of conductor

$$\sigma = \frac{1}{\rho} = \frac{ne^2\tau}{m}.$$

* On bending the wire on several places its resistance does not change because resistance of wire depends upon the free electron density (n) and the relaxation time (τ) which do not change on bending.

Solved Examples

Ex.5 The number density of conduction electron in copper is 8.5×10^{28} (m^{-3}) and the mean free time τ between collisions is 2.5×10^{-14} s. What is the conductivity of copper?

Sol. $\sigma = \frac{ne^2\tau}{m}$

$$= \frac{8.5 \times 10^{28} \times (1.6 \times 10^{-19})^2 \times 2.5 \times 10^{-14}}{9 \times 10^{-31}}$$

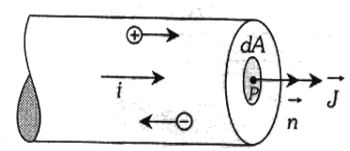
$$= \frac{8.5 \times 1.6 \times 1.6 \times 2.5}{9} \times 10^7$$

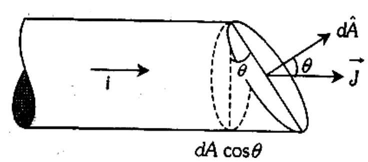
$\approx 6 \times 10^7$ mho/m or Siemen/metre

CURRENT DENSITY AND MOBILITY

* In case of flow of charge through a cross-sectional, current density is defined as a vector having magnitude equal to current per unit area surrounding that point. Remember area is normal to the direction of charge flow (or current passes) through that point.

Current density at point P is given by $\vec{J} = \frac{di}{dA}\vec{n}$





If the cross-sectional area is not normal to the current, the cross-sectinal area normal to current in accordance with following figure will be dA cosθ and so in this situation :

$$J = \frac{di}{dA\cos\theta} \quad \text{i.e. } di = JdA\cos\theta$$

or $di = \vec{J}.\vec{dA} \Rightarrow i = \int \vec{J}.\vec{dA}$

i.e., in terms of current density, current is the flux of current density.

Note : If current density $\vec{J}$ is uniform for a normal cross-sectional $\vec{A}$ then :

$$i = \int \vec{J}.\vec{ds} = \vec{J}.\int \vec{ds}$$

[as $\vec{J}$ contant]

or $i = \vec{J}.\vec{A} = JA\cos 0 = JA \Rightarrow J = \frac{i}{A}$

[as $\vec{dA} = \vec{A}$ and $\theta = 0°$]

* Current density $\vec{J}$ is a vector quantity having S.I. unit amp/m^2 & dimensions [$L^{-2}A$]
* Current density in terms of drift velocity

$$J = \frac{I}{A} = \frac{neAv_d}{A} = nev_d$$

but $v_d = \frac{eE}{m}\tau$

$\therefore$ $J = (ne)\left(\frac{eE}{m}\right)\tau = \frac{ne^2\tau}{m}E$

but $\frac{ne^2\tau}{m} = \sigma = \text{conductivity}$

$\therefore$ $J = \sigma E$.

This is another form of Ohm's law.

* The ratio of the applied electric field in the conductor to the current density is called resistivity, that is

$$\rho = \left(\frac{E}{J}\right)$$

* If J = 1, then $\rho = E$, that is, for a unit current density, the intensity of electric field is equivalent to the resistivity.
* More the resistivity of a conductor, more intensity of electric field will be required for maintaining its current density to a certain value.
* Mobility : Drift velocity is directly proportional to the applied electric field, so the ratio of the drift velocity to the external electric field is a constant and it is called mobility. Its unit is m^2 per volt-sec.

$\therefore$ $\text{Mobility}(\mu) = \frac{v_d}{E} = \frac{e}{m}\tau$ m^2/V-s

Solved Examples

Ex.6 One end of an aluminium wire whose diameter is 2.5 mm is welded to one end of a copper wire whose diameter is 2.0 mm. The composite wire carries a steady current of 6.25 A. What is the current density in each wire?

Sol. $J = \frac{I}{A}$

This for aluminium,

$$J = \frac{6.25}{\frac{\pi}{4}\times 2.5^2 \times 10^{-6}} = \frac{4\times 10^6}{\pi}$$

$= 1.27 \times 10^6 \text{ A/m}^2$

= 1.27 A/cm^2

For copper

$$J = \frac{6.25}{\frac{\pi}{4}\times 2^2 \times 10^{-6}} = \frac{6.25\times 10^6}{\pi}$$

$\approx 2 \times 10^6 \text{ A/m}^2$

$\approx 200 \text{ A/cm}^2$

Ex.7 What is drift velocity of conduction electrons in the copper wire of example 1, if $n = 8.5 \times 10^{28}\ m^{-3}$?

Sol. $v_d = \frac{J}{ne}$

$$= \frac{200\times 10^4}{8.5\times 10^{28}\times 1.6\times 10^{-19}}$$

$= \frac{200}{13.6}\times 10^{-5}$ m/s

$= 14.7 \times 10^{-5}$ m/s

≈ 0.15 mm/s

RESISTANCE AND RESISTIVITY OR SPECIFIC RESISTANCE

* When current is passed through a conductor, it resists the flow of current. The flow of free electrons due to applied electric field is obstructed by the random motion due to thermal effect and colisions with ions. This property of the conductor is called its resistance.
* The resistance of a conductor is equal to the ratio of the potential difference across the ends of the conductor and the current flowing through it.
* If V is the potential difference across the ends of the conductor and I is the current flowing through it, then

$$R = \left(\frac{V}{I}\right)$$

* Unit of resistance is ohm and ohm = volt/ampere
  It is represented by a symbol Ω (ohm)
* If the potential difference across the ends of the conductor is one volt and the current flowing through it is one ampere, then the resistance of the conductor is one ohm.
* 1 ohm $= \frac{1\text{volt}}{1\text{ampere}}$

$$= \frac{10^8 \text{ emu potential difference}}{10^{-1} \text{ emu current}}$$

or 1 ohm = 10^9 emu of resistance

Unit of resistance :

$$\text{ohm} = \frac{\text{volt}}{\text{ampere}} = \frac{\text{joule/coulomb}}{\text{ampere}}$$

$$= \frac{(\text{newton} \times \text{metre})/(\text{ampere} \times \text{second})}{\text{ampere}}$$

$$= \frac{Kg.m^2}{A^2.s^3}$$

Dimensions fo resistance = $ML^2T^{-3}A^{-2}$

* The resistance of a conductor depends upon :
  (a) The length (L) : Resistance is directly proportional to the length of the conductor i.e., $R \propto L$ [A is constant]
  (b) The thickness or the area of cross-section of a wire (A) : Resistance is inversely proportional to the area of cross-section, i.e.,

$$R \propto \frac{1}{A} \text{ or } R \propto \frac{1}{\pi r^2} \quad \text{[L is constant]}$$

where r is the radius of the wire.

(c) The nature of the material.

(d) The temperature of the conductor. Resistance of metal conductors increases with the increase of temperature and decreases with the decrease of temperature because on increasing the temperature the random motion of free electrons increases.

* Thus $R \propto \frac{L}{A}$

or $R = \rho\left(\frac{L}{A}\right)$

Where ρ is a constant, called specific resistance or resistivity.

$\therefore \quad \rho = \frac{RA}{L}$

If the conductor is in the form of a wire of radius r, then

$$\rho = R\left(\frac{\pi r^2}{L}\right)$$

* The specific resistance of a material is equal to the resistance of a wire of unit length and having unit area of cross-section or the specific resistance of a material is equal to the resistance of a cube of unit length, when the current flows through the opposite faces of the cube.
* Unit of specific resistance : Ohm-metre.
  Dimensions of specific resistance : $ML^3T^{-3}A^{-2}$
* The specific resistance depends upon the nature of the material of conductor and the temperature. It does not depend upon the length and thickness of the conductor.

* Value of specific resistance is very small in case of conductors, more in semiconductors and maximum in insulators.
* Specific resistance is also called resistivity.

Solved Examples

Ex.8 The resistance of a rectangular block of copper of dimensions 2 mm x 2 mm x 5 metre between two square faces is 0.02 ohm. What is the resistivity of copper?

