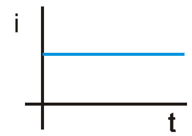
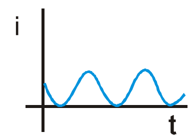
ALTERNATING CURRENT

AC and DC Current :

A current that changes its direction periodically is called alternating current (AC). If a current maintains its direction constant it is called direct current (DC).



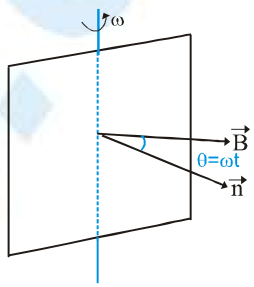
constant dc



periodic dc

Basic Principle of AC Generation

Alternating voltage is generated by rotating a coil of conducting wire in a strong magnetic field. The magnetic flux linked with the coil changes with time and an alternating emf is thus induced. Instantaneous flux linked with coil is



$$\phi = (\vec{A}.\vec{B})n$$

$$= ABn\cos(\omega t + \theta_0)$$

where A = area of the coil (in m^2)

B = magnetic field (in tesla) n = number of turns

ω = angular frequency = $\frac{2\pi}{T} = 2\pi f$ (in rad s^{-1})

f = frequency (in hertz) θ_0 = initial phase angle

With the change of time $\cos(\omega t + \theta_0)$ changes consequently an emf V is induced. According to Faraday's law

$$V = \frac{d\phi}{dt}$$

$$= -\frac{d}{dt}[Abn\cos(\omega t + \theta_0)] = Abn\ \omega\sin(\omega t + \theta_0) \Rightarrow V = V_m \sin(\omega t + \theta_0)$$

Here V_m = voltage amplitude of sinusoidal voltage or the peak value of ac voltage

where V_m - ABnω

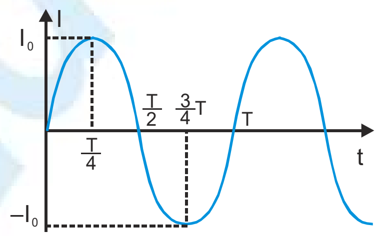
Alternating Current and Voltage

Voltage or current is said to be alternating if it is change continously in magnitude and perodically in direction with time. It can be represented by a sine curve or cosine curve

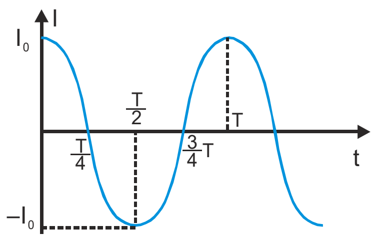
$I = I_0 \sin \omega t$ or $I = I_0 \cos \omega t$

where I = Instantaneous value of current at time t, I_0 = Amplitude or peak value

ω = Angular frequency $\omega = \frac{2\pi}{T} = 2\pi f$ T = time period f = frequency



I as a sine function of t



I as a cosine function of t

Amplitude of AC

The maximum value of current in either direction is called peak value or the amplitude of current. It is represented by I_0.   Peak to peak value = $2I_0$

Periodic Time

The time taken by alternating current to complete one cycle of variation is called periodic time or time period of the current.

Frequency

The number of cycle completed by an alternating current in one second is called the frequency of the current.

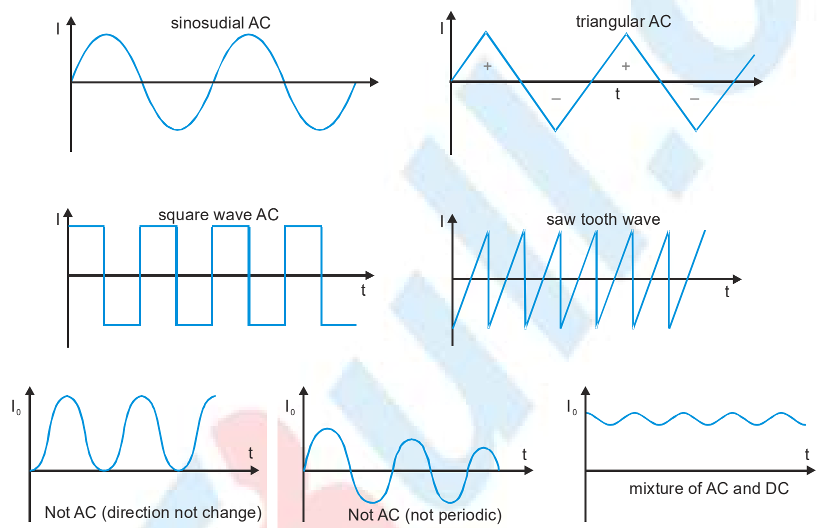
Unit : cycle/s ; (Hz)

In India : f = 50 Hz , supply voltage = 220 volt   **In USA :** f = 60 Hz ,supply voltage = 110 volt

Condition required for Current/ Voltage to be Alternating

(a) Amplitude is constant   **(b)** Alternate half cycle is positive and half negative

The alternating current continuously varies in magnitude and periodically reverses its direction.



Average Value or Mean Value

The mean value of A.C over any half cycle (either positive or negative) is that value of DC which would send same amount of charge through a circuit as is sent by the AC through same circuit in the same time.

average value of current for half cycle $< I > = \dfrac{\int_0^{T/2} I\,dt}{\int_0^{T/2} dt}$

Average value of I = I_0 sin ωt over the positive half cycle :

$$I_{av} = \frac{\int_0^{\frac{T}{2}} I_0 \sin \omega t \, dt}{\int_0^{\frac{T}{2}} dt} = \frac{2\,I_0}{\omega T}\left[-\cos \omega t\right]_0^{\frac{T}{2}} = \frac{2\,I_0}{\pi}$$

$< \sin\theta > = < \sin 2\theta > = 0$

$< \cos\theta > = < \cos 2\theta > = 0$

$< \sin\theta \cos\theta > = 0$

$< \sin^2\theta > = < \cos^2\theta > = \frac{1}{2}$

- For symmetric AC, average value over full cycle = 0, Average value of sinusoidal AC

Full cycle	(+ve) half cycle	(–ve) half cycle
0	$\frac{2I_0}{\pi}$	$\frac{-2I_0}{\pi}$

As the average value of AC over a complete cycle is zero, it is always defined over a half cycle which must be either positive or negative

Maximum Value

(a) $I = a\sin\theta \Rightarrow I_{Max} = a$

(b) $I = a + b\sin\theta \Rightarrow I_{Max} = a + b$ (if a and b > 0)

(c) $I = a\sin\theta + b\cos\theta \Rightarrow I_{Max} = \sqrt{a^2+b^2}$

(d) $I = a\sin^2\theta \Rightarrow I_{Max} = a$ (a > 0)

Root Mean square (rms) Value

It is value of DC which would produce same heat in given resistance in given time as is done by the alternating current when passed through the same resistance for the same time.

$$I_{rms} = \sqrt{\frac{\int_0^T I^2 dt}{\int_0^T dt}}$$ rms value = Virtual value = Apparent value

rms value of $I = I_0 \sin\omega t$;

$$I_{rms} = \sqrt{\frac{\int_0^T (I_0\sin\omega t)^2 dt}{\int_0^T dt}} = \sqrt{\frac{I_0^2}{T}\int_0^T \sin^2\omega t\, dt} = I_0\sqrt{\frac{1}{T}\int_0^T\left[\frac{1-\cos 2\omega t}{2}\right]dt} = I_0\sqrt{\frac{1}{T}\left[\frac{t}{2}-\frac{\sin 2\omega t}{2\times 2\omega}\right]_0^T} = \frac{I_0}{\sqrt{2}}$$

- If nothing is mentioned then values printed in a.c circuit on electrical appliances, any given or unknown values, reading of AC meters are assumed to be RMS.

Current	Average	Peak	RMS	Angular frequency
$I_1 = I_0\sin\omega t$	0	I_0	$\frac{I_0}{\sqrt{2}}$	ω
$I_1 = I_0\sin\omega t\cos\omega t = \frac{I_0}{2}\sin 2\omega t$	0	$\frac{I_0}{2}$	$\frac{I_0}{2\sqrt{2}}$	2ω
$I_1 = I_0\sin\omega t + I_0\cos\omega t$	0	$\sqrt{2}I_0$	I_0	ω

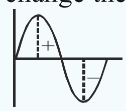
- For above varieties of current rms = $\frac{\text{Peak value}}{\sqrt{2}}$

(i) AC can't be used in
(a) Charging of battery or capacitor (as its average value = 0)
(b) Electrolysis and electroplating (Due to large inertia, ions can not follow frequency of A.C)

(ii) The rate of change of A.C. → Minimum, at that instant when they are near their peak values
→ Maximum, at that instant when they change their direction.

(iii) For alternating current $I_0 > I_{rms} > I_{av.}$

(iv) Average value over half cycle is zero if one quarter is positive and the other quarter is negative.

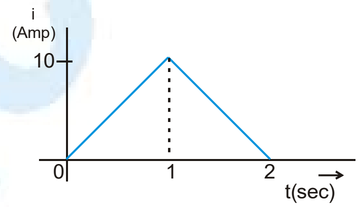


(v) Average value of symmetrical AC for a cycle is zero thats why average potential difference on any element in A.C circuit is zero.

(vi) The instrument based on heating effect of current are works on both A.C and D.C supply and also provides same heating for same value of A.C (rms) and D.C. that's why a bulb bright equally in D.C. and A.C. of same value.

(vii) If the frequency of AC is f then it becomes zero 2f times in one second and the direction of current changes 2f times in one second. Also it become maximum 2f times in one second.

Ex. Find the average value of current shown graphically, from t = 0 to t = 2 sec.



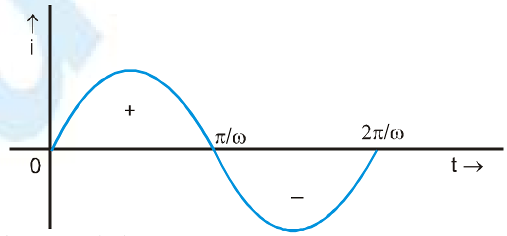
Sol. From the i – t graph, area from t = 0 to t = 2 sec

$$= \frac{1}{2} \times 2 \times 10 = 10 \text{ Amp. sec.}$$

$$\therefore \text{ Average Current} = \frac{10}{2} = 5 \text{ Amp.}$$

Ex. Find the average value of current from t = 0 to $t = \frac{2\pi}{\omega}$ if the current varies as $i = I_m \sin \omega t$.

