Electrostatic potential and Capacitance

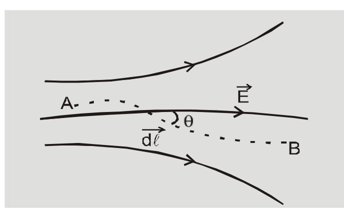
2

ELECTRIC POTENTIAL AND POTENTIAL ENERGY

Line integral of $\vec{E}$

The line integral of electric fields is defined as the integral

$$\int_A^B \vec{E}.\vec{d\ell}$$



The value of line integral depends only on the position of points A and B, and is independent of the path between A and B

$$\int_A^B \vec{E}.\vec{d\ell} = -\int_B^A \vec{E}.\vec{d\ell}$$

Line integral for a closed path is zero

$$\oint \vec{E}.\vec{d\ell} = 0 \quad \nabla \times \vec{E} = 0$$

such electric fields are called conservative fields.

Electric potential

Electric potential and potential energy are defined only for conservative fields.

Definition in terms of work done.

Potential at any point A is equal to the amount of work done (by external agent against electric field) in bringing a unit positive charge from infinity to that point.

$$V_A = \frac{W_{\infty A}}{q}$$

Unit of potential (V) = J/C or volt

Potential at a point is said to be one volt if the amount of work done in bringing one coulomb of positive charge from infinity to that point is one joule.

Since work and charge, both are scalars, the electric potential is a scalar quantity.

The dimensions of electric potential are $= \frac{ML^2T^{-2}}{TA}$

or $[V] = ML^2T^{-3}A^{-1}$

Potential difference

Potential difference between two points f (final and i (initial) is defined as equal to the amount of work done (by external agent) in moving a unit positive charge from point i (initial) to f (final)

$$V_f - V_i = \frac{W_{if}}{q}$$

If work done in carrying a unit positive charge from point 1 to point 2 is one joule then the potential difference $V_2 - V_1$ is said to be one volt.

Potential difference may be positive or negative.

The work done against electrical forces in transporting a charge q from point i (potential V_i) to point f (potential V_f) is $W = qV$ where $V = V_f - V_i$.

Relation between E and electric potential V

$$\Delta V = V_f - V_i = -\vec{E}\cdot\Delta\vec{\ell}$$

In one dimensions

$$E = -\frac{dV}{dr} \qquad ......(1)$$

$$V = -\int E\,dr \qquad ......(2)$$

[If electric potential is known, electric field can be determined from eq. (1) and if E is known, V can be determined from (2)]

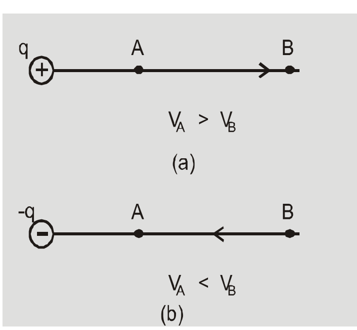
In general $\boxed{\vec{E} = -\nabla V}$

Electric field at any point is equal to negative of potential gradient at that point.

The electric field always points from higher potential to lower potential (see fig.)

A positive charge always moves from higher potential to lower potential.

A negative charge always moves from lower potential to higher potential.

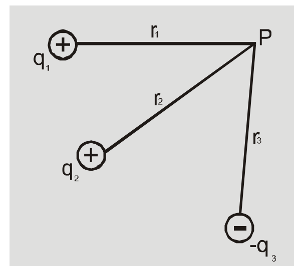


Electric potential due to point charges

One point charge $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$

Many point charges

Potential is a scalar quantity and adds like scalers. Thus potential at point P (see fig.) due to charges $q_1, q_2, -q_3$ is equal to (algebraic) sum of potentials due to individual charges.



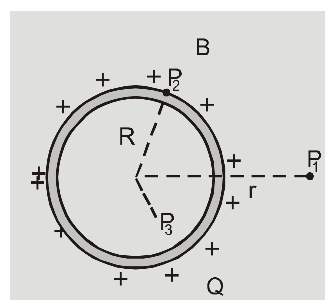
$$V = V_1 + V_2 + V_3 + ...$$

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{-q_3}{r_3} + ...\right)$$

Potential due to a charged spherical shell

The charge resides on the shell surface. The potential at P_1, outside point, is

$$\boxed{V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r}}$$



The potential at P_2, surface point is

$$\boxed{V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{R}}$$

The potential at P_3, inside point is

$$\boxed{V = V_{surface}} \qquad \boxed{V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}}$$

If is constant inside the shell (same at all points inside the shell)

Potential due to a charged conducting sphere

The charge always resides on the surface of a conducting sphere. Therefore, as far as electric field and potential, outside, at the surface and inside points are concerned the expressions are identical to those obtained for the spherical shell. Thus for points,

Outside the sphere $V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$

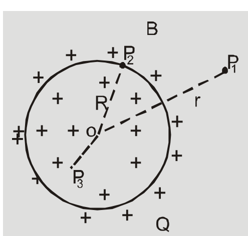
At the surface of sphere $V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$

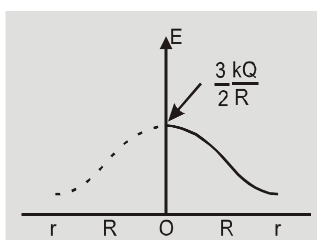
Inside the sphere $V = V_{surface} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$

The potential at the points inside a conducting sphere is constant. (The electric field inside is zero)

Potential due to a charged non-conducting sphere

The charge resides inside the sphere also, uniformly distributed over entire volume. For outside points it acts as a point charge located at O. The potential at points





Outside (e.g. point P_3) $V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$

At the surface (e.g. point P_2) $V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$

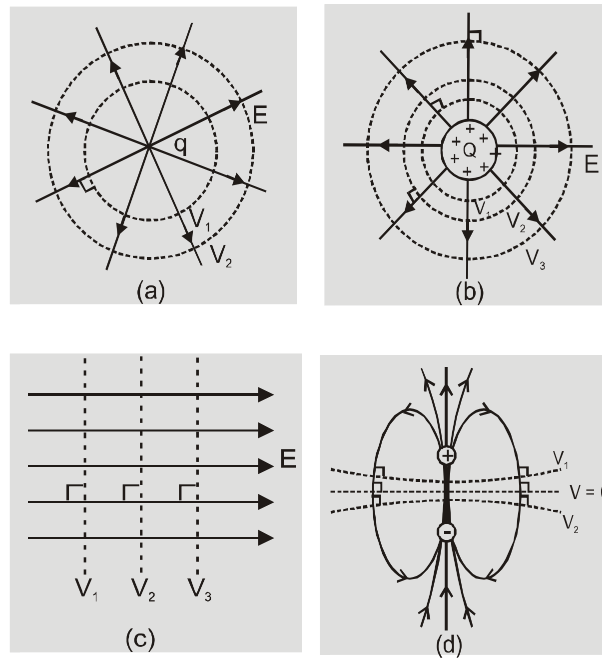
Inside the sphere (e.g. point P_1)

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q(3R^2 - r^2)}{2R^3}$$

The potential at the centre of the sphere is $V_{centre} = \frac{3}{2}\frac{kQ}{R}$. This is 1.5 times the potential at the surface of the sphere $V_{surface} = kQ/R$

Equipotential surface

A surface on which the potential is constant is called an equipotential surface. (A curve on which the potential is constant is called equipotential curve)



The electric field lines are perpendicular to equipotential surface (every where)

Since E = 0 inside a conductor, the entire conductor is at a constant potential.

For a point charge q and spherical charge distributions, the equipotential surfaces are spherical fig.a & b dotted lines)

For a uniform field equipotential surface is plane (see fig. c) dotted lines)

For a dipole, V = 0 surface is the equatorial plane. Other equipotential surfaces are curved.

Thus, in general equipotential surface can be of any shape.

When a charge is moved on an equipotential surfaces, work done is zero

Solved Examples

Ex.25 A uniform electric field of magnitude E_0 and directed along positive x-axis exist in a certain region of space. If at x = 0, then potential V is zero, then what is the potential at $x = +x_0$?

Sol. For a uniform field $\Delta V = -E\Delta r$. In this case $\Delta V = V - 0$, $\Delta r = x_0 - 0$, and $E = E_0$.

Thus $V - 0 = -E(x_0 - 0)$

or $V = -Ex_0$

(Note the negative sign. As one moves along the direction of electric field, the potential falls)

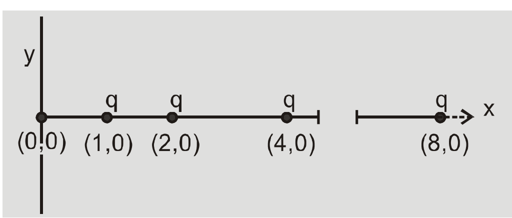
Ex.26 Electric field intensity is given by the relation $E = 100/x^2$ where x is in meters. Find potential difference between the points x = 10 m and x = 20 m.

Sol. $V = -\int_{x_1}^{x_2} E dx = -\int_{10}^{20} 100x^{-2} dx = 100\,[x^{-1}]_{10}^{20}$

$$= 100\left[\frac{1}{10} - \frac{1}{20}\right] = 5 \text{ volt.}$$

Ex.27 Infinite charges of magnitude q are placed at coordinates x = 1m, 2m, 8m respectively along the x-axis. Find the value of potential at x = 0 due to these charges.

Sol. Resultant potential at x = 0



$$V = kq\left[\frac{1}{1} + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + ...\right] = kq\left[\frac{1}{1 - 1/2}\right]$$

$= 2kq$ [$\therefore$ Sum of above G.P., S. $= \frac{a}{1-r}$]

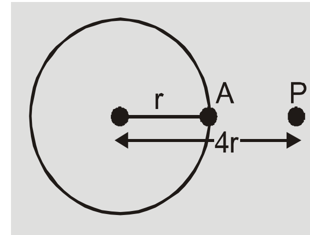
Ex.28 In the above question if alternate charges are positive and negative then find potential x = 0.

Sol. $V = kq\left[\frac{1}{1} - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} ....\right] = kq\left[\frac{1}{1 + 1/2}\right] = \frac{2}{3}kq$

Ex.29 A metallic charged sphere of radius r. V is the potential difference between point A on the surface and point P distance 4r from the centre of the sphere. Then the electric field at a point which is at a distance 4r from the centre of the sphere will be -

Sol. $V = V_A - V_P$

$$= \frac{kq}{r} - \frac{kq}{4r} = \frac{3kq}{4r}$$



Therefore for point P

$$E = \frac{kq}{(4r)^2} = \frac{V}{12r}$$

Ex.30 A ring of radius R has a charge +q. A charge q_0 is freed from the distance $\sqrt{3}$ R on its axis, when it reaches to the centre of the ring its kinetic energy becomes.

Sol. $V = \frac{1}{4\pi \in_0} \frac{q}{\sqrt{R^2 + r^2}}$

At $r = \sqrt{3}R$, $V_1 = \frac{kq}{2R}$

At $r = 0$, $V_2 = \frac{kq}{R}$

$KE = (V_2 - V_1)\, q_0 = \frac{kqq_0}{2R}$

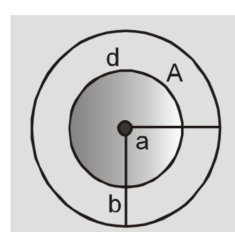
Ex.31 A solid spherical conductor carries a charge Q. It is surrounded by a concentric uncharged spherical shell. The potential difference between the surface of solid sphere and the shell in V. If a charge of –3Q is given to the shell. Then the new potential difference between the above two points will be -

Sol. Initial potential difference before charge is given to the shell

$$V_A - V_B = \frac{Q}{4\pi \in_0}\left[\frac{1}{a} - \frac{1}{b}\right] = V$$

(ii) Final potential difference after the charge –3Q is supplied to the shell

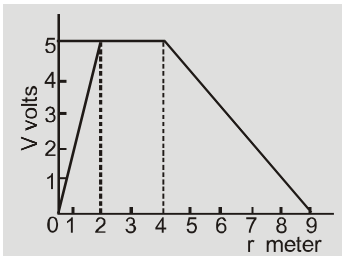
$$V_A' = \frac{1}{4\pi \in_0}\left(\frac{Q}{a} - \frac{3Q}{b}\right)$$



$$V_B' = \frac{1}{4\pi \in_0}\left(\frac{Q}{b} - \frac{3Q}{b}\right)$$

$$\therefore\ V_A' - V_B' = \frac{Q}{4\pi \in_0}\left[\frac{1}{a} - \frac{1}{b}\right] = V$$

Ex.32 In the following diagram the variation of potential with distance r is represented. The intensity of electric field in V/m at r = 3m will be



Sol. Because at r = 3m potential V = 5 volt = const.