Sol. $\rho = \frac{R.A}{\ell} = \frac{0.02 \times 4 \times 10^{-6}}{5}$

$= 1.6 \times 10^{-8}\ \Omega m$

Ex.9 What is the resistance between two rectangular faces of a block of dimensions 4 cm x 4 cm x 10 cm of manganin $(\rho = 48 \times 10^{-8}\ \Omega m)$?

Sol. $R = \frac{\rho.\ell}{A} = \frac{48 \times 10^{-8} \times 4 \times 10^{-2}}{4 \times 10 \times 10^{-4}} = 4.8\ \mu\Omega$

Ex.10 Two wires of the same material having lengths in the ratio of 1 : 2 and diameters in the ratio 2 : 1 are connected in series with a cell of emf 2 volt and internal resistance 1 ohm. What is the ratio of the potential difference across the two wires?

Sol. Since in series, same current flows, thus

$$\frac{V_1}{V_2} = \frac{IR_1}{IR_2} = \frac{R_1}{R_2}$$

$$= \frac{\ell_1}{\pi r_1^2} \times \frac{\pi r_2^2}{\ell_2} = \frac{\ell_1}{\ell_2} \times \frac{r_2^2}{r_1^2}$$

$$= \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right)^2 = \frac{1}{8}$$

or $V_1 : V_2 = 1 : 8$

Ex.11 Two wires of equal length one of Cu and other of manganin have the same resistances. Which wire is thicker?

Sol. For Cu, $R_1 = \rho_1 \frac{\ell_1}{A_1}$

For manganin $R_2 = \rho_2 \frac{\ell_2}{A_1}$

Given $\ell_1 = \ell_2, R_1 = R_2$, thus

$\frac{\rho_1}{A_1} = \frac{\rho_2}{A_2}$ or $\frac{A_2}{A_1} = \frac{\rho_2}{\rho_1}$

Since for manganin $\rho \sim 48 \times 10^{-8}\ \Omega$m.while for Cu $\rho \sim 1.7 \times 10^{-8}\ \Omega$m, $\rho_2 / \rho_1 > 1$. So $A_2 > A_1$. The manganin wire is thicker.

Ex.12 If a copper wire is stretched to make its radius decrease by 0.15%, then the percentage increase in resistance is approximately

(1) 0.15% (2) 0.40%
(3) 0.60% (4) 0.90%

Sol. Due to stretching resistance changes are in the ratio

$$\frac{R_2}{R_1} = \left(\frac{r_1}{r_2}\right)^4$$

or $R \propto r^{-4}$

or $\Delta R \propto -4r^{-5}\ \Delta r$

or $\frac{\Delta R}{R} = -4\frac{\Delta r}{r}$

$= 4 \times 0.15\% = 0.60\%$ Ans. **(3)**

EFFECT OF STRETCHING OF A WIRE ON RESISTANCE

In stretching, the density of wire usually does not change. Therefore

Volume before stretching = Volume after

$$\ell_1 A_1 = \ell_2 A_2$$

and $\frac{R_2}{R_1} = \frac{\ell_2}{\ell_1} \times \frac{A_1}{A_2}$

If information on lengths before and after stretching is given, then use $\frac{A_1}{A_2} = \frac{\ell_2}{\ell_1}$

$$\frac{R_2}{R_1} = \left(\frac{\ell_2}{\ell_1}\right)^2$$

If information on radius r_1 and r_2 is given then use

$\frac{\ell_2}{\ell_1} = \frac{A_1}{A_2}$ $\frac{R_2}{R_1} = \left(\frac{A_1}{A_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^4$

CONDUCTIVITY

(a) Reciprocal of resistivity of a conductor is called its conductivity. It is generally represented by σ.

(b) $\sigma = \frac{1}{\rho}$

(c) Unit : $ohm^{-1}.metre^{-1}$ or ohm per metre.

Ex.13 Find conductivity of copper.

Sol. $\sigma = \frac{1}{\rho} = \frac{1}{1.7 \times 10^{-8}}$

$= 5.9 \times 10^{7}$ Siemens m^{-1} or mho/metre

EFFECT OF TEMPERATURE ON RESISTANCE AND RESISTIVITY

* The resistance of a conductor depends upon the temperature. As the temperature increases, the random motion of free electrons also increases. If the number density of charge carrier electrons remains constant as in the case of a conductor, then the increase of random motion increases the resistivity. The variation of resistance will temperature is given by the following relation

$$R_t = R_0(1 + \alpha t + \beta t^2)$$

where R_t and R_0 are the resistance at t^0C and 0^0C respectively and α and β are constants. The constant β is very small so its may be assumed negligible.

$\therefore \quad R_t = R_0(1 + \alpha t)$

or $\quad \alpha = \frac{R_t - R_0}{R_0 \times t}$

This constant a is called as temperature coefficient of resistance of the substance.

* If R_0 = 1 ohm, t = 1^0C, then

$$\alpha = (R_t - R_0)$$

Thus, the temperature coefficient of resistance is equal to the increase in resistance of a conductor having a resistance of one ohm on raising its temperature by 1^0C. The temperature coefficient of resistance may be positive or negative.

* From calculations it is found that for most of the metals the value of α is nearly $\frac{1}{273} / ^0C$. Hence substituting α in the above equation

$$R_t = R_0\left(1 + \frac{t}{273}\right)$$

$$= R_0\left(\frac{273 + t}{273}\right) = R_0 \frac{T}{273}$$

where T is the absolute temperature of the condutor.

$\therefore \quad R_t \propto T$

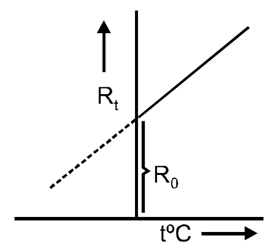
Thus, the resistance of a pure metallic wire is directly proportional to its absolute temperature.

NOTE :

(a) If R_1 and R_2 are the resistance at t_1°C and t_2ºC respectively then $\frac{R_1}{R_2} = \frac{1 + \alpha t_1}{1 + \alpha t_2}$

(b) The value of α is different at different temperature. Temperature coefficient of resitance averaged over the temperature range t_1ºC to t_2ºC is given by $\alpha = \frac{R_2 - R_1}{R_1(t_2 - t_1)}$ which gives $R_2 = R_1[1 + \alpha(t_2 - t_1)]$. This formula gives an approximate value.

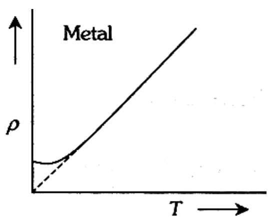
* The graph drawn between the resistance R_t and temperature t is found to be a straight line



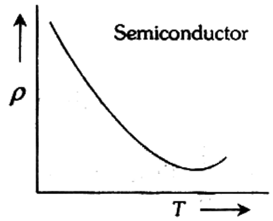
* The resistivity or specific resistance varies with temperature. This variation is due to change in resistance of a conductor with temperature. The dependence of the resistivity with temperature is represented by the following equation.

$$\rho_t = \rho_0(1 + \alpha t)$$

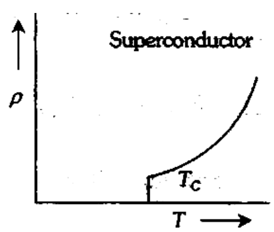
* With the rise of temperature the specific resistance or resistivity of pure metals increases and that of semi-conductors and insulators decrease. The resistivity of alloys increases with the rise of temperature but less than that of metals.



ρ increases with temperature



ρ decreases with temperature



ρ decreases with temperature and becomes zero at a certain temperature

* On applying pressure on pure metals, its resistivity decreases but on applying tension, the resistivity increases.

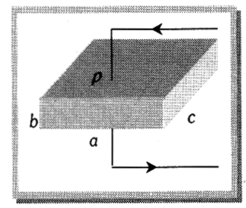
* The resistance of alloys such as eureka, manganin etc., increases in smaller amount with the rise in temperature. Their temperature coefficient of resistance is negligible. On account of their high resistivity and negligible temperature coefficient of resistance these alloys are used to make wires for resistance boxes, potentiometer, metre bridge etc.,

* The resistance of semiconductors, insulators, electrolytes etc., decreases with the rise in temperature. Their temperature coefficients of resistance are negative.

