TRIGONOMETRY

SYSTEM OF MEASUREMENT OF ANGLE

There are three system for measuring angles.

Sexagesimal or English system
Centesimal or French system
Circular system

Sexagesimal system :

The principal unit in this system is degree (^{o}). One right angle is divided into 90 equal parts and each part is called one degree (1^o). One degree is divided into 60 equal parts and each part is called one minute. Minute is denoted by (1'). One minute is equally divided into 60 equal parts and each part is called one second (1").

In Mathematical form :

One right angle = 90^o
1^o = 60'
1' = 60"

Centesimal system :

The principal unit in system is grade and is denoted by (^{g}). One right angle is divided into 100 equal parts, called grades, and each grade is subdivided into 100 minutes, and each minutes into 100 seconds.

In Mathematical Form :

One right angle = 100^g
1^g = 100'
1' = 100'

Relation between sexagesimal and Centesimal systems :

One right angle = 90^o (degree system) ...(1)

One right angle = 100^g (grade system) ...(2)

by (1) and (2),

$90^o = 100^g$

or, $\frac{D}{90} = \frac{G}{100}$

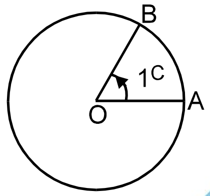
then we can say,

$$1^o = \left(\frac{100}{90}\right)^g$$

Circular system :

One radian, written as 1^C, is the measure of an angle subtended at the centre of a circle by an arc of length equal to the radius of the circle.

Consider a circle of radius r having centre at O. Let A be a point on the circle. Now cut off an arc AB whose length is equal to the radius r of the circle. Then by the definition the measure of $\angle AOB$ is 1 radian (1^C).



Relation between systems of measurement of angles :

$$\frac{D}{90} = \frac{G}{100} = \frac{2C}{\pi}$$

Fundamental Trigonometrical Identities :

(a) $\sin\theta = \frac{1}{\operatorname{cosec}\theta}$

(b) $\cos\theta = \frac{1}{\sec\theta}$

(c) $\cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$

(d) $1 + \tan^2\theta = \sec^2\theta$

or, $\sec^2\theta - \tan^2\theta = 1$

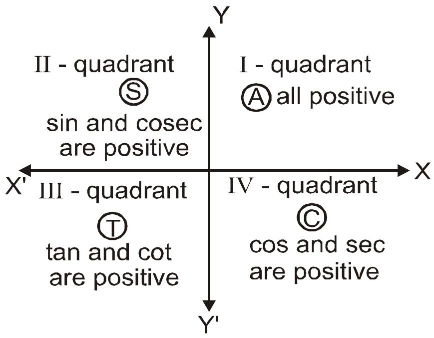
$(\sec\theta - \tan\theta) = \frac{1}{(\sec\theta + \tan\theta)}$

(e) $\sin^2\theta + \cos^2\theta = 1$

(f) $1 + \cot^2\theta = \operatorname{cosec}^2\theta$

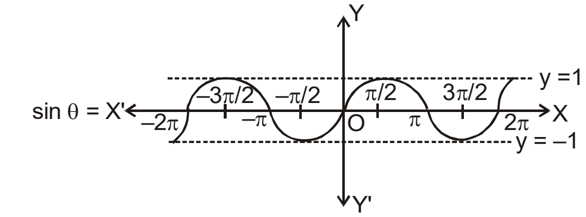
$(\operatorname{cosec}\theta - \cot\theta) = \frac{1}{\operatorname{cosec}\theta + \cot\theta}$

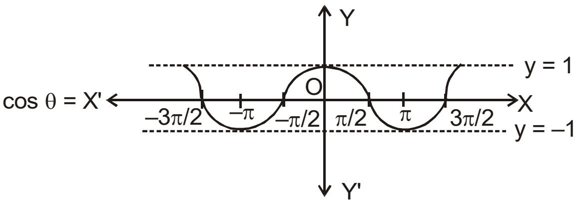
Signs of the trigonometrical ratios or functions:

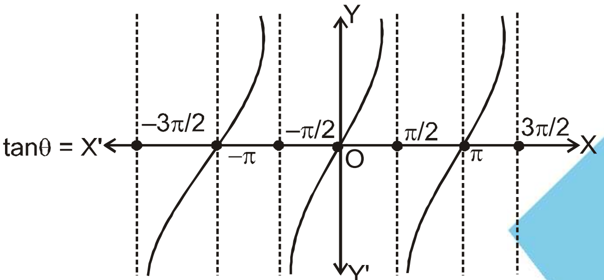


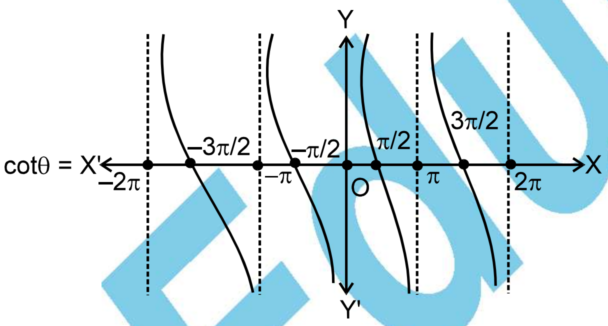
A crude aid to memorise the signs of trigonometrical ratio in different quadrant.

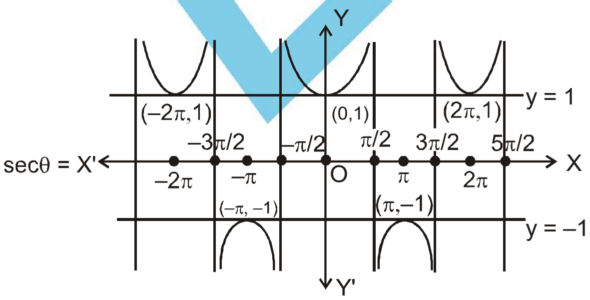
GRAPH OF DIFFERENT TRIGONOMETRICAL RATIOS



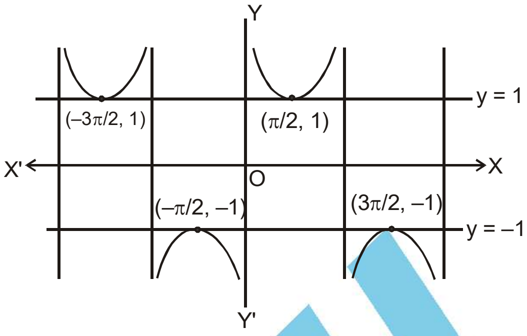








cosecθ =



TRIGONOMETRICAL RATIO OF ALLIED ANGLES

$\sin(90° - \theta) = \cos\theta$

$\cos(90° - \theta) = \sin\theta$

$\tan(90° - \theta) = \cot\theta$

$\cot(90° - \theta) = \tan\theta$

$\text{cosec}(90° - \theta) = \sec\theta$

$\sec(90° - \theta) = \text{cosec}\theta$

$\sin(90° + \theta) = \cos\theta$,

$\cos(90° + \theta) = -\sin\theta$

$\tan(90° + \theta) = -\cot\theta$

$\cot(90° + \theta) = -\tan\theta$

$\sec(90° + \theta) = -\text{cosec}\,\theta$

$\text{cosec}(90° + \theta) = \sec\theta$

Periodic Function :

