Units & Measurements

Physical Quantity

A quantity which can be measured is called **physical quantity** e.g length mass, time force. In order to measure a physical quantity a standard is required. This standard is called **unit of that physical quantity.**

A physical quantity is represented completely by its magnitude and unit

Physical quantity = magnitude x unit

$Q = nu \qquad n \propto \frac{1}{u}$

means larger the unit, smaller will be the magnitude.

Fundamental and Derived Quantities

The physical quantities which are independent of other quanties are called **Fundamental Quantities.** The quantities which are expressed in terms of fundamental quantities are called **Derived Quantities.**

Fundamental and Derived unit

Units of fundamental physical quantities are called **fundamental units** and units of derived physical quantities are called **derived unit.**

System of Units

(i) CGS system - In this system length is measured in centimeter, mass is measured in gram and time is measured in second.

(II) M.K.S, In this system length is measured in meter, mass in kilogram and time in second.

(III) F.P.S. In this system length is measured in foot, mass in pound and time in second.

(IV) S.I. System, It is a International System of units. In this system seven fundamental quantities are taken, which are given below :

	Fundamental Quantity	Unit	Symbol
1.	Length	metre	m
2.	Mass	Kilogram	kg
3.	Time	Second	S
4.	Electrier Current	ambere	A
5.	Temperature	Kelvin	K
6.	Amount of Substance	mole	mol
7.	Luminous inensity	candela	Cd.

Dimension

The powers to which fundamental quantities must be raised in order to express the given physical quantity are called its **dimension.**

e.g. force = mass x acceleration.

$$= \text{mass} \times \frac{\text{velocity}}{\text{time}}$$

$$= \frac{\text{mass} \times \text{length/ time}}{\text{time}}$$

$$= \text{mass} \times \text{length} \times (\text{time})^{-2}$$

Thus dimension of force are 1 in mass, 1 in length, and -2 in time.

For convenience mass, length and time are denoted by M,L and T. The electric current, the temperature, the amount of substance and Luiminous intensity are denoted by symbols I, K, mol and candela respectively. The physical quantity which is expressed in terms of base quantities are enclosed in square brackets.

Thus equation for force in written as [Force] = MLT^{-2}

Dimensional Formula

The dimensional formula of a physical quantity is the symbolic representation of its dimensions. Thus the dimensional formula of force is MLT^{-2}.

Dimensional Equation

The dimensional equation of a physical quantity is written as

[Quantity] = Dimensional formula.

Thus [Force] = $[MLT^{-2}]$ or [F] = $[MLT^{-2}]$

Important Dimensions in Physics

(a) Mechanics

S. No.	Physical quantity	Symbol	Dimensional formulae	Commonly used unit
1.	Speed	c,u,v,w	$[M^0LT^{-1}]$	ms^{-1}
2.	Velocity	c,u,v,w	$[M^0LT^{-1}]$	ms^{-1}
3.	Acceleration	a	$[M^0LT^{-2}]$	ms^{-2}
4.	Velocity gradient	$\frac{dv}{dr}$	$[M^0L^0T^{-1}]$	s^{-1}
5.	Area	S.A. a	$[M^0L^2T^0]$	m^2
6.	Angle	$\theta, \alpha, \beta, \gamma$	$[M^0L^0T^0]$	radian (rad)
7.	Solid angle	Ω	$[M^0L^0T^0]$	steradian (sr)
8.	Angular velocity	ω	$[M^0L^0T^{-1}]$	rad s^{-1}
9.	Angular frequency	ν	$[M^0L^0T^{-1}]$	s^{-1}
10.	Angular acceleration	α	$[M^0L^0T^{-2}]$	rad s^{-2}
11.	Momentum	P	$[MLT^{-1}]$	kg m s^{-1}
12.	Angular momentum	L	$[ML^2T^{-1}]$	kg m^2 s^{-1}
13.	Force	F	$[MLT^{-2}]$	newton (N)
14.	Torque, couple	τ	$[ML^2T^{-2}]$	Nm or joule (J)
15.	Impulse	I	$[MLT^{-1}]$	Ns or kg ms^{-1}
16.	Frequency	f, ν	$[M^0L^0T^{-1}]$	hertz (Hz)
17.	Time period	T	$[M^0L^0T^1]$	S
18.	Volume	V	$[M^0L^3T^0]$	m^3
19.	Density	ρ, d	$[ML^{-3}T^0]$	kg m^{-3}
20.	Pressure	p, P	$[ML^{-1}T^{-2}]$	pascal (pa) or $[Nm^{-2}]$
21.	Radius of gyration	K	$[M^0LT^0]$	m
22.	Moment of intrtia	I	$[ML^2T^0]$	kg m^2
23.	Strain	ε	$[M^0L^0T^0]$	–
24.	Stress	σ	$[ML^{-1}T^{-2}]$	pascal (pa) or $[Nm^{-2}]$