$\therefore \; E = -\frac{dV}{dr} = 0$ V/m

Ex.33 In the above example the value of E in V/m at r = 6m will be.

Sol. At r_2 = 7m, V_2 = – 2 volt

at r_1 = 5m, V_1 = 4 volt

$$E = -\frac{(V_2 - V_1)}{(r_2 - r_1)} = -\frac{[2-4]}{[7-5]} = \frac{2}{2} = 1 \text{ V/m}$$

Ex.34 Electric potential for a point (x, y, z) is given by $V = 4x^2$ volt. Electric field at point (1, 0, 2) is -

Sol. $E = -\frac{dV}{dx} = -8xE$ at (1, 0, 2) = –8V/m

$\therefore$ Magnitude of E = 8 V/m and direction is along x-axis.

Ex.35 Electric field is given by $E = \frac{100}{x^2}$. Find potential between x = 10 and x = 20 m.

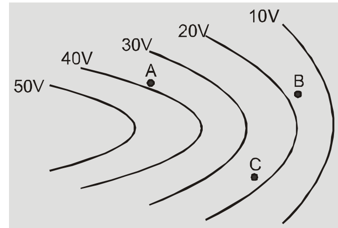
Sol. $E = -\frac{dV}{dx}$ or $dV = -Edx$

or $\int_A^B dV = -\int_A^B E.dx$

or $V_B - V_A = \int_{10}^{20} \frac{100}{x^2} = -5$ volts

Potential difference = 5volt.

Ex.36 Fig. shows lines of constant potential in a region in which an electric field is present. The value of potentials are written. At which the points A, B and C is the magnitude of the electric field greatest?

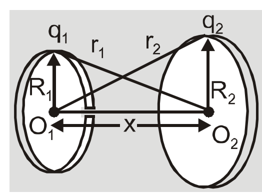


Sol. In an electric field, electric intensity E and potential V are related as

$$\vec{E} = -\frac{dV}{dr}\vec{n}, \quad \text{i.e.,} \quad E = -\frac{dV}{dr}$$

For a given line, V = constant and the potential difference between any two consecutive lines $dV = V_1 - V_2$ = 10 V = const.. So E will be maximum where the distance dr between the lines is minimum, i.e., at B (where the lines are closest)

Ex.37 Two circular loops of radius 0.05 m and 0.09 m respectively are put such that their axes coincide and their centres are 0.12 m apart. Charge of 10^{-6} coulomb is spread uniformly on each loop. Find the potential difference between the centre of loops.



Sol. The potential at the centre of a ring will be due to charge on both the rings and as every element of a ring is at a constant distance from the centre,

So, $V_1 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{R_1} + \frac{q_2}{\sqrt{R_2^2 + x}}\right]$

$= 9 \times 10^9 \left[\frac{10^{-4}}{5} + \frac{10^{-4}}{\sqrt{9^2 + 12^2}}\right]$

$= 2.40 \times 10^5$ V

Similarly, $V_2 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_2}{R_2} + \frac{q_1}{\sqrt{R_1^2 + x^2}}\right]$

or $V_2 = 9 \times 10^5 \left[\frac{1}{9} + \frac{1}{13}\right] = \frac{198}{117} \times 10^5$

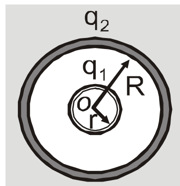
$= 1.69 \times 10^5$ V

So, $V_1 - V_2 = (2.40 - 1.69) \times 10^5 = 71$ kV

Ex.38 A charge Q is distributed over two concentric hollow spheres of radii r and R (> r) such that the surface densities are equal. Find the potential at the common centre.

Sol. If q_1 and q_2 are the charges on spheres of radii r and R respectively, then in accordance with 'conservation of charge' $q_1 + q_2 = Q$

According to given problem $\sigma_1 = \sigma_2$



or $\dfrac{q_1}{4\pi r^2} = \dfrac{q_2}{4\pi R^2}$

or $\dfrac{q_1}{q_2} = \dfrac{r^2}{R^2}$

So $q_1 = \dfrac{Qr^2}{(r^2+R^2)}$ and $q_2 = \dfrac{QR^2}{(r^2+R^2)}$

Now as potential inside a conducting sphere is equal to its surface, so potential at the common centre

$$V = V_1 + V_2 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r}+\frac{q_2}{R}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{Qr}{(R^2+r^2)}+\frac{QR}{(R^2+r^2)}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Q(R+r)}{(R^2+r^2)}$$

Electric strength (dielectric strength)

The electric strength of air is about 3×10^6 V/m or 3000 V/mm. This means that if the electric field exceeds this value sparking will occur in air. This sets a limit on maximum charge that can be given to a conducting sphere in air.

The electric strength sets a limit of the maximum charge that can be placed on a conductor.

Electric potential energy

The electric potential energy of a system of fixed point charges is equal to the work that must be done by an external agent to assemble the system, bringing each charge in from an infinite distance.

U is a scalar quantity.

Dimension of [U] = ML^2T^{-2}

Unit of [U] = joule

For two charges

$\boxed{U = \dfrac{kq_1q_2}{r}}$ $\boxed{U = q_2V_1}$ $\boxed{U = \dfrac{kq_1q_2}{r_{12}}}$

For three charges

$U = U_{12} + U_{23} + U_{13}$

$$= \frac{kq_1q_2}{r_{12}} + \frac{kq_2q_3}{r_{23}} + \frac{kq_1q_3}{r_{13}}$$

Solved Examples

Ex.39 Three charges, –q, Q, q are placed at equal distances on a straight line. If the total potential energy of the system of the three charges is zero, then Q : q = ...

Sol. Let d be the equal distance. The total potential energy of the system is,

$U = U_{12} + U_{23} + U_{31}$

or $U = \dfrac{1}{4\pi\epsilon_0}\left[\dfrac{q_1q_2}{d}+\dfrac{q_2q_3}{d}+\dfrac{q_3q_1}{2d}\right]$

or, $U = \dfrac{1}{4\pi\epsilon_0}\dfrac{q}{d}\left(-Q-Q+\dfrac{q}{2}\right)$ Since U = 0

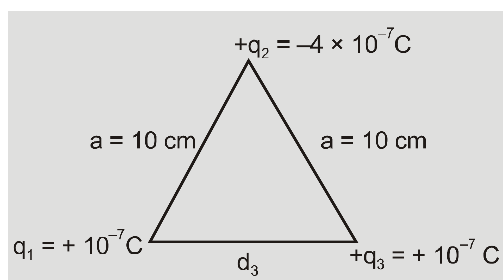
$-2Q + \dfrac{q}{2} = 0$ or, $-4Q + q = 0$

or, $4Q = q$ or, $\dfrac{Q}{q} = 1 : 4$

Ex.40 Three charges are arranged as shown in fig. Find the potential energy of the system.

Sol. The potential energy of the system is

$U = U_{12} + U_{23} + U_{31}$



$$= \frac{1}{4\pi\epsilon_0}\left[\frac{q_1(-q_2)}{a}+\frac{(-q)\times q_3}{a}+\frac{q_3q_1}{a}\right]$$

$$= \frac{1}{4\pi\epsilon_0}\left[\frac{4\times10^{-14}}{0.1}-\frac{8\times10^{-14}}{0.1}+\frac{2\times10^{-14}}{0.1}\right]$$

$$= \frac{1}{4\pi\epsilon_0}\left[\frac{10^{-14}}{0.1}(-4-8+2)\right]$$

or $U = 9 \times 10^9 \times 10^{-13}(-10)$

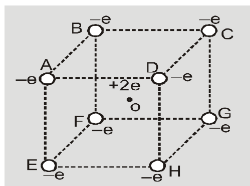
$= -9 \times 10^9 \times 10^{-12}$

$\Rightarrow -9 \times 10^{-3}$ joules

Ex.41 An electron (charge, –e) is placed at each of the eight corners of a cube of side a and α-particle (charge, +2e) at the centre of the cube. Compute the P.E of the system.

Sol. Fig., shows an electron placed at each to the eight corners of the cube and α particle at the centre. The total energy of the system is the sum of energies of each pair of charges. There are 12 pairs like A and B (separation a), 12 paris like A and C (separation, $\sqrt{2}a$), 4 pairs like A and G (separation $\sqrt{3}a$) and 8 paris like A and 0 (separation $\frac{\sqrt{3}}{2}$ a). Hence

$$U = \frac{1}{4\pi \epsilon_0}$$

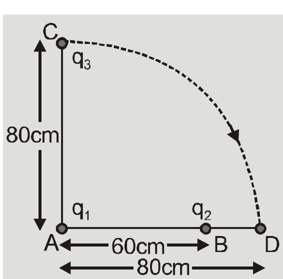


$$\left[12\frac{(-e)(-e)}{a} + 12\frac{(-e)(-e)}{\sqrt{2}a} + 4\frac{(-e)(-e)}{\sqrt{3}a} + \frac{8(-e)(2e)}{\frac{\sqrt{3}}{2}a}\right]$$

$$= (9 \times 10^9)\ \frac{e^2}{a}\ \left[12 + \frac{12}{\sqrt{2}} + \frac{4}{\sqrt{3}} - \frac{32}{\sqrt{3}}\right]$$

$$= (9 \times 10^9)\ \frac{e^2}{a}\ (4.32) = 3.9 \times 10^{10} \left(\frac{e^2}{a}\right) \text{ J.}$$

Ex.42 The value of q_1 and q_2 are 2×10^{-8} coulomb and 0.4×10^{-8} coulomb respectively as shown in fig. A third charge $q_3 = 0.2 \times 10^{-8}$ coulomb is moved from point C to point D along the arc of a circle. The change in the potential energy of charge will be -



Sol. Potential energy of q_3 at point C

$$U_C = k\left[\frac{q_1q_3}{0.8} + \frac{q_3q_2}{1}\right] \quad .....(1)$$

Potential energy of q_3 at point D

$$U_D = k\left[\frac{q_1q_3}{0.8} + \frac{q_2q_3}{0.2}\right] \quad .....(2)$$

$$\therefore\ U_D - U_C = kq_2q_3\left[\frac{1}{0.2} - \frac{1}{1}\right]$$

$$= 9 \times 10^9 \times 0.4 \times 10^{-8} \times 0.2 \times 10^{-8} \times 4$$

$$= 2.88 \times 10^{-7} \text{ joule}$$

Electron volt It is equal to the amount of energy gained by an electron when accelerated through a potential difference of one volt. It is unit of energy.

$$\boxed{1 \text{ eV} = 1.6 \times 10^{-19} \text{ joule}}$$

When a charged particle moves under the influence of an electric field, then,

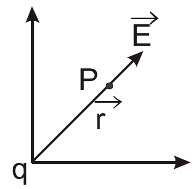
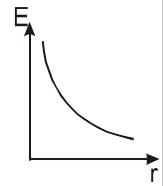
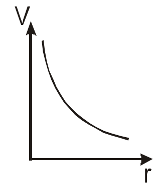
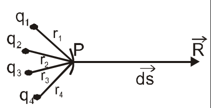
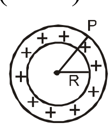
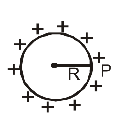
Kinetic energy gained = Potential energy lost

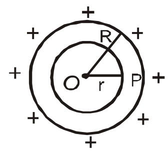
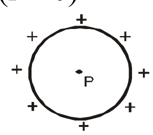
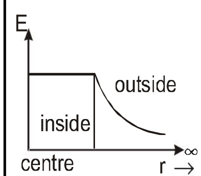
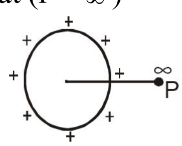
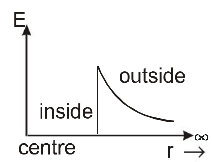
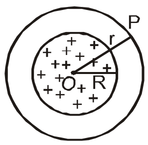
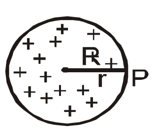
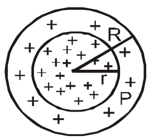
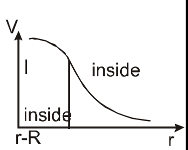
Energy density In electric field, energy stored per unit volume is called energy density. It is equal to