All the trigonometric function are periodic function. They repeat their value after a certain period

$\sin(2n\pi + \theta) = \sin\theta$

$\cos(2n\pi + \theta) = \cos\theta$

$\tan(n\pi + \theta) = \tan\theta$

SUM & DIFFERENCE FORMULAE

(a) $\sin(A + B)$

$= \sin A \cos B + \cos A \sin B$

(b) $\sin(A - B)$

$= \sin A \cos B - \cos A \sin B$

(c) $\cos(A + B)$

$= \cos A \cos B - \sin A \sin B$ (d)

$\cos(A - B)$

$= \cos A \cos B + \sin A \sin B$

(e) $\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

(f) $\tan(A - B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

(g) $\cot(A + B) = \dfrac{\cot A \cot B - 1}{\cot B + \cot A}$

(h) $\cot(A - B) = \dfrac{\cot A \cot B + 1}{\cot B - \cot A}$

Some More Results :

*(a) $\sin(A+B).\sin(A-B)$
$= \sin^2 A - \sin^2 B$
$= \cos^2 B - \cos^2 A$

*(b) $\cos(A+B).\cos(A-B)$
$= \cos^2 A - \sin^2 B$
$= \cos^2 B - \sin^2 A$

(c) $\sin(A+B+C) = \sin A \cos B \cos C + \cos A \sin B \sin C + \cos A \cos B \sin C - \sin A \sin B \sin C$

(d) $\cos(A+B+C) = \cos A \cos B \cos C - \cos A \sin B \sin C - \sin A \cos B \sin C - \sin A \sin B \cos C$

(e) $\tan(A+B+C)$

$$= \frac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - \tan A \tan B - \tan B \tan C - \tan C \tan A}$$

(Note : * Important)

FORMULAE FOR PRODUCT INTO SUM OR DIFFERENCE CONVERSION

We know that,

$\sin A \cos B + \cos A \sin B = \sin(A+B)$ ....(i)

$\sin A \cos B - \cos A \sin B = \sin(A-B)$ ......(ii)

$\cos A \cos B - \sin A \sin B = \cos(A+B)$ .....(iii)

$\cos A \cos B + \sin A \sin B = \cos(A-B)$ .....(iv)

Adding (i) and (ii),
$2 \sin A \cos B = \sin(A+B) + \sin(A-B)$

Subtracting (ii) from (i),
$2 \cos A \sin B = \sin(A+B) - \sin(A-B)$

Adding (iii) and (iv),
$2 \cos A \cos B = \cos(A+B) + \cos(A-B)$

Subtraction (iii) from (iv).
$2 \sin A \sin B = \cos(A-B) - \cos(A+B)$

Formulae :

(a) $2 \sin A \cos B = \sin(A+B) + \sin(A-B)$

(b) $2 \cos A \sin B = \sin(A+B) - \sin(A-B)$

(c) $2 \cos A \cos B = \cos(A+B) + \cos(A-B)$

(d) $2 \sin A \sin B = \cos(A-B) - \cos(A+B)$

FORMULAE FOR SUM OR DIFFERENCE INTO PRODUCT CONVERSION

We know that,

$\sin(A+B) + \sin(A-B)$
$= 2 \sin A \cos B$ ......(i)

Let $A + B = C$ and $A - B = D$

then $A = \frac{C+D}{2}$ and $B = \frac{C-D}{2}$

Substituting in (i),

(a) $\sin C + \sin D = 2 \sin\left(\frac{C+D}{2}\right).\cos\left(\frac{C-D}{2}\right)$

similarly other formula are,

(b) $\sin C - \sin D = 2 \cos\left(\frac{C+D}{2}\right).\sin\left(\frac{C-D}{2}\right)$

(c) $\cos C + \cos D$

$= 2 \cos\left(\frac{C+D}{2}\right).\cos\left(\frac{C-D}{2}\right)$

☞ (d) $\cos C - \cos D = 2 \sin\left(\frac{C+D}{2}\right).\sin\left(\frac{D-C}{2}\right)$

TRIGONOMETRICAL RATIOS OF MULTIPLE ANGLES :

(i) $\sin 2\theta = 2 \sin\theta \cos\theta = \frac{2\tan\theta}{1+\tan^2\theta}$

(ii) $\cos 2\theta = \cos^2\theta - \sin^2\theta$
$= 2\cos^2\theta - 1$
$= 1 - 2\sin^2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$

(iii) $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

(iv) $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

(v) $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

(vi) $\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}$

(vii) $\sin\theta/2 = \sqrt{\frac{1-\cos\theta}{2}}$

(viii) $\cos\theta/2 = \sqrt{\frac{1+\cos\theta}{2}}$

(ix) $\tan\theta/2 = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$

$= \frac{1-\cos\theta}{\sin\theta} = \frac{\sin\theta}{1+\cos\theta}$

THE GREATEST AND LEAST VALUE OF THE EXPRESSION [a sinθ + b cosθ]

Let $a = r\cos\alpha$ ...(1)
and $b = r\sin\alpha$ ...(2)
Squaring and adding (1) and (2)
then $a^2 + b^2 = r^2$

or, $r = \sqrt{a^2 + b^2}$

$\therefore$ a sinθ + b cosθ

= r (sinθ cosα + cosθ sinα)

= r sin(θ + α)

But $-1 \le \sin\theta \le 1$

so $-1 \le \sin(\theta + \alpha) \le 1$

then $-r \le r\sin(\theta + \alpha) \le r$

hence,

$-\sqrt{a^2+b^2} \le a\sin\theta + b\cos\theta \le \sqrt{a^2+b^2}$ then the greatest and least values of a sinθ + b cosθ are respectively

$\sqrt{a^2+b^2}$ and $-\sqrt{a^2+b^2}$

GENERAL SOLUTION OF STANDARD TRIGONOMETRICAL EQUATIONS

Since Trigonometrical functions are periodic functions, therefore, solutions of Trigonometrical equations can be generalised with the help of periodicity of Trigonometrical functions. The solution consisting of all possible solutions of a Trigonometrical equation is called its general solution.

