NEWTON LAW OF MOTION

1. FORCE

A pull or push which changes or tends to change the state of rest or of uniform motion or direction of motion of any object is called force. Force is the interaction between the object and the source (providing the pull or push). It is a vector quantity.

Effect of resultant force :

- may change only speed
- may change only direction of motion.
- may change both the speed and direction of motion.
- may change size and shape of a body

unit of force : newton and $\frac{kg.m}{s^2}$ (MKS System)

dyne and $\frac{g.cm}{s^2}$ (CGS System)

1 newton = 10^5 dyne

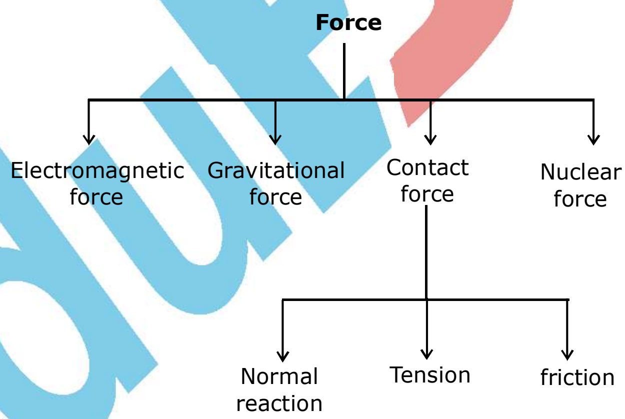
Kilogram force (kgf)

The force with which earth attracts a 1 kg body towards its centre is called kilogram force, thus

$$kgf = \frac{\text{Force in newton}}{g}$$

Dimensional Formula of force : [MLT⁻²]

- For full information of force we require
  - → Magnitude of force
  - → direction of force
  - → point of application of the force

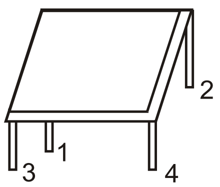


Contact force :

Forces which are transmitted between bodies by short range atomic molecular interactions are called contact forces. When two objects come in contact they exert contact forces on each other.

Normal force (N) :

It is the component of contact force perpendicular to the surface. It measures how strongly the surfaces in contact are pressed against each other. It is the electromagnetic force. A table is placed on Earth as shown in figure.



2. NEWTON'S FIRST LAW OF MOTION :

According to this law "A system will remain in its state of rest or of uniform motion unless a net external force act on it.

1[st] law can also be stated as "If the net external force acting on a body is zero, only then the body remains at rest."

- The word external means external to the system (object under observation), interactions within the system has not to be considered.
- The word net means the resultant of all the forces acting on the system.
- Newton's first law is nothing but Galileo's law of inertia.
- Inertia means inability of a body to change its state of motion or rest by itself.
- The property of a body that determines its resistance to a change in its motion is its mass (inertia). Greater the mass, greater the inertia.
- An external force is needed to set the system into motion, but no external force is needed to keep a body moving with constant velocity in its uniform motion.
- Newton's laws of motion are valid only in a set of frame of references, these frames of reference are known as inertial frames of reference.
- Generally, we take earth as an inertial frame of reference, but strictly speaking it is not an inertial frame.
- All frames moving uniformly with respect to an inertial frame are themselves inertial.
- We take all frames at rest or moving uniformly with respect to earth, as inertial frames.

3. NEWTON'S SECOND LAW OF MOTION :

Newton's second law states, "The rate of change of a momentum of a body is directly proportional to the applied force and takes place in the direction in which the force acts"

i.e., $\vec{F} \propto \frac{d\vec{p}}{dt}$ or $\vec{F} = k\frac{d\vec{p}}{dt}$

where k is a constant of proportionality.

$\vec{p} = m\vec{v}$, So $\vec{F} = k\frac{(dm\vec{v})}{dt}$

For a body having constant mass,

$\Rightarrow \quad \vec{F} = km\frac{d\vec{v}}{dt} = km\vec{a}$

From experiments, the value of k is found to be 1.

So, $\vec{F}_{net} = m\vec{a}$

- Force can't change the momentum along a direction normal to it, i.e., the component of velocity normal to the force doesn't change.
- Newton's 2[nd] law is strictly applicable to a single point particle. In case of rigid bodies or system of particles or system of rigid bodies, $\vec{F}$ refers to total external force acting on system and $\vec{a}$ refers to acceleration of centre of mass of the system. The internal forces, if any, in the system are not to be included in $\vec{F}$.
- Acceleration of a particle at any instant and at a particular location is determined by the force (net) acting on the particle at the same instant and at same location and is not in any way depending on the history of the motion of the particle.

4. APPLICATIONS :

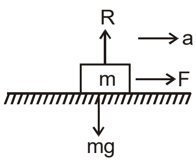
4.1 Motion of a Block on a Horizontal Smooth Surface.

Case (i) : When subjected to a horizontal pull :

The distribution of forces on the body are shown. As there is no motion along vertical direction, hence,

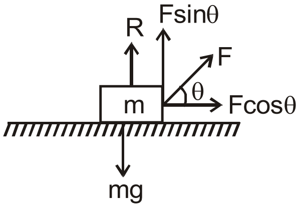
R = mg

For horizontal motion F = ma or a = F/m



Case (ii) : When subjected to a pull acting at an angle (θ) to the horizontal :

Now F has to be resolved into two components, F cosθ along the horizontal and F sin θ along the vertical direction.



For no motion along the vertical direction.
we have R + F sin θ = mg
or R = mg – F sin θ

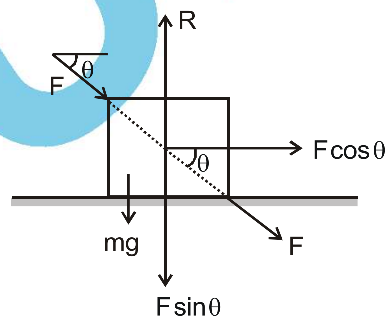
: Hence $R \neq mg$. R < mg

For horizontal motion

$F \cos\theta = ma, \; a = \dfrac{F\cos\theta}{m}$

Case (iii) : When the block is subjected to a push acting at an angle θ to the horizontal : (down ward)

The force equation in this case
R = mg + F sin θ



: $R \neq mg$, R > mg

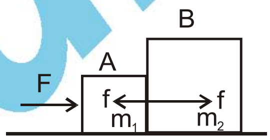
For horizontal motion

$F \cos\theta = ma, \; a = \dfrac{F\cos\theta}{m}$

4.2 Motion of bodies in contact.

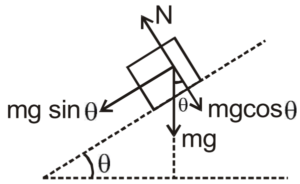
Case (i) : Two body system :

Let a force F be applied on mass m_1



4.3 Motion of a body on a smooth inclined plane :

Natural acceleration down the plane = g sin θ
Driving force for acceleration a up the plane, F=m(a+ gsinθ)
and for an acceleration a down the plane, F=m(a – gsinθ)



5. NEWTONS' 3RD LAW OF MOTION :

Statement : "To every action there is equal and opposite reaction".

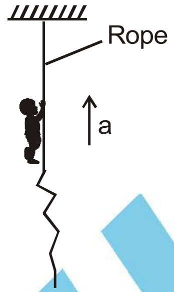
But what is the meaning of action and reaction and which force is action and which force is reaction? Every force that acts on body is due to the other bodies in environment. Suppose that a body A experiences a force $\vec{F}_{AB}$ due to other body B. Also body B will experience a force $\vec{F}_{BA}$ due to A. According to Newton third law two forces are equal in magnitude and opposite in direction Mathematically we write it as

$$\vec{F}_{AB} = -\vec{F}_{BA}$$

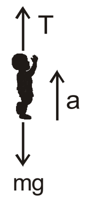
Here we can take either $\vec{F}_{AB}$ or $\vec{F}_{BA}$ as action force and other will be the reaction force.