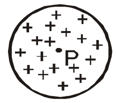
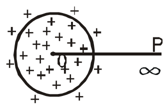
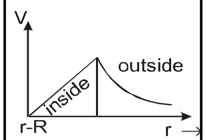
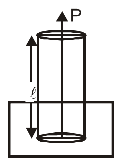
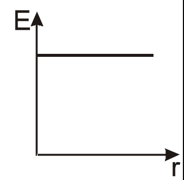
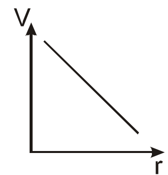
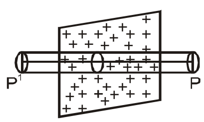
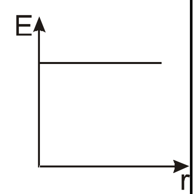
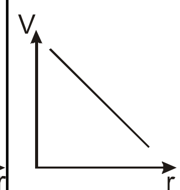
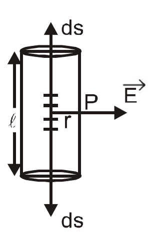
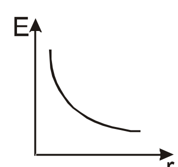
$$\boxed{u = \frac{1}{2}\varepsilon_0 E^2}$$

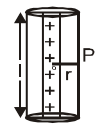
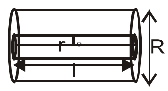
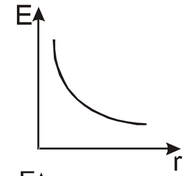
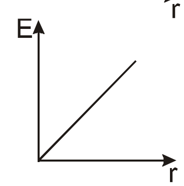
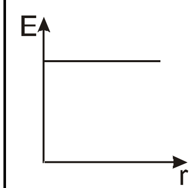
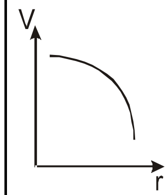
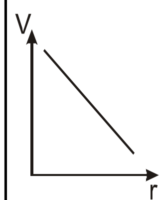
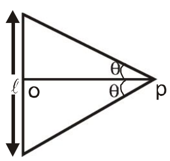
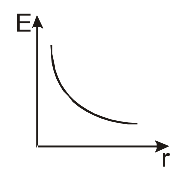
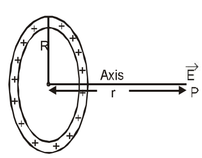
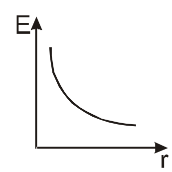
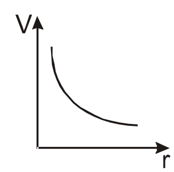
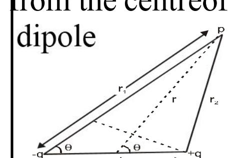
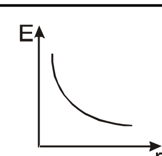
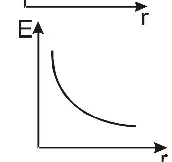
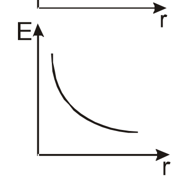
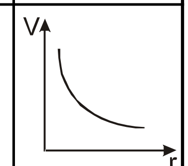
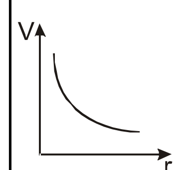
6. ELECTRIC FIELD AND POTENTIAL DUE TO DIFFERENT BODIES

Graphical representation of changes electric field and electric potential

S. No.	Charge distribution	Point observation	Gaussian surface	Eleectric ficid	Electric potential	E-r curves	V-r curves
1.	Point charge		Spherical	$E = \frac{Kq}{r^2}\hat{r}$	$V = \frac{kq}{r}$		
2.	Group of source chariges	Point Pdistant r_1, r_2, r_3 ...from q_1, q_2, q_3 ... respectvely 	Spherical	$V = \sum_{i=I}^{n} \frac{Kq_i}{r_i^2}$	$V = \sum_{i=I}^{n} \frac{Kq_i}{r_i}$		
3.	Uniform distribution of charges			$E = K\int \frac{dq}{r^2}$ $E = K\int \frac{\rho dV}{r^2}$	$V = k\int \frac{dq}{r}$ $V = k\int \frac{\rho dV}{r}$		
4.	Charged conducting sphere	(i) Point is situated outside the conducting sphere (r > R)  (a) (ii) Point is situated on the surface of conducting sphere  (b)	Spherical Spherical	$E = \frac{Kq}{r^2}\hat{r}$ $E = \frac{Kq}{R^2}\hat{r}$	$V = \frac{Kq}{r}$ $V = \frac{Kq}{r}$		

		(iii) Point is situated inside the conducting sphere (r<R)  (c)	Spherical	$E = 0$	$V = \frac{Kq}{R}$		
		(iv) point is situated of the centre of sphere (r = 0)  (d)	Spherical	$E = 0$	$V = \frac{Kq}{R}$		
		(v) point is situated at ($r = \infty$)  (e)	Spherical	$E = 0$	$V = 0$ 		
5.	Uniformly charged non conducting sphere	(i) r>R  (a)	Spherical	$E = \frac{Kq}{r^2}\hat{r}$	$V = \frac{Kq}{r}$		
		(ii) r = R  (b)	Spherical	$E_s = \frac{Kq}{R^2}$	$V_s = \frac{Kq}{R}$		
		(iii) r < R  (c)	Spherical	$E_i = \frac{Kq}{R^3}\vec{r}$	$V = \frac{K[3R^2 - r^2]}{2R^3}$		

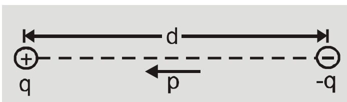
		(iv) r = o  (d)	Spherical	E = 0	$V=\frac{3Kq}{2R}=\frac{3}{2}V_s$		
		(v) r = ∞  (e)	Spherical	E = 0	V = 0	 	
6.	Charged conducting plate	Outside plate 	Cylindrical	$\vec{E}=\frac{\sigma}{\varepsilon_0}\hat{n}$ $=\frac{q}{\varepsilon_0 A}\hat{n}$	$V=\frac{-\sigma r}{\varepsilon_0}+C$		
7.	Infinite sheet of Charge	Outside the sheet 	Cylindrical	$\vec{E}=\frac{\sigma}{2\varepsilon_0}\hat{n}$ $=\frac{q}{2A\varepsilon_o}\hat{n}$	$V=-\frac{\sigma r}{2\varepsilon_0}+C$		
8.	Infinite time of Charge	At a distancer from the axis of cylinderical gaussian surface 	Cylindrical	$E=\frac{2\lambda K}{r}\hat{r}$ $E=\frac{\lambda}{2\pi\varepsilon_o r}\hat{r}$ $E=\frac{2qK}{\ell r}\hat{r}$ $E=\frac{q}{2\pi\varepsilon_o r\ell}\hat{r}$	$V=\frac{-\lambda}{2\pi\varepsilon_o}\log_e r+c$ $V=\frac{-q}{2\pi\varepsilon_o l}\log r+c$		

9.	Charged cyllndrical conductor	(i) At an out side point r>R  (a) (ii) At an inside point r < R  (b) (iii) At the surface r = R	Cylindrica Cylindrica Cylindrical	$E=\frac{\lambda}{2\pi\varepsilon_o r}\hat{r}$ $E=\frac{\lambda r}{2\pi\varepsilon_o R^2}$ $E=\frac{\lambda}{2\pi\varepsilon_o R}$	$V=\frac{-\lambda}{2\pi\varepsilon_o}\log_e r+c$ $V=\frac{-\lambda r^2}{4\pi\varepsilon_o R^2}+c$ $V=\frac{-\lambda}{2\pi\varepsilon_o}\log_e r+c$	  	 
10.	Linear charge distribution of length ℓ		Cylindrical	$E=\frac{\lambda\sin\theta\hat{r}}{2\pi\varepsilon_o r}$	$V=\frac{\lambda}{2\pi\varepsilon_o}\log\left[\frac{\sqrt{r^2+\ell^2}-1}{\sqrt{r^2+r^2}+1}\right]$		
11.	Charged circular ring	(a) On the axis of ring at a distancer from its centre  (b) at the centre of ring		$E=\frac{\lambda R\hat{r}}{2\pi\varepsilon_o\left[R^2+r^2\right]^{\frac{3}{2}}}$ $E = 0$	$V=\frac{\lambda}{2\pi\varepsilon_o\sqrt{R^2+r^2}}$  $V =$ constant		
12.	Electric dipole	(i) On the axis of dipole at a dista cer from is its centre (ii) On the equatorial axis at a distancer from its centre (iii) at point (r,θ) from the centreof dipole 	Cylindrical	$E_{axial}=\frac{2KP}{r^3}$ $E_{eqatorial}=\frac{KP}{r^3}$ $E=\frac{KP}{r^3}\sqrt{1+3\cos^2\theta}$	$V=\frac{KP}{r^2}$ $V = 0$ $V=\frac{K\vec{P}.\vec{r}}{r^3}$	  	 

ELECTRIC DIPOLE

Two equal and opposite charges separated by a small distance is called an electric dipole.

The dipole moment $\vec{p}$ is a vector quantity whose magnitude is equal to the product of magnitude of one charge and the distance between the two charges. It is directed from negative charge to positive charge.

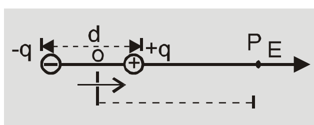


Unit of dipole moment (p) = coulomb × metre = C.m

Dimensions of $p = M^0L^1T^1A^1$

Electric field at an axial point

$$\vec{E} = \frac{k2\vec{p}r}{[r^2-(d/2)^2]^2}$$



The direction is along the axis, parallel to $\vec{p}$. The magnitude is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{2pr}{[r^2-(d/2)^2]^2}$$

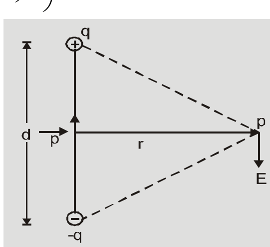
For r >> d,

$$E = \frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3}$$

Electric field at an equatorial point

The electric field at an equatorial point P is

$$\vec{E} = -\frac{k\vec{p}}{\left(r^2+\left(\frac{d}{2}\right)^2\right)^{3/2}}$$



The field is directed opposite to $\vec{p}$ and the magnitude is $E = \frac{1}{4\pi\varepsilon_0}\frac{p}{(r^2+(d/2)^2)^{3/2}}$

For r >> d

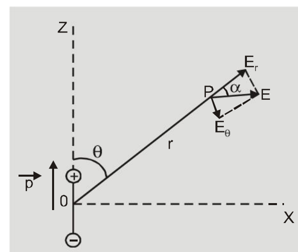
$$E = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}$$

Electric field at (r, θ) point

For r >> d, the radial component E_r and angular component E_θ of electric field due to a dipole, are

$$E_r = k\frac{2p\cos\theta}{r^3}$$

$$E_\theta = k\frac{p\sin\theta}{r^3}$$



The resultant field is

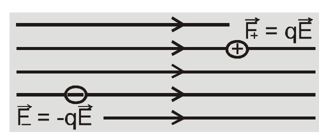
$$E = \sqrt{E_r^2+E_\theta^2}$$

$$= \frac{kp}{r^3}\sqrt{1+3\cos^2\theta}$$

The angle α in figure is such that

$$\tan\alpha = \frac{E_\theta}{E_r} = \frac{1}{2}\tan\theta$$

Force and torque on a dipole placed in an electric field



(a) A positive charge +q experiences a force parallel to the electric field $\vec{F} = q\vec{E}$

(b) A negative charge –q experiences a force in a direction opposite to that of the electric field $\vec{F} = -q\vec{E}$

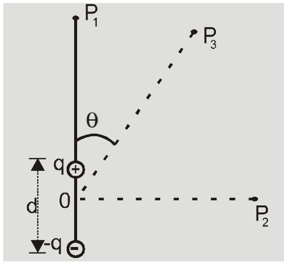
(c) The total force on a dipole placed in an uniform electric field is zero $\vec{F} = q\vec{E}+(-q\vec{E}) = 0$

(d) The torque on a dipole placed in uniform electric field is $\vec{\tau} = \vec{p}\times\vec{E} \Rightarrow \tau = pE\sin\theta$

Electric potential due to a dipole

The potential at an axial point (P_1) is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^2-(d/2)^2}$$



If d << r then

$$\boxed{V = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^2}}$$

where p = qd is the dipole moment

The potential at an equatorial point (P_2)

$$\boxed{V = 0}$$

The potential at any arbitrary point (P_3, located at r, θ coordinates) is (for r >> d)

$$\boxed{V = \frac{1}{4\pi\varepsilon_0}.\frac{p\cos\theta}{r^2}}$$

Potential at a point which is equidistant from +q and –q charge is zero. Thus potential at all points lying on the equatorial plane is zero for a dipole.