Sol.
$$\langle i \rangle = \frac{\int_0^{2\pi/\omega} I_m \sin \omega t\, dt}{\frac{2\pi}{\omega}} = \frac{\frac{I_m}{\omega}\left(1 - \cos \omega \frac{2\pi}{\omega}\right)}{\frac{2\pi}{\omega}} = 0$$



It can be seen graphically that the area of i – t graph of one cycle is zero.
$\therefore$ < i > in one cycle = 0.

Ex. Show graphically that the average of sinusoidally varying current in half cycle may or may not be zero

Sol.

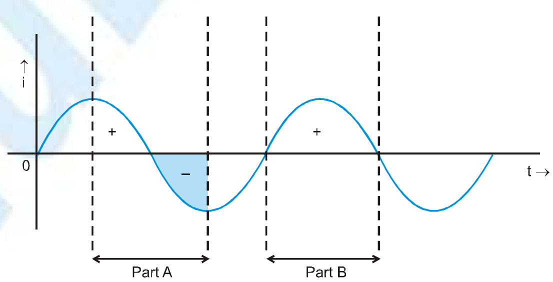


Figure shows two parts A and B, each half cycle. In part A we can see that the net area is zero
$\therefore$ < i > in part A is zero.
In part B, area is positive hence in this part $\langle i \rangle \neq 0$.

Ex. Find the average value of current $i = I_m \sin \omega t$ from **(i)** $t = 0$ to $t = \frac{\pi}{\omega}$ **(ii)** $t = \frac{\pi}{2\omega}$ to $t = \frac{3\pi}{2\omega}$.

Sol. **(i)** $\langle i \rangle = \dfrac{\int_0^{\pi/\omega} I_m \sin \omega t dt}{\frac{\pi}{\omega}} = \dfrac{\frac{I_m}{\omega}\left(1 - \cos \omega \frac{\pi}{\omega}\right)}{\frac{\pi}{\omega}} = \dfrac{2I_m}{\pi}$ **(ii)** $\langle i \rangle = \dfrac{\int_{\pi/2\omega}^{3\pi/2\omega} I_m \sin \omega t dt}{\frac{\pi}{\omega}} = 0.$

Ex. Current in an A.C. circuit is given by $i = 2\sqrt{2}\ \sin(\pi t + \pi/4)$, then the average value of current during time $t = 0$ to $t = 1$ sec is:

Sol. $\langle i \rangle = \dfrac{\int_0^1 i\, dt}{1} = 2\sqrt{2} \int_0^1 \sin\left(\pi t + \frac{\pi}{4}\right) = \dfrac{4}{\pi}$ **Ans.**

Root Mean Square Value :

Root Mean Square Value of a function, from t_1 to t_2, is defined as $f_{rms} = \sqrt{\dfrac{\int_{t_1}^{t_2} f^2 dt}{t_2 - t_1}}$.

Ex. Find the rms value of current from $t = 0$ to $t = \frac{2\pi}{\omega}$ if the current varies as $i = I_m \sin \omega t$.

Sol. $i_{rms} = \sqrt{\dfrac{\int_0^{2\pi/\omega} I_m^{\ 2} \sin^2 \omega t dt}{\frac{2\pi}{\omega}}} = \sqrt{\dfrac{I_m^{\ 2}}{2}} = \dfrac{I_m}{\sqrt{2}}$

Ex. Find the rms value of current $i = I_m \sin \omega t$ from **(i)** $t = 0$ to $t = \frac{\pi}{\omega}$ **(ii)** $t = \frac{\pi}{2\omega}$ to $t = \frac{3\pi}{2\omega}$.

Sol. **(i)** $i_{rms} = \sqrt{\dfrac{\int_0^{\pi/\omega} I_m^{\ 2} \sin^2 \omega t dt}{\frac{\pi}{\omega}}} = \sqrt{\dfrac{I_m^{\ 2}}{2}} = \dfrac{I_m}{\sqrt{2}}$ **(ii)** $\langle i \rangle = \sqrt{\dfrac{\int_{\pi/2\omega}^{3\pi/2\omega} I_m^2 \sin^2 \omega t\, dt}{\frac{\pi}{\omega}}} = \sqrt{\dfrac{I_m^{\ 2}}{2}} = \dfrac{I_m}{\sqrt{2}}$

Note:
- The r m s values for one cycle and half cycle (either positive half cycle or negative half cycle) is same.
- From the above two Ex.s note that for sinusoidal functions rms value (Also called effective value)

$= \dfrac{\text{peak value}}{\sqrt{2}}$ or $I_{rms} = \dfrac{I_m}{\sqrt{2}}$

Ex. Find the effective value of current $i = 2 \sin 100\pi t + 2 \cos(100\pi t + 30°)$.

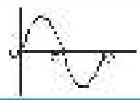
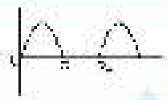
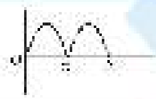
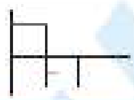
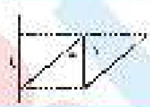
Sol. The equation can be written as $i = 2 \sin 100\pi t + 2 \sin(100\pi t + 120°)$

So phase difference $\phi = 120°$

$$(I_m)_{res} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$$

$$= \sqrt{4 + 4 + 2 \times 2 \times 2\left(-\frac{1}{2}\right)} = 2, \text{ so effective value or rms value} = 2/\sqrt{2} = \sqrt{2}\text{ A}$$

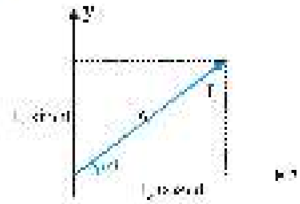
Some Important Wave Forms and their RMS and Average Value

Nature of wave form	Wave form	RMS Value	Average or mean
Sinusoidal		$\frac{I_0}{\sqrt{2}} = 0.707\, I_0$	$\frac{2I_0}{\pi} = 0.637\, I_0$
Half wave rectified		$\frac{I_0}{2} = 0.5\, I_0$	$\frac{I_0}{\pi} = 0.318\, I_0$
Full wave rectified		$\frac{I_0}{\sqrt{2}} = 0.707\, I_0$	$\frac{2I_0}{\pi} = 0.637\, I_0$
Square or Rectangular		I_0	I_0
Saw Tooth wave		$\frac{I_0}{\sqrt{3}}$	$\frac{I_0}{2}$

Phasor Diagrams

Generally currents and voltages in ac circuits are represented in the form of phasors or anticlockwise rotating vectors. The length of arrow represents the peak value of the quantity and its projection on x and y axis gives its instantaneous value.

For example let $I = I_0 \sin\omega t$, then it will be represented as shown in the figure. Length of arrow is I_0 which represents the peak value of I. Its projection on y axis is $I_0 \sin\omega t$ which represents the instantaneous value. ωt is the phase angle which increases with time.



Note : If the equation of current were in cosine form as $I = I_0 \cos\omega t$ then projection on x axis will represent the instantaneous value.

AC Circuits

Basic AC circuit elements are resistors, indicators and capacitor we will discuss the behaviour of each of them when connected in ac circuits.

AC Voltage Applied to a Resistor

A resistor connected to a source ε of ac voltage as shown in the circuit diagram. The symbol for an ac source on a circuit diagram is ~. For simplicity, we consider a source which produces sinusoidally varying potential difference across its terminals. Let this potential difference, also called ac voltage, be given by

$$V = V_0 \sin\omega t \qquad \text{....... (i)}$$

where V_0 is the amplitude of the sinusoidal voltage and ω is its angular frequency.

AC voltage applied to a resistor

The instantaneous potential drop across the resistor R is

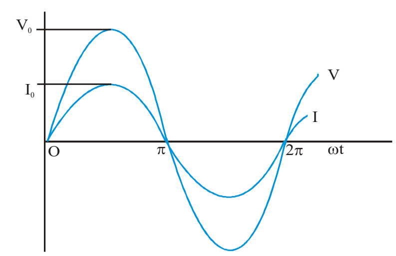
$$V_0 \sin\omega t = IR$$

or
$$I = \frac{V_0}{R}\sin\omega t$$

$$I = I_0 \sin\omega t \qquad \text{....... (ii)}$$

where I is the instantaneous current and the current amplitude I0 is given by

$$I_0 = \frac{V_0}{R} \qquad \text{....... (iii)}$$



Equation **(iii)** is just Ohm's law which for resistors work equally well for both ac and dc voltaghes. The voltage across a pure resistor and the current through it, given by equation **(i)** and **(ii)** are ploted as a function of time in figure. Note, in particular that both V and I reach zero, minimum and maximum values at the same time. Clearly, the voltage and current are in phase for a circuit containing pure resistance.

We see that,. like the applied voltage, the current varies sinusoidally and has corresponding positive and negative values during each cycle. Thus, the sum of the instantaneous current valuesover one complete cycle is zero, and the average current is zero. The fact that the average current is zero, however, does not mean that the average power is zero and that there is no dissipation of electrical energy. As you know, joule heating is given by I^2R and depends on I^2 (which is always positive whether I is positive or negative) and not on I. Thus there is Joule heating and dissipation of electrical energy when an ac current passes through a resistor.

The instantaneous power dissipated in the resistor is

$$P = I^2R = I_0^2 R \sin^2\omega t \qquad \text{....... (iv)}$$

The average value of Power P over a cycle is

$$\bar{P} = \frac{1}{2} I_0^2 R = I_{rms}^2 R \qquad \text{...... (v)}$$

Where the bar over a letter (here, P) denotes its average value.

To express ac power in the same fprm as dc power ($P=I^2R$), as special value of current is used. It is called, root mean square (rms) or effective current and is denoted by I_{rms}.

similarly, we define the rms voltage or effective voltage

From equation **(iii)**, we have

$V_0 = I_0R$ ...... **(vi)**

or $\frac{V_0}{\sqrt{2}} = \frac{I_0}{\sqrt{2}}R$ ...... **(vii)**

or $V_{rms} = I_{rms}R$ ...... **(viii)**

In terms of rms values, the equation for power and relation between current and voltage in ac circuits are essentially the same as those for the dc case.

In fact, the I_{rms} or rms current is the equivalent dc current that would produce same average power loss as tghe alternating current. Equation **(v)** can also be written as

$P = V_{rms}^2 / = I_{rms}^2 R$ (since $V_{rms} = I_{rms} R$) .... **(ix)**

Ex. A bulb is rated 60 W at 220 V/30 Hz. Find the maximum value of instantaneous current through the filament ?