: (i) Action-Reaction pair acts on two different bodies.

(ii) Magnitude of force is same.
(iii) Direction of forces are in opposite direction.
(iv) For action-reaction pair there is no need of contact



F.B.D of man



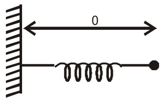
5.1 Climbing on the Rope :

Now three condition arises.
if $T > mg \Rightarrow$ man accelerates in upward direction
$T < mg \Rightarrow$ man accelerates in downward direction
$T = mg \Rightarrow$ man's acceleration is zero

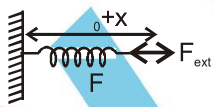
* Either climbing or decending on the rope man exerts force downward

6. SPRING FORCE :

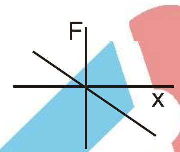
Every spring resists any attempt to change its length; when it is compressed or extended, it exerts force at its ends. The force exerted by a spring is given by $F = -kx$, where x is the change in length and k is the stiffness constant or spring constant (unit Nm^{-1})
When spring is in its natural length, spring force is zero.



$F = 0$



$F = -kx$



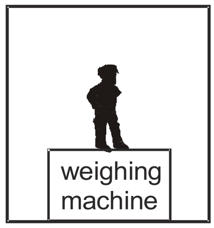
Graph between spring force v/s x

7. WEIGHING MACHING :

A weighing machine does not measure the weight but measures the force. exerted by object on its upper surface or we can say weighing machine measure normal force on the man.

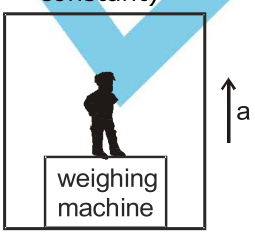
7.1 Motion in a lift :

(A) If the lift is unaccelerated ($v = 0$ or constant)
In this case no pseudo force act on the man

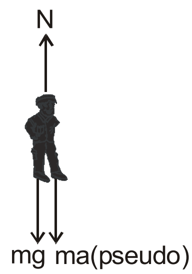


In this case the F.B.D. of the man
$N = mg$
In this case machine read the actual weight

(B) If the lift is accelerated upward.
(where a = constant)



F.B.D of man with respect to lift



So weighing machine read
$N = m(g + a)$
Apparent weight
$N >$ Actual weight (mg)

(c) If the lift is accelerated down ward.

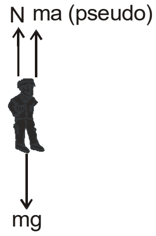
F.B.D of man with respect to lift

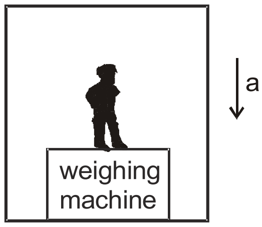
So weighing machine read

$N = m(g - a)$

Apparent weight

$N <$ Actual weight (mg)





Note :

(i) If $a = g \Rightarrow N = 0$

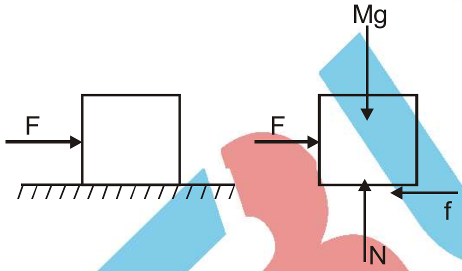
Thus in a freely falling lift, the man will experience a state of weightlessness

(ii) If the lift is accelerated downwards such that $a > g$: So the man will be accelerated upward and will stay at the ceiling of the lift.

(iii) Apparent weight is greater than or less than actual weight only depends on the direction and magnitude of acceleration. Magnitude and direction of velocity doesn't play any roll in apparent weight.

8. FRICTION :

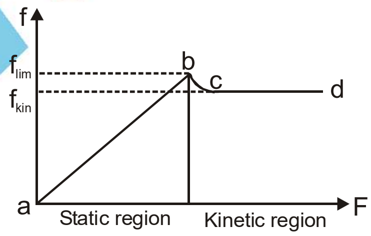
Friction is a contact force that opposes the relative motion or tendency of relative motion of two bodies.



Consider a block on a horizontal table as shown in the figure. If we apply a force, acting to the right, the block remains stationary if F is not too large. The force that counteracts F and keeps the block in rest from moving is called frictional force. If we keep on increasing the force, the block will remain at rest and for a particular value of applied force, the body comes to state of about to move. Now if we slightly increase the force from this value, block starts its motion with a jerk and we observe that to keep the block moving we need less effort than to start its motion.

So from this observation, we see that we have three states of block, first, block does not move, second, block is about to move and third, block starts moving. The friction force acting in three states are called static frictional force, limiting frictional force and kinetic frictional force respectively. If we draw the graph between applied force and frictional force for this observation its nature is as shown in figure.

8.1 Static frictional force



When there is no relative motion between the contact surfaces, frictional force is called static frictional force. It is a self-adjusting force, it adjusts its value according to requirement (of no relative motion). In the taken example static frictional force is equal to applied force. Hence one can say that the portion of graph ab will have a slope of 45º.

The Direction of Static Friction

The direction of static friction on a body is such that the total force acting on it keeps it at rest with respect to the body in contact.

The direction of static friction is as follows. For a moment consider the surfaces to be frictionless. In absence of friction the bodies will start slipping against each other. One should then find the direction of friction as opposite to the velocity with respect to the body applying the friction.

8.2 Limiting Frictional Force

This frictional force acts when body is about to move. This is the maximum frictional force that can exist at the contact surface.

(i) The magnitude of limiting frictional force is proportional to the normal force at the contact surface.

$f_{lim} \propto N \Rightarrow f_{lim} = \mu_s N$

Here μ_s is a constant the value of which depends on nature of surfaces in contact and is called as 'coefficient of static firction'.

8.3 Kinetic Frictional Force

Once relative motion starts between the surface in contact, the frictional force is called as kinetic frictional force. The magnitude of kinetic frictional force is also proportional to normal force.

$f_k = \mu_k N$

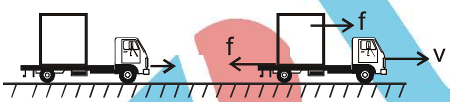
From the previous observation we can say that $\mu_k < \mu_s$

Although the coefficient of kinetic friction varies with speed, we shall neglect any variation i.e., when relative motion starts a constant frictional force starts opposing its motion.

Direction of Kinetic Friction

The kinetic friction on a body A slipping against another body B is opposite to the velocity of A with respect to B.

It should be carefully noted that the velocity coming into picture is with respect to the body applying the force of friction.



Suppose we have a long truck moving on a horizontal road. A small block is placed on the truck which slips on the truck to fall from the rear end. As seen from the road, both the truck and the block are moving towards right, of course the velocity of the block is smaller than that of the truck. What is the direction of the kinetic friction acting on the block due to the truck ? The velocity of the block as seen from the truck is towards left. Thus, the friction on the block is towards right. The friction acting on the truck due to the block is towards left.

9. MINIMUM FORCE REQUIRED TO MOVE THE PARTICLE :

A body of mass m rests on a horizontal floor with which it has a coefficient of static friction μ. It is desired to make the body slide by applying the minimum possible force F.