Potential energy of a dipole

The work done in rotating a dipole placed in a uniform electric field E, from initial angle θ_1 to final angle θ_2 is

$$W = \int_{\theta_1}^{\theta_2} \tau d\theta \quad = pE(\cos\theta_1 - \cos\theta_2)$$

where U_2, U_1 are the potential energy of the dipole in the two orientations.

Potential energy

$$U_2 - U_1 = -pE\cos\theta_2 - (-pE\cos\theta_1)$$
$$= -pE(\cos\theta_2 - \cos\theta_1)$$

The zero of the potential is taken at θ = 90º . Thus, potential energy of the dipole is

$$U = -\vec{p}.\vec{E} \quad = -pE\cos\theta$$

$U_{min} = -pE$, $U_{max} = +pE$

Work done in rotating a dipole from θ = 0 (aligned parallel to E), to θ = 180º (aligned antiparallel to E) is $\boxed{W = 2pE}$

Solved Examples

Ex.43 The work required to turn an electric dipole end for end in a uniform electric field when the initial angle between $\vec{p}$ and $\vec{E}$ is θ_0 is -

Sol. $W = pE(\cos\theta_1 - \cos\theta_2)$, in this case $\theta_1 = \theta_0$ and $\theta_2 = \pi = \theta_0$. Thus

$W = pE\{\cos\theta_0 - \cos(\pi + \theta_0)\}$

$= 2pE\cos\theta_0$

Ex.44 Calculate the electric intensity due to a dipole of length 10 cm and having a charge of 500 μC at a point on the axis distance 20 cm from one of the charges in air .

Sol. The electric intensity on the axial line of the dipole

$$E = \frac{1}{4\pi\epsilon_0}\frac{2pd}{(d^2-\ell^2)^2}$$

2t = 10 cm $\therefore \ell = 5 \times 10^{-2}$ m

d = 20 + 5 = 25 cm = 25×10^{-2} m

$p = 2q\ell = 2 \times 500 \times 10^{-6} \times 5 \times 10^{-2} \Rightarrow 5 \times 10^{-3} \times 10^{-2} = 5 \times 10^{-5}$ C-m

$$\therefore \quad E = \frac{9\times10^9\times2\times5\times10^{-5}\times25\times10^{-2}}{10^{-8}[25^2-5^2]^2} \Rightarrow 6.25 \times 10^7 \text{ N.C.}$$

Ex.45 Calculate the electric intensity due to an electric dipole of length 10 cm having charges of 100 μC at a point 20 cm from each charge.

Sol. The electric intensity on the equatorial line of an electric dipole is $E = \frac{1}{4\pi\epsilon_0}\frac{p}{(d^2+\ell^2)^{3/2}}$

$p = 2\ell q$ C-m

$= 10 \times 10^{-2} \times 100 \times 10^{-6}$

$= 10^{-5}$ C-m

$d^2 + \ell^2 = (20 \times 10^{-2})^2 = 4 \times 10^{-2}$

$$\therefore \quad E = \frac{9\times10^9\times10^{-5}}{(4\times10^{-2})^{3/2}}$$

$$= \frac{9\times10^9\times10^{-5}}{10^{-3}\times8} = \frac{9}{8}\times10^7 = 1.125 \times 10^7 \text{ N/C}$$

Ex.46 Find out the torque on dipole in N-m given :

Electric dipole moment $\vec{P} = 10^{-7}(5\hat{i} + \hat{j} - 2\hat{k})$ coulomb metre and electric field $\vec{E} = 10^{7}(\hat{i} + \hat{j} + \hat{k})$ Vm^{-1} is -

Sol. $\vec{\tau} = \vec{P} \times \vec{E} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & 1 & -2 \\ 1 & 1 & 1 \end{vmatrix}$

$= \hat{i}(1+2) + \hat{j}(-2-5) + \hat{k}(5-1) = 3\hat{i} - 7\hat{j} + 4\hat{k}$

$|\vec{\tau}| = 8.6$ N-m

CHARGED LIQUID DROP

If n small drops each of radius r coalesce to form a big drop of radius R, then

(i) $\frac{4}{3}\pi R^3 = n\frac{4\pi}{3}r^3$

$\therefore \quad R = rn^{1/3}$

(ii) If each small drop has a charge q, then the charge on the big drop

$q' = nq$

(iii) If V is the potential of the small drop, then the potential of the big drop will be

$$V' = \frac{Kq'}{r} = \frac{Knq}{rn^{1/3}} = Vn^{2/3}$$

(iv) If E is the electric field intensity at the surface of the small drop, then the electric field intensity at the surface of the big drop will be

$$E' = \frac{Kq'}{R^2} = \frac{Knq}{R^2} = \frac{Kqn^{1/3}}{r^2} = n^{1/3}E$$

Solved Examples

Ex.47 1000 equal drops of radius 1cm, and charge 1×10^{-6} C are fused to form one bigger drop. The ratio of potential of bigger drop to one smaller drop, and the electric field intensity on the surface of bigger drop will be respectively -

Sol. Let the potential of one smaller drop be B then potential of bigger drop, is $V' = n^{2/3}$ V

$\Rightarrow \frac{V'}{V} = n^{2/3} = (1000)^{2/3} = 100$

$\therefore$ V' : V = 100 : 1

Also let the electric field on the surface of smaller drop be E then electric field on bigger drop is

$E' = n^{1/3}E = n^{1/3}\frac{kq}{r^2} = (1000)^{1/3}\frac{9\times10^9 \times 1\times10^{-6}}{(1\times10^2)^2}$

$= 9 \times 10^8$ V/m

FORCE ON A CHARGED SURFACE

(i) If we consider an element of the charged surface, then the charge on the element experiences a repulsive force due to the charge on the remaining part. As a result, a resultant force acts on it perpendicular to the surface in the outward direction.

(ii) The charged surface behaves as a stretched membrane.

(iii) If σ is the surface charge density, then the electric field intensity at external points close to the surface is $\frac{\sigma}{\epsilon_0}$ and at internal points close to the surface the field is zero. Thus the average intensity at the surface.

$$E = \frac{\sigma}{2\epsilon_0}$$

(iv) The repulsive force acting on a unit area of the surface will be.

$$F = E\sigma = \frac{\sigma^2}{2\epsilon_0} \text{ N/m}^2$$

(v) The repulsive force acts in the outwards direction. The force acting on a unit area of the surface is electrical pressure.

$$\therefore \quad P_{elec} = \frac{\sigma^2}{2\epsilon_0}$$

$= 2\pi K\sigma^2$ N/m^2

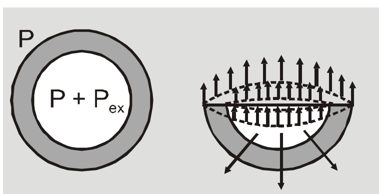
EQUILIBRIUM OF A CHARGED SOAP BUBBLE

(i) In equilibrium the air pressure inside a soap bubble is greater than the atmospheric pressure. This excess pressure is produced due to surface tension of the soap solution. If r is the radius of the bubble and T is its surface tension, then for the uncharged bubble in equilibrium,

Force due to excess pressure = force produced due to surface tension

$$p_{ex} \times \pi r^2 = T(2 \times 2\pi r)$$

$$\therefore \quad p_{ex} = \frac{4T}{r}$$



(ii) If a bubble is charged, then electrical pressure due to charge acts in outwards direction on the bubble.

$$p_{elec} = \frac{\sigma^2}{2 \epsilon_0}$$

where σ is the surface charge density.

(iii) In equilibrium, the force produced due to surface tension is equal to the sum of forces due to excess air pressure inside the bubble and the electrical pressure due to charge, i.e.

$$(p_{ex} + p_{elec})\pi r^2 = T(2 \times 2\pi r)$$

$$\text{or} \quad p_{ex} + p_{elec} = \frac{4T}{r}$$

$$\text{or} \quad p_{ex} + \frac{\sigma^2}{2 \epsilon_0} = \frac{4T}{r}$$

(iv) For charged bubble,

$$p_{ex} = \frac{4T}{r} - \frac{\sigma^2}{2 \epsilon_0}$$

(v) If the air pressure inside the bubble is equal to the atmospheric pressure outside, i.e., $p_{ex} = 0$ then

$$\frac{\sigma^2}{2 \epsilon_0} = \frac{4T}{r}$$

$$\text{or} \quad \sigma = \sqrt{\frac{8 \epsilon_0 T}{r}} = \sqrt{\frac{2T}{\pi K r}}$$

(vi) If charge given to the soap bubble is q, then

$$\sigma = \frac{q}{4\pi r^2}$$

∴ The charge on the bubble

$$q = 4\pi r^2 \sigma$$

$$= 4\pi r^2 \sqrt{\frac{8 \epsilon_0 T}{r}} = 8\pi\sqrt{2 \epsilon_0 T r^3} = \sqrt{32\pi T r^3 / K}$$

(vii) The intensity of electric field at the surface of the bubble

$$E = \sqrt{32\pi T K / r} = \sqrt{8T / \epsilon_0 r}$$

(viii) The electric potential at the surface of bubble

$$V = \sqrt{32\pi r T K} = \sqrt{8 T r / \epsilon_0}$$

(ix) On charging a bubble the air pressure inside it decreases because the radius of the bubble increased due to charging.

(x) A soap bubble always expands on giving any kind of charge (positive or negative)

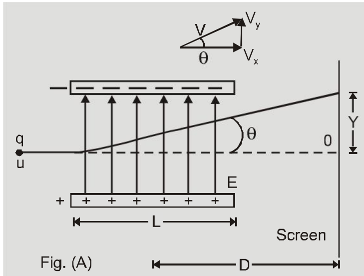
(xi) When charge is given to the soap bubble, Boyle's law holds during this process because mass of air inside the bubble remains constant.

MOTION OF A CHARGED PARTICLE IN ELECTRIC FIELD

(i) A charged particle at rest or moving experiences a force $\vec{F} = q\vec{E}$ in the presence of electric field. The acceleration, velocity and displacement are given by

$$\vec{a} = \frac{q\vec{E}}{m}$$

$$\vec{v} = \vec{u} + \vec{a}t$$



$$\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$$

(ii) For a charged particle with initial velocity perpendicular to the electric field

Note that $F_x = 0$, $a_x = 0$, $V_x = u$ at all times

$$F_y = qE,\ a_y = \frac{qE}{m},\ V_y = \frac{qE}{m}t$$

The displacement components are $x = ut$

$$y = \frac{1}{2}\frac{qE}{m}t^2$$

Eliminating t, $\boxed{y = \frac{1}{2}\frac{qE}{mu^2}x^2}$ which is the equation of a parabola.

(iii) The path of a charged particle entering a region of electric field with initial velocity perpendicular to the field follows a parabolic trajectory.