Sol. $V_{max} = 220\sqrt{2} = 311$ V

$$R = \frac{220^2}{P} = \frac{220 \times 220}{60} = \frac{2420}{2} = 806.67\Omega$$

$$I = \frac{V_{max}}{R} = \frac{311}{806.67} = 0.39A$$

Ex. A light bulb is rated at 200 W for a 220 V supply. Find

(a) The resistance of the bulb

(b) The peak voltage of the source; and

(c) The rms current through the bulb.

Sol. **(a)** We are given P = 100 W and V = 220 V. The resistance of the bulb is

$$R = \frac{V_{rms}^2}{P} = \frac{(220V)^2}{200W} = 242\Omega$$

(b) The peak voltage of the source is

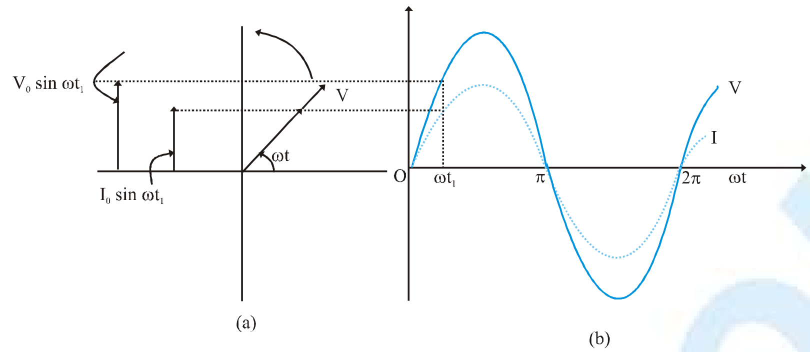
$$V_m = \sqrt{2}V_{max} = 311V$$

(c) Since, $P = I_{rms} V_{rms}$

$$I_{rms} = \frac{P}{V_{rms}} = \frac{200W}{220V} = 0.90A$$

Representation of AC Current and Voltage by Rotating Vectors - Phasors

In the previous section, we saw that the current through a resistor is in phase with the ac voltage. But this is not so in the case of an inductor, a capacitor or a combination. In order to show phase relationship betwen voltage and current in an ac circuit, we use the motion of PHASORS. The analysis of an ac circuit is faciliated by the use of a phasor diagram. a phasor is a vector which rotates about the origin with angular speed ω, as shown in figure. The vertical components of phasors V and I represent the sinusoidally varying quantities V and I. The magnetic of phasors V and I represent the amplitudes or the peak values V_0 and I_0 of these oscillating quantities. Figure **(a)** shows the voltage and their relationship at time t_1 i.e., corresponding to the circuit show in figure for the case of an ac source connected to a resistor. The projection of voltage and current phasors on vertical axis, i.e., $V_0 \sin \omega t$ and $I_0 \sin \omega t$, respectively represent the instaneous value of voltage and current at that instant. As they rotate with frequency ω, curves in figure **(b)** are generated which represent the sinusoidal variation of voltage and current with time.



(a) A phasor diagram for the circuit in figure
(b) Graph of V and I versus ω

AC Voltage Applied to an Inductor :

An ac source connected to an inductor as shown in the circuit below. Usually, inductors have appreciable resistance in their windings but we shall assume that this is ideal inductor (having zero resistance). Thus, the circuit is a purely inductive ac circuit. Let the voltage across the source be $V = V_0 \sin \omega t$. Using the loop equation $\sum \varepsilon(t) = 0$, and since there is no resistor in the circuit.

$$V - L\frac{dI}{dt} = 0 \quad \text{.......... (x)}$$

An AC source connected to an inductor

where the second term is the self-induced emf in the inductor, and L is the self-inductance of the coil. Combining equation (i) and (x), we have

$$\frac{dI}{dt} = \frac{V}{L} = \frac{V_0}{L}\sin \omega t \quad \text{........... (xi)}$$

$$dt = \frac{V_0}{L}\sin \omega t \text{ dt} \quad \Rightarrow \quad I = -\frac{V_0}{\omega L}\cos(\omega t)$$

Using $-\cos(\omega t) = \sin\left(\omega t - \frac{\pi}{2}\right)$, we have

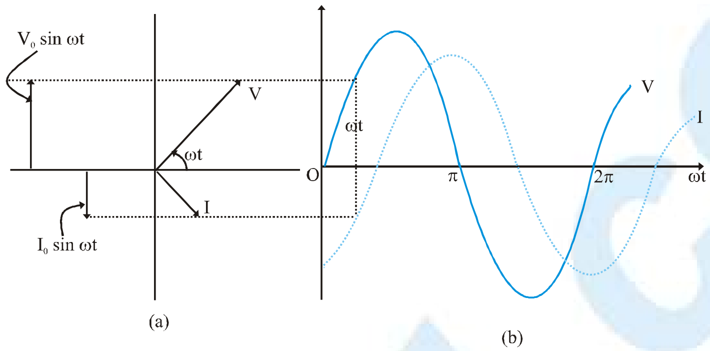
$$i = I_0 \sin\left(\omega t - \frac{\pi}{2}\right) \quad \text{........... (xii)}$$

where $I = -\frac{V_0}{\omega L}$ is the amplitude of the current. The quantity ωL is analogous to the resistance and is called inductive reactance, denoted by X_L :

$$X_L = \omega L = 2\pi f L \quad \text{........... (xiii)}$$

The dimension of inductive reactance is the same as that of resistance and SI unit is ohm(Ω). The inductive reactance limits the current in a pure inductive circuit in the same way as does the resistance in a pure resistive circuit. The inductive resistance is the directly proportional to the inductance frequency of the voltage source.

A comparision of equation (i) and (ii) for the source voltage and the current in an inductor shows that the current lags the voltage by $\frac{\pi}{2}$ or one-quarter $\left(\frac{1}{4}\right)$ cycle. Figure a shows the voltage and the current phasors in the present case at instant t. The current phasor is 1 is $\frac{\pi}{2}$ behind the voltage phasor V. When rotated with frequency ω counter-clockwise, they generate the voltage and current given by equation (1) and (xii), respectively and as shown in figure (b).



(a) A phasor diagram for the circuit in figure
(b) Graph of V and I versus ωt

We see that current reaches its maximum value later than the voltage by one-fourth of a period $\left[\frac{T}{4}=\frac{\pi/2}{\omega}\right]$. You have seen that an inductor has reactance that limits current similar to resistance in a dc circuit. Does it also consume power like a resistance ? Let us try to find out.
The instantaneous power supplied to the inductor is

$$P_L = IV = I_m \sin\left(\omega t - \frac{\pi}{2}\right)V_0 \sin(\omega t) = -I_0V_0\cos(\omega t).\sin(\omega t) = -\frac{I_0V_0}{2}\sin(2\omega t) \quad \text{...... (xiv)}$$

So, the average power over a complete cycle is zero
since the average of sin(2ωt) over a complete cycle is zero.

Thus, the average power supplied to an inductor over one complete cycle is zero.

Physically, this result means the follows, During the first quarter of each current cycle, the flux through the inductor builds up and sets uo a magnetic field and energy is stored in the inductor. In the next quarter of cycle, as the current decrease, the flux decreases and the stored energy is returned to the source. Thus, in each half cycle, the energy which is withdrawn from the source is returned to it without any dissipation of power.

Ex. A pure inductor of 50.0 mH is connected to a source of 220 V. Find the inductive resistance and rms current in the circuit if the frequency of the source is 50 Hz.

Sol. The inductive reactance.

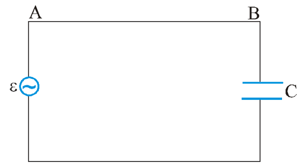
$$X_L = 2\pi fL = 2\times3.14\times50\times50\times10^{-3}\,\Omega = 15.7\,\Omega$$

The rms current in the circuit is

$$I_{rms} - \frac{V_{rms}}{X_L} = \frac{220V}{15.7\Omega} = 14.01\text{A}$$

AC Voltage Applied to A Capacitor

An ac source ε connected to a capacitor only, a purely capacitive ac circuit is as shown.



An AC source connected to a capacitor

When the capacitor is connected to an ac source, as in figure, it limits or regulates the current, but does not completely prevent the flow of charge. The capacitor is alternately charged and discharged as the current reverses each half cycle. Let 1(t) be the charge on the capacitor at any time t. The instantaneous voltage V(t) across the capacitor is

$$V(t)=\frac{q(t)}{C} \qquad \text{....... (xv)}$$

To find the current, we use the relation $I=\frac{dq}{dt}$

$$I=\frac{d}{dt}(V_0 C\sin\omega t)=\omega C V_0\cos(\omega t)$$

Using the relation, $\cos(\omega t)=\sin\left(\omega t+\frac{\pi}{2}\right)$, we have

$$I=I_0\sin\left(\omega t+\frac{\pi}{2}\right) \qquad \text{....... (xvi)}$$

where the amplitude of the oscillating current is

$$I_0=\frac{V_0}{(I/\omega C)}$$

Comparing it to $I_0=\frac{V_0}{R}$ for a purely resistive circuitmm we find that (I/ωC) plays the role of resistance.

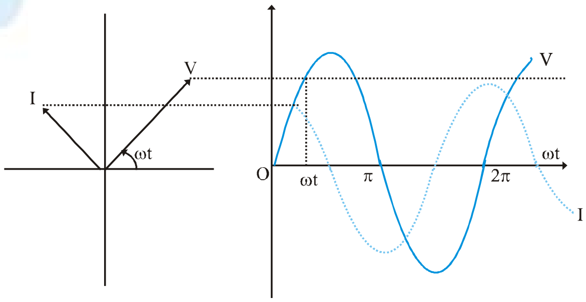
It is called capacitive reactance and is denoted by X_c,

$$X_c = 1/\omega C = 1/2\pi fC \qquad \text{........... (xvii)}$$

So that the amplitude of the current is

$$I_0=\frac{V_0}{X_c} \qquad \text{........... (xviii)}$$

The dimension of capacitive reactance is the same as that of resistance and its SI unit is Ohm (Ω). The capacitive reactance limits the amplitude of the current in a purely capacitive circuit in the same way as does the resistance in a purely resistive circuit. But it is inversely proportional to the frequency and the capacitance.



AS comparision of equation (xvii) with the equation of source voltage equation (i) shows that the current in a capacitor leads the voltage by $\pi/2$. Figure shows the phasor diagram at an instant t. here the current phasor I is $\frac{\pi}{2}$ rad ahead of the voltage phasor V as they rotate counter clockwise. Figure shows the variation of voltage and current with time. We see that the current reaches its maximum value earlier than the voltage by one-fourth of a period.