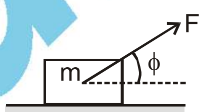
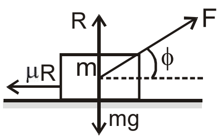


Fig. A

Let the applied force F be at angle ϕ with the horizontal



R = Normal force

Fig. B

For vertical equilibrium,

$R + F\sin\phi = mg$ or, $R = (mg - F\sin\phi)$...(i)

For horizontal equilibrium i.e. when the block is just about to slide,

$F\cos\phi = \mu R$ ...(ii)

Substituting for R,

$F\cos\phi = \mu\,(mg - F\sin\phi)$ or $F = \mu mg / (\cos\phi + \mu\sin\phi)$

for minimum F $(\cos\phi + \mu\sin\phi)$ is maximum,

$\Rightarrow$ Let $x = \cos\phi + \mu\sin\phi$

$$\frac{dx}{d\phi} = -\sin\phi + \mu\cos\phi$$

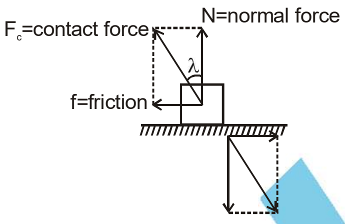
for maximum of x, $\frac{dx}{d\phi} = 0$

$\tan\phi = \mu$ and at this value of ϕ

$$F_{min} = \frac{\mu mg}{\sqrt{1+\mu^2}}$$

10.FRICTION AS THE COMPONENT OF CONTACT FORCE :

When two bodies are kept in contact, electromagnetic forces act between the charged particles at the surfaces of the bodies. As a result, each body exerts a contact force on other The magnitudes of the contact forces acting on the two bodies are equal but their directions are opposite and hence the contact forces obey Newton's third law.



The direction of the contact force acting on a particular body is not necessarily perpendicular to the contact surface. We can resolve this contact force into two components, one perpendicular to the contact surface and the other parallel to it. The perpendicular component is called the normal contact force or normal force and parallel component is called friction.

Contact force = $\sqrt{f^2 + N^2}$

$F_{c\,min}$ = N {when f_{min} = 0}

$F_{c\,max}$ = $\sqrt{\mu^2N^2 + N^2}$ {when $f_{max} = \mu N$}

$N \le F_c \le \sqrt{(\mu^2+1)}N$

$0 \le \lambda \le \tan^{-1}\mu$

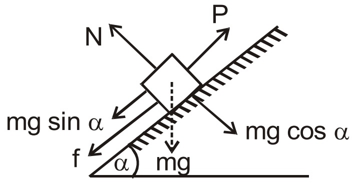
11.MOTION ON A ROUGH INCLINED PLANE

Suppose a motion up the plane takes place under the action of pull P acting parallel to the plane

$N = mg\cos\alpha$

Frictional force acting down the plane

$F = \mu N = \mu\, mg\cos\alpha$



Appling Newton's second law for motion up the plane

$P - (mg\sin\alpha + f) = ma$

$P - mg\sin\alpha - \mu\, mg\cos\alpha = ma$

If P = 0 the block may slide downwards with an acceleration a. The frictional force would then act up the plane

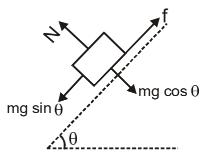
$mg\sin\alpha - F = ma$

or, $mg\sin\alpha - \mu\, mg\cos\alpha = ma$

12.ANGLE OF REPOSE :

Consider a rough inclined plane whose angle of inclination θ with ground can be changed. A block of mass m is resting on the plane. Coefficient of (static) friction between the block and plane is μ.

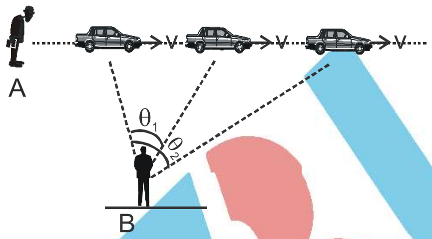
For a given angle θ, the FBD (Free body diagram) of the block is



Where f is force of static friction on the block. For normal direction to the plane, we have N=mg cosθ
As θ increases, the force of gravity down the plane, mg sin θ, increases. Friction force resists the slide till it attains its maximum value.
$f_{max} = \mu N = \mu$ mg cos θ
Which decreases with θ (because cos θ decreases as θ increases)
Hence, beyond a critical value $\theta = \theta_c$, the blocks starts to slide down the plane. The critical angle is the one when mg sin θ is just equal of f_{max}, i.e., when
mg sin $\theta_c = \mu$ mg cosθ_c
or tan $\theta_c = \mu$
where θ_C is called angle of repose
If $\theta > \theta_c$, block will slide down.For $\theta < \theta_c$ the block stays at rest on the incline.

13.CIRCULAR MOTION

When a particle moves in a plane such that its distance from a fixed (or moving) point remains constant then its motion is called as the circular motion with respect to that fixed (or moving) point. That fixed point is called centre and the distance between fixed point and particle is called radius.



The car is moving in a straight line with respect to the man A. But the man B continuously rotate his face to see the car. So with respect to man A $\frac{d\theta}{dt} = 0$

But with respect to man B $\frac{d\theta}{dt} \neq 0$

Therefore we conclude that with respect to A the motion of car is straight line but for man B it has some angular velocity

14.KINEMATICS OF CIRCULAR MOTION :

14.1 Variables of Motion :

(a) Angular Position :
The angle made by the position vector with given line (reference line) is called angular position Circular motion is a two dimensional motion or motion in a plane. Suppose a particle P is moving in a circle of radius r and centre O. The position of the particle P at a given instant may be described by the angle θ between OP and OX. This angle θ is called the angular position of the particle. As the particle moves on the circle its angular position θ change. Suppose the point rotates an angle $\Delta\theta$ in time Δt.

(b) Angular Displacement :
Definition :
Angle rotated by a position vector of the moving particle in a given time interval with some reference line is called its angular displacement.

Important point :

- It is dimensionless and has proper unit SI unit radian while other units are degree or revolution 2π rad = 360° = 1 rev
- Infinitely small angular displacement is a vector quantity but finite angular displacement is not because the addition of the small angular displacement is cummutative while for large is not.

$$d\vec{\theta}_1 + d\vec{\theta}_2 = d\vec{\theta}_2 + d\vec{\theta}_1 \text{ but } \theta_1 + \theta_2 \neq \theta_2 + \theta_1$$

- Direction of small angular displacement is decided by right hand thumb rule. When the fingers are directed along the motion of the point then thumb will represents the direction of angular displacement.
- Angular displacement can be different for different observers

(c) Angular Velocity ω

(i) Average Angular Velocity

$$\omega_{av} = \frac{\text{Total Angle of Rotation}}{\text{Total time taken}} \;;$$

$$\omega_{av} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}$$

where θ_1 and θ_2 are angular position of the particle at time t_1 and t_2 respectively.

(ii) Instantaneous Angular Velocity

The rate at which the position vector of a particle with respect to the centre rotates, is called as instantaneous angular velocity with respect to the centre.