The time spent in electric field is $t = \frac{L}{u}$ (see fig. A)

The y component of velocity when it emerges out of the field region is $V_y = \frac{qEL}{mu}$

The resultant velocity

$$V = \sqrt{V_x^2 + V_y^2} = \sqrt{u^2 + \left(\frac{qEL}{mu}\right)^2}$$

The angle at which the particle emerges

$$\tan\theta = \frac{V_y}{V_x} = \frac{qEL}{mu^2}$$

The height Y at which the particle hits the screen (see fig. A)

$$Y = D\tan\theta$$

$$Y = \frac{qELD}{mu^2} \Rightarrow Y = \frac{qELD}{2K}$$

(where K is initial kinetic energy)

Solved Examples

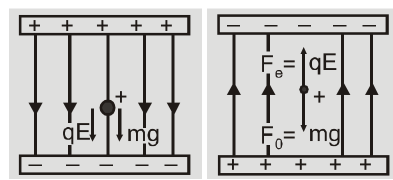
Ex.48 A positively charged oil droplet remains stationary in the electric field between two horizontal plates separated by a distance of 1cm. If the charge on the drop is 9.6×10^{-10} esu and the mass of droplet is 10^{-11} g, what is the potential difference between two plates ? Now if the polarity of the plates is reversed what is the instantaneous acceleration of the droplet? [$g = 9.8\ m/s^2$]

Sol. As the droplet is at rest, its weight W = mg will be balanced by electric force F = qE

i.e., qE = mg

or $V = \frac{mgd}{q}$ $V = \frac{(10^{-14})(9.8)(1\times10^{-2})}{(9.6\times10^{-10}/3\times10^{9})}$

= 3062.5 volt



Now if the polarity of the plates is reversed, both electrical and gravitational force will act downward

so, F = mg + qE = 2mg [as mg = qE]

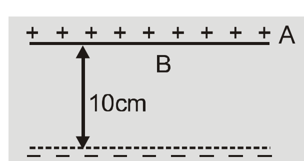
And hence instantaneous acceleration of drop :

$$a = \frac{F}{m} = \frac{2mg}{m} = 2g = 19.6\ m/s^2$$

Ex.49 An oil drop 'B' has charge 1.6×10^{-19} C and mass 1.6×10^{-14} kg. If the drop is in equilibrium position, then what will be potential diff. between the plates. [The distance between the plates is 100mm]

Sol. For equilibrium, electric force = weight of drop

$$\Rightarrow qE = mg \quad \text{or} \quad q = \frac{V}{d} = mg$$



$$\Rightarrow V = \frac{mgd}{q} = \frac{1.6\times10^{-14}\times9.8\times10\times10^{-3}}{1.6\times10^{-19}} = 10^4 \text{ volt}$$

CAPACITANCE

CAPACITANCE OF A CONDUCTOR

* A capcitor is a divice which stores electric energy. It is also named as condenser.
* When charge is given to a conductor, its potential increases in the ratio of given charge.

  The charge given to the conductor is directly proportional to its potential.

  $Q \propto v$ or $Q = cv$

  where C is a constant called electrical capacitance of the conductor.

  If V=1 volt, then Q = C; i.e., the electrical capacitance of the conductor is equal to that amount of charge which increases the potential of the conductor by a unit amount.
* The electrical capacitance of the conductor depends upon the following:

  (a) The size of the conductor:

  The capacitance is directly proportional to the area of the surface of the conductor, i.e., $C \propto A$ (area)

  (b) The medium around the conductor: If a conductor is placed in a medium other than air or vacuum, its capacitance will increase.

  $$C_{medium} = KC_{air\,or\,vacuum}$$

  where K is a constant called dielectric constant of the medium. This constant is always greater than 1.
* Theoretically the charge can be given to any isolated conductor upto an infinite amount. When a very large amount of charge is given to the conductor, its potential increases to such an extent that dielectric breaks down and the charge starts leaking from the object and electrical discharge takes place between the conductor and nearby objects or earth. This decreases the potential of the conductor.
* Unit of Capacitance:

  (a) In M.K.S. system unit of capacitance is farad.

  $$\text{farad} = \frac{\text{coulomb}}{\text{volt}}$$

  Dimensions of the capacitance are $M^{-1}L^{-2}T^4A^2$

  (b) If a charge of one coulomb given to a conductor increases the potential by one volt, then its capacitance is one farad.

  (c) In practice, submultiples of farad are used as units of capacitance.

  1 micro farad = $1\mu F = 10^{-6} F$

  1 nano farad = $1nF = 10^{-9} F$

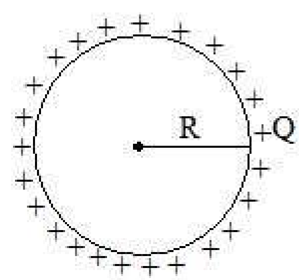
  1 micro-micro farad or pico farad = $1\mu\mu F = 1pF = 10^{-12} F$

  (d) Electrostatic unit of electrical capacitance is stat farad.

  1 farad = 9×10^{11} statfarad

CAPACITANCE OF SPHERICAL CONDUCTOR

When a charge Q is given to a spherical conductor of radius R, then



* The potential of the surface of the spherical conductor will be $V = \frac{Q}{4\pi \epsilon_0 R}$
* Electrical capacitance of spherical conductor in M.K.S. system will be

  $$C = \frac{Q}{V} = \frac{Q}{Q/4\pi \epsilon_0 R} = 4\pi \epsilon_0 R$$

  In C.G.S. system, $C = R$ because K = 1 (in C.G.S unit)
* The electrical capacitance of a spherical conductor is directly proportional to its radius i.e., $C \propto R$
* The electrical capacitance of a spherical conductor does not depend on the charge given to a conductor.

POTENTIAL ENERGY OF A CHARGED CONDUCTOR OR STORED ENERGY

* The work done in charging a conductor is equal to its potential energy or the stored energy.
* This energy resides in the form of energy of the electric field created.
* The potential energy of a charged conductor:

$$U = \int_0^Q V dq = \frac{1}{C}\int_0^Q q dq = \frac{Q^2}{2C} \text{ or } U = \frac{1}{2}QV = \frac{1}{2}CV^2$$

* Unit of U is joule
* Energy stored in the conductor depends upon the given charge, its potential and its capacitance.
* Stored energy depends upon the capacitance of the capacitor and the given charge or the potential difference. It does not depend upon the shape of the capacitor.
* If the area of the plates are A and the thickness of dielectric constant is d, then energy stored per unit volume i.e., energy density of the medium will be

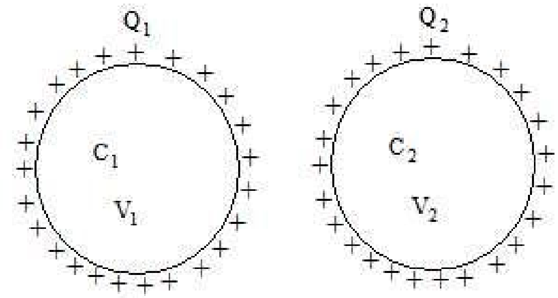
$$u = \frac{1}{2} \in E^2 = \frac{1}{2} \in_0 \in_r E^2$$

where E is the electric field.
* The energy of the charged capacitor resides in the electric field between its plates.

Note : In charging a capacitor by battery half the energy supplied is stored in the capacitor and remaining half energy $\left(\frac{1}{2}QV\right)$ is lost in the form of heat.

REDISTRIBUTION OF CHARGE ON JOINING THE CHARGED CONDUCTORS AND ENERGY LOSS

* When two charged conductors are joined together by a conducting wire of negligible capacity, the charge flows from higher potential to lower potential.
* When the potentials of both conductors become equal, the flow of charge stops.
* Law of conservation of charge holds good in the process i.e., total charge on the two conductors will be same after redistribution.
* Let the amounts of charge on the conductors be Q_1 and Q_2 and their electrical capacitances be C_1 and C_2 respectively. If their potentials are V_1 and V_2, then



$Q_1 = C_1V_1$ and $Q_2 = C_2V_2$

Total charge $Q = Q_1 + Q_2$

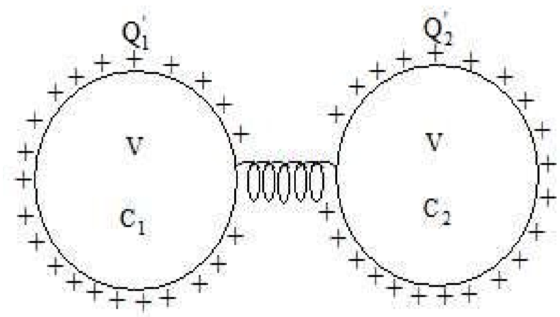
$= C_1V_1 + C_2V_2$

Total capacitance $C = C_1 + C_2$

* On joining the conductors, the common potential becomes V, then

$$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

The charge on the conductors after joining them will be



$$\therefore Q_1' = C_1V = \frac{C_1(C_1V_1 + C_2V_2)}{(C_1 + C_2)}$$

$$Q_2' = C_2V = \frac{C_2(C_1V_1 + C_2V_2)}{(C_1 + C_2)}$$

Charge transferred $\Delta Q = \frac{C_1C_2}{C_1 + C_2}(V_1 - V_2)$

* The ratio of the amounts of charge after redistribution

$$\frac{Q_1'}{Q_2'} = \frac{C_1}{C_2}$$

* In the process of redistribution of charge some work is to be done. Total amount of charge remains constant in this process but their total potential energy decreases i.e., there is loss of energy. This loss of energy is due to transformation of electrical energy into heat in the connecting wire and in electrical discharge.

* Energy loss due to redistribution of charge

$$\Delta U = U_{initial} - U_{final}$$

$$= \frac{1}{2}\left(\frac{C_1C_2}{C_1 + C_2}\right)(V_1 - V_2)^2,$$

which is always positive.

(a) If $V_1 = V_2$, then $\Delta U = 0$ that is, there is no loss of energy. Hence no loss of energy occurs in joining the conductors of equal potentials.

(b) If $V_1 > V_2$ or $V_2 > V_1$, then $(V_1 - V_2)^2$ will remain positive. Thus energy will be lost, i.e., there is always loss of energy in joining the conductors at unequal potentials.

Solved Examples

Ex1. A 20μ capacitor is charged to potential of 500V and then connected in parallel to another capacitor of capacity $10\mu F$. If the potential of $10\mu F$ capacitor is 200 volt then the common potential of two will be

[1] 100 V [2] 200 V
[3] 300 V [4] 400 V

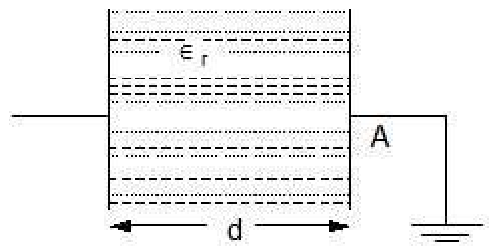
Sol. $$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2} = \frac{20\times10^{-6}\times500 + 10\times10^{-6}}{20\times10^{-6} + 10\times10^{-6}} = 400\,V$$

Hence the correct answer will be (4)

CAPACITANCE OF CAPACITORS OF DIFFERENT SHAPES

(i) Parallel plate capacitor : It consists of two parallel plates separated by a small distance d with a dielectric material in between.

If A is area of each plate, d is the distance between the plates and $\in_r$ is the dielectric constant of the material placed between the plates, then the capacitance of parallel plate capacitor will be



$$C = \frac{\in_0 A}{d}$$

(for air or vacuum as medium)

where $\in_0 = 8.85\times10^{-12} F/m$ and $C = \frac{\in_0 \in_r A}{d}$

(for medium of dielectric constant $\in_r$)

Note : (i) If two plates are placed side by side then three capacitors are formed. One between distant earthed bodies and the first face of the first plate, the second between the two plates and the third between the second face of the second plate and distant earthed objects. However the capacitances of the first and third capacitors are negligibly small in comparison to that between the plates which is the main capacitance.

* The capacitance of a capacitor depends upon the following:

(a) Area of the plates : The capacitance is directly proportional to the area of either plate, i.e., $C \propto A$.

(b) Distance between the plates : The capacitance of a capacitor decreases as the distance between the plates increases i.e., $C \propto \frac{1}{d}$

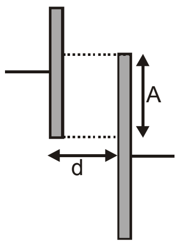
(c) Dielectric medium between the plates : If any medium other than air or vacuum is between the plates, the capacitance of the capacitor increases by $\in_r$ times where $\in_r$ is called relative permittivity or dielectric constant of the medium.

* On keeping the dielectric medium between the plates, the charge on the plates remains unchanged but the potential difference between the plates decreases.

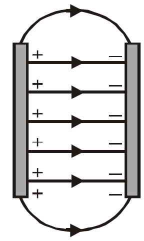
Note : (ii) It is a very common misconception that a capacitor stores charge but actually a capacitor stores electric energy in the electrostatic field between the plates. The energy density of the field

$u = \frac{1}{2} \in E^2$

(iii) Two plates of unequal area can also form a capacitor because effective overlapping area is considered.