* On increasing the temperature of semi conductors a large number of electrons get free after breaking their bonds. These electrons reach the conduction band from valence band. Thus conductivity increases or resistivity decreases with the increase of free electron density.

NOTE : Resistance according to potential difference :

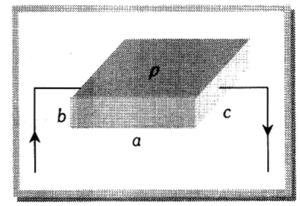
Resistance of a conducting body is not unique but depends on its length and area of cross-sectional i.e. how the potential difference is applied. See the following figures



Length = b

Area of cross-section = $a \times c$

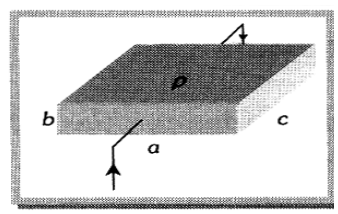
Resistance $R = \rho\left(\frac{b}{a \times c}\right)$



Length = a

Area of cross-section = $b \times c$

Resistance $R = \rho\left(\frac{a}{b \times c}\right)$



Length = c

Area of cross-section = $a \times b$

Resistance $R = \rho\left(\frac{c}{a \times b}\right)$

Solved Examples

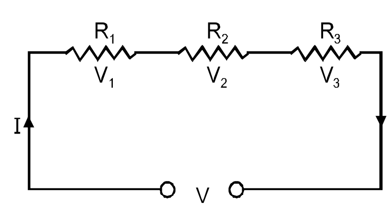
Ex.14 A wire has a resistance of 2 ohm at 273 K and a resistance of 2.5 ohm at 373 K. What is the temperature coefficient of resistance of the material?

Sol. $\alpha = \frac{R - R_0}{R_0(T - T_0)} = \frac{2.5 - 2}{2 \times (373 - 273)}$

$= \frac{0.5}{200} = 2.5 \times 10^{-3} / {}^0K$

COMBINATION OF RESISTANCES

(a) Series Combination :



(i) Resistance are connected in series in the following way:

(ii) Same amount of current flows through each resistance.

(iii) The potential difference across each resistance depends upon the value of resistance.

(iv) The sum of potential differences across the resistances is equal to the voltage applied in the circuit, ie.,

$$V = V_1 + V_2 + V_3$$

(v) Total resistance of the circuit is

$$R = R_1 + R_2 + R_3$$

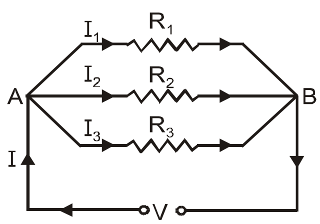
Thus, the equivalent resistance of the resistances connected in series is equal to the sum of all resistances.

(vi) If identical resistances of resistance R' are connected in series, then total resistance will be

$$R = nR'$$

(vii) On joining the resistances in series total resistance in the circuit increases and current decreases in the circuit.

(b) Parallel Combination :



(i) Resistances are connected in parallel in the following way:

(ii) Potential difference across each resistance is same.

(iii) Current flowing through each resistance is inversely proportional to the resistance.

(iv) The sum of currents flowing through different resistances is equal to the total current flowing in the circuit, ie., $I = I_1 + I_2 + I_3$

(v) If R is the equivalent resistance of the circuit, then

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Thus, the reciprocal of equivalent resistance of the resistances connected in parallel is equal to the sum of the reciprocals of those resistances.

(vi) The equivalent resistance of the resistances connected in parallel is less than the smallest resistance among those resistances.

(vii) If n identical resistances of resistance R' are connected in parallel, then the equivalent resistance of the circuit will be

$$R = \frac{R'}{n}$$

(viii) On joining the resistances in parallel, the total resistance in the circuit decreases and the current taken from the cell increases.

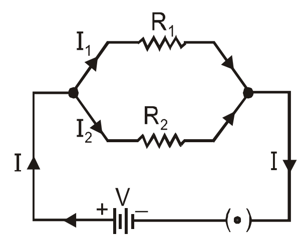
(ix) When two resistances are connected in parallel, the current flowing through these resistances is inversely proportional to the resistance.

Thus, $I_1 = \frac{V}{R_1}$

and $I_2 = \frac{V}{R_2}$

and $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

$\therefore$ $\frac{I_1}{I_2} = \frac{R_2}{R_1}$



and equivalent resistawnce

$$R = \frac{R_1 R_2}{R_1 + R_2}$$

Thus, $V = IR$

$$\therefore \quad I_1 = \frac{IR}{R_1} = \frac{I}{R_1}\left(\frac{R_1R_2}{R_1+R_2}\right) = \frac{IR_2}{(R_1+R_2)}$$

$$\text{and} \quad I_2 = \frac{IR}{R_2} = \frac{I}{R_2}\left(\frac{R_1R_2}{R_1+R_2}\right) = \frac{IR_1}{(R_1+R_2)}$$

Solved Examples

Ex.15 The resistance of two conductors in series is 40 ohm and their resistance becomes 7.5 ohm when connected in parallel. What are the resistances?

Sol. Series $R_1 + R_2 = 40\ \Omega$ .... (1)

Parallel $\frac{R_1R_2}{R_1 + R_2} = 7.5\ \Omega$

$\therefore R_1R_2 = 7.5 \times 40 = 300\ \Omega$

Since $(R_1 - R_2)^2 = (R_1 + R_2)^2 - 4R_1R_2$

$= 40^2 - 1200$

$= 400$

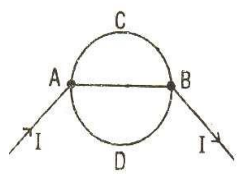
$R_1 - R_2 = 20$ .... (2)

Solve (1) and (2), to get $R_1 = 30\Omega$ and $R_2 = 10\ \Omega$

Ex.16 A wire of resistance $0.5\Omega m^{-1}$ is bent into a circle of radius 1 m. The same wire is connected across a diameter AB as shown in figure. The equivalent resistance is (in ohms)

(1) π (2) $\pi + 1$

(3) $\frac{\pi}{(\pi+2)}$ (4) $\frac{\pi}{(\pi+4)}$



Sol. There are three resistances, ACB, AB and ADB, that are in parallel. Length ACB = $\pi r = \pi$, length ADB = π and AB = 2 (because r = 1m). The resistances are $R_{ACB} = 0.5 \times \pi, R_{ADB} = 0.5\pi$, and $R_{AB} = 0.5 \times 2 = 1$. The equivalent resistance is

$$\frac{1}{R} = \frac{1}{R_{ACB}} + \frac{1}{R_{ADB}} + \frac{1}{R_{AB}}$$

$$= \frac{1}{0.5\pi} + \frac{1}{0.5\pi} + \frac{1}{1} = \frac{4+\pi}{\pi}$$

or $R = \frac{\pi}{\pi + 4}$ The correct Answer is (**4**)

Ex.17 Resistance R (R + 1), (R + 2) ....(R + n) Ω are connected in series, their resultant resistance will be -

(1) $(n + 1)\left[R + \frac{n}{2}\right]$ (2) $(n - 1)\left[R - \frac{n}{2}\right]$

(3) n (R + n) (4) n (R – n)

Sol. Suppose the resultant resistance of the given resistance be R', then

R' = R (R + 1) + (R + 2) + ... (R + n)

$= \frac{(n+1)}{2}$ [2R + (n + 1) – 1] = $\frac{(n+1)}{2}$

[2R + n] = (n + 1) $\left[R + \frac{n}{2}\right]$

[$\because$ sum of n terms in arithmetic series is

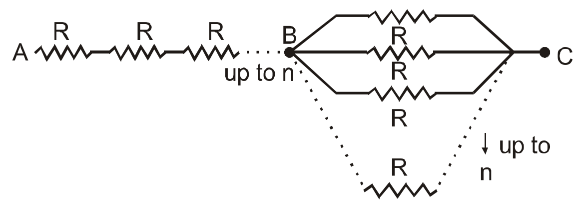
$S_n = \frac{n}{2}$ [2a + (n – 1) d]