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

Important points :

- It is an axial vector with dimensions $[T^{-1}]$ and SI unit rad/s.
- For a rigid body as all points will rotate through same angle in same time, angular velocity is a characteristic of the body as a whole, e.g., angular velocity of all points of earth about its own axis is $(2\pi/24)$ rad/hr.
- If a body makes 'n' rotations in 't' seconds then angular velocity in radian per second will be

$$\omega_{av} = \frac{2\pi n}{t}$$

If T is the period and 'f' the frequency of uniform circular motion

$$\omega_{av} = \frac{2\pi \times 1}{T} = 2\pi f$$

- If $\theta = a - bt + ct^2$ then $\omega = \frac{d\theta}{dt} = -b + 2ct$

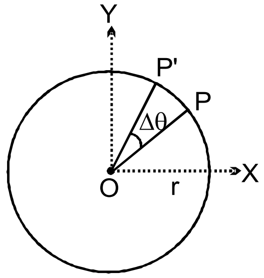
Relation between speed and angular velocity :

$$\omega = \lim_{\Delta t \to \theta} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

The rate of change of angular velocity is called the angular acceleration (α). Thus,

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

The linear distance PP' travelled by the particle in time Δt is



$$\Delta s = r\Delta\theta \quad \text{or} \quad \lim_{\Delta t \to 0} \frac{\Delta S}{\Delta t}$$

$$= r \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} \quad \text{or} \quad \frac{\Delta s}{\Delta t} = r\frac{d\theta}{dt} \quad \text{or} \quad v = r\omega$$

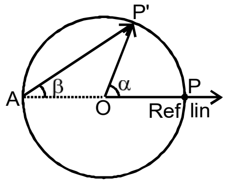
Here, v is the linear speed of the particle

It is only valid for circular motion

$v = r\omega$ is a scalar quantity $\left(\vec{\omega} \neq \frac{\vec{v}}{r}\right)$

(d) Relative Angular Velocity

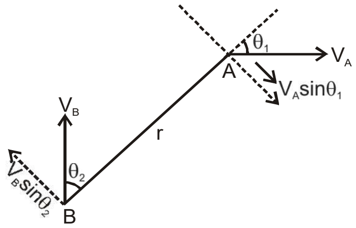
Angular velocity is defined with respect to the point from which the position vector of the moving particle is drawn Here angular velocity of the particle w.r.t. 'O' and 'A' will be different



$\omega_{PO} = \frac{d\alpha}{dt}$; $\omega_{PA} = \frac{d\beta}{dt}$

Definition :

Relative angular velocity of a particle 'A' with respect to the other moving particle 'B' is the angular velocity of the position vector of 'A' with respect to 'B'. That means it is the rate at which position vector of 'A' with respect to 'B' rotates at that instant



$\omega_{AB} = \frac{(V_{AB})_\perp}{r_{AB}}$ here $V_{AB\perp}$ = Relative velocity $\perp$ to position vector AB

$= \frac{\text{Relative velocity of A w.r.t. B perpendicular to line AB}}{\text{Seperation between A and B}}$

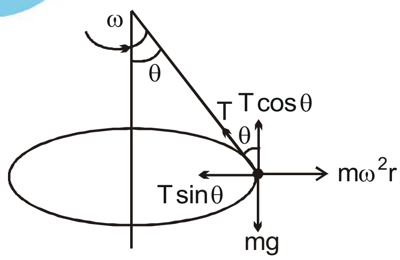
$(V_{AB})_\perp = V_A \sin\theta_1 + V_B \sin\theta_2$

$r_{AB} = r$

$\omega_{AB} = \frac{V_A \sin\theta_1 + V_B \sin\theta_2}{r}$

15.CIRCULAR MOTION IN HORIZONTAL PLANE

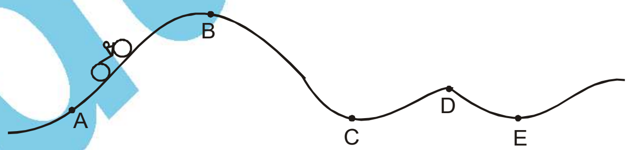
A ball of mass m attached to a light and inextensible string rotates in a horizontal circle of radius r with an angular speed ω about the vertical. If we draw the force diagram of the ball. We can easily see that the component of tension force along the centre gives the centripetal force and component of tension along vertical balances the gravitation force. Such a system is called a conical pendulum.



FBD of ball w.r.t ground

16.MOTION OF A MOTORCYCLIST ON A CURVED PATH.

A cylist having mass m move with constant speed v on a curved path as shown in figure.



We divide the motion of cyclist in four parts :

(1) from A to B (2) from B to C (3) from C to D (4) from D to E

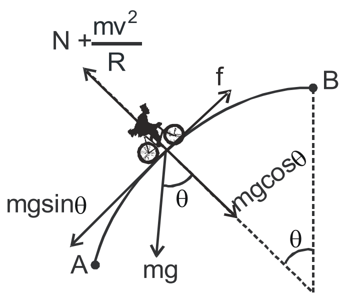
(1 and 3 are same type of motion)

(A) Motion of cyclist from A to B

$N + \frac{mv^2}{R} = mg\cos\theta$

$\Rightarrow$ $N = mg\cos\theta - \frac{mv^2}{R}$ ...(1)

$f = mg\sin\theta$ ..(2)



(1) As cyclist move upward

$\because$ θ decreases & $\cos\theta$ increases

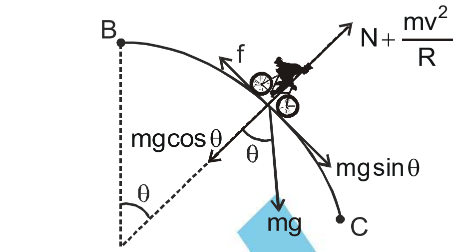
$\therefore$ N increases

and

$\because$ θ decreases sin θ decreases

$\therefore$ friction force required to balance mg sin θ (As cyclist is moving with constant speed) also decreases

(B) Motion of cyclist from B to C



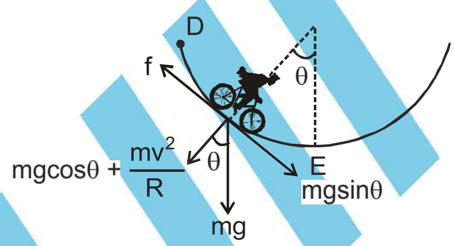
$$N + \frac{mv^2}{R} = mg\cos\theta$$

$$\Rightarrow N = mg\cos\theta - \frac{mv^2}{R} \qquad \text{...(1)}$$

f = mg sinθ ...(2)

Therefore from B to C Normal force decrease but friction force increase becuse θ increases.

(C) Motion of cyclist from D to E



$$N = \frac{mv^2}{R} + mg\cos\theta \qquad f = mg\sin\theta$$

from D to E θ decreases

therefore mg cos θ increase

So N increase but f decreases

17. CIRCULAR TURNING ON ROADS :

When vehicles go through turnings, they travel along a nearly circular arc. There must be some force which will produce the required centripetal acceleration. If the vehicles travel in a horizontal circular path, this resultant force is also horizontal. The necessary centripetal force is being provided to the vehicles by following three ways.

1. By Friction only

2. By Banking of Roads only

3. By Friction and Banking of Roads both.

In real life the necessary centripetal force is provided by friction and banking of roads both. Now let us write equations of motion in each of the three cases separately and see what are the constant in each case.

17.1 By Friction Only

Suppose a car of mass m is moving at a speed v in a horizontal circular arc of radius r. In this case, the necessary centripetal force to the car will be provided by force of friction f acting towards center

Thus, $f = \frac{mv^2}{r}$

Further, limiting value of f is μN

or $f_L = \mu N = \mu mg$ (N = mg)

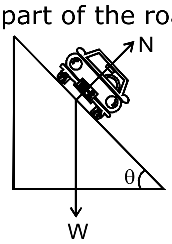
Therefore, for a safe turn without sliding $\frac{mv^2}{r} \le f_L$

or $\frac{mv^2}{r} \le \mu mg$ or $\mu \ge \frac{v^2}{rg}$ or $v \le \sqrt{\mu rg}$

Here, two situations may arise. If μ and r are known to us, the speed of the vehicle should not exceed $\sqrt{\mu rg}$ and if v and r are known to us, the coefficient of friction should be greater than $\frac{v^2}{rg}$.

17.2 By Banking of Roads Only

Friction is not always reliable at circular turns if high speeds and sharp turns are involved to avoid dependence on friction, the roads are banked at the turn so that the outer part of the road is some what lifted compared to the inner part.