The instantaneous power supplied to the capacitor is

$$P_c = IV = I_0 \cos(\omega t).\ V_0 \sin(\omega t)$$

$$= I_0 V_0 \cos(\omega t) \sin(\omega t)$$

$$= \frac{I_0 V_0}{2} \sin(2\omega t) \qquad \text{...... (xiv)}$$

So, as in the case of an inductor, the average power

Since average of sin 2ωt over a complete cycle is zero. As discussed in the case of an inductor, the energy stored by a capacitor in each quarter period is returned to the source in the next quarter period.

Thus, we see that in the case of an inductor. The current lags the voltage by 90° and in the case of a capacitor, the current leads the voltage by 90°.

Ex. 30.0 μF capacitor is connected to a 220 V, 50 Hz source. Find the capacitive resistance and the current (rms and peak) in the circuit. If the frequency is doubled, what happens to the capacitive reactance and the current.

Sol. The capacitive reactance is

$$X_c = \frac{1}{2\pi fC} = 106\Omega$$

The rms current is

$$i_{rms} = \frac{V_{rms}}{X_c} = 2.08\text{A}$$

The peak current is

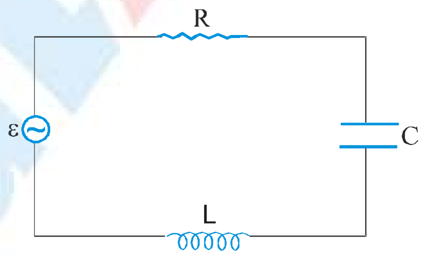
$$I_0 = \sqrt{2} I_{rms} = 2.96\text{A}$$

This current oscillates betwen 2.96A and –2.96A and is ahead of the voltage by 90°.

If the frequency is doubled, the capacitive reactance is halved and consequently, the current is doubled.

AC Voltage Applied to a Series LCR Circuit

Figure shows a series LCR circuit connected to an ac source ε. As usual, we take the voltage of the source to be $V = V_0 \sin\omega t$.



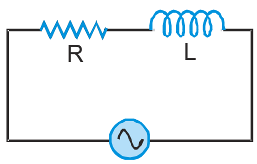
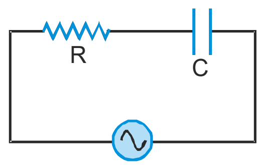
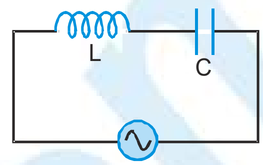
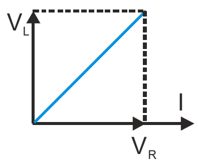
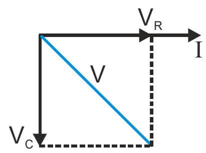
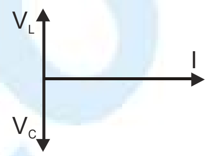
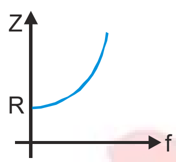
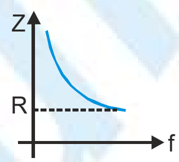
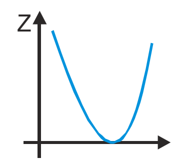
An series LCR circuit connected to an ac source

If q is the charge on the capacitor and I the current, at time t, we have, from Kirchhoff's loop rule :

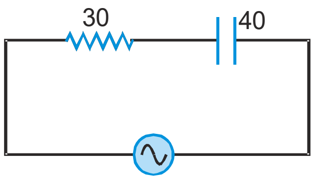
$$L\frac{dI}{dt} + IR + \frac{q}{C} = V \qquad \text{....... (xx)}$$

We want to determine the instantaneous current I and its phase relationship to the applied alternating voltage V. We shall use the technique of phasors so solve equation (xx) to obtain the time - dependence of I.

Combination of Components (R-L or R-C or L-C)

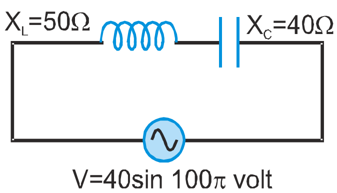
Term	R-L	R-C	L-C
Circuit	 I is same in R & L	 I is same in R & C	 I is same in L & C
Phasor diagram	 $V^2 = V_R^2 + V_L^2$	 $V^2 = V_R^2 + V_C^2$	 $V = V_L - V_C$ $(V_L > V_C)$ $V = V_C - V_L$ $(V_C > V_L)$
Phase difference in between V & I	V leads I ($\phi = 0$ to $\frac{\pi}{2}$) V leads I ($\phi = +\frac{\pi}{2}$, if $X_L > X_C$)	V lags I ($\phi = -\frac{\pi}{2}$ to 0) Impedance $Z = \sqrt{R^2 + X_L^2}$	V lags I ($\phi = -\frac{\pi}{2}$, if $X_C > X_L$) $Z = \sqrt{R^2 + (X_C)^2}$ $Z = \lvert X_L - X_c \rvert$
Variation of Z with f At very low f At very high f	as f↑, Z ↑  $Z \simeq R$ $(X_L \to 0)$ $Z \simeq X_L$	as f↑, Z↓  $Z \simeq X_C$ $Z \simeq R$ $(X_C \to 0)$	as f↑, Z first ↓ then↑  $Z \simeq X_C$ $Z \simeq X_L$

Ex. Calculate the impedance of the circuit shown in the figure.



Sol. $Z = \sqrt{R^2 + (X_c)^2} = \sqrt{(30)^2 + (40)^2} = \sqrt{2500} = 50\ \Omega$

Ex. If $X_L = 50\ \Omega$ and $X_C = 40\ \Omega$ Calculate effective value of current in given circuit.

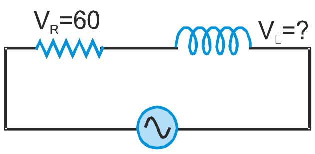


V=40sin 100π volt

Sol. $Z = X_L - X_C = 10\ \Omega$

$I_0 = \frac{V_0}{Z} = \frac{40}{10} = 4\ A \Rightarrow I_{rms} = \frac{4}{\sqrt{2}} = 2\sqrt{2}\ A$

Ex. In given circuit calculate, voltage across inductor

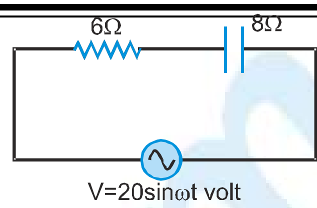


V=100√2 sinωt volt

Sol. ∵ $V^2 = V_R^2 + V_L^2$ ∴ $V_L^2 = V^2 - V_R^2$

$V_L \sqrt{V^2 - V_R^2} = \sqrt{(100)^2 - (60)^2} = \sqrt{6400} = 80\ V$

Ex. In given circuit find out **(i)** impedance of circuit **(ii)** current in circuit



Sol. **(i)** $Z = \sqrt{R^2 + X_C^2} = \sqrt{(6)^2 + (8)^2} = 10\,\Omega$

(ii) $V = IZ \Rightarrow I = \frac{V_0}{Z} = \frac{20}{10} = 2\,A$ So $I_{rms} = \frac{2}{\sqrt{2}} = \sqrt{2}\,A$

Ex. When 10V, DC is applied across a coil current through it is 2.5 A, if 10V, 50 Hz A.C. is applied current reduces to 2 A. Calculate reactance of the coil.

Sol. For 10 V D.C. $\Rightarrow V = IR$ $\quad \therefore$ Resistance of coil $R = \frac{10}{2.5} = 4\Omega$ For 10 V A.C. : $V = IZ \Rightarrow Z = \frac{V}{I} = \frac{20}{10} = 5\Omega$

$\Rightarrow Z = \sqrt{R^2 + X_L^2} = 5 \Rightarrow R^2 + X_L^2 = 25 \Rightarrow X_L^2 = 5^2 - 4^2 \Rightarrow X_L = 3\,\Omega$

Ex. When an alternating voltage of 220V is applied across a device X, a current of 0.5 A flows through the circuit and is in phase with the applied voltage. When the same voltage is applied across another device Y, the same current again flows through the circuit but it leads the applied voltage by π/2 radians.

(a) Name the devices X and Y.

(b) Calculate the current flowing in the circuit when same voltage is applied across the series combination of X and Y.

Sol. **(a)** X is resistor and Y is a capacitor

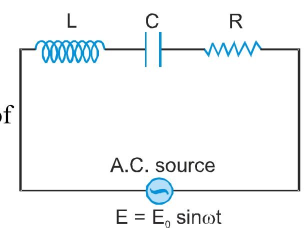
(b) Since the current in the two devices is the same (0.5A at 220 volt)

When R and C are in series across the same voltage then

$$R = X_C = \frac{220}{0.5} = 440\,\Omega \Rightarrow I_{rms} = \frac{V_{rms}}{\sqrt{R^2 + X_C^2}} = \frac{220}{\sqrt{(440)^2 + (440)^2}} = \frac{220}{440\sqrt{2}} = 0.35A$$

Inductance, Capacitance and Resistance in Series (L-C-R series circuit)

A circuit containing a series combination of an resistance R, a coil of inductance L and a capacitor of capacitance C, connected with a source of alternating e.m.f. of peak value of E_0, as shown in fig.



Phasor Diagram For Series L-C-R circuit

Let in series LCR circuit applied alternating emf is $E = E_0 \sin \omega t$.

As L,C and R are joined in series, therefore, current at any instant through the three elements has the same amplitude and phase.

However voltage across each element bears a different phase relationship with the current.

Let at any instant of time t the current in the circuit is I

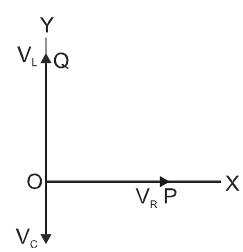
Let at this time t the potential differences across L, C, and R

$V_L = IX_L$, $V_C = IX_C$ and $V_R = IR$

Now, V_R is in phase with current I but V_L leads I by 90°

While V_C legs behind I by 90°.