25.	Young's modulus	Y	$[ML^{-1}T^{-2}]$	Pa or Nm^{-2}
26.	Bulk modulus	B	$[M^0L^{-1}T^{-2}]$	Pa or Nm^{-2}
27.	Modulus of rigidity	η	$[ML^{-1}T^{-2}]$	Pa or Nm^{-2}
28.	Poisson's ratio	σ	$[M^0L^0T^0]$	-
29.	Surface Tension	T, σ	$[ML^0T^{-2}]$	Nm^{-1} or Jm^{-2}
30.	Planck's constant	h	$[ML^2T^{-1}]$	Js

(b) Electricity

Quantity	**Unit**	**Dimension**
Electric Charge (q)	Coulomb	$[M^0L^0T^1A^1]$
Electric current (I)	Ampere	$[M^0L^0T^0A^1]$
Electric potential (V)	Joule/ coulomb	$[M^1L^2T^{-3}A^{-1}]$
Capacitance (C)	Coulomb/ volt or Farad	$[M^{-1}L^{-2}T^4A^2]$
Permittivity of free space (ε_0)	$\frac{\text{Coulomb}^2}{\text{Newton-metre}^2}$ or ferad per meter	$[M^{-1}L^{-3}T^4A^2]$
Dielectric constant (K) Resistance (R)	Unitless Volt/Ampere or ohm	$[M^0L^0T^0]$ $[M^1L^3T^{-3}A^{-2}]$
Resisvity or Specific	Ohm-metre	$[M^1L^3T^{-3}A^{-2}]$
resistance (p)		
Coefficient of Self-induction	$\frac{\text{volt}-\text{second}}{\text{ampere}}$ or henry or ohm-second	$[M^1L^2T^{-2}A^{-2}]$
Magnetic flux (ϕ)	Volt-second or weber	$[M^1L^2T^{-2}A^{-1}]$
Magnetic induction (B)	$\frac{\text{newton}}{\text{ampere}-\text{metre}}$ $\frac{\text{Joule}}{\text{ampere}-\text{metre}^2}$ $\frac{\text{volt}-\text{second}}{\text{metre}^2}$ orTesla	$[M^1L^0T^{-2}A^{-1}]$

(c) Thermodynamics

Quantity	**Unit**	**Dimension**
Temperature (T)	Kelvin	$[M^0L^0T^0\theta^1]$
Heat (Q)	Joule	$[ML^2T^{-2}]$
Specific Heat (c)	Joule/kg-K	$[M^0L^2T^{-2}\theta^{-1}]$
Thermal capacity	Joule/K	$[M^1L^2T^{-2}\theta^{-1}]$
Latent heat (L)	Joule/Kg	$[M^0L^2T^{-2}]$

Gas constant (R)	Joule/mol-K	$[M^1L^2T^{-2}\theta^{-1}]$
Boltzmann constant (k)	Joule/K	$[M^1L^2T^{-2}\theta^{-1}]$
Coefficient of thermal conductivity (K)	Joule/m-s-K	$[M^1L^1T^{-3}\theta^{-1}]$
Stefan's constant (σ)	Watt/m^2-K^4	$[M^1L^0T^{-3}\theta^{-4}]$
Wien's constant (b)	Metre-Kelvin	$[M^0L^1T^0\theta^1]$
Planck's constant (h)	Joule-s	$[M^1L^2T^{-1}]$
Coefficient of Linear Expansion (α)	$Kelvin^{-1}$	$[M^0L^0T^0\theta^{-1}]$
Mechanical equivalent of Heat (J)	Joule/Calorie	$[M^0L^0T^0]$
Vander wall's constant (a)	Newton-m^4	$[ML^5T^{-2}]$
Vander wall's constant (b)	m^3	$[M^0L^3T^0]$