General Solution of the equation sin θ = 0 :

when sin θ = 0 is

$\theta = n\pi$: $n \in I$ i.e. n = 0, ± 1, ± 2 ....

General solution of cos θ = 0 :

when cosθ = 0

$\theta = (2n + 1)\,\pi/2$, $n \in I$.

i.e. n = 0, ± 1, ± 2 ....

General solution of tan θ = 0 :

when tanθ = 0 it same as of sinθ = 0

so that, general solution of tan θ = 0 is

$\theta = n\pi$; $n \in I$

Note : General solution of secθ = 0 and cosecθ =0 does not exist because secθ and cosecθ can never be equal to 0.

General solution of the equation

sin θ = sin α :

$\theta = n\pi + (-1)^n\alpha$; $n \in I$

General solution of the equation

cos θ = cos α :

$\theta = 2n\pi - \alpha$; $n \in I$ and

$\theta = 2n\pi + \alpha$; $n \in I$

for the general solution of cos θ = cos α, combine these two result which gives.

$\theta = 2n\pi \pm \alpha$, $n \in I$

General solution of the equation

tan θ = tan α :

$\theta = n\pi + \alpha$; $n \in I$

GENERAL SOLUTION OF SQUARE OF THE TRIGONOMETRICAL EQUATIONS :

$\theta = n\pi \pm \alpha$; $n \in I$

General solution of $\cos^2\theta = \cos^2\alpha$

$\theta = n\pi \pm \alpha$; $n \in I$

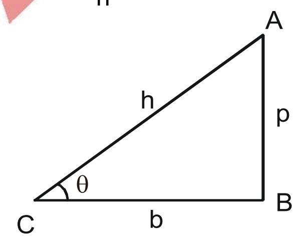
General solution of $\tan^2\theta = \tan^2\alpha$:

$\Rightarrow \theta = n\pi \pm \alpha$; $n \in I$

SOME USEFUL RESULTS

In a triangle ABC,

$\sin\theta = \frac{p}{h}$, $\cos\theta = \frac{b}{h}$

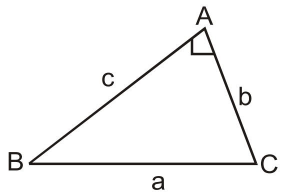


In any triangle ABC,

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ [By sine rule]

or cosine formula

i.e., $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$



SOLVED PROBLEMS

Ex.1 Find the value of the expression -

$$1 - \frac{\sin^2 y}{1+\cos y} + \frac{1+\cos y}{\sin y} - \frac{\sin y}{1-\cos y}$$

Sol. $1 - \frac{\sin^2 y}{1+\cos y} + \frac{1+\cos y}{\sin y} - \frac{\sin y}{1-\cos y}$

$$= \frac{1+\cos y-\sin^2 y}{1+\cos y} + \frac{1-\cos^2 y-\sin^2 y}{\sin y\,(1-\cos y)}$$

$$= \frac{\cos y+\cos^2 y}{1+\cos y} + 0 = \cos y$$

Ex.2 If cosec θ –sinθ =m and secθ–cosθ = n then find $(m^2n)^{2/3} + (n^2m)^{2/3}$

Sol. cosec θ – sin θ = m

$$m = \frac{1}{\sin\theta} - \sin\theta = \frac{\cos^2\theta}{\sin\theta} \quad \ldots(i)$$

$$n = \frac{1}{\cos\theta} - \cos\theta = \frac{\sin^2\theta}{\cos\theta} \quad \ldots(ii)$$

$$m \times n = \frac{\cos^2\theta}{\sin\theta} \cdot \frac{\sin^2\theta}{\cos\theta} = \sin\theta\cos\theta$$

from (i) and (ii)

from (i) $\cos^2\theta = m \cdot \sin\theta$

or $\cos^3\theta = m\sin\theta\cos\theta$

$= m \cdot (mn) = m^2n$

Similarly $\sin^3\theta = n^2m$

since $\sin^2\theta + \cos^2\theta = 1$

$(n^2m)^{2/3} + (m^2n)^{2/3} = 1$

Ex.3 Find the value of $\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\frac{5\pi}{8}\right)\left(1+\cos\frac{7\pi}{8}\right)$

Sol. $\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\left(\pi-\frac{3\pi}{8}\right)\right)\left(1+\cos\left(\pi-\frac{\pi}{8}\right)\right)$

$$= \left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1-\cos\frac{3\pi}{8}\right)\left(1-\cos\frac{\pi}{8}\right)$$

$$= \left(1-\cos^2\frac{\pi}{8}\right)\left(1-\cos^2\frac{3\pi}{8}\right)$$

$$= \frac{1}{4}\left(2-1-\cos\frac{\pi}{4}\right)\left(2-1-\cos\frac{3\pi}{4}\right)$$

$$= \frac{1}{4}\left(1-\cos\frac{\pi}{4}\right)\left(1-\cos\frac{3\pi}{4}\right)$$

$$= \frac{1}{4}\left(1-\frac{1}{\sqrt{2}}\right)\left(1+\frac{1}{\sqrt{2}}\right) = \frac{1}{4}\left(1-\frac{1}{2}\right) = \frac{1}{8}$$

Ex.4 Find the value of sin 20° sin 40° sin 60° sin 80°

Sol. sin 20° sin 40° sin 60° sin 80°

$$= \frac{\sqrt{3}}{2}\sin 20°\sin(60° - 20°)\sin(60° + 20°)$$

$$= \frac{\sqrt{3}}{2}\sin 20°\,(\sin^2 60° - \sin^2 20°)$$

$$= \frac{\sqrt{3}}{2}\sin 20°\,\left(\frac{3}{4} - \sin^2 20°\right)$$

$$= \frac{\sqrt{3}}{8}(3\sin 20° - 4\sin^3 20°)$$

$$= \frac{\sqrt{3}}{8}\sin 60°$$

$$= \frac{\sqrt{3}}{8} \cdot \frac{\sqrt{3}}{2} = \frac{3}{16}$$

Ex.5 Find $\cos^4\frac{\pi}{8} + \cos^4\frac{3\pi}{8} + \cos^4\frac{5\pi}{8} + \cos^4\frac{7\pi}{8}$