The vector OP represents V_R (which is in phase with I) the vector OQ represent VL (which leads I by 90°)

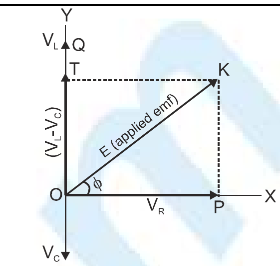


and the vector OS represents V_C (which legs behind I by 90°)

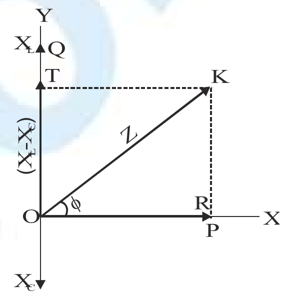
V_L and V_C are opposite to each other.

If $V_L > V_C$ (as shown in figure) the their resultant will be $(V_L - V_C)$ which is represented by OT.

Finally, the vector OK represents the resultant of V_R and $(V_L - V_C)$, that is, the resultant of all the three = applied e.m.f.



Thus $$E = \sqrt{V_R^2 + (V_L - V_C)^2} = I\sqrt{R^2 + (X_L - X_C)^2} \Rightarrow I = \frac{E}{\sqrt{R^2 + (X_L - X_C)^2}}$$

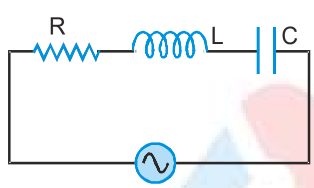
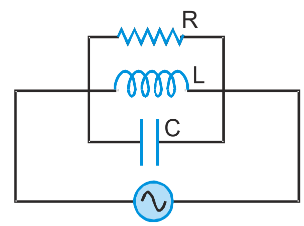
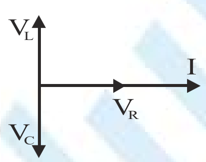
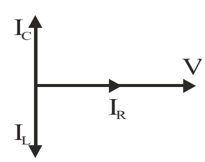
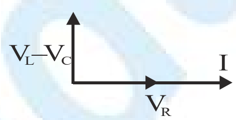
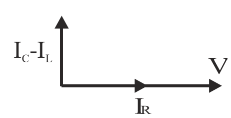
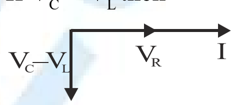
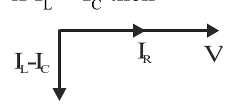


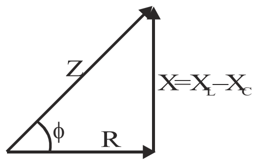
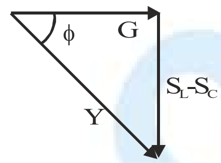
Impedance $$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

The phasor diagram also shown that in LCR circuit the applied e.m.f.

leads the current I by a phase angle ϕ $\quad \tan\phi = \frac{X_L - X_C}{R}$

Series LCR and parallel LCR combination

Series L-C-R circuit	Parallel L-C-R circuit
1. Circuit diagram  I same for R, L & C	 V same for R, L & C
2. Phasor diagram 	
(i) If $V_L > V_C$ then 	**(i)** if $I_C > I_L$ then 
(ii) If $V_C > V_L$ then 	**(ii)** if $I_L > I_C$ then 

(iii) $V = \sqrt{V_R^2 + (V_L - V_C)^2}$ Impedance $Z = \sqrt{R^2 + (X_L - X_C)^2}$ $\tan\phi = \frac{X_L - X_C}{R} = \frac{V_L - V_C}{V_R}$ **(iv)** Impedance triangle 	**(iii)** $I = \sqrt{I_R^2 + (I_L - I_C)^2}$ Admittance $Y = \sqrt{G^2 + (S_L - S_C)^2}$ $\tan\phi = \frac{S_L - S_C}{G} = \frac{I_L - I_C}{I_R}$ **(iv)** Admittance triangle 

(i)

Series

(a) if $X_L > X_C$ then V leads I,ϕ (positive) circuit nature inductive

(b) if $X_C > X_L$ then V lags I,ϕ (negative) circuit nature capacitive

Parallel

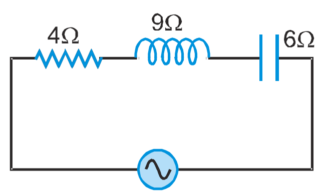
(a) if $S_L > S_C$ ($X_L < X_C$ then V leads I,ϕ (positive) circuit nature inductive

(b) if $S_C > S_L$ ($X_C < X_L$) then V lags I, ϕ (negative) circuit nature capacitive

(ii) In A.C. circuit voltage for L or C may be greater than source voltage or current but it happens only when circuit contains L and C both and on R it never greater than source voltage or current.

(iii) In parallel A.C.circuit phase difference between I_L and I_C is π

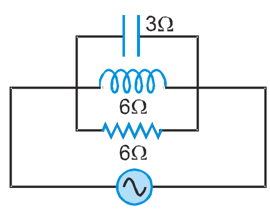
Ex. Find out the impedance of given circuit.



Sol. $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{4^2 + (9-6)^2} = \sqrt{4^2 + 3^2} = \sqrt{25} = 5\Omega$

(∵ $X_L > X_C$ ∴ Inductive)

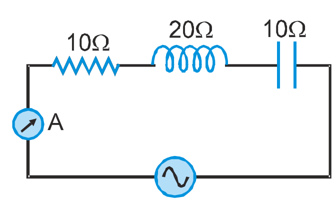
Ex. Find out impedance of given circuit.



Sol. $Y^2 = G^2 + (S_L - S_C)^2 \ \frac{1}{36} + \left[\frac{1}{6} - \frac{1}{3}\right]^2$

$Y = \frac{\sqrt{2}}{6}\Omega \quad \Rightarrow \quad Z = \frac{6}{\sqrt{2}}\ \Omega$ (capacitive, because $X_L > X_C$)

Ex. Find out reading of A C ammeter and also calculate the potential difference across, resistance and capacitor.



E=100sinωt100πt volt

Sol. $Z = \sqrt{R^2 + (X_L - X_C)^2} = 10\sqrt{2}\ \Omega \quad \Rightarrow \quad I_0 = \frac{V_0}{Z} = \frac{100}{10\sqrt{2}} = \frac{10}{\sqrt{2}}$ A

∵ ammeter reads RMS value, so its reading = $\frac{10}{\sqrt{2}\sqrt{2}} = 5$A

So $V_R = 5 \times 10 = 50$ V and $V_C = 5 \times 10 = 50$ V

Ex. In LCR circuit with an AC source R = 300 Ω , C = 20 μF, L = 1.0 H, E_{rms} = 50V and f = 50/π Hz. Find RMS current in the circuit.

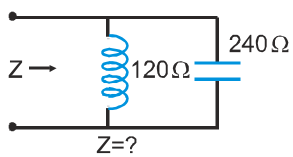
Sol.

$$I_{rms} = \frac{E_{rms}}{Z} = \frac{E_{rms}}{\sqrt{R^2 + \left[\omega L - \frac{1}{\omega C}\right]^2}} = \frac{50}{\sqrt{300^2 + \left[2\pi \times \frac{50}{\pi} \times 1 - \frac{1}{20 \times 10^{-6} \times 2\pi \times \frac{50}{\pi}}\right]^2}}$$

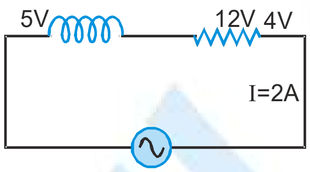
$$\Rightarrow \quad I_{rms} = \frac{50}{\sqrt{(300)^2 + \left[100 - \frac{10^3}{2}\right]^2}} = \frac{50}{100\sqrt{9+16}} = \frac{1}{10} = 0.1\,A$$

Ex. Calculate impedance of the given circuit :

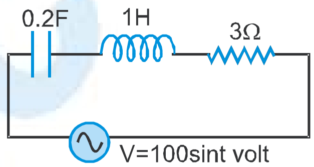
(i)



(ii)



(iii)



Sol. **(i)** It is parallel circuit so Y is evaluated

$$Y = S_L - S_C = \frac{1}{120} - \frac{1}{240} = +\frac{1}{240} \quad \Rightarrow \quad Z = 240\ \Omega \text{ (inductive)}$$

(ii) $V_s^{\,2} = 5^2 + 12^2 = 169 \quad \Rightarrow \quad V_s = 13$ volt therefore $Z = \frac{V_s}{I} = \frac{13}{2} = 6.5\Omega$

(iii) $R = 3\Omega,\ X_L = \omega L = 1$ as $(\omega = 1)$

$$X_C = \frac{1}{\omega C} = \frac{1}{(0.2)1} = 5\Omega \quad \text{so} \quad Z^2 = R^2 + (X_L - X_C)^2 = 3^2 + (1-5)^2 = 25 \Rightarrow Z = 5\,\Omega$$

Resonance

A circuit is said to be resonant when the natural frequency of circuit is equal to frequency of the applied voltage. For resonance both L and C must be present in circuit.

There are two types of resonance : **(i)** Series Resonance **(ii)** Parallel Resonance

Series Resonance

(a) At Resonance

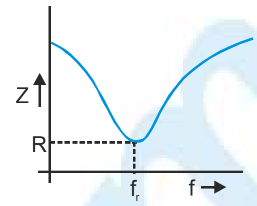
(i) $X_L = X_C$ **(ii)** $V_L = V_C$ **(iii)** $\phi = 0$ (V and I in same phase)

(iv) $Z_{min} = R$ (impedance minimum) **(v)** $I_{max} = \frac{V}{R}$ (current maximum)

(b) Resonance frequency

$$X_L = X_C \Rightarrow \omega_r L = \frac{1}{\omega_r C} \Rightarrow \omega_r^2 = \frac{1}{LC} \Rightarrow \omega_r = \frac{1}{\sqrt{LC}} \Rightarrow f_r = \frac{1}{2\pi\sqrt{LC}}$$

(c) **Variation of Z with f**



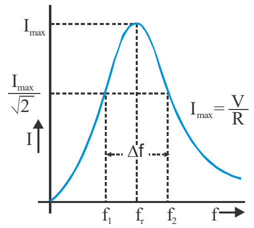
(i) If $f < f_r$ then $X_L < X_C$ circuit nature capacitive, ϕ (negative)

(ii) At $f = f_r$ then $X_L = X_C$ circuit nature, Resistive, ϕ = zero

(iii) If $f > f_r$ then $X_L > X_C$ circuit nature is inductive, ϕ (positive)

Variation of I with f as f increase, Z first decreases then increase

(d)



as f increase, I first increase then decreases

At resonance impedance of the series resonant circuit is minimum so it is called 'acceptor circuit' as it most readily accepts that current out of many currents whose frequency is equal to its natural frequency. In radio or TV tuning we receive the desired station by making the frequency of the circuit equal to that of the desired station.