(iv) The distance between the plates is kept small to avoid fringing or edge effect (non-uniformity of the field) at the boundaries of the plates.



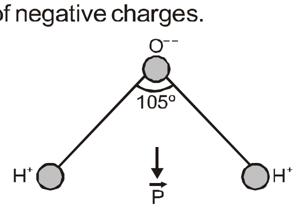
Dielectric Medium and Dielectric Constant:

Note : (v) Dielectrics are insulating (non-conducting) materials which transmits electric effect without conducting we know that in every atom, there is a positively charged nucleus and a negatively charged electron cloud surrounding it. The two oppositely charged regions have their own centres of charge. The centre of positive charge is the centre of mass of positively charged protons in the nucleus. The centre of negative charge is the centre of mass of negatively charged electrons in the atoms/molecules.

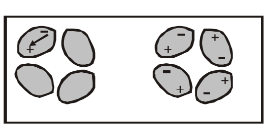
(1) Type of Dielectrics : Dielectrics are of two types -

(i) Polar dielectrics : Like water, Alcohol, CO_2, NH_3, HCl etc. are made of polar atoms/molecules

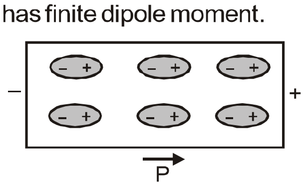
In polar molecules when no electric field is applied centre of positive charges does not coincide with the centre of negative charges.



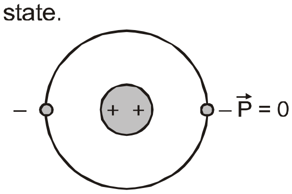
A polar molecule has permanent electric dipole moment (p) in the absence of electric field also. But a polar dielectric has net dipole moment is zero in the absence of electric field because polar molecules are randomly oriented as shown in figure.



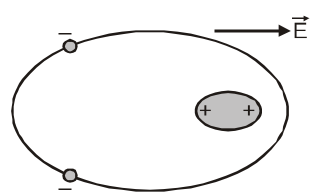
In the presence of electric field polar molecules tends to line up in the direction of electric field, and the substance has finite dipole moment.



(ii) Non polar dielectric : Like N_2, O_2, Benzene, Methane etc. are made of non-polar atoms/moelcules. In non-polar molecules, when no electric field is applied the centre of positive charge coincides with the centre of negative charge in the molecule. Each molecule has zero dipole moment in its normal state.



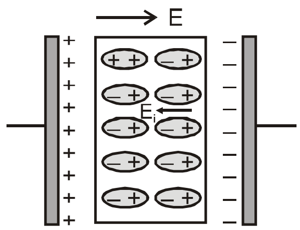
When electric field is applied, positive charge experiences a force in the direction of electric field and negative charge experiences a force in the direction opposite to the field i.e., molecules becomes induced electric dipole.



Note : In general, any non-conducting, material can be called as a dielectric but broadly non conducting material having non polar molecules referred to as dielectric because induced dipole moment is created in the non polar molecule.

(2) Polarization of a dielectric slab : It is the process of inducing equal and opposite charges on the two faces of the dielectric on the application of electric field.

Suppose a dielectric slab is inserted between the plates of a capacitor, as shown in figure.



Induced electric field inside the dielectric is E_1, hence this induced electric field decreases the main field E to $E - E_i$ i.e., New electric field between the plates will be $E' = E - E_i$.

(3) Dielectric constant : After placing a dielectric slab in an electric field. The net field is decreased in that region hence.

If E = Original electric field and E' = Reduced electric field. Then $\frac{E}{E'} = K$ where K is called dielectric constant K is also known as relative permittivity (ϵ_r) of the material or **SIC** ***(specific inductive capacitance)***

The value of K is always greater than one. For vacuum there is no polarization and hence E = E' and K = 1

(4) Dielectric breakdown and dielectric strength : If a very high electric field is created in a dielectric the outer electrons may get detached from their parent atoms. The dielectric then behaves like a conductor. This phenomenon is known as ***dielectric breakdown.***

The maximum value of electric field (or potential gradient) that a dielectric material can tolerate without it's electric breakdown is called it's ***dielectric strength.***

S.I. unit of dielectric strength of a material is $\frac{V}{m}$ but practical unit is $\frac{kV}{mm}$.

* On the basis of electrical behavior there are three types of medium:

(a) conductor (b) semiconductor (c) insulator

* Free charge carriers do not exist in insulators and their conductivity is of the order of 10^{-16} mho/metre or resistivity of the order of 10^{16} ohm-m.

* In insulators there is microscopic local displacement of charges under the influence of electric field. Such materials are called dielectrics.

* Electrical behaviour of a dielectric medium is represented by a dimensionless constant called dielectric constant.

$$\text{Dielectric constant} = \frac{\text{Permittivity of medium}}{\text{Permittivity of vacuum or free space}}$$

$$\epsilon_r = \frac{\epsilon}{\epsilon_0}$$

Dielectric constant is also called relative permittivity or specific inductive capacity of that medium.

* Dielectric constants are different for different media.

* Following effects are observed when a dielectric medium is placed between the plates of the capacitor.

(a) The capacitance of the capacitor increases while the potential difference between the plates decreases.

(b) The capacitor can be charged upto a higher potential than without the dielectric.

(c) Electrostatic potential energy of the capacitor decreases.

(d) The plates of the capacitor can be placed very close to each other without touching each other.

* The capacitance C, electric field E, potential difference V and the charge q are effected due to introduction of dielectric medium as follows

(a) $C = C_0 \epsilon_r$

(b) $E = \frac{E_0}{\epsilon_r}$ for a given charge

(c) $V = \frac{V_0}{\epsilon_r}$ for a given charge

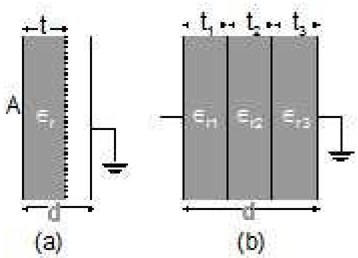
(d) $q' = \epsilon_r q$ for a given potential difference.

(a) If capacitor is partially filled with dielectric between the plates, then

$$C = \frac{\epsilon_0 A}{\left[d - t + \frac{1}{\epsilon_r}\right]}$$

where t is the thickness of the dielectric medium.

(b) If several slabs of dielectrics of different thicknesses are placed between the plates, then



$$C = \frac{\epsilon_0 A}{\left[\frac{t_1}{\epsilon_{r_1}} + \frac{t_2}{\epsilon_{r_2}} + \frac{t_3}{\epsilon_{r_3}} + \ldots..\right]}$$

(c) If a slab of metal $(\epsilon_r = \infty)$ of thickness t is placed between the plates, then $C = \frac{\epsilon_0 A}{(d-t)}$

Solved Examples

Ex.2 Two dielectrics of equal size and of constant 2 and 3 respectively fill up space between two plates of a condenser. The ratio of capacities in two possible arrangements will be

[1] $\frac{24}{25}$ [2] $\frac{25}{24}$

[3] $\frac{4}{5}$ [4] $\frac{5}{4}$

Sol. $C_1 = \frac{2K_1K_2}{[K_1 + K_2]}$

$C_2 = \frac{[K_1 + K_2]}{2}$

$\therefore \frac{C_1}{C_2} = \frac{4K_1K_2}{(K_1 + K_2)^2}$

or $\frac{C_1}{C_2} = \frac{4 \times 2 \times 3}{5^2} = \frac{24}{25}$

Hence the correct answer will be (1)

Ex.3 The capacity of a parallel plate capacitor in air is $50\mu F$ and on immersing it into oil it becames $110\mu F$. The dielectric constant of oil is

[1] 0.45 [2] 0.55

[3] 1.10 [4] 2.20

Sol. $K = \frac{C}{C_0}$

$K = \frac{110}{50} = \frac{11}{5} = 2.20$

Hence the correct answer will be (4)

Ex.4 A parallel plate capacitor with air between the plates has a capacitance of 8 pF. What will be the capacitance if the distance between the plates is reduced by half, and the space between them is filled with a substance of dielectric constant 6?

[1] 86 pF [2] 96 pF

[3] 90 pF [4] 80 pF

Sol. For parallel plate capacitor

$C = \frac{K\epsilon_0 A}{d}$, with K = 1 for air

If distance between plates is reduced to half and K = 6, the new capacitance is

$$C' = 6 \times \frac{\epsilon_0 A}{\left(\frac{d}{2}\right)} = 12.\frac{\epsilon_0 A}{d}$$

= 12 × 8 pF = 96 pF

Ex.5 In a parallel plate capacitor with air between the plates, each plate has an area of $6 \times 10^{-3} m^2$ and the distance between the plates is 3 mm, in the capacitor a 3 mm thick mica sheet of dielectric constant = 6, is inserted between the plates, while the voltage supply remains connected. What is the charge on each plate now

[1] 1.08×10^{-8} Coulomb [2] 2.08×10^{-8} Coulomb

[3] 3.08×10^{-8} Coulomb [4] 4.08×10^{-8} Coulomb

Sol. If dielectric is inserted, the capactiance becomes

C' = KC

$\approx$ 6 × 18 pF

The charge on plates is Q = C' V

= 108 × 10^{-12} × 100

= 1.08 × 10^{-8} coulomb

Ex.6 A parallel plate condenser is charged to a certain potential and then disconnected. The separation of the plates is now increased by 2.4 mm and a plate of thickness 3 mm is inserted into it keeping its potential constant. The dielectric constant of the medium will be

[1] 5 [2] 4
[3] 3 [4] 2

Sol. As charge and potential of the condenser both are constant in two cases, hence its capacity must also remain constant

$\therefore C_0 = C$

or $\frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0 A}{d' - t\left[1-\frac{1}{K}\right]}$

or $d = d' - t\left[1-\frac{1}{K}\right]$

or $(d' - d) = t\left[1-\frac{1}{K}\right]$

or $2.4 \times 10^{-3} = 3 \times 10^{-3}\left[1-\frac{1}{K}\right]$

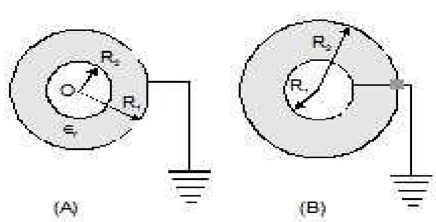
or $1-\frac{1}{K} = 0.8$ or $\frac{1}{K} = 0.2$

$\therefore$ K = 5

Hence the correct answer will be (1)

(ii) Spherical Capacitor: It consists of two concentric spheres. The space between the spherical surfaces is filled by a dielectric.

(a) When outer sphere is earthed and inner is given the charge, then



$$C = 4\pi \in_0 \in_r \frac{R_1 R_2}{R_2 - R_1}$$

(b) When inner sphere is earthed and outer one is given the charge, then

$$C = 4\pi \in_0 \in_r \frac{R_1 R_2}{R_2 - R_1} + 4\pi \in_0 R_2$$

In order to increase the capacitance of a spherical capacitor:

(1) both spheres should be very close to each other i.e., $(R_2 - R_1)$ should be small.

(2) medium between the spheres should be such that its dielectric constant is high.

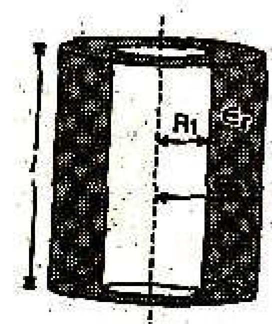
(3) the radii of both spheres should be large.

(iii) Cylindrical capacitor : It consists of two coaxial cylinders with the space between them filled by a dielectric.