Sol. $= \cos^4\frac{\pi}{8} + \cos^4\frac{3\pi}{8} + \cos^4\frac{5\pi}{8} + \cos^4\frac{7\pi}{8}$

$$= \cos^4\frac{\pi}{8} + \cos^4\frac{3\pi}{8} + \cos^4\frac{3\pi}{8} + \cos^4\frac{\pi}{8}$$

$$= 2\left(\cos^4\frac{\pi}{8} + \cos^4\frac{3\pi}{8}\right)$$

$$= \frac{1}{2}\left[\left(2\cos^2\frac{\pi}{8}\right)^2 + \left(2\cos^2\frac{3\pi}{8}\right)^2\right]$$

$$= \frac{1}{2}\left[\left(1+\cos\frac{\pi}{4}\right)^2 + \left(1+\cos\frac{3\pi}{4}\right)^2\right]$$

$$= \frac{1}{2}\left[\left(1+\frac{1}{\sqrt{2}}\right)^2 + \left(1-\frac{1}{\sqrt{2}}\right)^2\right]$$

$$= \frac{1}{2}[2+1] = \frac{3}{2}$$

Ex.6 If $A + B + C = \frac{3\pi}{2}$, then prove that $\cos 2A + \cos 2B + \cos 2C = 1 - 4\sin A \sin B \sin C$

Sol. $\cos 2A + \cos 2B + \cos 2C$

$= 2\cos(A + B)\cos(A - B) + \cos 2C$

$= 2\cos\left(\frac{3\pi}{2} - C\right)\cos(A - B) + \cos 2C$

$\because A + B + C = \frac{3\pi}{2}$

$= -2\sin C\cos(A - B) + 1 - 2\sin^2 C$

$= 1 - 2\sin C\,[\cos(A - B) + \sin C)$

$= 1 - 2\sin C\left[\cos(A-B) + \sin\left(\frac{3\pi}{2} - (A+B)\right)\right]$

$= 1 - 2\sin C$
$[\cos(A - B) - \cos(A + B)]$

$= 1 - 4\sin A\sin B\sin C$

Ex.7 In any triangle ABC, $\sin A - \cos B = \cos C$, then find angle B

Sol. We have, $\sin A - \cos B = \cos C$

$\sin A = \cos B + \cos C$

$2\sin\frac{A}{2}\cos\frac{A}{2} = 2\cos\left(\frac{B+C}{2}\right)\cos\left(\frac{B-C}{2}\right)$

$2\sin\frac{A}{2}\cos\frac{A}{2}$

$= 2\cos\left(\frac{\pi - A}{2}\right)\cos\left(\frac{B-C}{2}\right)$

$\because A + B + C = \pi$, $2\sin\frac{A}{2}\cos\frac{A}{2}$

$= 2\sin\frac{A}{2}\cos\left(\frac{B-C}{2}\right)$

$\cos\frac{A}{2} = \cos\frac{B-C}{2}$

or $A = B - C$

But $A + B + C = \pi$

Therefore $2B = \pi \Rightarrow B = \pi/2$

Ex.8 Find $\tan 9° - \tan 27° - \tan 63° + \tan 81°$

Sol. $\tan 9° + \tan 81° - (\tan 27° + \tan 63°)$

$(\tan 9° + \cot 9°) - (\tan 27° + \cot 27°)$

$= \left(\frac{\sin 9°}{\cos 9°} + \frac{\cos 9°}{\sin 9°}\right) - \left(\frac{\sin 27°}{\cos 27°} + \frac{\cos 27°}{\sin 27°}\right)$

$= \frac{1}{\sin 9° \cos 9°} - \frac{1}{\cos 27° \sin 27°}$

$= \frac{2}{\sin 18} - \frac{2}{\sin 54°} = \frac{2}{\sin 18°} - \frac{2}{\sin 36°}$

$= \frac{2\times 4}{\sqrt{5}-1} - \frac{2\times 4}{\sqrt{5}+1} = 8\left[\frac{\sqrt{5}+1-\sqrt{5}+1}{(\sqrt{5}-1)(\sqrt{5}+1)}\right]$

$= \frac{16}{4} = 4$

Ex.9 If $\sin 3\theta = \sin\theta$, then find the general value of θ

Sol. $\sin 3\theta = \sin\theta$

or, $3\theta = m\pi + (-1)^m\theta$

For (m) even i.e. $m = 2n$,

then $\theta = \frac{2n\pi}{2} = n\pi$

and for (m) odd

i.e. $m = (2n + 1)$

or, $\theta = (2n + 1)\frac{\pi}{4}$

Ex.10 Find the number of solutions of equation, $\sin 5x \cos 3x = \sin 6x \cos 2x$, in the interval $[0, \pi]$

Sol. The given equation can be written as

$\frac{1}{2}(\sin 8x + \sin 2x) = \frac{1}{2}(\sin 8x + \sin 4x)$

or, $\sin 2x - \sin 4x$

$\Rightarrow - 2\sin x\cos 3x = 0$

Hence $\sin x = 0$ or $\cos 3x = 0$.

That is, $x = n\pi$ ($n \in I$), or

$3x = k\pi + \frac{\pi}{2}$

($k \in I$). Therefore, since $x \in [0, \pi]$, the given equation is satisfied

if $x = 0, \pi, \frac{\pi}{6}, \frac{\pi}{2}$ or $\frac{5\pi}{6}$.

Ex.11 Find the general solution of

$\tan\left(\frac{\pi}{2}\sin\theta\right) = \cot\left(\frac{\pi}{2}\cos\theta\right)$

Sol. We have, $\tan\left(\frac{\pi}{2}\sin\theta\right) = \cot\left(\frac{\pi}{2}\cos\theta\right)$

$\Rightarrow \tan\left(\frac{\pi}{2}\sin\theta\right) = \tan\left(\frac{\pi}{2} - \frac{\pi}{2}\cos\theta\right)$

$\Rightarrow \frac{\pi}{2}\sin\theta = r\pi + \frac{\pi}{2} - \frac{\pi}{2}\cos\theta$,
$r \in Z$

$\Rightarrow \sin\theta + \cos\theta = (2r + 1)$, $r \in Z$

$\Rightarrow \frac{1}{\sqrt{2}}\sin\theta + \frac{1}{\sqrt{2}}\cos\theta = \frac{2r+1}{\sqrt{2}}$,
$r \in Z$

$\Rightarrow \cos\left(\theta - \frac{\pi}{4}\right) = \frac{2r+1}{\sqrt{2}}$, $r \in Z$

$\Rightarrow \cos\left(\theta - \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$ or $-\frac{1}{\sqrt{2}}$

$\Rightarrow \theta - \frac{\pi}{4} = 2r\pi \pm \frac{\pi}{4}$, $r \in Z$

$\Rightarrow q = 2r\pi \pm \frac{\pi}{4} + \frac{\pi}{4}$, $r \in Z$

$\Rightarrow \theta = 2r\pi,\ 2r\pi + \frac{\pi}{2}$, $r \in Z$

But $\theta = 2r\pi + \frac{\pi}{2}$, $r \in Z$ gives extraneous roots as it does not satisfy the given equation. Therefore $\theta = 2r\pi$, $r \in Z$

Ex.12 Find the general solution of the equation $\sec 4\theta - \sec 2\theta = 2$