Half power frequencies

The frequencies at which, power become half of its maximum value called half power frequencies

Band width $= \Delta f = f_2 - f_1$

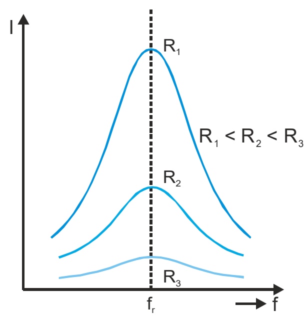
Quality factor Q : Q-factor of AC circuit basically gives an idea about stored energy & lost energy.

$$Q = 2\pi \frac{\text{maximum energy stored per cycle}}{\text{maximum energy loss per cycle}}$$

(i) It represents the sharpness of resonance. **(ii)** It is unit less and dimension less quantity

(iii) $Q = \frac{(X_L)_r}{R} = \frac{(X_C)_r}{R} = \frac{2\pi f_r L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{f_r}{\Delta f} = \frac{f_r}{\text{band width}}$

Magnification



At resonance V_L or $V_C = QE$ (where E = supplied voltage)

So at resonance Magnification factor = Q-factor

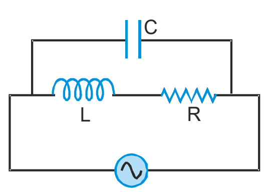
Sharpness

Sharpness ∝ Quality factor ∝ Magnification factor

R decrease ⇒ Q increases ⇒ Sharpness increases

Parallel Resonance

(a) **At resonance**



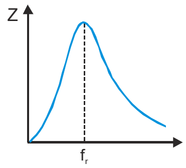
(i) $S_L = S_C$ **(ii)** $I_L = I_C$ **(iii)** $\phi = 0$

(iv) $Z_{max} = R$ (impedance maximum)

(v) $I_{min} = \frac{V}{R}$ (current minimum)

(b) **Resonant frequency** $f_r = \dfrac{1}{2\pi\sqrt{LC}}$

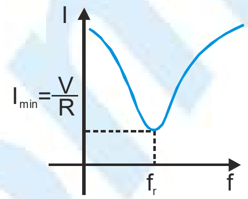
(c)



Variation of Z with f as f increases , Z first increases then decreases

◎ If $f < f_r$ then $S_L > S_C$, ϕ (positive), circuit nature is inductive

◎ If $f > f_r$ then $S_C > S_L$, ϕ (negative), circuit nature capacitive.



(d) **Variation of I with f** as f increases , I first decreases then increases

Note : For this circuit $f_r = \dfrac{1}{2\pi}\sqrt{\dfrac{1}{LC} - \dfrac{R^2}{L^2}} \Rightarrow Z_{max} = \dfrac{L}{RC}$ For resonance $\dfrac{1}{LC} > \dfrac{R^2}{L^2}$

(i) Series resonance circuit gives voltage amplificaltion while parallel resonance circuit gives current amplification.

(ii) At resonance current does not depend on L and C, it depends only on R and V.

(iii) At half power frequencies : net reactance = net resistance.

(iv) As R increases ,bandwidth increases

(v) To obtain resonance in a circuit following parameter can be altered :

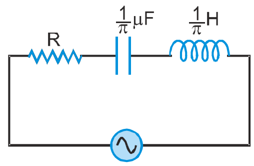
(i) L **(ii)** C **(iii)** frequency of source.

(vi) Two series LCR circuit of same resonance frequency f are joined in series then resonance frequency of series combination is also f

(vii) The series resonance circuit called acceptor whereas parallel resonance circuit called rejector circuit.

(viii) Unit of $\sqrt{LC}$ is second

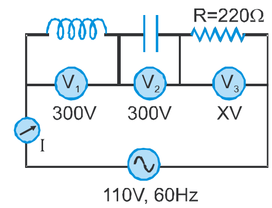
Ex. For what frequency the voltage across the resistance R will be maximum.



Sol. It happens at resonance

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{\frac{1}{\pi}\times 10^{-6} \times \frac{1}{\pi}}} = 500 \text{ Hz}$$

Ex. A capacitor, a resistor and a 40 mH inductor are connected in series to an AC source of frequency 60Hz, calculate the capacitance of the capacitor, if the current is in phase with the voltage. Also calculate the value of X and I.



Sol. At resonance

$$\omega L = \frac{1}{\omega C},\ C = \frac{1}{\omega^2 L} = \frac{1}{4\pi^2 f^2 L} = \frac{1}{4\pi^2 \times (60)^2 \times 40 \times 10^{-3}} = 176\,\mu F$$

$V = V_R \Rightarrow X = 110\text{ V}$ and $I = \dfrac{V}{R} = \dfrac{110}{220} = 0.5\text{ A}$

Ex. A coil, a capacitor and an A.C. source of rms voltage 24 V are connected in series, By varying the frequency of the source, a maximum rms current 6 A is observed, If this coil is connected to a bettery of emf 12 V, and internal resistance 4Ω , then calculate the current through the coil.

Sol. At resonance current is maximum. $I = \frac{V}{R} \Rightarrow$ Resistance of coil $R = \frac{V}{I} = \frac{24}{6} = 4\,\Omega$

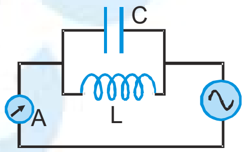
When coil is connected to battery, suppose I current flow through it then $I = \frac{E}{R+r} = \frac{12}{4+4} = 1.5\text{ A}$

Ex. Radio receiver receives a message at 300m band, If the available inductance is 1 mH, then calculate required capacitance

Sol. Radio receives EM waves. (velocity of EM waves $c = 3 \times 10^8$ m/s)

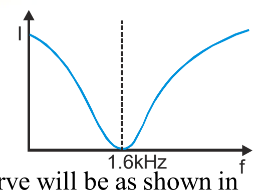
$$\therefore \quad c = f\lambda \Rightarrow f = \frac{3\times10^8}{300} = 10^6\text{ Hz} \quad \text{Now} \quad f = \frac{1}{2\pi\sqrt{LC}} = 1\times10^6 \Rightarrow C = \frac{1}{4\pi^2 L\times10^{12}} = 25\text{ pF}$$

Ex. In a L–C circuit parallel combination of inductance of 0.01 H and a capacitor of 1 μF is connected to a variable frequency alternating current source as shown in figure. Draw a rough sketch of the current variation as the frequency is changed from 1kHz to 3kHz.



Sol. L and C are connected in parallel to the AC source,

so resonance frequency $f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.01\times10^{-6}}} = \frac{10^4}{2\pi}\ ;\ 1.6\text{kHz}$



In case of parallel resonance, current in L–C circuit at resonance is zero, so the I-f curve will be as shown in figure.

Power in AC Circuit

The average power dissipation in LCR ac circuit

Let $V = V_0 \sin\omega t$ and $I = I_0 \sin(\omega t - \phi)$

Instantaneous power $P = (V_0 \sin\omega t)(I_0 \sin(\omega t - \phi) = V_0 I_0 \sin\omega t\,(\sin\omega t\cos\phi - \sin\phi\cos\omega t)$

$$\text{Average power } \langle P\rangle = \frac{1}{T}\int_0^T (V_0 I_0 \sin^2 \omega t\cos\phi - V_0 I_0 \sin\omega t\cos\omega t\sin\phi)dt$$

$$= V_0 I_0\left[\frac{1}{T}\int_0^T \sin^2\omega t\cos\phi\,dt - \frac{1}{T}\int_0^T \sin\omega t\cos\omega t\sin\phi\,dt\right] = V_0 I_0\left[\frac{1}{2}\cos\phi - 0\times\sin\phi\right]$$

$$\Rightarrow \quad \langle P\rangle = \frac{V_0 I_0\cos\phi}{2} = V_{rms} I_{rm,s}\cos\phi$$

Instantaneous power	Average power/actual power/ dissipated power/power loss	Virtual power/ apparent Power/rms Power	Peak power
$P = VI$	$P = V_{rms} I_{rms}\cos\phi$	$P = V_{rms} I_{rms}$	$P = V_0 I_0$

(i) $I_{rms}\cos\phi$ is known as active part of current or wattfull current, workfull current. It is in phase with voltage.

(ii) $I_{rms}\sin\phi$ is known as inactive part of current, wattless current, workless current. It is in quadrature (90°) with voltage.

Power factor :

Average power $\overline{P} = E_{rms} I_{rms} \cos\phi = \text{r m s power} \times \cos\phi$

Power factor $(\cos\phi) = \dfrac{\text{Average power}}{\text{r m s Power}}$ and $\cos\phi = \dfrac{R}{Z}$

Power factor : **(i)** is leading if I leads V **(ii)** is lagging if I lags V

(i) $P_{av} \le P_{rms}$.

(ii) Power factor varies from 0 to 1

(iii)

Pure/Ideal power	ϕ	V	Power factor = $\cos\phi$	Average
R	0	V, I same Phase	1 (maximum)	$V_{rms} . I_{rms}$
L	$+\dfrac{\pi}{2}$	V leads I	0	0
C	$-\dfrac{\pi}{2}$	V lags I	0	0
Choke coil	$+\dfrac{\pi}{2}$	V leads I	0	0

(iv) At resonance power factor is maximum ($\phi = 0$ so $\cos\phi = 1$) and $P_{av} = V_{rms} I_{rms}$

Ex. A voltage of 10 V and frequency 10^3 Hz is applied to $\dfrac{1}{\pi}$ μF capacitor in series with a resistor of 500Ω. Find the power factor of the circuit and the power dissipated.