Applying Newton's second law along the radius and the first law in the vertical direction.

$$N\sin\theta = \frac{mv^2}{r} \quad \text{or} \quad N\cos\theta = mg$$

from these two equations, we get

$$\tan\theta = \frac{v^2}{rg} \quad \text{or} \quad v = \sqrt{rg\tan\theta}$$

17.3 By Friction and Banking of Road Both

If a vehicle is moving on a circular road which is rough and banked also, then three forces may act on the vehicle, of these force, the first force, i.e., weight (mg) is fixed both in magnitude and direction.

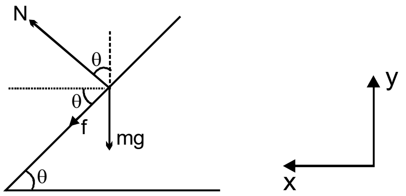


Figure (ii)

The direction of second force i.e., normal reaction N is also fixed (perpendicular or road) while the direction of the third i.e., friction f can be either inwards or outwards while its magnitude can be varied upto a maximum limit ($f_L = \mu N$). So the magnitude of normal reaction N and directions plus magnitude of friction f are so adjusted that the resultant of the three forces mentioned above is $\frac{mv^2}{r}$ towards the center. Of these m and r are also constant. Therefore, magnitude of N and directions plus magnitude of friction mainly depends on the speed of the vehicle v. Thus, situation varies from problem to problem. Even though we can see that :

(i) Friction f will be outwards if the vehicle is at rest v = 0. Because in that case the component weight mg $\sin\theta$ is balanced by f.

(ii) Friction f will be inwards if

$$v > \sqrt{rg\tan\theta}$$

(iii) Friction f will be outwards if

$$v < \sqrt{rg\tan\theta} \text{ and}$$

(iv) Friction f will be zero if

$$v = \sqrt{rg\tan\theta}$$

(v) For maximum safe speed (figure (ii))

$$N\sin\theta + f\cos\theta = \frac{mv^2}{r} \qquad \text{...(i)}$$

$$N\cos\theta - f\sin\theta = mg \qquad \text{...(ii)}$$

As maximum value of friction

$$f = \mu N$$

$$\therefore \frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta} = \frac{v^2}{rg} \qquad \therefore v_{max} = \sqrt{\frac{rg(\mu + \tan\theta)}{(1 - \mu\tan\theta)}}$$

Similarly ; $v_{min} = \sqrt{\frac{rg(\mu - \tan\theta)}{(1 + \mu\tan\theta)}}$

: • The expression $\tan\theta = \frac{v^2}{rg}$ also gives the angle of banking for an aircraft, i.e., the angle through which it should tilt while negotiating a curve, to avoid deviation from the circular path.

• The expression $\tan\theta = \frac{v^2}{rg}$ also gives the angle at which a cyclist should lean inward, when rounding a corner. In this case, θ is the angle which the cyclist must make with the vertical to negotiate a safe turn.

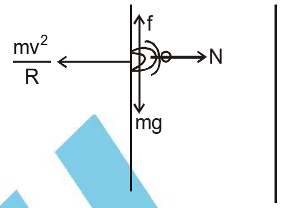
18. DEATH WELL :

A motor cyclist is driving in a horizontal circle on the inner surface of vertical cyclinder of radius R. Friction coefficient between tyres of motorcyclist and surface of cylinder is μ. Find out the minimum velocity for which the motorcyclist can do this. v is the speed of motor cyclist and m is his mass.

$$N = \frac{mv^2}{R}$$

$$f = mg$$

$$f_{max} = \frac{\mu mv^2}{R}$$

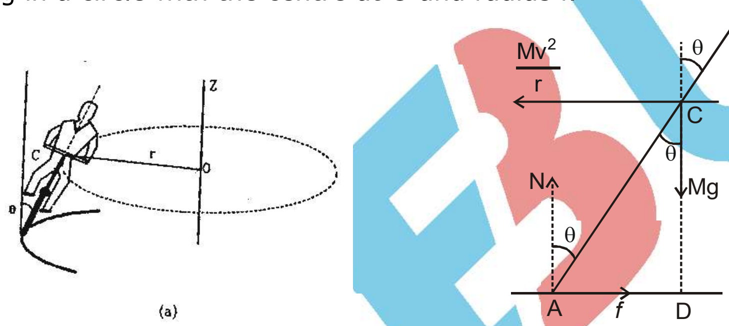


Cyclist does not drop down when

$$f_{max} \geq mg \Rightarrow \frac{\mu mv^2}{R} \geq mg \qquad v \geq \sqrt{\frac{gR}{\mu}}$$

19. MOTION OF A CYCLIST ON A CIRCULAR PATH :

Suppose a cyclist is going at a speed v on a circular horizontal road of radius r which is not banked. Consider the cycle and the rider together as the system. The centre of mass C (figure shown) of the system is going in a circle with the centre at O and radius r.



(a) (b)

Let us choose O as the origin, OC as the X-axis and vertically upward as the Z-axis. This frame is rotating at an angular speed $\omega = \frac{v}{r}$ about the Z-axis. In this frame the system is at rest. Since we are working from a rotating frame of reference, we will have to apply a centrifugal force on each particle. The net centrifugal force on the system will be $M\omega^2 r = Mv^2/r$, where M is the total mass of the system. This force will act through the centre of mass. Since the system is at rest in this frame, no other pseudo force is needed. Figure in shows the forces. The cycle is bent at an angle θ with the vertical. The forces are

(i) weight Mg, (ii) normal force N (iii) friction *f* and (iv) centrifugal force $\frac{Mv^2}{r}$

In the frame considered, the system is at rest. Thus, the total external force and the total external torque must be zero. Let us consider the torques of all the forces about the point A. The torques of N and f about A are zero because these forces pass through A. The torque of Mg about A is Mg(AD) in the clockwise direction and that of $\frac{Mv^2}{r}$ is $\frac{Mv^2}{r}$(CD) in the anticlockwise direction. For rotational equilibrium,

$$Mg(AD) = \frac{Mv^2}{r}(CD)$$

or, $\frac{AD}{CD} = \frac{v^2}{rg}$ or, $\tan\theta = \frac{v^2}{rg}$ ...(10.9)

Thus, the cyclist bends at an angle $\tan^{-1}\left[\frac{v^2}{rg}\right]$ with the vertical

SOLVED EXAMPLE

Ex.1 ***An astronaut accidentally gets separated out of his small spaceship baccelerating in inter stellar space at a constant rate of 100 m s^{-2}. What is the acceleration of the astronaut the instant after he is outside the spaceship ? (Assume that there are no nearby stars to exert gravitational force on him.)***

Ans. Since there are no nearby stars to exert gravitational force on him and the small spaceship exerts negligible gravitational attraction on him, the net force acting on the astronaut, once he is out of the spaceship, is zero. By the first law of motion the acceleration of the astronaut is zero.

Ex.2 ***A bullet of mass 0.04 kg moving with a speed of 90 m s-1 enters a heavy wooden block and is stopped after a distance of 60 cm. What is the average resistive force exerted by the block on the bullet?***

Ans. The retardation 'a' of the bullet (assumed constant) is given by a

$$= \frac{-90 \times 90}{2 \times 0.6} \text{ m/s}^2$$

$$= -6750 \text{ m s}^{-2}$$

Ex.3 ***The motion of a particle of mass m is described by y = $ut + \frac{1}{2}gt^2$ Find the force acting on the particle.***

Ans. We know – $y = ut + \frac{1}{2}gt^2$

Now, $v = \frac{dy}{dt} = 4 + g$

Then the force is given by

$F = ma = mg$

Thus the given equation describes the motion of a particle under acceleration due to gravity and y is the position coordinate in the direction of g.