Where a → is first term

d → is common difference

In the given question total terms are (n + 1)

Ex.18 In the following figure the resultant. Resistance between A and C will be -



(1) $R\left(\frac{n^2+1}{n}\right)$ (2) $R\left(\frac{n+1}{n}\right)$

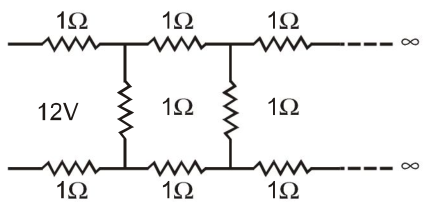
(3) $R\left(\frac{n^2-1}{n}\right)$ (4) $R\frac{n-1}{n}$

Sol. The resistance are connected in series between the points A and B and those between B and C are in parallel. Let R_1 and R_2 to be the resultant to these two combinations, then

$R_1 = nR$ and $R_2 = R/n$

$R' = R_1 + R_2 = nR + \frac{R}{n} = R\left(\frac{n^2+1}{n}\right)$

Ex19 In the following fig, the current drawn by the battery of 12 V supply (in amp) will be -

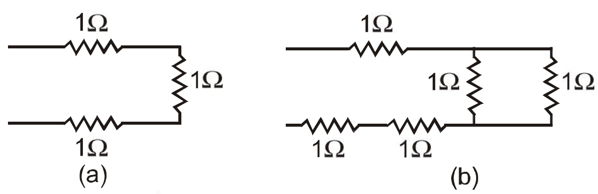


(1) $6(\sqrt{3}-1)$ (2) $6(\sqrt{3}+1)$

(3) $12(\sqrt{3}-1)$ (4) $12(\sqrt{3}+1)$

Sol. Let x the resultant resistance. If in the following small combination (a) is added, the value of x will remain unaffected. Hence the resultant circuit will be as shown in fig.

from fig (b) the resultant resistance

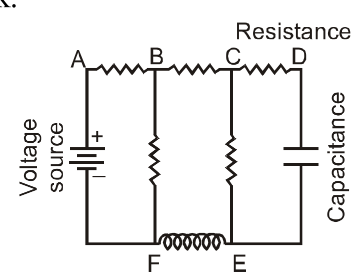


$x = 1 + \frac{1.x}{1+x} + 1 = 2 + \frac{x}{1+x}$ $x = 1 + \sqrt{3}$

Hence current $I = \frac{V}{R} = 12/(1+\sqrt{3}) = 6(\sqrt{3}-1)A$

TERMS RELATED WITH ELECTRICAL CIRCUITS

(a) Net work : A group of circuits connected to each other and formed by connecting electric source and different components such as resistance, inductor, capacitor, diode, transistor etc., is called network.



In the following figure a network has been shown :

(b) Branch : A part of the network through which same current flows, is called branch of the network. In the figure, AB, BC, CD, BF and BE are branches of the network.

(c) Node : The junction at which two or more branches of the network meet, is called node. In the figure B, C, E etc., are nodes.

(d) Loop : The closed path of the current in the circuit consisting of some branches, is called loop. In the figure ABFA, BCEFB are loops.

SOURCE OF EMF

It is a device which maintains a potential difference between two points in the circuit. Example. Primary cell, battery, solar cells, electric generators, Thermopile.

(i) EMF (Electromotive force)

In the interior of source of emf, positive charges move from a point of low potential (negative terminal) to a point of higher potential (positive terminal). The source of emf by doing work on the charges maintains a potential difference between its terminals. The amount of work done per unit charge is equal to the emf of the source.

$$E = \frac{W}{Q}$$

Unit of emf = volt = joule / coulomb

Dimensions $ML^2T^{-3}A^{-1}$

The emf of a cell is the potential difference between the two electrodes in an open circuit, i.e, when no current is drawn from the cell.

The source of energy required for moving charges from negative to positive terminal may be chemical (as in a cell or a battery), mechanical (in a generator), thermal (in a thermopile) or radiation (in a solar cell).

(ii) Internal Resistance (r) :

It is the resistance offered by the electrolyte of the cell to flow of charges (ions). The internal resistance can not be separated from the cell. It reduces the current that the emf can supply to the external circuit.

$$I = \frac{E}{R+r}$$

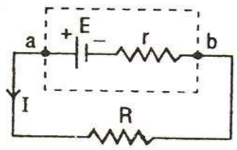
(iii) Terminal Potential Difference (V) :

The potential difference between the two electrodes of a cell in a closed circuit (when current is drawn from the cell) is called terminal potential difference (V). For figure 5, the terminal potential difference

$V = V_a - V_b$

$V = IR$

$= E - Ir$



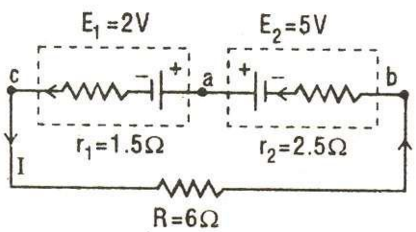
Ex.20 A battery of emf 10 V and internal resistance 3 ohm is connected to a resistor. If the current in the circuit is 0.5 A, what is the resistance of resistor? What is the terminal voltage of the battery when the circuit is closed?

Sol. (i) $I = \dfrac{E}{R+r}$ or $0.5 = \dfrac{10}{3+R}$

Solve to get $R = 17\ \Omega$

(ii) $V = E - Ir = 10 - 0.5 \times 3 = 8.5$ Volt

Ex.21 See Fig., (i) What is the current in the circuit,



(ii) What is the potential difference between points a and b,

(iii) What is the potential difference between points a and c.

Sol. (i) $I = \dfrac{E_2 - E_1}{r_1 + r_2 + R}$

(ii) Imagine going from a to E_2 to r_2 to b.

or $V_a - E_2 + Ir_2 = V_b$

or $V_a - V_b = E_2 - Ir_2$

(When the cell is beign discharged. The terminal potential difference is V = E – Ir)

$V_a - V_b = 5 - 0.3 \times 2.5$

$= 4.25$ Volt

[Alternatively, go from a to E_1 to r_1 to c to R to b, you will get the same answer,]

$V_a - E_1 - Ir_1 - IR = V_b$

or $V_a - V_b = E_1 + I(r_1 + R)$

$= 4.25$ Volt

(iii) Similarly, for $V_a - V_c$, go from a to E_1 to r_1 to c, then

$V_a - E_1 - Ir_1 = V_c$

(when the cell is being charged, then the terminal potential difference V = E + Ir)

$V_a - V_c = E_1 + Ir_1$

$= 2 + 0.45$

$= 2.45$ volt

GROUPING OF CELLS

(a) Each cell has definite emf E and internal resistance r.

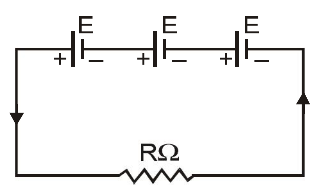
(b) Cells can be grouped in the following three ways

(i) Series grouping

(ii) Parallel grouping

(iii) MIxed grouping

(c) Series grouping : Total emf of all cells connected in series is equal to the sum of the emfs of individual cells and the total internal resistance is equal to the sum of the internal resistances of individual cells.



If n identical cells grouped in series are connected with an external resistance R, then the current

$I = \dfrac{nE}{R + nr}$

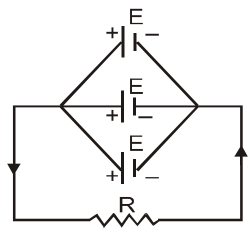
If r is negligible, then

$I = \dfrac{nE}{R}$

= n x current obtained from one cell.

When the internal resistance is negligible, the current becomes n times the current due to one cell.