If R_1 and R_2 are the radii of inner and outer cylinders respectively and ℓ is the length of the cylinder, then

$$C = \frac{2\pi \in_0 \in_r l}{log_e\left(\frac{R_2}{R_1}\right)}$$

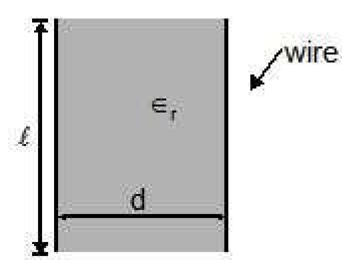
$$= \frac{2\pi \in_0 \in_r l}{2.303\, log_{10}\left(\frac{R_2}{R_1}\right)}$$



(iv) Capacitance of two parallel transmission lines: If the radius of each wire is r, d is the distance between them (d > > r) and ℓ is the length of the wires, then

$$C = \frac{\pi \in_0 \in_r l}{log_e\left(\frac{d}{r}\right)}$$

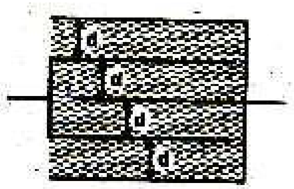
$$= \frac{\pi \in_0 \in_r l}{2.3\, log_{10}\left(\frac{d}{r}\right)}$$



As medium between the wires is usually air so

$$C = \frac{\pi \in_0 l}{log_e\left(\frac{d}{r}\right)}$$

(v) Multiplate capacitor and capacitance of a variable capacitor : If n is the number of plates, A is the area of each plate, $\in_r$ is the dielectric constant of medium and d is the distance between two successive plates, then



$$C = (n-1)\frac{\in_0 \in_r A}{d}$$

In order to obtain signals of a desired frequency in electronic instruments such as radio, T.V. etc., a parallel resonance circuit is used. In this circuit a special type of parallel plate capacitor called gang condenser is connected in parallel with an inductance coil. It consists of two sets of semi circular plates. The plates of one set are stationary and the plates of other set are rotated by means of a knob. By changing the common area of the plates, the resultant capacitance can be changed.

COMBINATIONS OF CAPACITORS

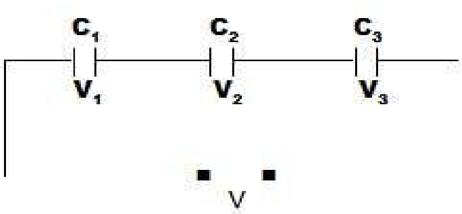
Capacitors can be combined in two ways:

(a) Series combination

(b) Parallel combination

(a) Series combination:

* Combination of capacitors on series is shown in following diagram



* Amount of charge is same on each capacitor, i.e.,

$$Q = C_1V_1 = C_2V_2 = C_3V_3 = ...$$

* Potential difference across each capacitor will be different and is inversely proportional to its capacitance.

$$V_1 = \frac{Q}{C_1}, V_2 = \frac{Q}{C_2}, V_3 = \frac{Q}{C_3}$$

In this combination the potential difference between the plates of the capacitor of least capacitance is maximum.

* The potential difference applied in this combination is the sum of potential differences on individual capacitors.

$$V = V_1 + V_2 + V_3 + ...$$

* If C is the equivalent capacitance of this combination, then

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ...$$

That is, if several capacitors are connected in series,then the reciprocal of the capacitors equivalent capacitance is equal to the sum of the reciprocals of capacitances of the individual capacitors.

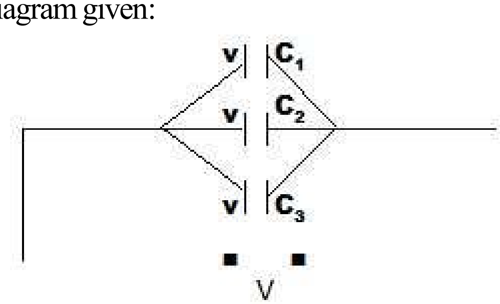
* On combining the capacitors in series, the total capacitance of the circuit decreases and the equivalent capacitance is less than the lowest capacitance connected in series.
* If n identical capacitors each of capacitance C' are connected in series, then the equivalent capacitance will be : $C = \frac{C'}{n}$
* **This combination is used when:**

(a) a capacitance less than the lowest value of the given capacitance is needed,

(b) a high voltage is to be divided on several capaacitors.

(b) Parallel combination :

* Combination of capacitors in parallel is shown in the diagram given:



* The potential differance across each capacitors is equal to the applied potential difference in the circuit.
* The amount of charge is different on each capacitor and the charge is directly proportional to the capacitance of the capacitor.

$$Q_1 = C_1V, Q_2 = C_2V, Q_3 = C_3V....$$

* Total charge in the circuit :

$$Q = Q_1 + Q_2 + Q_3 + ...$$

or $Q = C_1V + C_2V + C_3V + ...$

* If C is equivalent capacitance of this combination, then:

$$C = C_1 + C_2 + C_3 + ...$$

That is, the equivalent capacitance of the capacitors connected in parallel is equal to the sum of their capacitances.

* On combining the capacitances in parallel the equivalent capacitance of the circuit increases.
* If n identical capacitors each of capacitance C are connected in parallel, then equivalent capacitance will be :

$$C' = nC$$

* This combination is used when :

(a) The capacitance in the circuit is to be increased.

(b) Higher capacitance is required at low potential.

Solved Examples

Ex.7 Three capacitors of same capacitance are connected in parallel. When they are connected to a cell of 2 volt, total charge of $1.8\mu C$ is accumulated on them. Now they are connected in series and then charged by the same cell. The total charge stored in them will be

[1] $1.8\mu C$ [2] $0.9\mu C$

[3] $0.6\mu C$ [4] $0.2\mu C$

Sol. If the capacitance of each capacitors is C, then equivalent capacitance on joining them en parallel will be 3C.

$\therefore$ Stored charge Q = 3CV

$$\therefore\ C = \frac{Q}{3V} = 3 \times 2$$

Let C' be the equivalent capacitance on joining them in series, then

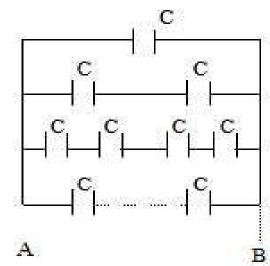
$$\frac{1}{C'} = \frac{1}{0.3} + \frac{1}{0.3} + \frac{1}{0.3} = \frac{3}{0.3}$$

$$C' = \frac{0.3}{3} = 0.1\ \mu F$$

Now $Q' = C'V = 0.1 \times 2 = 0.2\ \mu C$

$\therefore$ Answer will be (4)

Ex.8 In the adjoining figure, if the capacitance of each capacitor is $1\mu F$, then the equivalent capacitance between A and B will be



[1] $2\mu F$

[2] $2.5\mu F$

[3] $4.5\mu F$

[4] $5\mu F$

Sol. If equivalent capacitances of the rows of capacitors are C_1, C_2, C_3....., then

$C_1 = 1\ \mu F$ $C_2 = \frac{1}{2}\ \mu F$

$C_3 = \frac{1}{4}\mu F$ $C_4 = \frac{1}{8}\ \mu F$ etc.

$\therefore$ Equivalent capactance between A and B

$$C = C_1 + C_2 + C_3 + C_4 + ...$$

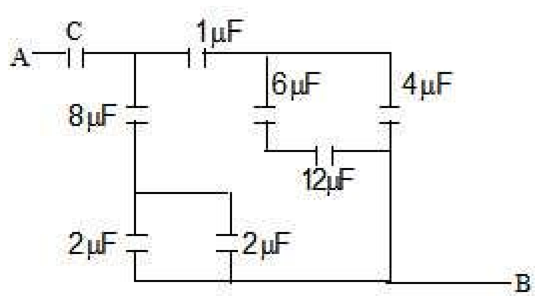
$$= 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8}$$

This is a geometric series whose first term is a = 1 and common ratio $r = \frac{1}{2}$

$$\therefore\ C = \frac{a}{1-r} = \frac{1}{1-\frac{1}{2}} = 2\mu F$$

$\therefore$ Answer will be (1)

Ex.9 The effective capacitance between points A and B is $1\mu F$. What is the capacitance of C?



[1] $1.39\mu F$ [2] $2.46\mu F$

[3] $2.23\mu F$ [4] None of the above

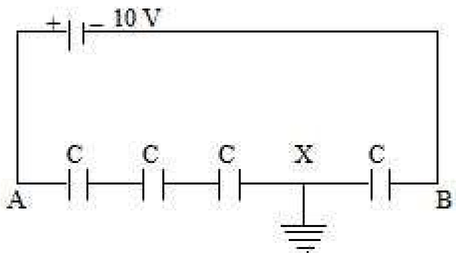
Sol. The circuit is equivalent to fig, Thus, given data implies that

$$\frac{(32C+9)}{(C+32/9)} = 1$$

$C = 32/23 = 1.39\ \mu F$

The correct answer is (1)

Ex.10 Four indentical capacitors are connected in series with a battery of emf 10 V. The point X is earthed. T hen the potential of point A is



[1] 10 V [2] 7.5 V

[3] –7.5 V [4] 0 V

Sol. The equivalent capacitance of the circuit is C/4 and the charge on each capacitor is

$$Q = C'V = \frac{C}{4}\times 10 = 2.5C$$

Thus potential difference acros each capacitor is

$$V = \frac{Q}{C} = 2.5 \text{ volt.}$$

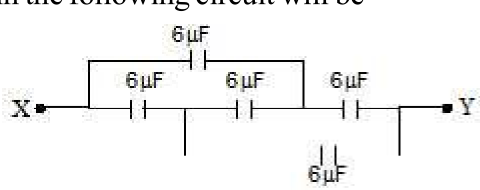
Starting, from grounded point X, we go to point A, with change in p.d of V across each capacitor.

Thus $V_A - V_X = 7/5$ V

since $V_x = 0$, thus $V_A = 7.5$ volt.

The correct anser is (2)

Ex.11 The equivalent capacity between the points X and Y in the following circuit will be



[1] 6μF [2] 1μF

[3] 24μF [4] 3μF

Sol. Because the bridge is balanced, hence the central capacitance between Z and T is ineffective.

C_1 and C_2 are connected in series, hence their resultant $C' = \frac{C}{2} = 3\mu F$

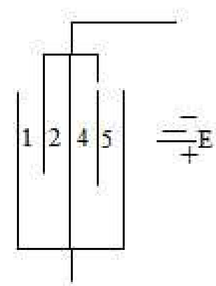
C_3 and C_4 are connected in series, hence their resultant $C'' = \frac{C}{2} = 3\mu F$

Now the two branches are connected in parallel

$\therefore C_{eq} = 3 + 3 = 6\mu$ F

Hence the correct answer will be (1)

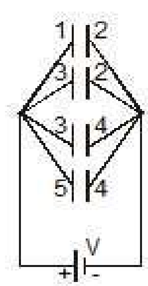
Ex.12 Five similar condenser plates, each of area A, are placed at equal distance d apart and are connected to a source of e.m.f. E as shown in the following diagram. The charge on the plates 1 and 4 will be



[1] $\frac{\epsilon_0 A}{d}, \frac{-2\epsilon_0 A}{d}$ [2] $\frac{\epsilon_0 AV}{d}, \frac{-2\epsilon_0 AV}{d}$

[3] $\frac{\epsilon_0 AV}{d}, \frac{-3\epsilon_0 AV}{d}$ [4] $\frac{\epsilon_0 AV}{d}, \frac{-4\epsilon_0 AV}{d}$

Sol. Equivalent circuit diagram charge on first plate



$Q = CV$

$$Q = \frac{\epsilon_0 AV}{d}$$

charge on fourth plate

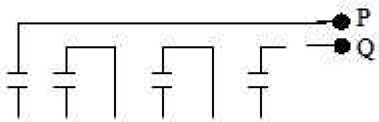
$$Q' = \frac{-\epsilon_0 AV}{d}$$

As okate 4 us reoeated twice, hence charge on 4 will be Q'' = 2Q'

Hence $Q' = -\frac{2\epsilon_0 AV}{d}$

Hence the correct answer will be (2)

Ex.13 Four condensers of equal capacity are connected as shown in the figure. If the capacity of the system between P and Q is $1\mu F$ then the capacity of each condenser will be



[1] $1\mu F$ [2] $2\mu F$

[3] $3\mu F$ [4] $4\mu F$

Sol. All the four condensers are connected in series between P and Q and capacity of each is say C. Hence resultant capacity of the combination

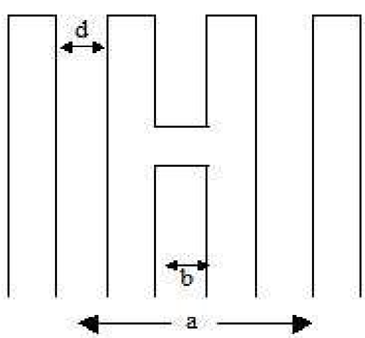
$C' = \frac{C}{4}$

But $C' = 1\ \mu f$

$C = \mu f$

Hence the correct answer will be (2)

Ex.14 Two condensers are joined as shown in the figure. Their central rigid part is movable. The capacity of the combination will be



[1] $\frac{\in_0 A}{a+b}$ [2] $\frac{2\in_0 A}{a+b}$

[3] $\frac{\in_0 A}{a-b}$ [4] $\frac{(a-b)}{\in_0 A}$

Sol. For the first condenser

$C_1 = \frac{\in_0 A}{d}$

For second condenser

$C_2 = \frac{\in_0 A}{a-(b+d)}$

C_1 and C_2 are connected in series

$$\therefore C = \frac{C_1C_2}{C_1+C_2} = \frac{\frac{\in_0 A}{d} \cdot \frac{\in_0 A}{a-(b+d)}}{\frac{\in_0 A}{d}+\frac{\in_0 A}{a-(b+d)}}$$

$= \frac{\in_0 A}{(a-b)}$

Hence the correct answer will be (3)

USE OF CAPACITORS

(a) In storing charge .