Ex.4 ***A batsman hits back a ball straight in the direction of the bowler without changing its initial speed of 12 m s–1. If the mass of the ball is 0.15 kg, determine the impulse imparted to the ball. (Assume linear motion of the ball)***

Ans. Change in momentum

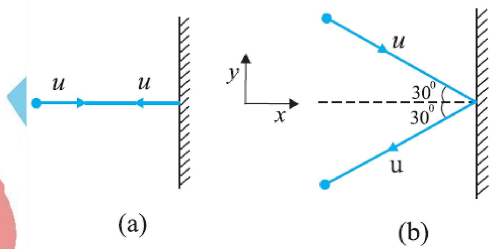
$= 0.15 \times 12 - (-0.15 \times 12)$

$= 3.6$ N s,

Impulse = 3.6 N s,

in the direction from the batsman to the bowler.

Ex.5 ***Two identical billiard balls strike a rigid wall with the same speed but at different angles, and get reflected without any change in speed, as shown in Fig. What is (i) the direction of the force on the wall due to each ball? (ii) the ratio of the magnitudes of impulses imparted to the balls by the wall ?***



Ans. An instinctive answer to (i) might be that the force on the wall in case (a) is normal to the wall, while that in case (b) is inclined at 30° to the normal. This answer is wrong. The force on the wall is normal to the wall in both cases. How to find the force on the wall? The trick is to consider the force (or impulse) on the ball due to the wall using the second law, and then use the third law to answer (i). Let u be the speed of each ball before and after collision with the wall, and m the mass of each ball. Choose the x and y axes as shown in the figure, and consider the change in momentum of the ball in each case :

Case (a)

$$(p_x)_{\text{initial}} = mu \qquad (p_y)_{\text{initial}} = 0$$

$$(p_x)_{\text{final}} = -mu \qquad (p_y)_{\text{final}} = 0$$

Impulse is the change in momentum vector.

Therefore, x-component of impulse = – 2 m u

y-component of impulse = 0 Impulse and force are in the same direction.

Clearly, from above, the force on the ball due to the wall is normal to the wall, along the negative x-direction. Using Newton's third law of motion, the force on the wall due to the ball is normal to the wall along the positive x-direction. The magnitude of force cannot be ascertained since the small time taken for the collision has not been specified in the problem.

Case (b)

$$(p_x)_{initial} = m\,u\cos 30^\circ,\ (p_y)_{initial} = -\,m\,u\sin 30^\circ$$

$$(p_x)_{final} = -\,m\,u\cos 30^\circ,\ (p_y)_{final} = -\,m\,u\sin 30^\circ$$

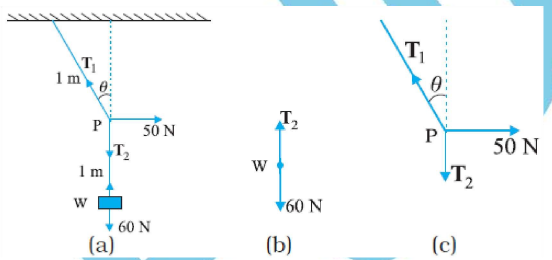
Note, while p_x changes sign after collision, p_y does not. Therefore, x-component of impulse $= -2\,m\,u\cos 30^\circ$ y-component of impulse = 0

The direction of impulse (and force) is the same as in (a) and is normal to the wall along the negative x direction. As before, using Newton's third law, the force on the wall due to the ball is normal to the wall along the positive x direction.

The ratio of the magnitudes of the impulses imparted to the balls in (a) and (b) is

$2\,mu/(2\,mu\cos 30^o) = \dfrac{2}{\sqrt{3}} \approx 1.2$

Ex.6 ***A mass of 6 kg is suspended by a rope of length 2 m from the ceiling. A force of 50 N in the horizontal direction is applied at the midpoint P of the rope, as shown. What is the angle the rope makes with the vertical in equilibrium ? (Take g = 10 m s^{-2}). Neglect the mass of the rope.***



Ans. Consider the equilibrium of the weight W. Clearly, $T_2 = 6 \times 10 = 60$ N.

Consider the equilibrium of the point P under the action of three forces - the tensions T_1 and T_2, and the horizontal force 50 N. The horizontal and vertical components of the resultant force must vanish separately :

$T_1 \cos\theta = T_2 = 60$ N

$T_1 \sin\theta = 50$ N

which gives that

$$\tan\theta = \frac{5}{6} \text{ or } \theta = \tan^{-1}\left(\frac{5}{6}\right) = 40^\circ$$

Ex.7 ***Determine the maximum acceleration of the train in which a box lying on its floor will remain stationary, given that the co-efficient of static friction between the box and the train's floor is 0.15.***

Ans. Since the acceleration of the box is due to the static friction, the static friction,

$ma = f_s \le \mu_s N = \mu_s m g$

i.e. $a \le \mu_s g$

$\therefore$ $a_{max} = \mu_s g = 0.15 \times 10$ m s^{-2}

$= 1.5$ m s^{-2}

Ex.8 ***A mass of 4 kg rests on a horizontal plane. The plane is gradually inclined until at an angle θ = 15° with the horizontal, the mass just begins to slide. What is the coefficient of static friction between the block and the surface ?***

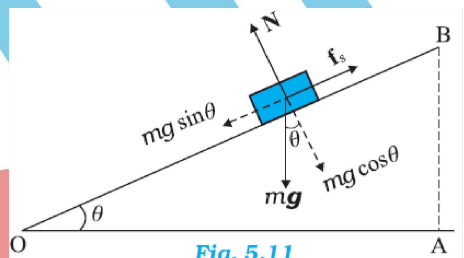


Fig. 5.11

Ans. The forces acting on a block of mass m at rest on an inclined plane are (i) the weight mg acting vertically downwards (ii) the normal force N of the plane on the block, and (iii) the static frictional force f_s opposing the impending motion. In equilibrium, the resultant of these forces must be zero. Resolving the weight mg along the two directions shown, we have

$m\,g \sin\theta = f_s$, $m\,g\cos\theta = N$

As θ increases, the self-adjusting frictional force f_s increases until at $\theta = \theta_{max}$, f_s achieves its maximum value, $(f_s)_{max}$ $f = \mu_s N$.

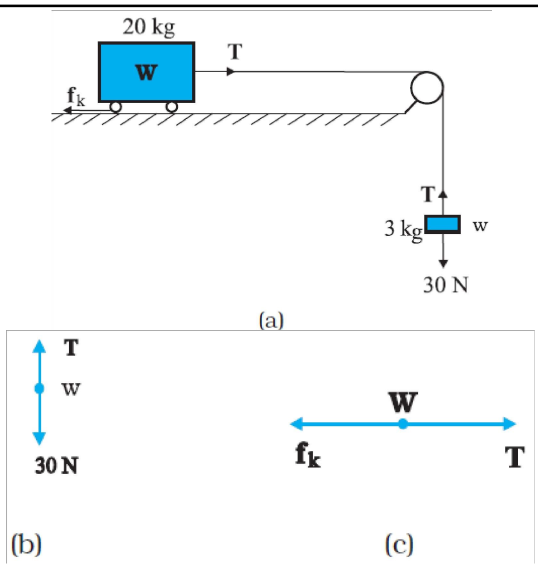
Therefore,

$\tan\theta_{max} = \mu_s$ or $\theta_{max} = \tan^{-1}\mu_s$

When θ becomes just a little more than θ_{max} , there is a small net force on the block and it begins to slide. Note that θ_{max} depends only on μ_s and is independent of the mass of the block.

For $\theta_{max} = 15°$, $\mu_s = \tan 15° = 0.27$

Ex.9 **What is the acceleration of the block and trolley system shown in a diagram (a), if the coefficient of kinetic friction between the trolley and the surface is 0.04? What is the tension in the string? (Take g = 10 m s^{-2}). Neglect the mass of the string.**



Ans. As the string is inextensible, and the pully is smooth, the 3 kg block and the 20 kg trolley both have same magnitude of acceleration. Applying second law to motion of the block

30 – T = 3a

Apply the second law to motion of the trolley

$T - f_k = 20$ a.