(d) Parallel grouping : In parallel combination fo identical cells the total emf is equivalent to the emf E of one cell but equivalent internal resistance r' can be obtained as



$$\frac{1}{r'} = \frac{1}{r} + \frac{1}{r} + ... + \frac{1}{r} = \frac{n}{r}$$

$$\therefore \qquad r' = \frac{r}{n}$$

The current flowing from this parallel combination of cells through external resistance R

$$I = \frac{E}{R + \frac{r}{n}} = \frac{nE}{nR + r}$$

If $\frac{r}{n} > R$, the parallel combination is advantageous because the current becomes n times the current due to one cell.

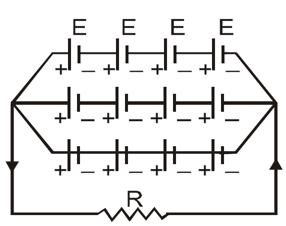
(e) Mixed grouping : If m rows of N cells are formed, then in each row there will be $n = \frac{N}{m}$ cells connected in series.

Equivalent emf = nE

Equivalent internal resistance = $\frac{nr}{m}$

The current through external resistance R

$$I = \frac{nE}{R + \frac{nr}{m}}$$



$$= \frac{mnE}{mR + nr}$$

$$= \frac{mnE}{\left(\sqrt{nr} - \sqrt{mR}\right)^2 + 2\sqrt{nmrR}}$$

Current will be maximum when nr = mR

or $\qquad R = \frac{nr}{m}$

Maximum current will be $I_{max} = \frac{e\sqrt{N}}{2\sqrt{rR}}$

Maximum Power Transfer from a D.C. Source :

* If an D.C. source is connected to a load resistance, then power is transfered from source to load. When load is varied, the current flowing through it changes and the power transferred to the load also changes. There is always internal resistance in each electric source. Due to variation of current the magnitude of power dissipation inside the source also changes. In order to provide more power to the load, there is more power dissipation in the source. Hence the power transfer to the load can be increased upto a certain limit.

* Power transfer to the load from a D.C. source is maximum when the resistance of the load is equal to the internal resistance of the source. This is called maximum power transfer theorem.

* If the internal reistance of a D.C. source of emf E is X and variable load resistance is R, then the current in the circuit

$$I = \frac{E}{X + R}$$

and the potential difference across the resistance R

$$V = IR = \frac{ER}{X + R}$$

$\therefore$ The power given to the load

$$P = I^2R = IV = \frac{V^2}{R}$$

or $$P = \frac{E^2R}{(X + R)^2}$$

* In case of maximum transfer of power

$$\frac{dP}{dR} = 0$$

or $$\frac{dP}{dR} = E^2\left[\frac{-2R}{(x + R)^3} + \frac{1}{(X + R)^2}\right] = 0$$

$$= E^2\frac{(X - R)}{(X + R)^3} = 0$$

$\therefore$ R = X

Maximum power delivered = $\frac{E^2}{4X}$

* The process of obtaining maximum power by varying load resistance is called matching of load and the load in this condition is called matched load.

Solved Examples

Ex.22 Twelve cells each having the same emf and negligible internal resistanc;e are kept in a closed box. Some of the cells are connected in the reverse order. This battery is connected in series with an ammeter, an external resistance R and two cells of the same type as in the box. The current when they aid the battery is 3 ampere and when they oppose, it is 2 ampere. How many cells in the battery are connected in reverse order?

Sol. Let n cells are connected in reverse order. Then emf of the battery is

$$E' = (12-n)E - nE$$

$$= (12-2n)E$$

In case (i)

$$I = \frac{E' + 2E}{R} = 3$$

or $E' + 2E = 3R$,

or $(14-2n)\ E = 3R$ .... (1)

In case (ii)

$$I = \frac{E' - 2E}{R} = 2$$

or $E' - 2E = 2R$,

or $(10-2n)\ E = 2R$ .... (2)

Dividing (1) and (2)

$$\frac{14-2n}{10-2n} = \frac{3}{2}$$

or $n = 1$

One cell is connected in reverse order.

Ex.23 8 cells are grouped to obtained the maximum current through a resistance of 2 ohm. If the emf of each cell is 2 volt and internal resistance is 1 ohm. Grouping of cells will have :

(1) all cells in series

(2) all cells in parallel

(3) two rows of four cells

(4) four rows of two cells

Sol. For maximum current, the number of rows

$$m = \sqrt{\frac{Nr}{R}} = \sqrt{\frac{8\times 1}{2}} = 2$$

∴ They will be grouped in two rows of 4 cells.

Answer will be (**3**)

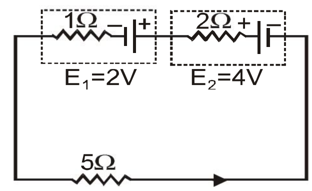
Ex.24 Current flowing in the following circuit will be

(1) 7.5

(2) 0.75 A

(3) 2.5 A

(4) 0.25 A



Sol. $i = \frac{E_2 - E_1}{1+2+5} = \frac{4-2}{8}$

$$= \frac{2}{8} = 0.25\ A$$

KIRCHHOFF'S LAWS

Kirchhoff proposed two laws by application of which the distribution of current among the conductors of complex electrical circuit or networks can be found out. These are:

(a) Current law, (b) Voltage law.

(a) Current law :

(i) In an electric circuit, the algebraic sum of the currents meeting at any junction is zero. ie.,

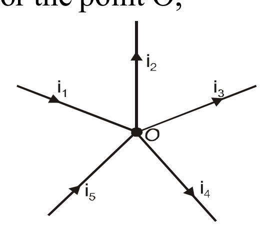
$$\Sigma i = 0$$

(ii) While applying this law following sign convention is used : The current going towards the junction is taken as positive while that going away from the junction is taken as negative.

From Kirchhoff's law for the point O,

$$i_1 - i_2 - i_3 - i_4 + i_5 = 0$$

or $i_1 + i_5 = i_2 + i_3 + i_4$



(iii) In other words, the sum of the currents flowing towards the junction is equal to the sum of currents flowing away from the junction.

(iv) This law represents the law of conservation of charge, ie., when a constant current flows in a circuit, the charge does not accummulate at a junction or at any point of the circuit.

(b) Voltage law :

(i) The algebraic sum of the products of the current and the resistance in a closed circuit is equal to the algebraic sum of the applied emf ie.,

$$\Sigma iR = \Sigma \text{emf}$$

or $$\Sigma iR - \Sigma \text{emf} = 0$$

(ii) This law is a general form of Ohm's law.

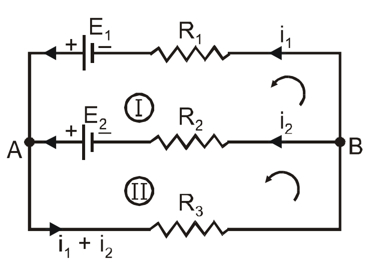
(iii) This law is applicable only in closed circuits.

(iv) Sign convention is as follows:

(A) When we traverse in the direction of current the product of the current and the resistance is taken as positive while that in the opposite direction of flow of current it is taken as negative.

(B) When we traverse from negative electrode to positive electrode of a cell the emf is taken as positive while that in the opposite direction emf is taken as negative.

(v) In the following circuit :



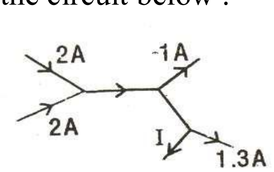
For (I) mesh : $i_1R_1 - i_2R_2 = E_1 - E_2$

For (II) mesh : $i_2R_2 + (i_1 + i_2)R_3 = E_2$

Solved Examples

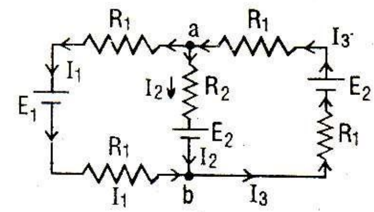
Ex.25 Find the value of I in the circuit below :

Sol. I = 4 – 1 – 1.3
= 1.7 A



Ex.26 Figure shows a circuit whose elements have the following vaues : $E_1 = 2V$, $E_2 = 6V$, $R_1 = 1.5$ ohm and $R_2 = 3.5$ ohm. Find the curents in the three branches of the circuit.