Sol. Given equation is,

$\sec 4\theta - \sec 2\theta = 2$

or, $\frac{1}{\cos 4\theta} - \frac{1}{\cos 2\theta} = 2$,

$\cos 4\theta \neq 0$, $\cos 2\theta \neq 0$

or, $\cos 2\theta - \cos 4\theta = 2 \cos 4\theta \cos 2\theta$

or, $\cos 2\theta - \cos 4\theta = \cos 6\theta + \cos 2\theta$

or, $\cos 6\theta + \cos 4\theta = 0$

or $2 \cos 5\theta \cos \theta = 0$

$\therefore$ either $\cos 5\theta = 0$ or $\cos \theta = 0$

If $\cos 5\theta = 0$, then $5\theta = (2n + 1) \pi/2$

or, $\theta = (2n + 1) \pi/10$, where $n \in I$.

and if $\cos \theta = 0$, then $\theta = (2n + 1) \pi/2$, where $n \in I$.

obviously for $\theta = (2n + 1) \pi/2$ and $\theta = (2n + 1) \pi/10$, $\cos 2\theta$ or $\cos 4\theta$ are not zero

Hence $\theta = (2n + 1) \pi/2$, $(2n + 1) \pi/10$ are the general solutions of the given equation.

Ex.13 A tower subtends an angle of 30º at a point on the same level as its foot, and at a second point h m above the first, the depression of the foot of tower is 60º .Find the height of the tower

Sol. Let OP be the tower of height x., A the point on the same level as the foot O of the tower and B be the point h m above A (see Fig.). Then $\angle AOB = 60º$ and $\angle PAO = 30º$. From right-angled triangle AOP, we have

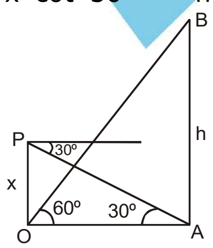
OA = x cot 30º

and from right-angled triangle OAB, we have

OA = h cot 60º

Therefore, from (1) and (2), we get

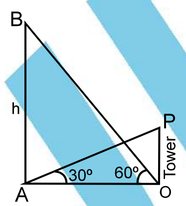
x cot 30º = h cot 60º



$\sqrt{3}\,x = \frac{1}{\sqrt{3}}h$, $\quad x = \frac{1}{3}h$

Ex.14 A tower subtends an angle of 30º at a point on the same level as the foot of the tower . At a second point, h metre above first, the depression of the foot of the tower is 60º, find the horizontal distance of the tower from the points

Sol. Let the tower OP subtend an angle of 30º at A.a point on the same level as the foot of the tower. From a point B at the height h, vertically above A, from where the angle of depression of foot O of the tower is 60º i.e., $\angle AOP = 60º$.



From Δ OAP ,

OA = h cot 60º .

Also from Δ OAP ,

OA = OP cot 30º .

$\therefore$ OA = h cot 60º = OP cot 30º

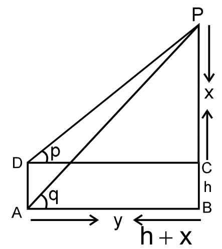
$\Rightarrow$ OP = h/3.

$\therefore$ OA = (h/3) cot 30º,

which is given in (B)

Ex.15 The top of a hill observed from the top and bottom of a building of height h is at angles of elevation p and q respectively. Fnd the height of the hill

Sol. Let AD be the building of height h and BP be the hill. Then



$\tan q = \frac{h+x}{y}$ and $\tan p = \frac{x}{y}$

$\Rightarrow \tan q = \frac{h+x}{x \cot p}$

$\Rightarrow x \cot p = (h + x) \cot q$

$x = \frac{h \cot q}{\cot p - \cot q}$

$\Rightarrow h + x = h + \frac{h \cot q}{\cot p - \cot q}$

$= \frac{h \cot p}{\cot p - \cot q}$

EXERCISE

Q.1 If $\cos\theta = \frac{-\sqrt{3}}{2}$ and θ lies in Quadrant III, find the value of all the other five trigonometric functions.

Q.2 If $\sin\theta = \frac{-1}{2}$ and θ lies in Quadrant IV, find the values of all the other five trigonometric functions.

Q.3 If $\text{cosec}\theta = \frac{5}{3}$ and θ lies in Quadratic II, find the values of all the other five trigonometric functions.

Q.4 If $\sec\theta = \sqrt{2}$ and θ lies in Quadrant IV, find the values of all the other trigonometric functions.

Q.5 If $\sin x = \frac{-2\sqrt{6}}{5}$ and x lies in Quadrant III, find the values of cos x and cot x.

Q.6 If $\cos x = \frac{-\sqrt{15}}{4}$ and $\frac{\pi}{2} < x < \pi$, find the value of sin x.

Q.7 If sec x = –2 and $\pi < x < \frac{3\pi}{2}$, find the values of all the other five trigonometric functions.

Q.8 Find the value of

(i) $\sin\left(\frac{31\pi}{3}\right)$ (ii) $\cos\left(\frac{17\pi}{2}\right)$ (iii) $\tan\left(\frac{-25\pi}{3}\right)$

(iv) $\cot\left(\frac{13\pi}{4}\right)$ (v) $\sec\left(\frac{-25\pi}{3}\right)$ (vi) $\text{cosec}\left(\frac{-41\pi}{4}\right)$

Q.9 Find the value of

(i) sin 405° (ii) sec(–1470°) (iii) tan(–300°)

(iv) cot(585°) (v) cosec (–750°) (vi) cos(–2220°)

Q.10 Prove that

(i) $\tan^2\frac{\pi}{3} + 2\cos^2\frac{\pi}{4} + 3\sec^2\frac{\pi}{6} + 4\cos^2\frac{\pi}{2} = 8$

(ii) $\sin\frac{\pi}{6}\cos 0 + \sin\frac{\pi}{4}\cos\frac{\pi}{4} + \sin\frac{\pi}{3}\cos\frac{\pi}{6} = \frac{7}{4}$

(iii) $4\sin\frac{\pi}{6}\sin^2\frac{\pi}{3} + 3\cos\frac{\pi}{3}\tan\frac{\pi}{4} + \text{cosec}^2\frac{\pi}{2} = 4$

Q.11 Find the value of

(i) cos 840° (ii) sin870° (iii) tan(–120°)

(iv) sec(–420°) (v) cosec(–690°) (vi) tan(225°)

(vii) cot(–315°) (viii) sin(–1230°) (ix) cos(495°)

Q.12 Find the values of all trigonometric functions of 135°.