Now $f_k = \mu k$ N,

Here $\mu k = 0.04$,

N = 20 x 10 = 200 N.

Thus the equation for the motion of the trolley is T – 0.04 x 200 = 20 a Or T – 8 = 20a.

These equations give a = 22 23 m s^{-2} = 0.96 m s^{-2} and T = 27.1 N.

Ex.10 A cyclist speeding at 18 km/h on a level road takes a sharp circular turn of radius 3 m without reducing the speed. The co-efficient of static friction between the tyres and the road is 0.1. Will the cyclist slip while taking the turn ?

Ans. On an unbanked road, frictional force alone can provide the centripetal force needed to keep the cyclist moving on a circular turn without slipping. If the speed is too large, or if the turn is too sharp (i.e. of too small a radius) or both, the frictional force is not sufficient to provide the necessary centripetal force, and the cyclist slips. The condition for the cyclist not to slip is given by Eq. (5.18) :

$v^2 \le \mu_s R g$

Now, R = 3 m, g = 9.8 m s-2, $\mu_s = 0.1$. That is, $\mu_s R g$ = 2.94 m2 s^{-2}. v = 18 km/h = 5 m s^{-1}; i.e., $v^2 = 25\ m^2\ s^{-2}$. The condition is not obeyed. The cyclist will slip while taking the circular turn.

Ex.11 A circular racetrack of radius 300 m is banked at an angle of 15°. If the coefficient of friction between the wheels of a race-car and the road is 0.2, what is the (a) optimum speed of the racecar to avoid wear and tear on its tyres, and (b) maximum permissible speed to avoid slipping ?

Ans. On a banked road, the horizontal component of the normal force and the frictional force contribute to provide centripetal force to keep the car moving on a circular turn without slipping. At the optimum speed, the normal reaction's component is enough to provide the needed centripetal force, and the frictional force is not needed. The optimum speed vo is given by

$$v_0 = (R\ g\ \tan\theta)^{1/2}$$

Here R = 300 m, $\theta = 15°$, g = 9.8 m s-2;

we have

$$v_0 = 28.1\ m\ s^{-1}.$$

The maximum permissible speed υ_{max} is given

by $\upsilon_{max} = \left(Rg\dfrac{u_s + \tan\theta}{1 - u_s\tan\theta}\right) = 38.1\ ms^{-1}$

Exercise - I UNSOLVED PROBLEMS

Q.1 Give the magnitude and direction of the net force acting on
(a) a drop of rain falling down with a constant speed.
(b) a cork of mass 10 g floating on water.
(c) a kite skillfully held stationary in the sky,
(d) a car moving with a constant velocity of 30 km/h on a rough road,
(e) a high-speed electron in space far from all gravitating objects, and free of electric and magnetic fields.

Q.2 A pebble of mass 0.05 kg is thrown vertically upwards. Give the direction and magnitude of the net force on the pebble.
(a) during its upward motion,
(b) during its downward motion,
(c) at the highest point where it is momentarily at rest. Do your answer change if the pebble was thrown at an angle of say 45° with the horizontal direction ? Ignore air resistance.

Q.3 Give the magnitude and direction of the net force acting on a stone of mass 0.1 kg.
(a) just after it is dropped from the window of a stationary train.
(b) just after it is dropped from the window of a train running at a constant velocity of 35 km/h.
(c) just after it is dropped from the window of a train accelerating with 1 m s^{-2},
(d) lying on the floor of a train which is accelerating with 1 m s^{-2}, the stone being at rest relative to the train. Neglect air resistance throughout.

Q.4 One end of a string of length l is connected to a particle of mass m and the other to a small peg on a smooth horizontal table. If the particle moves i a circle with speed v the net force on the particle (directed towards the centre) is.

(i) T, (ii) $T - \frac{mv^2}{\ell}$ (iii) $T + \frac{mv^2}{\ell}$ (iv) 0

T is the tension in the string.
[Choose the correct alternative].

Q.5 A constant retarding force of 50 N is applied to a body of mass 20 kg moving initially with a speed of 15 m s^{-1}. How long does the body take to stop ?

Q.6 A constant force acting on a body of mass 3.0 kg changes its sped from 2.0 m s^{-1} to 3.5 s^{-1} in 25 s. The direction of the motion of the body remains unchanged. What is the magnitude and direction of the force ?

Q.7 A body of mass 5 kg is acted upon by two perpendicular forces 8 N and 6 N. Give the magnitude and direction of the acceleration of the body.

Q.8 The driver of a three-wheeler moving with a speed of 36 km/h sees a child standing in the middle of the road and brings his vehicle to rest in 4.0 s just in time to save the child. What is the average retarding force on the vehicle ? The mass of the three-wheeler is 400 kg and the mass of the driver is 65 kg.

Q.9 A rocket with a lift-off mass 20,000 kg is blasted upwards with an initial acceleration of 5.0 m s^{-2}. Calculate the initial thrust (force) of the blast.

Q.10 A particle of mass 0.40 kg moving initially with a constant speed of 10 m s^{-1} to the north is subject to a constant force of 8.0 N directed towards the south for 30 s. Take the instant the force is applied to be t = 0, the position of the particle at that time to be x = 0, and predict its position at t = – 5 s, 25 s, 100 s.

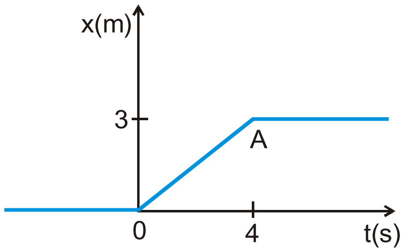
Q.11 A truck starts from rest and accelerates uniformly with 2.0 m s^{-2}. At t = 10 s, a stone is dropped by a person standing on the top of the truck (6 m high from the ground). What are the (a) velocity, and (b) acceleration of the stone at t = 11 s ? (Neglect air resistance.)

Q.12 A bob of mass 0.1 kg hung from the ceiling of a room by a string 2 m long is set into oscillation. The speed of the bob at its mean position is 1 m s^{-1}. What is the trajectory of the bob if the string is cut when the bob is (a) at one of its extreme positions, (b) at its mean position.

Q.13 A man of mass 70 kg stands on a weighing scale in a lift which is moving
(a) upwards with a uniform speed of 10 m s^{-1},
(b) downwards with a uniform acceleration of 5 m s^{-2},
(c) upwards with a uniform acceleration of 5 m s^{-2}
What would be the readings on the scale in each case ?

(d) What would be the reading if the lift mechanism failed and it hurtled down freely under gravity ?

Q.14 Figure shows the position-time graph of a particle of mass 4 kg. What is the (a) force on the particle for $t < 0$, $t > 4$ s, $0 < t < 4$ s ? (b) impulse at $t = 0$ and $t = 4$ s ? (Consider one-dimensional motion only).



Q.15 A horizontal force of 600 N pulls two masses 10 kg and 20 kg (lying on a frictionless table) connected by a light string. What is the tension in the string ? Does the answer depend on which mass end the pull is applied ?

Q.16 Two masses 8 kg and 12 kg are connected at the two ends of a light inextensible string that goes over a frictionless pulley. Find the acceleration of the masses, and the tension in the string when the masses are released.

Q.17 A nucleus is at rest i the laboratory frame of reference. Show that if it disintegrates into two smaller nuclei the products must be emitted in opposite directions.

Q.18 Two billiard balls each of mass 0.05 kg moving in opposite directions with speed 6 m s^{-1} collide and rebound with the same speed. What is the impulse imparted to each ball due to the other ?

Q.19 A shell of mass 0.020 kg is fired by a gun of mass 100 kg. If the muzzle speed of the shell is 80 m s^{-1}, what is the recoil speed of the gun ?