Sol. From junction rule at **a**



$I_3 = I_1 + I_2$ .... (1)

For the left hand loop **aE₁ba,**

$$I_1R_1 - E_1 - I_1R_1 + E_2 + I_2R_2 = 0$$

or $E_2 - E_1 = 2I_1R_1 - I_2R_2$

or $4 = 3I_1 - 3.5I_2$ .... (2)

For the loop on right hand side starting from **a** (clockwise)

$$I_3R_1 - E_2 + I_3R_1 + E_2 + I_2R_2 = 0$$

$$2I_3R_1 + I_2R_2 = 0$$

Use $I_3 = I_1 + I_2$ Eq. (1)

$$2I_1R_1 + I_2(2R_1 + R_2) = 0$$

$3I_1 + 6.5I_2 = 0$ .... (3)

From (2), use $3I_1 = 4 + 3.5I_2$, in (3), to get

$$4 + 3.5I_2 + 6.5I_2 = 0$$

or $I_2 = -0.4\text{ Amp}$

Substitute it in (2), to get

$$I_1 = 0.87\text{ Amp}$$

Therefore, $I_3 = I_1 + I_2 = 0.87 + (-0.4)$

$= 0.47\text{ Amp}$

Ex.27 What is the potential difference between points **a** and **b** in the circuit of above figure.

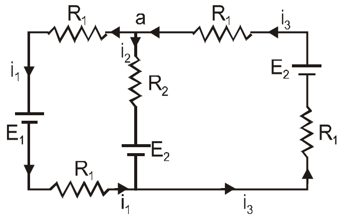
Sol. In going from a (potential V_a) to b (potential V_b), we have

$$V_a - I_2R_2 - E_2 = V_b$$

$$V_a - V_b = E_2 + I_2R_2$$

$$= 6 + (-0.4) \times (3.5)$$

$$= 6 - 1.4 = 4.6 \text{ volt}$$



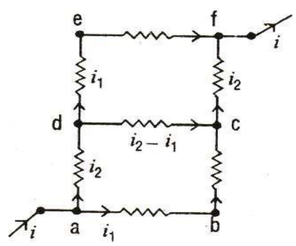
Ex.28 The current in a,b for the circuit given here is (each resistance = R ohm).

(1) i/5

(2) 2i/5

(3) 3i/5

(4) i



Sol. From symmetry the currents in various branches are as shown. Now for path abc, $V_a - V_c = 2Ri_1$, and for path adc, $V_a - V_c = 2Ri_2 - Ri_1$. Therefore,

$$2Ri_1 = 2Ri_2 - Ri_1$$

or $\quad 3i_1 = 2i_2$

Further $i_1 + i_2 = i$. Thus $i_1 + \left(\frac{3}{2}\right)i_1 = i$ or

$i_1 = \left(\frac{2}{5}\right)i$. The answer is (2) 2/5 i irrespective of the value of R]

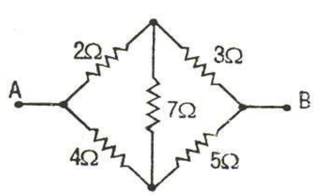
Ex.29 Five resistances are connected as shown in the figure. The equivalent resistance between the point A and B will be

(1) 3 ohm

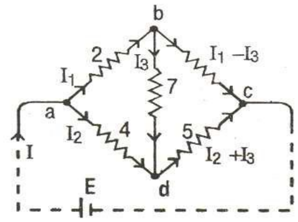
(2) 3.2 ohm

(3) 2.5 ohm

(4) 4 ohm



Sol. It is unbalanced Wheatstone bridge with current distributions shown in figure. Consider loops (i) abda, (ii) bcdb and (iii) adcEa



$2I_1 - 4I_2 + 7I_3 = 0$ ....(1)

$3I_1 - 5I_2 - 15I_3 = 0$ ....(2)

$E = 9I_2 + 5I_3$ ....(3)

from (1) $I_3 = \frac{-2}{7}I_1 + \frac{4}{7}I_2$ ....(4)

use it in (2) and solve

$51I_1 = 95\ I_2$

use (4) and (5) in (3), solve to obtain

$E = (3283/357)I_2$

use $I = I_1 + I_2$ with $I_1 = (95I_2 / 5I)$

It given $I = (146/51)\ I_2$ use it in (6)

$$E = \frac{3283}{357} \times \frac{51}{146}\ I$$

$$= 3.2\ I$$

The equvialent resistance is E/I = 3.2 ohm. The correct answer is (2)

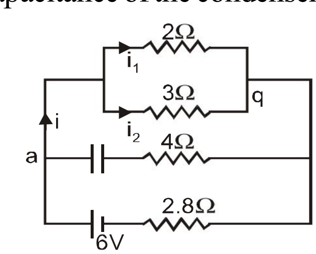
Ex.30 The steady state current in a 2 Ω resistor shown in fig will be-(The internal resistance of the battery is negligible and the capacitance of the condenser C is 0.2μF).

(1) 1.5 A

(2) 0.9 A

(3) 1.2 A

(4) 1.3 A



Sol. In steady state the branch containing capacitance acts as the open circuit since capacitance offers infinite resistance to d.c. The capacitance simply collects charge. The effective resistance of 2Ω and 3Ω resistors connected in parallel is

$$R' = \frac{R_1R_2}{R_1 + R_2} = \frac{2 \times 3}{2 + 3} = \frac{6}{5} = 1.2\Omega$$

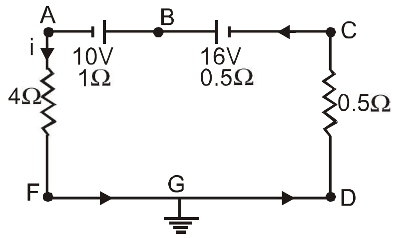
current drawn from cell, $i = \frac{E}{R} = \frac{6}{4} = 1.5\,A$

Potential difference across pq = iR' = 1.5 × 1.2

= 1.8V

Current in 2Ω resistor, $i_1 = \frac{V}{2} = \frac{1.8}{2} = 0.9\,A$

Ex.31 From the fig. determine



(i) potential at A,

(ii) potential at C, and

(iii) reading of the voltmeter connected across the 10V bettery-

Sol. The current in circuit is (consider loop (CBAFGDC)

$$I = \frac{E_2 - E_1}{r_1 + r_2 + R_1 + R_2} = \frac{16-10}{1+0.5+4+0.5} = \frac{6}{6} = 1A$$

(i) $V_A - V_F = IR = 4$ volt

Because $V_F = 0$ (grounded), therefore $V_A = 4$ volt

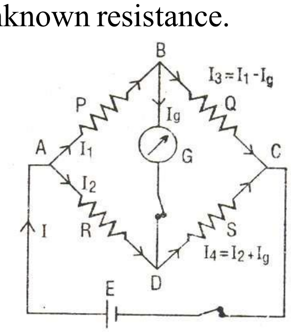
(ii) $V_D - V_C = 1 \times 0.5 = 0.5$ volt

∴ $V_D = 0$ (grounded), So $V_C = -0.5$ volt

(iii) The 10V battery is being charged therefore V = E + Ir = 10 + 1 × 1 = 11 volt

WHEATSTONE'S BRIDGE

Four resistances P, Q, R S connected as in figure forms a Wheatstone bridge. Conventionally P, Q are called ratio arms, R is a variable resistance and S is some unknown resistance.



Principle

When no current flows in the galvanometer G(I_g = 0), then the potentials of points B and D are equal. Then, the bridge is said to be balanced, and

$$\frac{P}{Q} = \frac{R}{S}$$

Knowledge of any three, say, P, Q, R determines the fourth S.

Proof : From Kirchhoff's second rule, for loop ABDA,

$$-I_1P + 0 \times G + I_2R = 0, \text{ or}$$

$$I_1P = I_2R$$

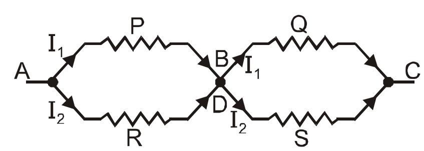
For loop BCDB, $(\because I_g = 0, I_3 = I_1, I_4 = I_2)$,

$$= I_1Q + I_2S + 0 \times G = 0, \text{ or}$$

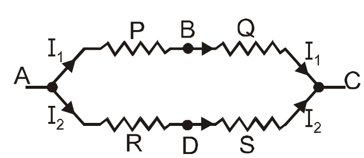
$$I_1Q = I_2S$$

Thus $\frac{P}{Q} = \frac{R}{S}$

Comment (View-I) Since B and D are at the same potential for a balanced bridge, they can be thought to be the same point, so that Wheatstone bridge appears as in figure.