Quantities having Same Dimensions

Dimension	Quantiy
$[M^0L^0T^{-1}]$	Frequency, angular frequency, angular velocity, velocity gradient and decay constant
$[M^1L^2T^{-2}]$	Work, internal energy, potential energy, kinetic energy, torque, moment of force
$[M^1L^{-1}T^{-2}]$	Pressure, stress, Young's modulus, bulk modulus, modulus of rigidity, energy density
$[M^1L^1T^{-1}]$	Momentum, impulse
$[M^0L^1T^{-2}]$	Acceleration due to gravity, gravitational field intensity
$[M^1L^1T^{-2}]$	Thrust, force, weight, energy gradient
$[M^1L^2T^{-1}]$	Angular momentum and Planck's constant
$[M^1L^0T^{-2}]$	Surface tension, Surface energy (energy per unit area), spring constant
$[M^0L^0T^0]$	Strain, refractive index, relative density, angle, solid angle, distance gradient, relative permittivity (dielectric constant), relative permeability etc
$[M^0L^2T^{-2}]$	Latent heat and gravitational potential
$[ML^2T^{-2}\theta^{-1}]$	Thermal capacity gas constant, Boltzmann constant and entropy
$[M^0L^0T^1]$	$\sqrt{l/g}, \sqrt{m/k}, \sqrt{R/g}$, where l= length g = acceleration due to gravity, m = mass, k = spring constant, R= Radius of earth
$[M^0L^0T^1]$	L/R, $\sqrt{LC}$, RC where L= inductance, R= resistance, C= capacitance
$[ML^2T^{-2}]$	$I^2Rt, \frac{V^2}{R}t$, Vlt, qV, $LI^2, \frac{q2}{C}, CV^2$ where I = current, t= time, q= charge, L= inductance, C= capacitance, R=Resistance

Application of Dimensional Analysis

(i) **Convert a physical quantity from one system to other** The measure of a physical quantity is nu = constant If a physicall quantity X has dimensional formula $\left[M^{a}L^{b}T^{c}\right]$ and if (derived) units of that physical quantity in two systems are $\left[M_1^{a}L_1^{b}T_1^{c}\right]$ and $\left[M_2^{a}L_2^{b}T_2^{c}\right]$ respectively and n_1 and n_2 be the numerical value in the two systems respectively, then

$$N_1[u_1] = n_2[u_2]$$

$$\Rightarrow n_1\left[M_1^{a}L_1^{b}T_1^{c}\right] = n_2\left[M_2^{a}L_2^{b}T_2^{c}\right]$$

$$\Rightarrow n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$$

where M_1 L_1 and T_1 are fundamental units of mass, length and time in the first (known) system and M_2, L_2 and T_2 are fundamental units of mass, length and time in the second (unknown) system, Thus knowing the values of fundamental units in two systems and numerical value in one system, the numerical value in other system may be evaluted.

(ii) **To check the dimensional correctness of a given physical relation** This is based on the 'principle of homogeneity'. According to this principle the dimensions of each term on both sides of an equation must be the same.

If $X = A \pm (BC)^2 \pm \sqrt{DEF}$,

Then according to principle of homogeneity

$$[X] = [A] = \left[(BC)^2\right] = \left[\sqrt{DEF}\right]$$

If the dimensions of each term on both sides are same, the equation is dimensionally correct, otherwise not. **A dimensionally correct equation may or may not be physically correct.**

(iii) **Deducing relations among the Physical Quantities** If one knows the dependency of a physical quantity on other quantities and if the dependency is of the product type, then using the method of dimensional analysis, relation between the quantities can be derived.

Limitation of Dimensional Analysis :

The dimensional method is a simple and a very convenient way of finding the dependence of physical quantity on other quantities but it has its own limitations. Some of which are listed as follows:

(i) In more complicated situations, it is often not easy to find out the factors on which a physical quantity will depend, In such cases, one has to make a guess which may or may not work.

(ii) This method gives no information about the dimensionless constant which has to be determined either by experiment or by a complete mathematical derivation.

(iii) This method is used only if a physical quantity varies as the product of other physical quantities. It fails if a physical quantity depends on the sum or difference of two quantities.

(iv) This method will not work if a quantity depends on another quantity as sin or cos of an angle, i.e. if the dependence is on a trigonometric function. Similar is the case with logarithmic and exponential function. The method works only if the dependence is on proper functions only.

(v) This method does not give a complete information in cases where a physical quantity depends on more than three quantities, because by equating the powers of M,L and T, we can obtain only three equations for the exponents.