Q.20 a batsman deflects a ball by an angle of 45° without changing its initial speed which is equal to 54 km/h. What is the impulse imparted to the ball ? (Mass of the ball is 0.15 kg.)

Q.21 A stone of mass 09.25 kg tied to the end of a string is whirled round i a circle of radius 1.5 m with a speed of 40 rev./min in a horizontal plane. What is the tension in the string ? What is the maximum speed with which the stone can be whirled around if the string can withstand a maximum tension of 200 N ?

Q.22 If in 5.21 the speed of the stone is increased beyond the maximum permissible value, and the string breaks suddenly, which of the following correctly describes the trajectory of the stone after the string breaks :

(A) the stone jerks radially outwards.

(b) the stone flies off tangentially from the instant the string breaks,

(c) the stone flies off at an angle with the tangent whose magnitude depends on the speed of the particle ?

Q.23 For ordinary terrestrial experiments, which of the observers below are inertial and which non-inertial

(a) a child revolving in a " giant wheel".

(b) a driver in a sports car moving with a constant high speed of 200 km/h on a straight road,

(c) the pilot of an aeroplane which is taking off,

(d) the pilot of an aeroplane which is taking off,

(d) a cyclist negotiating a sharp turn.

(e) the guard of a train which is slowing down to stop at a station ?

Q.24 One often comes across the following type of statement concerning circular motion : 'A particle moving uniformly along a circle experiences a force directed towards the centre (centripetal force) and an equal and opposite force directed away from the centre (centrifugal force). The two forces together keep the particle in equilibrium'. Explain what is wrong with this statement.

Q.25 Explain why

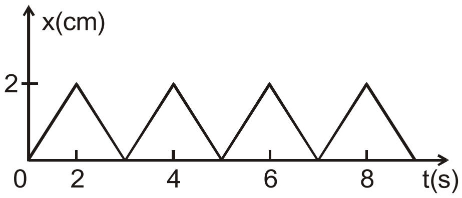
(a) a horse cannot pull a cart and run in empty space,

(b) passengers are thrown forward from their seats when a speeding bus stops suddenly.

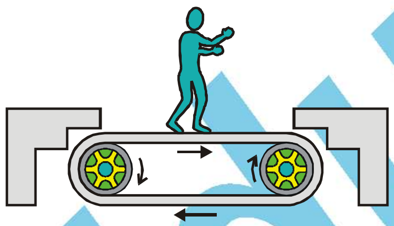
(c) it is easier to pull a lawn mower than to push it,

(d) a cricketer moves his hands backwards while holding a catch.

Q.26 Figure shows the position-time graph of a particle of mass 0.04 kg. Suggest a suitable physical context for this motion. What is the time between two consecutive impulses received by the particle ? What is the magnitude of each impulse ?



Q.27 figure shows a man standing stationary with respect to a horizontal conveyor belt that is accelerating with 1 m s^{-2}. What is the net force on the man ? If the coefficient of static friction between the mans shoes and the belt is 0.2, up to what acceleration of the belt can the man continue to be stationary relative to the belt ?
(Mass of the man = 65 kg.)



Q.28 A stone of mass m tied to the emf of a string is revolved in a vertical circle of radius R. The net forces at the lowest and highest points of the circle directed vertically downwards are : [Choose the correct alternative].

Lowest Point	Highest Point
(a) $mg - T_1$	$mg + T_2$
(b) $mg + T_1$	$mg - T_2$
(c) $mg + T_1 - (mv_1^2)/R$	$mg - T_2 + (mv_1^2)/R$
(d) $mg - T_1 - (mv_1^2)/R$	$mg + T_2 + (mv_1^2)/R$

Here T_1, T_2 (and v_1, v_2) denote the tension in the string (and the speed of the stone) at the lost and the highest point respectively.

Q.29 A helicopter of mass 1000 kg rises with a vertical acceleration of 15 m s^{-2}. The crew and the passengers weigh 300 kg. give the magnitude and direction of the
(a) force on the floor by the crew and passengers,
(b) action of the rotor of the helicopter on the surrounding air,
(c) force on the helicopter due to the surrounding air.

Q.30 A steam of water flowing horizontally with a speed of 15 m s^{-1} gushes out of a tube of cross-sectional area 10^{-2} m^2, and hits at a vertical wall nearby. What is the force exerted on the wall by the impact of water, assuming it does not rebound ?

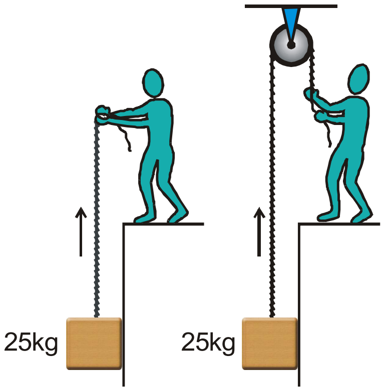
Q.31 Ten one-rupee coins are put on top of each other on a table. Each coin has a mass m kg. give the magnitude and direction of
(a) the force on the 7th coin (counted from the bottom) due to all the coins on its top.
(b) the force on the 7th coin by the eighth coin,
(c) the reaction of the 7th coin on the 7th coin.

Q.32 a aircraft executes a horizontal loop at a speed of 720 km/h with its wings banked at 15°. What is the radius of the loop ?

Q.33 A train rounds an unbanked circular bend of radius 30 m at a speed of 54 km/h. The mass of the train is 10^6 kg. What provides the centripetal force required for this purpose ? The engine or the rails ? What is the angle of banking required to prevent wearing out of the rail ?

Q.34 "It is reasonable to consider the Earth as an inertial frame for any laboratory scale terrestrial experiments, but the Earth is a non-inertial frame of reference for astronomical observations". Explain this statement. Do you see how the same frame of reference is approximately inertial for one purpose, and non-inertial for the other ?

Q.35 A block of mass 25 kg is raised by a 50 kg man in two different ways as shown in figure 5.22. What is the action on the floor by the man in the two cases ? If the floor yields to a normal force of 700 N, which mode should the man adopt to lift the block without the floor yielding ?



Q.36 A monkey of mass 40 kg climbs on a rope as shown in figure which can bear a maximum tension of 600 N. In which of the following cases will the rope break : The monkey

(a) climbs up with an acceleration of 6 m s^{-2}

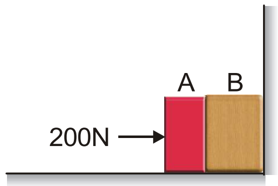
(b) climbs down with an acceleration of 4 m s^{-2}

(c) climbs up with a uniform speed of 5 m s^{-1}

(d) falls down the rope nearly freely under gravity ?

(Ignore the mass of the rope)

Q.37 Two bodies A and B of masses 5 kg and 10 kg in contact with each other rest on a table against a rigid partition (figure). The co-efficient of friction between the bodies and the table is 0.15. A force of 200 N is applied horizontally at A. What are (a) the reaction of the partition (b) the action-reaction forces between A and B? What happens when, the partition is removed ? Does the answer to (b) change, when the bodies are in motion ? Ignore the difference between μ_s and μ_k.



Q.38 A block of mass 15 kg is placed on a long trolley. The co-efficient of static friction between the block and the trolley is 0.18. The trolley accelerates from rest with 0.5 m s^{-2} for 20 s and then moves with uniform velocity. Discuss the motion of the block as viewed by (a) a stationary observer on the ground, (b) an observer moving with the trolley.

Q.39 The rear side of a truck is open and a box of 40 kg mass is placed 5 m away from the open end as shown in Fig. The co-efficient of friction between the box and the truck below it is 0.15. On a straight road, the truck starts from rest and accelerates with a constant acceleation of 2 m s^{-2}. At what distance from the starting point does the box fall off the truck ? (Ignore the size of the box)