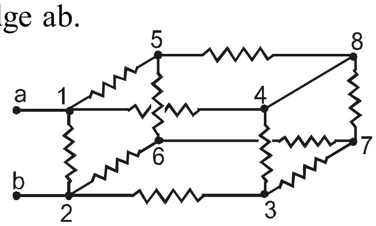
(View-2) Since no current flows through the galvanometer, we can ignore the G arm (as if it is not connected). Then, the balanced Wheatstone bridge, appears as in figure.



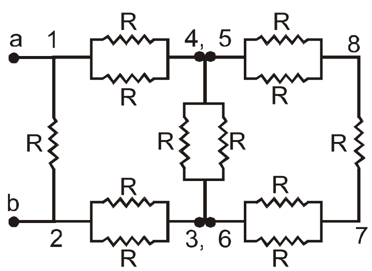
These views help a great deal in determining equivalent resistance.

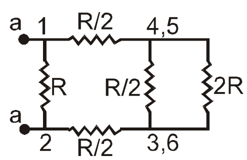
Solved Examples

Ex.32 Figure shows a cube made of 12 resistances, each of resistance R. Find the equivalent resistance across a cube edge ab.



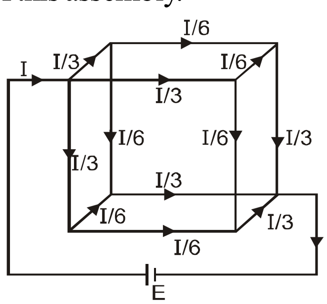
Sol. From the consideration of symmetry alone we notice that points 4 and 5 must be at the same potential. So must be points 3 and 6. This implies that the circuit can be redrawn with points 4 and 5 connected, and points 3 and 6 connected, as shown in figure. This figure further reduces to the combination shown in figure.





The equivalent resistance between a and b is (calculate) = $\left(\frac{7}{12}\right)R$

Ex.33 Twelve equal wires, each of resistance R ohm are connected so as to form a skeleton cube. An electric current enters this cube from one corner and leaves out the diagonally opposite corner. Calculate the total resistance of this assembly.



Sol. Let ABCDEFGH be skeleton cube formed of twelve equal wires each of resistance R. Let a battery of e.m.f. E be connected across A and G. Let the total current entering at the corner A and leaving the diagonally opposite corner G be I. By symmetry the distribution of currents in wires of cube, according to Kirchoff's I law is shown in fig. Applying Kirchoff's II law to mesh ADCGEA, weget

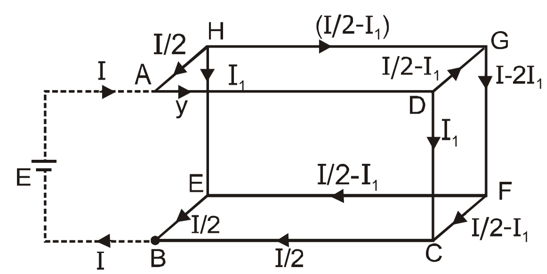
$$-\frac{1}{3}R-\frac{1}{6}R-\frac{1}{3}R+E=0$$

or $\quad E=\frac{5}{6}IR \quad$ ....(1)

If R_{AB} is equivalent resistance between corners A and B, then from ohm's law comparing (1) and (2), we get

$$IR_{AB}=\frac{5}{6}IR$$

Ex.34 Eleven equal wires each of resistance 2Ω form the edges of an incomplete skeleton cube. Find the total resistance between points A and B of the vacant edge.



Sol. Let a battery of e.m.f. E' is applied between points A and B.

Let a current I, enter through point A.

If R_{AB} is equivalent resistance between points A and B, then from ohm's law $\quad R_{AB}\,I = E$

The distribution of currents, keeping in mind symmetry condition, is shown is fig.

Let R = (2Ω) be the resistance of each wire.

Applying Kirchoff's II law to mesh DGFC, we get

$$\left(\frac{1}{2}-I_1\right)R+(1-2I_1)R+\left(\frac{1}{2}-I_1\right)R-I_1R=0$$

or $$2\left(\frac{1}{2}-I_1\right)+(1-2I_1)-I_1=0$$

or $$2I-5I_1=0 \quad \text{or} \quad I_1=\frac{2}{5}I \quad ....(2)$$

Applying Kirchoff's II law to external circuit AHEBE', we get

$$\frac{1}{2}R+I_1R+\frac{1}{2}R=E^1$$

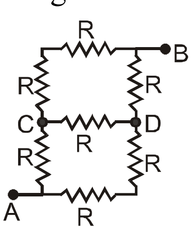
$$IR+\frac{2}{5}\,IR=E' \text{ [Using (2)]}$$

or $$\frac{7}{5}\,IR=E' \quad ....(3)$$

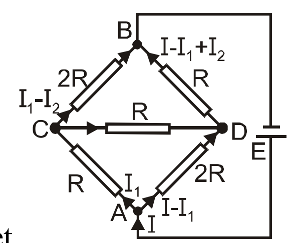
Comparing (1) and (3), we get $R_{AB}\,I=\frac{7}{5}IR$

i.e. $R_{AB}=\frac{7}{5}R=\frac{7}{5}\times 2=2.4\Omega$

Ex.35 Find the equivalent resistance between points A and B in the following circuit.



Sol. This is Wheatstone bridge but is unbalanced. To find equivalent resistance, we imagine, that a cell of emf E is connected between points A and B. Then the combination looks like figure.



For the loop **ACDA**

$$\boxed{2I=3I_1+I_2}$$

For the loop **BDCB**

$$\boxed{I=3I_1-4I_2}$$

Solving the two we get

I_1 (3/5) I, I_2 = (1/5)I

Consider loop **ADBEA,** then

$E = (I - I_1)\,2R + (I - I_1 + I_2)R$

$= (4R/5)I + (3R/5)I$

$$\boxed{E=(7R/5)I}$$

Therefore the effective resistance is

$R_{eq} = E/I$

$= 7R/5$

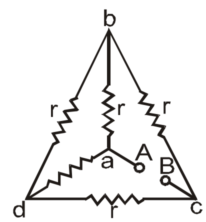
Ex.36 In the adjoining network of resistors, each is of resistance r ohm, the equivalent resistance between points A and B is

(1) 5r

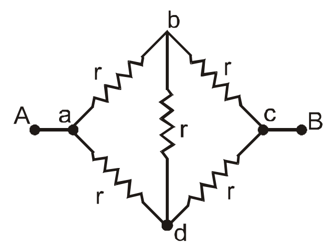
(2) 2r/3

(3)r

(4) r/2



Sol. Imagine, Aa being pulled on the left side, then abcd becomes a balanced Wheatstone bridge (figure).



The arm bd can be ignored.

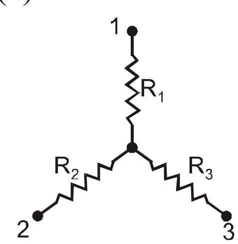
Then resistance between A, B becomes = r.

The answer is (3)

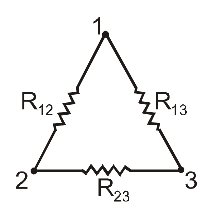
UNBALANCED WHEATSTONE BRIDGE

(i) STAR DELTA CONVERSIONS

The combination of resistances shown in figure (a) is called a star connection and that shown in figure (b) is called a delta connection.



(a)



(b)

These two arrangements are electrically equivalent for the resistances measured between any pair of terminals. A star connection can be replaced by a delta, and a delta can be replaced by a star.