Q.13 Prove that

(i) $\sin 80^\circ \cos 20^\circ - \cos 80^\circ \sin 20^\circ = \frac{\sqrt{3}}{2}$

(ii) $\cos 45^\circ \cos 15^\circ - \sin 75^\circ \sin 15^\circ = \frac{1}{2}$

(iii) $\cos 75^\circ \cos 15^\circ + \sin 75^\circ \sin 15^\circ = \frac{1}{2}$

(iv) $\sin 40^\circ \cos 20^\circ + \cos 40^\circ \sin 20^\circ = \frac{\sqrt{3}}{2}$

(v) $\cos 130^\circ \cos 40^\circ + \sin 130^\circ \sin 40^\circ = 0$

Q.14 Prove that

(i) $\sin(50^\circ+\theta)\cos(20^\circ+\theta) - \cos(50^\circ+\theta)\sin(20^\circ+\theta) = \frac{1}{2}$

(ii) $\cos(70^\circ+\theta)\cos(10^\circ+\theta) + \sin(70^\circ+\theta)\sin(10^\circ+\theta) = \frac{1}{2}$

Q.15 Prove that

(i) cos(n+2)xcos(n+1)x+sin(n+2)xsin(n+1)x=cos x

(ii) $\cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right) - \sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right) = \sin(x+y)$

Q.16 Prove that $\frac{\tan\left(\frac{\pi}{4}+4\right)}{\tan\left(\frac{\pi}{4}-x\right)} = \left(\frac{1+\tan x}{1-\tan x}\right)^2$.

Q.17 Prove that

(i) $\sin 75^\circ = \frac{(\sqrt{6}+\sqrt{2})}{4}$

(ii) $\frac{\cos 135^\circ - \cos 120^\circ}{\cos 135^\circ + \cos 120^\circ} = (3 - 2\sqrt{2})$

(iii) tan 15° + cot 15° = 4

Q.18 Prove that

(i) $\cos 15^\circ - \sin 15^\circ = \frac{1}{\sqrt{2}}$

(ii) cot 105° – tan 105° = $2\sqrt{3}$

(iii) $\frac{\tan 69^o + \tan 66^o}{1 - \tan 69^o \tan 66^o} = -1$

Q.19 Prove that $\frac{\cos 9^o + \sin 9^o}{\cos 9^o - \sin 9^o}$ = tan 54°.

Q.20 Prove that $\frac{\cos 8^o - \sin 8^o}{\cos 8^o + \sin 8^o}$ = tan 37°.

Q.21 Prove that $\frac{\cos(\pi+\theta)\cos(-\theta)}{\cos(\pi-\theta)\cos\left(\frac{\pi}{2}+\theta\right)}$ = –cotθ

Q.22 Prove that

$$\frac{\cos\theta}{\sin(90^o+\theta)} + \frac{\sin(-\theta)}{\sin(180^o+\theta)} - \frac{\tan(90^o+\theta)}{\cot\theta} = 3.$$

Q.23 Prove that

$$\frac{\sin(180^o+\theta)\cos(90^o+\theta)\tan(270^o-\theta)\cot(360^o-\theta)}{\sin(360^o-\theta)\cos(360^o+\theta)\operatorname{cosec}(-\theta)\sin(270^o+\theta)} = 1$$

Q.24 If θ. and φ lie in the first Quadrant such that sin θ. = $\frac{8}{17}$ and cos φ = $\frac{12}{13}$, find the values of

(i) sin(θ–φ) (ii) cos(θ+φ) (iii) tan(θ–φ)

Q.25 If x and y are acute angles such that sin x = $\frac{1}{\sqrt{5}}$ and sin y = $\frac{1}{\sqrt{10}}$, prove that (x + y)= $\frac{\pi}{4}$

Q.26 If x and y are acute such that cos x = $\frac{13}{14}$ and cos y = $\frac{1}{7}$, prove that (x – y) = $-\frac{\pi}{3}$.

Q.27 If sinx=$\frac{12}{13}$ and siny =$\frac{4}{5}$, where $\frac{\pi}{2}$<x<π and o<y<$\frac{\pi}{2}$, find the values of

(i) sin(x+y) (ii) cos(x+y) (iii) tan(x–y)

Q.28 If cosx=$\frac{3}{5}$ and cosy=$\frac{-24}{25}$, where $\frac{3\pi}{2}$<x<2π and π<y<$\frac{3\pi}{2}$, find the values of

(i) sin(x+y) (ii) cos(x–y) (iii) tan(x+y)

Q.29 Prove that

(i) $\cos\left(\frac{\pi}{3}+x\right) = \frac{1}{2}$(cos x - $\sqrt{3}$ sin x)

(ii) $\sin\left(\frac{\pi}{4}+x\right)$ + $\sin\left(\frac{\pi}{4}-x\right)$ = $\sqrt{2}$ cos x

(iii) $\frac{1}{\sqrt{2}}$ cos $\left(\frac{2\pi}{3}+x\right)$ + cos$\left(\frac{2\pi}{3}-x\right)$ = 0

Q.30 Prove that

(i) 2sin$\frac{5\pi}{12}$ sin$\frac{\pi}{12} = \frac{1}{2}$ (ii) 2cos $\frac{5\pi}{12}$ cos$\frac{\pi}{12} = \frac{1}{2}$

Prove that

Q.31 sin(150°+x)–sin(150°–x) =cosx

Q.32 cosx+cos(120°–x)+cos(120°+x) =0

Q.33 sin$\left(x-\frac{\pi}{6}\right)$+cos$\left(x-\frac{\pi}{3}\right)$=$\sqrt{3}$ sin x

Q.34 $\tan\left(\frac{\pi}{4}+x\right) = \frac{1+\tan x}{1-\tan x}$

Q.35 $\tan\left(\frac{\pi}{4}-x\right) = \frac{1-\tan x}{1+\tan x}$

Q.36 Express each of the following as a product

(i) sin10x+sin6x (ii) sin7x – sin3x

(iii) cos 7x + cos 5x (iv) cos 2x – cos 4x

Q.37 Express each of the following as an algebraic sum of sines or cosines :

(i) 2sin6x cos4x (ii) 2cos5x sin3x

(iii) 2cos7x cos3x (iv) 2sin8x sin2x

Prove that

Q.38 $\frac{\sin x + \sin 3x}{\cos x - \cos 3x} = \cot x$

Q.39 $\frac{\sin 7x - \sin 5x}{\cos 7x + \cos 5x} = \tan x$

Q.40 $\frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x} = \tan 4x$

Q.41 $\frac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x} = \frac{-\sin 2x}{\cos 10x}$

Q.42 $\dfrac{\sin x+\sin 3x+\sin 5x}{\cos x+\cos 3x+\cos 5x}=\tan 3x$

Q.43 $\dfrac{(\sin 7x+\sin 5x)+(\sin 9x+\sin 3x)}{(\cos 7x+\cos 5x)+(\cos 9x+\cos 3x)}=\tan 6x$