(b) In storing energy .

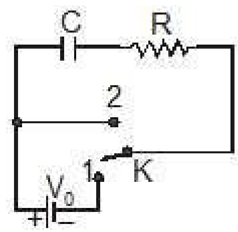
(c) In electronic devices for tuning.

(d) In electrical appliances as circuit elements, for smoothing rectified current, in timing circuits etc.

(e) In scientific studies.

CHARGING AND DISCHARGING OF A CAPACITOR THROUGH A RESISTANCE

(i) Charging process:



(a) When a capacitor of capacitance C connected in series with a resistance R is charged from a source of voltage (V_0), then the voltage across the capacitor increases from zero to the maximum value.The voltage at any time t is given by

$v_t = v_0\ (1 - e^{-t/CR})$

(b) In charging, initially the current is maximum. Afterwards the current decreases exponentially with time as :

$I = I_0\left(e^{-t/CR}\right)$

where I_0 = maximum current

$= \frac{V_0}{R}$

(c) The charge on the capacitor at time t is

$q = Q\left(1 - e^{-t/CR}\right)$

* In charging a capaciator the voltage and the charger rise exponentially whereas the current decreases exponentially.

* When $t = \infty$, the current becomes zero and the potential difference across the capacitor becomes equal to the voltage of the applied voltage source.

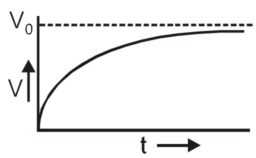
* RC is called time constant of the circuit. Time constant is the time in which the charge on the capacitor and the voltage across the capacitor become equal to 0.632 or 63.2% times their final values,i.e., at t=RC

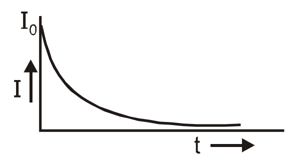
$V = 0.632V_0$ and $q = 0.632Q$

* Time constant is also the time in which the current in the circuit reaches a value 0.37 or 37% of its initial maximum value i.e., at t =RC

$I = 0.37I_0$

* In charging, the variation of voltage and current with time are shown in the following figures:





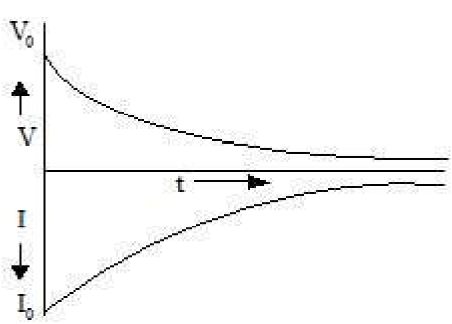
(ii) **Discharging process**:

(a) In discharging the potential difference decreases exponentially with time as : $V = V_0 e^{-t/RC}$

(b) The charge also decreases exponentially with time as : $q = Qe^{-t/RC}$

(c) In discharging the current decreases exponentially from maximum with time, as $I = I_0 e^{-t/RC}$

* In discharging the variation of the voltage and the current with time are shown in the following figure:



Solved Examples

Ex.15 A capacitor of capacity $10\mu F$ is charged through a resistance of 100 $k\Omega$ by a dc source of 2 volts. The potential difference across the capacitor after 2 sec will be

[1] 2.0 V [2] 1.73 V

[3] 1.52 V [4] 1.0 V

Sol. Time constant of the RC circuit

$\tau = RC = 100 \times 10^3 \times 10 \times 10^{-6}$

$= 1$ sec

During the process of charging

$V = V_0 (1 - e^{-t/RC})$, $(e = 2.718)$

$= 2 (1 - e^{-2/1}) = 2 (1 - 0.135)$

$= 2 \times 0.865 = 1.73$ volts

$\therefore$ Answer will be (2)

Ex.16 A $2500\mu F$ capacitor is charged through a $1 K\Omega$ resistor by a 12V d.c. source. What is the voltage across the capacitor after 5 sec?

[1] 10.38 volt [2] 11.38 volt

[3] 12.38 volt [4] 13.38 volt

Sol. The time constant of the circuit is

$\tau = RC$

$= 10^3 \times 2500 \times 10^{-6}$

$= 2.5$ sec

For charging

$V = V_0 (1 - e^{-t/RC})$

put t = 5 sec, and r = 2.5 sec, then

$V = 12 (1 - e^{-2})$

$= 12 (1 - 0.135)$

$= 12 \times 0.865$

$= 10.38$ volt

SOME IMPORTANT POINTS REGARDING CAPACITORS

* An arrangement of conductors in which capacitance can be increased without changing the size of the conductor is called condenser or capacitor. It has two conductors, one is charged and other is earthed. A suitable dielectric material may be placed between them.

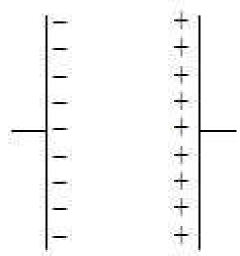
* There are several types of capacitors based on the dielectric such as paper capacitor, electrolytic capacitor, ceramic capacitor etc. Similarly there are several types of capacitors based on the shape of the conductors such as parallel plate capacitor, spherical capacitor, cylindrical capacitor etc.

* Alternating current flows easily through a capacitor while direct current does not flow through the capacitor.

* Both plates of the charged capacitor have equal and opposite charges. Force of attraction acts between these plates and it is equal to:

$$F = \frac{1}{2} \in AE^2$$

* If C is the capacitance of the capaacitor,V is the potential difference and d is the distance between the plates, then the attracative force between the plates:



$$F = \frac{1}{2}\frac{CV^2}{d}$$

* The relation between the voltage and the current is :

$$i = \left(\frac{dQ}{dt}\right) = \frac{d}{dt}(VC) = C\left(\frac{dV}{dt}\right)$$

Following conclusions can be drawn from the above relations.

(a) Since $Q = CV$, therefore voltage is directly proportional to the charge but not current.

(b) Capacitor has the ability to accumulate charge, therefore it has the capability to store information.

(c) There can be potential difference across the capacitor even when current is not flowing through the capacitor.

(d) Capacitor acts as an open circuit for direct current. Instantaneous current flows during charging or discharging of the capacitor.

(e) In the above relation $\frac{dV}{dt} = \frac{1}{C}$, that is, the rate of change of voltage is inversely proportional to the capacitance of the capacitor. If rate of change of voltage is less, then C will be more. Thus the voltage in the capacitor does not change suddenly.

* **Comparison of the energy of charged capacitor and the potential energy of the spring**:

(a) Comparing the relation for energy $\left(U = \frac{1}{2}\frac{q^2}{C}\right)$ of the capacitor and the energy of the spring $\left(U = \frac{1}{2}kx^2\right)$, the charge q is equivalent to displacement x.

(b) The reciprocal of the capacitance of the capacitor $\frac{1}{C}$ is equivalent to the force constant k of the spring.

(c) The potential difference between the plates of the capacitor $V = \frac{q}{C}$ is equivalent to the restoring force $F = kx$ of the spring.

* Metallic plate can not be used as a dielectric in the capacitor because it will short circuit the plates.

* The dielectric constant of a metal is infinite.

POINT TO BE REMEMBER

* **Capacity or Capacitance of a Conductor :**

$$C = \frac{Q}{V},$$

Unit is farad $= \frac{\text{coulomb}}{\text{volt}}$

$1\mu F = 10^{-6}$ F, 1 pF $= 10^{-12}$ F

* **Capacitance of a spherical conductor :** $C = 4\pi\epsilon_0 R$,

ϵ_0 is permittivity of vacuum.

$C_m = 4\pi\epsilon R = \epsilon_r\ (4\epsilon_0 R)$

ϵ is permittivity of medium and ϵ_r is dielectric constant of the medium.

* **Work done in charging or energy stored :**

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

* **Common potential when two charged conductors are connected :**
$C = C_1 + C_2$, $Q = Q_1 + Q_2 = C_1V_1 + C_2V_2$
Common potential
$$V = \frac{C_1V + C_2V_2}{C_1 + C_2}$$
Charge transferred
$$\Delta Q = \frac{C_1C_2}{C_1 + C_2}(V_1 - V_2)$$
Energy loss
$$\Delta U = \frac{1}{2} = \frac{C_1C_2}{C_1 + C_2}(V_1 - V_2)^2$$

* **Capacitor or condenser :** An arrangement of conductors for increasing the capacitance. It has two conductors placed nearby, one is charged and the other is earthed.

Capacity of a condenser $C = \frac{Q}{V}$

* **Parallel plate capacitor :**
$$C = \frac{\epsilon_0 A}{d}, C_m = \epsilon_r \left(\frac{\epsilon_0 A}{d}\right) = \epsilon_r C$$
If a dielectric of thickness t is placed in between, then
$$C = \frac{\epsilon_0 A}{\left(d - t + \frac{t}{\epsilon_r}\right)}$$
If slabs of thicknesses t_1, t_2, t_3 ... t_n of dielectric constants $\epsilon_{r_1}, \epsilon_{r_2}, ... \epsilon_{r_n}$ are placed in between
$$C = \frac{\epsilon_0 A}{\left(\frac{t_1}{\epsilon_{r_1}} + \frac{t_2}{\epsilon_{r_2}} + ... + \frac{t_n}{\epsilon_{r_n}}\right)}$$
and $d = t_1 + t_2 + ... + t_n$

* **Spherical condenser :**
When outer spherical shell is earthed
$$C = 4\pi \epsilon_0 \epsilon_r \frac{R_1R_2}{R_2 - R_1}$$
When inner sphere is earthed
$$C = 4\pi \epsilon_0 \epsilon_r \frac{R_1R_2}{R_2 - R_1} + 4\pi \epsilon_0 R_2$$

* **Cylindrical capacitor :**
$$C = \frac{2\pi \epsilon_0 \epsilon_r \ell}{\log_e (R_2/R_1)}$$
$$= \frac{2\pi \epsilon_0 \epsilon_r \ell}{2.303 \log_{10}(R_2/R_1)}$$

* **Multiplate capacitor :**
$$C = (n - 1) \frac{\epsilon_0 \epsilon_r A}{d}$$

* **Energy stored in a capacitor :**
$$U = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{1}{2}\frac{Q^2}{C}$$
This energy resides in electric field. Energy density of electric field
$$U = \frac{1}{2} \in E^2$$

* **Combination of capacitors :**

Series combination :
$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + ... + \frac{1}{C_n}, Q_1 = Q_2 = ... = Q$$

Parallel combination :
$$C = C_1 + C_2 + ... + C_n, V_1 = V_2 = ... = V$$

* **Charging and discharging of a capacitor through a resistance :**

Charging $q = Q\ (q - e^{-t/RC})$
$V = V_0\ (1 - e^{-t/RC})$
$$I = I_0 e^{-t/RC}, I_0 = \frac{V_0}{R}$$

Discharging $q = Qe^{-t/RC}$
$V = V_0\ e^{-t/RC}$
$I = -I_0 e^{-t/RC}$

Time constant :
τ = RC, time in which q = 0.63 Q, V = 0.63 V_0 and I = 0.37 I_0, while charging, q = 0.37 Q, V = 0.37 V_0, I = 0.37I_0 while discharging.

* **Force of attraction between the plates of a capacitor :**
$$F = \frac{1}{2} \in E^2 A = \frac{\sigma^2}{2 \in} A$$
$$= \frac{Q^2}{2 \in A} = \frac{1}{2}\frac{CV^2}{d}$$