STAR to DELTA

If resistances R_1, R_2 and R_3 are known and connected in star configuration (as in figure (a)) then it can be replaced by a delta configuration with following resistances.

$$R_{12} = R_1 + R_2 + \frac{R_1R_2}{R_3}$$

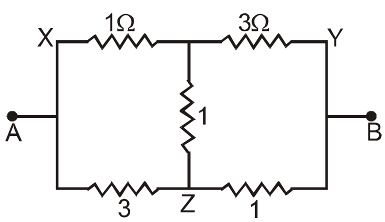
$$R_{23} = R_2 + R_3 + \frac{R_2R_3}{R_1}$$

$$R_{13} = R_1 + R_3 + \frac{R_1R_3}{R_2}$$

(Note $R_{12} = R_{21}, R_{13} = R_{31}, R_{23} = R_{32}$)

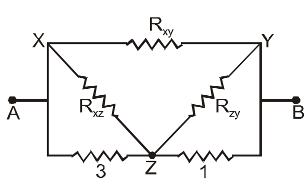
Solved Examples

Ex.37 Consider the unbalanced wheatstone bridge shown in Fig. (a) Find its equivalent resistance between A and B (all values are in ohm)

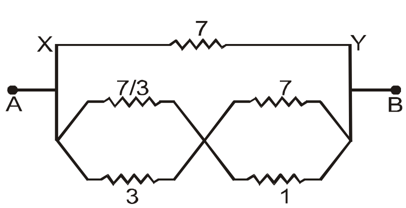


(a)

Sol. Consider the star combination between XYZ. It can be reduced into a delta combination. Then the (see fig.a) looks like (see fig.b) with



(b)



(c)

$$R_{xy} = 1 + 3 + \frac{1\times 3}{1} = 7$$

$$R_{xz} = 1 + 1 + \frac{1\times 1}{3} = 7/3$$

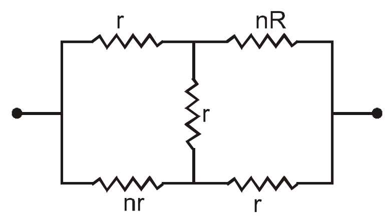
$$R_{zy} = 1 + 3 + \frac{1\times 3}{1} = 7$$

Thus the equivalent diagram now looks like Fig. (c). The resistance between ends A and B is then easily determined. Its value is.

$$R_{AB} = \frac{5}{3}$$

Thus the answer is $R_{AB} = 5/3$ ohm. (This problem can also be solved by delta to star conversion, see example 39).

Comment. For the circuit of Fig., memorize the following formula for equivalent

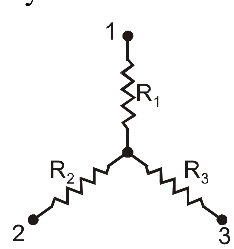


resistance. (If you are aspiring for engineering, prove it by using star-delta conversion).

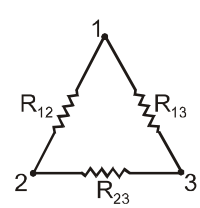
$$\boxed{R_{equivalent} = \left(\frac{3n+1}{n+3}\right)r}$$

(ii) DELTA to STAR Conversion

Consider Fig. (a) and (b) again. If resistance R_{12}, R_{23} and R_{13} are known and connected in a delta configuration as in Fig. (b), then it can be replaced by a star connection with the following resistances.



(a)



(b)

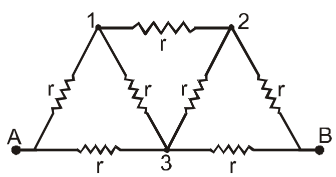
$$R_1 = \frac{R_{12}R_{13}}{R_{12}+R_{13}+R_{23}}$$

$$R_2 = \frac{R_{23}R_{21}}{R_{12}+R_{13}+R_{23}}$$

$$R_3 = \frac{R_{31}R_{32}}{R_{12}+R_{13}+R_{23}}$$

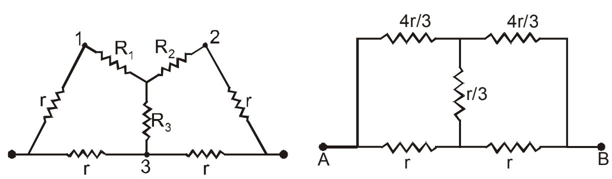
(note $R_{13} = R_{31}$, $R_{23} = R_{32}$, $R_{12} = R_{21}$)

Ex.38 Each resistance in the network is of r ohm. Then calculate the equivalent resistance between the terminals A and B.



(a)

Sol. Consider the delta connection between points 123. Converting it into a star connection, the Fig. (a) now looks like Fig. (b), with resistances.



(b) (c)

$$R_1 = \frac{r \times r}{r+r+r} = \frac{r}{3} \quad R_2 = \frac{r}{3} \quad R_3 = \frac{r}{3}$$

Thus Fig. (b), reduce to a balanced wheatstone bridge see fig. (c), whose equivalent resistance is

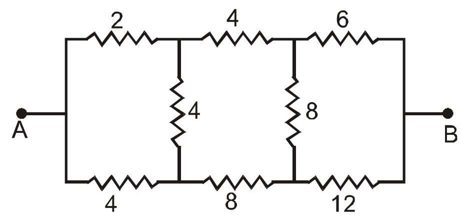
$$\boxed{R_{eq} = \frac{8}{7}r}$$ (Memorize the result)

Comment – The Star-delta conversion is useful if you do not wish to use Krichhoff's laws for solving equivalent resistance determination problems. However, if numerical values are not given, at first sight, it leads to tedious algebra, which then leads to simpler expressions. If you have patience verify the following two results.

(i) The equivalent resistance for the circuit of Fig. is

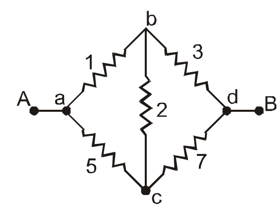
$$R_{eq} = \frac{r(3R+r)}{3r+R}$$ (Memorize it)

(ii) The equivalent resistance for the circuit of Fig.is (all resistance are in ohm),



R_{AB} = 8 ohm.

Ex.39 Consider the unbalanced wheatstone bridge shown in Fig. (a). Find the equivalent resistance between the points A and B (All resistances are in ohms).



(a)

Ans. Consider one of the delta combination, say abc. Then converting it into

equivalent star combination, we find fig. (b). A direct use of the conversion formula give.

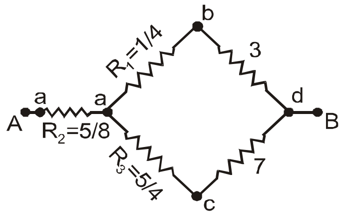
$$R_1 = \frac{1 \times 2}{1+2+5} = \frac{1}{4}$$

$$R_2 = \frac{5 \times 1}{8} = \frac{5}{8}$$

$$R_3 = \frac{2 \times 5}{8} = \frac{5}{8}$$

The R_{AB}, now can be calculated from the simplied Fig. (b). It gives

$$R_{AB} = \frac{5}{8} + \frac{13}{4} || \frac{33}{4}$$



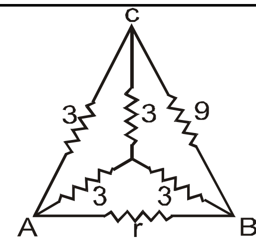
(b)

$$= \frac{5}{8} + \frac{429}{184} = \frac{544}{184}$$

or **$R_{AB} = 2.96\ \Omega$**

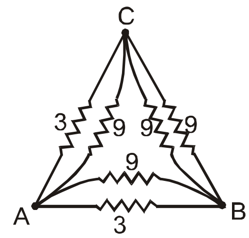
Solve example 37 by the above procedure

Ex.40 Consider the resistance combination shown in Fig.(a) Then determine the equivalent resistance betwee the terminals A and B
(all resistances are in ohms).



(a)

Sol. In Fig. (a), there is a star connection at the centre. This can be easily converted into a delta connection, with



(b)

$R_{12} = R_{13} = R_{23} = 3 + 3 + \frac{3 \times 3}{3} = 9$ the resulting diagram is then shown in Fig. (b). Therefore the equivalent resistance between terminal is (solve)

$R_{AB} = \frac{27}{16}$ ohm.