Q.44 cot 4x(sin 5x + sin 3x) = cot x(sin 5x –sin3x)

Q.45 (sin3x + sinx)sinx +(cos3x – cosx) cos x = 0

Q.46 $(\cos x - \cos y)^2 + (\sin x - \sin y)^2 = 4\sin^2\left(\dfrac{x-y}{2}\right)$

Q.47 $\dfrac{\sin 2x-\sin 2y}{\cos 2y-\cos 2x}=\cot(x+y)$

Q.48 $\dfrac{\cos x+\cos y}{\cos y-\cos x}=\cot\left(\dfrac{x+y}{2}\right)\cot\left(\dfrac{x-y}{2}\right)$

Q.49 $\dfrac{\sin x+\sin y}{\sin x-\sin y}=\tan\left(\dfrac{x+y}{2}\right)\cot\left(\dfrac{x-y}{2}\right)$

Q.50 $\sin 3x + \sin 2x - \sin x = 4\sin x \cos\dfrac{x}{2}\cos\dfrac{3x}{2}$

Q.51 $\dfrac{\cos 4x\sin 3x-\cos 2x\sin x}{\sin 4x\sin x+\cos 6x\cos x}=\tan 2x$

Q.52 $\dfrac{\cos 2x\sin x+\cos 6x\sin 3x}{\sin 2x\sin x+\sin 6x\sin 3x}=\cot 5x$

Q.53 $\sin 10^\circ \sin 30^\circ \sin 50^\circ \sin 70^\circ = \dfrac{1}{16}$

Q.54 $\sin 20^\circ \sin 40^\circ \sin 60^\circ \sin 80^\circ = \dfrac{3}{16}$

Q.55 $\cos 10^\circ \cos 30^\circ \cos 50^\circ \cos 70^\circ = \dfrac{3}{16}$

Q.56 If $\cos x+\cos y = \dfrac{1}{3}$ and $\sin x+\sin y = \dfrac{1}{4}$

provethat $\tan\left(\dfrac{x+y}{2}\right)=\dfrac{3}{4}$.

Q.57 Prove that $2\cos 45^\circ \cos 15^\circ = \dfrac{(\sqrt{3}+1)}{2}$

Q.58 If $\sin x = \dfrac{\sqrt{5}}{3}$ and $0 < x < \dfrac{\pi}{2}$, find the values of

(i) sin2x (ii) cos2x (iii) tan2x

Q.59 If $\cos x = \dfrac{-3}{5}$ and $\pi<x<\dfrac{3\pi}{2}$, find the values of

(i) sin2x (ii) cos2x (iii) tan2x

Q.60 If $\tan x = \dfrac{-5}{12}$ and $\dfrac{\pi}{2}<x<\pi$, find the values of

(i) sin2x (ii) cos2x (iii) tan2x

Q.61 (i) If $\sin x = \dfrac{1}{6}$, find the value of sin 3x.

(ii) If $\cos x = \dfrac{-1}{2}$, find the value of cos 3x.

Prove that (Q.62 to Q.77)

Q.62 $\dfrac{\cos 2x}{\cos x-\sin x}=\cos x+\sin x$

Q.63 $\dfrac{\sin 2x}{1+\cos 2x}=\tan x$

Q.64 $\dfrac{\sin 2x}{1-\cos 2x}=\cot x$

Q.65 $\dfrac{\tan 2x}{1+\sec 2x}=\tan x$

Q.66 sin2x(tanx + cotx) = 2

Q.67 cosec 2x + cot 2x = cot x

Q.68 $\cos 2x + 2\sin^2 x = 1$

Q.69 $(\sin x - \cos x)^2 = 1 - \sin 2x$

Q.70 cot x – 2cot2x = tanx

Q.71 $(\cos^4 x + \sin^4 x) = \dfrac{1}{2}(2 - \sin^2 2x)$

Q.72 $\dfrac{\cos^3 x-\sin^3 x}{\cos x-\sin x}=\dfrac{1}{2}(2+\sin 2x)$

Q.73 $\dfrac{1-\cos 2x+\sin x}{\sin 2x+\cos x}=\tan x$

Q.74 $\cos x \cos 2x \cos 4x \cos 8x = \dfrac{\sin 16x}{16\sin x}$

Q.75 Prove that $2\sin 22\dfrac{1^o}{2}\cos 22\dfrac{1^o}{2}=\dfrac{1}{\sqrt{2}}$

Q.76 Prove that $\sin^2 24^\circ - \sin^2 6^\circ = \dfrac{(\sqrt{5}-1)}{8}$

Q.77 Prove that $\tan 6^\circ \tan 42^\circ \tan 66^\circ \tan 78^\circ = 1$.

Q.78 If $\tan\theta = \dfrac{a}{b}$ prove that $a\sin 2\theta + b\cos 2\theta = b$

Q.79 If $\sin x = \frac{\sqrt{5}}{3}$ and $\frac{\pi}{2} < x < \pi$, find the values of

(i) $\sin\frac{x}{2}$ (ii) $\cos\frac{x}{2}$ (iii) $\tan\frac{x}{2}$

Q.80 If $\cos x = \frac{-3}{5}$ and $\frac{\pi}{2} < x < \pi$, find the values of

(i) $\sin\frac{x}{2}$ (ii) $\cos\frac{x}{2}$ (iii) $\tan\frac{x}{2}$

Q.81 If $\sin x = \frac{-1}{2}$ and x lies in Quadrant IV, find the values of

(i) $\sin\frac{x}{2}$ (ii) $\cos\frac{x}{2}$ (iii) $\tan\frac{x}{2}$

Q.82 If $\cos\frac{x}{2} = \frac{12}{13}$ and x lies in Quadrant I, find the values of

(i) sin x (ii) cos x (iii) cot x

Q.83 If $\sin x = \frac{3}{5}$ and $0 < x < \frac{\pi}{2}$, find the value of $\tan\frac{x}{2}$.