Sol. $X_C = \dfrac{1}{2\pi f C} = \dfrac{1}{2\pi \times 10^3 \times \dfrac{10^{-6}}{\pi}} = 500\Omega$ $\therefore$ $Z = \sqrt{R^2 + X_C^2} = \sqrt{(500)^2 + (500)^2} = 500\sqrt{2}\ \Omega$

Power factor $\cos\phi = \dfrac{R}{Z} = \dfrac{500}{500\sqrt{2}} = \dfrac{1}{\sqrt{2}}$, Power dissipated $= V_{rms} I_{rms} \cos\phi = \dfrac{V_{rms}^2}{Z} \cos\phi = \dfrac{(10)^2}{500\sqrt{2}} \times \dfrac{1}{\sqrt{2}} = \dfrac{1}{10}$ W

Ex. If V = 100 sin 100 t volt and I = 100 sin $(100\,t + \dfrac{\pi}{3})$ mA for an A.C. circuit then find out

(a) phase difference between V and I

(b) total impedance, reactance, resistance

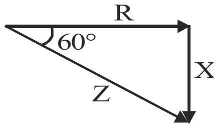
(c) power factor and power dissipated

(d) components contains by circuits

Sol. **(a)** Phase difference $\phi = -\dfrac{\pi}{3}$ (I leads V)

(b) Total impedance $Z = \dfrac{V_0}{I_0} = \dfrac{100}{100 \times 10^{-3}} = 1\text{k}\Omega$ Now resistance $R = Z\cos 60° = 1000 \times \dfrac{1}{2} = 500\Omega$

reactance $X = Z\sin 60° = 1000 \times \dfrac{\sqrt{3}}{2} = \dfrac{500}{\sqrt{3}}\ \Omega$



(c) $\phi = -60°$ $\Rightarrow$ Power factor = $\cos\phi = \cos(-60°) = 0.5$ (leading)

Power dissipated $P = V_{rms} I_{rms} \cos\phi = \dfrac{100}{\sqrt{2}} \times \dfrac{0.1}{\sqrt{2}} \times \dfrac{1}{2} = 2.5$ W

(d) Circuit must contains R as $\phi \ne \dfrac{\pi}{2}$

Ex. If power factor of a R-L series circuit is $\frac{1}{2}$ when applied voltage is V = 100 sin 100πt volt and resistance of circuit is 200Ω then calculate the inductance of the circuit.

Sol. $\cos\phi = \frac{R}{Z} \Rightarrow \frac{1}{2} = \frac{R}{Z} \Rightarrow Z = 2R \Rightarrow \sqrt{R^2 + X_L^2} = 2R \Rightarrow X_L = \sqrt{3}\ R$

$$\omega L = \sqrt{3}\ R \Rightarrow L = \frac{\sqrt{3}R}{\omega} = \frac{\sqrt{3}\times 200}{100\pi} = \frac{2\sqrt{3}}{\pi}H$$

Ex. A circuit consisting of an inductance and a resistance joined to a 200 volt supply (A.C.). It draws a current of 10 ampere. If the power used in the circuit is 1500 watt. Calculate the wattless current.

Sol. Apparent power = 200 × 10 = 2000 W

$$\therefore \text{Power factor } \cos\phi = \frac{\text{True power}}{\text{Apparent power}} = \frac{1500}{2000} = \frac{3}{4}$$

$$\text{Wattless current} = I_{rms} \sin\phi = 10\sqrt{1-\left(\frac{3}{4}\right)^2} = \frac{10\sqrt{7}}{4}A$$

Ex. A coil has a power factor of 0.866 at 60 Hz. What will be power factor at 180 Hz.

Sol. Given that cos φ = 0.866, ω = 2πf = 2π × 60 = 120π rad/s, ω' = 2πf' = 2π × 180 = 360π rad/s

Now, cos φ = R/Z ⇒ R = Z cos φ = 0.866 Z

$$\text{But } Z = \sqrt{R^2 + (\omega L)^2} \Rightarrow \omega L = \sqrt{Z^2 - R^2} = \sqrt{Z^2 - (0.866\ Z)^2} = 0.5\ Z \therefore L = \frac{0.5Z}{\omega} = \frac{0.5Z}{120\pi}$$

When the frequency is changed to ω' = 2π × 180 = 3 × 120π = 300 rad/s, then

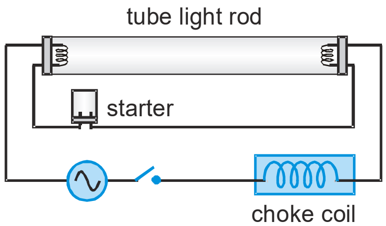
inductive reactance ω' L = 3 ωL = 3 × 0.5 Z = 1.5

$$\therefore \text{New impedance } Z' = \sqrt{[R'+(\omega' L)^2]} = \sqrt{(0.866\ Z)^2 + (1.5\ Z)^2} = Z\sqrt{[(0.866)^2 + (1.5)^2]} = 1.732Z$$

$$\therefore \text{New power factor} = \frac{R}{Z'} = \frac{0.866\ Z}{1.732\ Z} = 0.5$$

Choke Coil

In a direct current circuit, current is reduced with the help of a resistance.



Hence there is a loss of electrical energy $I^2 R$ per sec in the form of heat in the resistance. But in an AC circuit the current can be reduced by choke coil which involves very small amount of loss of energy. Choke coil is a copper coil wound over a soft iron laminated core. This coil is put in series with the circuit in which current is to be reduced. It also known as ballast.

Circuit with a choke coil is a series L-R circuit. If resistance of choke coil = r (very small)

The current in the circuit $I = \frac{E}{Z}$ with $Z = \sqrt{(R+r)^2 + (\omega L)^2}$ So due to large inductance L of the coil, the current in the circuit is decreased appreciably. However, due to small resistance of the coil r,

The power loss in the choke $P_{av} = V_{rms} I_{rms} \cos\phi \to 0$ ✈ $\cos\phi = \frac{r}{Z} = \frac{r}{\sqrt{r^2 + \omega^2 L^2}} \approx \frac{r}{\omega L} \to 0$

(i) Choke coil is a high inductance and negligible resistance coil.

(ii) Choke coil is used to control current in A.C. circuit at negligible power loss

(iii) Choke coil used only in A.C. and not in D.C. circuit

(iv) Choke coil is based on the principle of wattless current.

(v) Iron cored choke coil is used generally at low frequency and air cored at high frequency.

(vi) Resistance of ideal choke coil is zero

Ex. A choke coil and a resistance are connected in series in an a.c circuit and a potential of 130 volt is applied to the circuit. If the potential across the resistance is 50 V. What would be the potential difference across the choke coil.

Sol. $V = \sqrt{V_R^2 + V_L^2} \Rightarrow \quad V_L = \sqrt{V^2 - V_R^2} = \sqrt{(130)^2 - (50)^2} = 120\,V$

Ex. An electric lamp which runs at 80V DC consumes 10 A current. The lamp is connected to 100 V – 50 Hz ac source compute the inductance of the choke required.

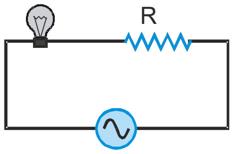
Sol. Resistance of lamp $R = \dfrac{V}{I} = \dfrac{80}{10} = 8\,\Omega$

Let Z be the impedance which would maintain a current of 10 A through the Lamp when it is run on 100 Volt a.c. then. $Z = \dfrac{V}{I} = \dfrac{100}{10} = 10\,\Omega$ but $Z = \sqrt{R^2 + (\omega L)^2}$

$\Rightarrow \quad (\omega L)^2 = Z^2 - R^2 = (10)^2 - (8)^2 = 36 \Rightarrow \omega L = 6 \Rightarrow L = \dfrac{6}{\omega} = \dfrac{6}{2\pi \times 50} = 0.02H$

Ex. Calculate the resistance or inductance required to operate a lamp (60V, 10W) from a source of (100 V, 50 Hz)

Sol. **(a)** Maximum voltage across lamp = 60V

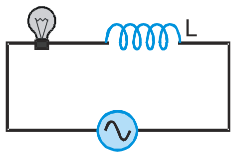


$\Rightarrow \quad V_{Lamp} + V_R = 100 \quad \therefore V_R = 40V$

Now current througth Lamp is $= \dfrac{\text{Wattage}}{\text{voltage}} = \dfrac{10}{60} = \dfrac{1}{6}\,A$

But $\quad V_R = IR \Rightarrow 40 = \dfrac{1}{6}(R) \Rightarrow R = 240\,\Omega$

(b) Now in this case $(V_{Lamp})^2 + (V_L)^2 = (V)^2$

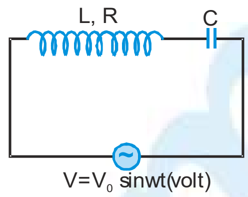


$(60)^2 + (V_L)^2 = (100)^2 \Rightarrow V_L = 80\,V$

Also $\quad V_L = IX_L = \dfrac{1}{6}X_L$ so $X_L = 80 \times 6 = 480\,\Omega = L\,(2\pi f) \Rightarrow L = 1.5\,H$

A capacitor of suitable capacitance replace a choke coil in an AC circuit, the average power consumed in a capacitor is also zero. Hence, like a choke coil, a capacitor can reduce current in AC circuit without power dissipation. Cost of capacitor is much more than the cost of inductance of same reactance that's why choke coil is used.

Ex. A choke coil of resistance R and inductance L is connected in series with a capacitor C and complete combination is connected to a.c. voltage, Circuit resonates when angular frequency of supply is $\omega = \omega_0$.



(a) Find out relation betwen ω_0, L and C

(b) What is phase difference between V and I at resonance, is it changes when resistance of choke coil is zero.

Sol. **(a)** At resonance condition $X_L = X_C \Rightarrow \omega_0 L = \frac{1}{\omega_0 C} \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$

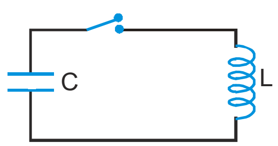
(b) ✈ $\cos\phi = \frac{R}{Z} = \frac{R}{R} = 1 \quad \therefore \phi = 0°$ No, It is always zero.

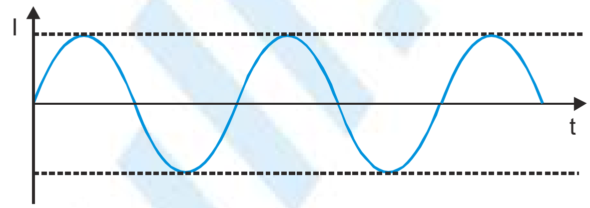
LC Oscillation

The oscillation of energy between capacitor (electric field energy) and inductor (magnetic field energy) is called LC Oscillation.

Undamped Oscillation

When the circuit has no resistance, the energy taken once from the source and given to capacitor keeps on oscillating between C and L then the oscillation produced will be of constant amplitude. These are called undamped oscillation.





After switch is closed

$$\frac{Q}{C} + L\frac{di}{dt} = 0 \Rightarrow \frac{Q}{C} + L\frac{d^2Q}{dt^2} = 0 \Rightarrow \frac{d^2Q}{dt^2} + \frac{1}{LC}Q = 0$$

By comparing with standard equation of free oscillation $\left[\frac{d^2x}{dt^2} + \omega^2 x = 0\right]$

$\omega^2 = \frac{1}{LC}$ Frequency of oscillation $f = \frac{1}{2\pi\sqrt{LC}}$

Charge varies sinusoidally with time $q = q_m \cos \omega t$

current also varies periodically with t $\quad I = \frac{dq}{dt} = q_m \omega \cos\left(\omega t + \frac{\pi}{2}\right)$

If initial charge on capacitor is q_m then electrical energy strored in capacitor is $U_E = \frac{1}{2}\frac{q_m^2}{C}$

At t = 0 switch is closed, capacitor is starts to discharge.

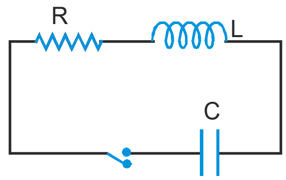
As the capacitor is fully discharged, the total electrical energy is stored in the inductor in the form of magnetic energy.

$U_B = \frac{1}{2}LI_m^2$ where I_m = max. current

$$(U_{max})_{EPE} = (U_{max})_{MPE} \Rightarrow \frac{1}{2}\frac{q_m^2}{C} = \frac{1}{2}LI_m^2$$

Damped Oscillation

Practically, a circuit can not be entirely resistanceless, so some part of energy is lost in resistance and amplitude of oscillation goes on decreasing. These are called damped oscillation.

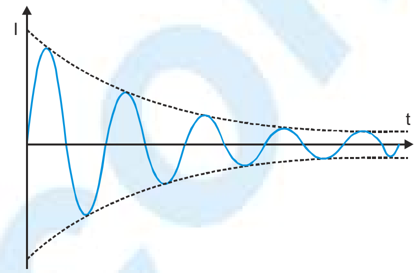


Angular frequency of oscillation $\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$

frequency of oscillation $f = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$

oscillation to be real if $\frac{1}{LC} - \frac{R^2}{4L^2} > 0$

Hence for oscilation to be real $\frac{1}{LC} > \frac{R^2}{4L^2}$



(i) In damped oscillation amplitude of oscillation decreases exponentially with time.

(ii) At $t = \frac{T}{4}, \frac{3T}{4}, \frac{5T}{4}$..... energy stored is completely magnetic.

(iii) At $t = \frac{T}{8}, \frac{3T}{8}, \frac{5T}{8}$..... energy is shared equally between L and C

(iv) Phase difference between charge and current is $\frac{\pi}{2}$ [when charge is maximum, current minimum / when charge is minimum, current maximum]

Ex. An LC circuit contains a 20mH inductor and a 50μF capacitor with an initial charge of 10mC. The resistance of the circuit is negligible. Let the instant the circuit is closed to be t = 0.

(a) What is the total energy stored initially.

(b) What is the natural frequency of the circuit.

(c) At what time is the energy stored is completely magnetic.

(d) At what times is the total energy shared equally between inductor and the capacitor.

Sol. **(a)** $U_E = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2} \times \frac{(10 \times 10^{-3})^2}{50 \times 10^{-6}} = 1.0J$

(b) $\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{20 \times 10^{-3} \times 50 \times 10^{-6}}} = 10^3$ rad/sec $\Rightarrow$ f = 159 Hz

(c) ✈ $q = q_0 \cos \omega t$

Energy stored is completely magnetic (i.e. electrical energy is zero, q = 0)

at $t = \frac{T}{4}, \frac{3T}{4}, \frac{5T}{4}$........ where $T = \frac{1}{f} = 6.3$ ms

(d) Energy is shared equally between L and C when charge on capacitor become $\frac{q_0}{\sqrt{2}}$

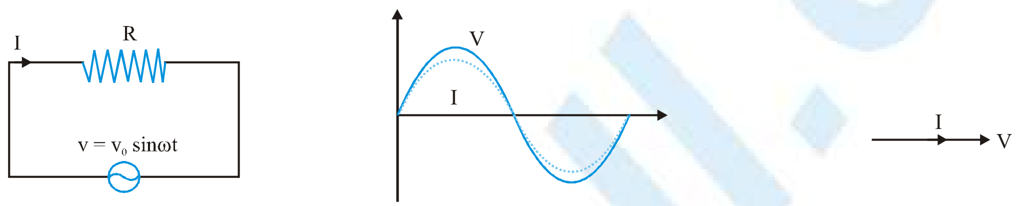
So, at $t = \frac{T}{8}, \frac{3T}{8}, \frac{5T}{8}$......

1. **Average value** $I_{av} = \dfrac{\int_0^T I\,dt}{\int_0^T dt} = \dfrac{1}{T}\int_0^T I\,dt$ **RMS value** $I_{rms} = \sqrt{\dfrac{\int_0^T I^2\,dt}{\int_0^T dt}}$

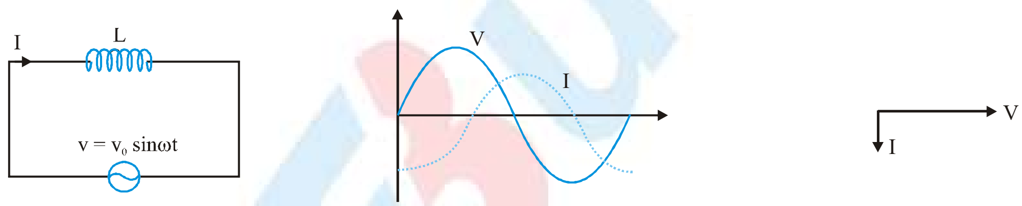
For sinusoidal voltage $V = V_0 \sin \omega t$: $V_{av} = \dfrac{2V_0}{\pi}$ & $V_{rms} = \dfrac{V_0}{\sqrt{2}}$

For sinusoidal current $I = I_0 \sin(\omega t + \phi)$: $I_{av} = \dfrac{2I_0}{\pi}$ & $I_{rms} = \dfrac{I_0}{\sqrt{2}}$

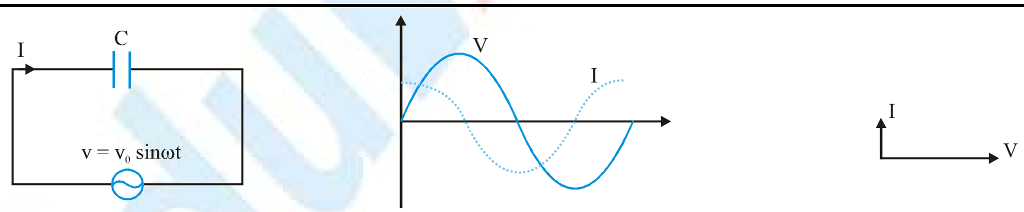
2. **AC Circuits**



$I = \dfrac{V_0}{R}\sin \omega t$



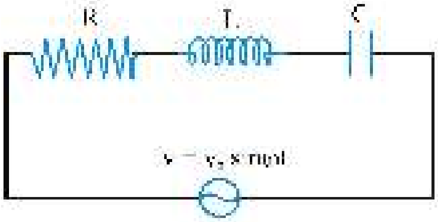
$I = \dfrac{V_0}{\omega L}\sin\left(\omega t - \dfrac{\pi}{2}\right)$



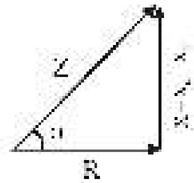
$I = V_0 \omega C \sin\left(\omega t + \dfrac{\pi}{2}\right)$

3. **Impendance :** $Z = \sqrt{R^2 + X^2}$ where X = reactance

4. Series LCR Circuit



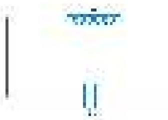
$Z = \sqrt{R^2 + (X_L - X_C)^2}$;



$V = \sqrt{V_R^2 + (V_L - V_C)^2}$

5. Power Factor $= \cos\phi = R/Z$ At resonance : $X_L = X_C \rightarrow Z = R, V = V_R$

6. LC Oscillation



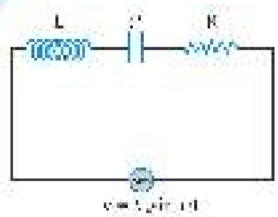
$q = q_0 \sin(\omega t + \theta)$, $I = I_0 \cos(\omega t + \theta)$ $I_0 = q_0\omega$

Energy $= \frac{1}{2}LI^2 + \frac{q^2}{2C} = \frac{q_0^2}{2C} = \frac{1}{2}LI_0^2 =$ constant

Comparison with SHM $q \rightarrow x, I \rightarrow v, L \rightarrow m, C \rightarrow \frac{1}{K}$

7. Comparison of Damped Mechanical & Electrical Systems

(a) Series LCR circuit :



$$\frac{d^2q}{dt^2} + \frac{R}{L}\frac{dq}{dt} + \frac{1}{LC}q = \frac{V_0}{L}\cos\omega t$$

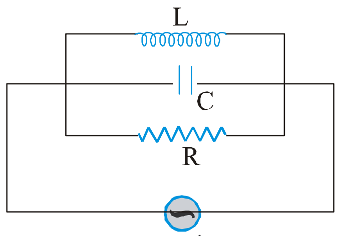
compare with mechanical damped system equation.

$$\frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = \frac{F_0}{m}\cos\omega t$$

where b = damping coefficient.

Mechanical system	Electrical system (series RLC)
Displacement (x)	Charge (q)
Driving force (F)	Driving voltage (V)
Kinetic energy $\left(\frac{1}{2}mv^2\right)$	Electromagnetic energy of moving charge $\frac{1}{2}L\left(\frac{dq}{dt}\right)^2 = \frac{1}{2}LI^2$
Potential energy $\frac{1}{2}kx^2$	Energy of static charge $\frac{q^2}{2C}$
mass (m)	(1/C)
Power P = Fv	Power P = VI
Damping (b)	Resistance (R)

(b) Parallel LCR circuit : In this case



$$I = I_L + I_C + I_R = \frac{\phi}{L} + \frac{d}{dt}C\left(\frac{d\phi}{dt}\right) + \frac{1}{R}\frac{d\phi}{dt} \Rightarrow \frac{d^2\phi}{dt^2} + \frac{1}{RC}\frac{d\phi}{dt^2} + \frac{1}{LC}\phi = \frac{V_0}{ZC}\sin\omega t$$

Displacement (x)	⇔	Flux linkage (ϕ)
Velocity $\left(\frac{dx}{dt}\right)$	⇔	Voltage $\left(\frac{d\phi}{dt}\right)$
Mass (m)	⇔	Capacitance (C)
Spring constant (k)	⇔	Reciprocal Industance (1/L)
Damping coefficient (b)	⇔	Reciprocal resistance (1/R)
Driving force (F)	⇔	Current (i)