Prove that (Q.84 to Q.88)

Q.84 $\cot\frac{x}{2} - \tan\frac{x}{2} = 2\cot x$

Q.85 $\tan\left(\frac{\pi}{4} + \frac{x}{2}\right) = \tan x + \sec x$

Q.86 $\sqrt{\frac{1+\sin x}{1-\sin x}} = \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$

Q.87 $\tan\left(\frac{\pi}{4} + \frac{x}{2}\right) + \tan\left(\frac{\pi}{4} - \frac{x}{2}\right) = 2\sec x$

Q.88 $\frac{\sin x}{1+\cos x} = \tan\frac{x}{2}$

IN ANY ΔABC, PROVE THAT (Q.89 TO Q.107)

Q.89 $a\sin A - b\sin B = c\sin(A - B)$

Q.90 $a^2\sin(B - C) = (b^2 - c^2)\sin A$

Q.91 $\frac{\sin(A-B)}{\sin(A+B)} = \frac{(a^2-b^2)}{c^2}$

Q.92 $\frac{c - b\cos A}{b - c\cos A} = \frac{\cos B}{\cos C}$

Q.93 $\frac{(a-b)}{(a+b)} = \frac{\tan\frac{1}{2}(A-B)}{\tan\frac{1}{2}(A+B)}$

Q.94 $a\cos B - b\cos A = (a^2 - b^2)$

Q.95 $\frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos C}{c} = \frac{(a^2+b^2+c^2)}{2abc}$

Q.96 $2(bc\cos A + ca\cos B + ab\cos C) = (a^2 + b^2 + c^2)$

Q.97 $\frac{a^2\sin(B-C)}{\sin A} + \frac{b^2\sin(C-A)}{\sin B} + \frac{c^2\sin(A-B)}{\sin C} = 0$

Q.98 $\frac{\cos 2A}{a^2} - \frac{\cos 2B}{b^2} = \left(\frac{1}{a^2} - \frac{1}{b^2}\right)$

Q.99 $\frac{\cos A}{(c\cos B + b\cos C)} + \frac{\cos B}{(a\cos C + c\cos A)} + \frac{\cos C}{(a\cos B + b\cos A)} = \frac{(a^2+b^2+c^2)}{2abc}$

Q.100 $2\left(b\cos^2\frac{C}{2} + c\cos^2\frac{B}{2}\right) = (a + b + c)$

Q.101 $4\left(bc\cos^2\frac{A}{2} + ca\cos^2\frac{B}{2} + ab\cos^2\frac{C}{2}\right) = (a+b+c)^2$

Q.102 $(b+c)\cos A + (c+a)\cos B + (a+b)\cos C = (a+b+c)$

Q.103 $\frac{a\sin(B-C)}{(b^2-c^2)} = \frac{b\sin(C-A)}{(c^2-a^2)} = \frac{c\sin(A-B)}{(a^2-b^2)}$

Q.104 $\frac{a^2\sin(B-C)}{\sin B + \sin C} + \frac{b^2\sin(C-A)}{\sin C + \sin A} + \frac{c^2\sin(A-B)}{\sin A + \sin B} = 0$

Q.105 $(c^2-a^2+b^2)\tan A = (a^2-b^2+c^2)\tan B = (b^2-c^2+a^2)\tan C$

Q.106 $\frac{\cos^2 B - \cos^2 C}{b+c} + \frac{\cos^2 C - \cos^2 A}{c+a} + \frac{\cos^2 A - \cos^2 B}{a+b} = 0$

Q.107 $a^2(\cos^2 B - \cos^2 C) + b^2(\cos^2 C - \cos^2 A) + c^2(\cos^2 A - \cos^2 B) = 0$

Q.108 If in a ΔABC, ∠C = 90° then prove that

$$\sin(A - B) = \frac{(a^2-b^2)}{(a^2+b^2)}$$

Q.109 If $\frac{\sin A}{\sin C} = \frac{\sin(A-B)}{\sin(B-C)}$, prove that a^2, b^2, c^2 are in AP.

Q.110 In a ΔABC, if $\sin^2 A + \sin^2 B = \sin^2 C$, show that the triangle is right angled.

Q.111 In a ΔABC, if $\frac{\cos A}{a} = \frac{\cos B}{b}$, show that the triangle is isosceles.

Q.112 In a ΔABC, if $\cos A = \sin B - \cos C$, show that it is a right-angled triangle.

Q.113 Solve the triangle in which c = 3.4 cm, $\angle A = 25^\circ$ and $\angle B = 85^\circ$.

Q.114 Solve the triangle in which a = 2 cm, b = 1 cm and c = $\sqrt{3}$ cm.

Q.115 Solve the triangle in which a = 72 cm, $\angle B = 108^\circ$ and $\angle A = 25^\circ$.

Q.116 Solve the triangle in which a = $(\sqrt{3} + 1)$, b = $(\sqrt{3} - 1)$ and $\angle C = 60^\circ$.

Q.117 In a ΔABC, if a = 13 cm, b = 14 cm and c = 15 cm, find the values of

(i) cos A (ii) $\sin\frac{A}{2}$ (iii) $\cos\frac{A}{2}$ (iv) $\tan\frac{A}{2}$

(v) ar(ΔABC) (vi) sin A (vi) tan A

Q.118 In a ΔABC, if a = 3 cm, b = 5 cm and c = 6 cm, find the values of

(i) cos B (ii) $\sin\frac{B}{2}$ (iii) $\cos\frac{B}{2}$

(iv) $\tan\frac{B}{2}$ (v) ar(ΔABC) (vi) sin B (vii) tan B

Q.119 In a ΔABC, if a = 25 cm, b = 52 cm and c = 63 cm, find the values of

(i) cos C (ii) $\sin\frac{C}{2}$ (iii) $\cos\frac{C}{2}$ (iv) $\tan\frac{C}{2}$

(v) ar(ΔABC) (vi) sin C (vii) tan C

In a ΔABC, prove that (Q.120 to Q.133)

Q.120 $\Delta = s(s-a)\tan\frac{A}{2} = s(s-b)\tan\frac{B}{2} = s(s-c)\tan\frac{C}{2}$

Q.121 $\Delta\left(\cot\frac{A}{2} + \cot\frac{B}{2} + \cot\frac{C}{2}\right) = s^2$

Q.122 $\Delta^2 = (abcs)\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

Q.123 $(a+b+c)\sin\frac{A}{2} = 2a\cos\frac{B}{2}\cos\frac{C}{2}$

Q.124 $(a+b+c)\left(\tan\frac{A}{2} + \tan\frac{B}{2}\right) = 2c\cot\frac{C}{2}$

Q.125 $4\Delta\cot A = b^2 + c^2 - a^2$

Q.126 (i) $\left(1 - \tan\frac{B}{2}\tan\frac{C}{2}\right) = \frac{2a}{(a+b+c)}$

(ii) $\frac{\left(\tan\frac{C}{2} - \tan\frac{A}{2}\right)}{\left(\tan\frac{C}{2} + \tan\frac{A}{2}\right)} = \frac{(c-a)}{b}$

Q.127 $(b+c-a)\left(\cot\frac{B}{2} + \cot\frac{C}{2}\right) = 2a\cot\frac{A}{2}$

Q.128 $(a+b-c)\cot\frac{B}{2} = (a-b+c)\cot\frac{C}{2}$

Q.129 $(b-c)\cot\frac{A}{2} + (c-a)\cot\frac{B}{2} + (a-b)\cot\frac{C}{2} = 0$

Q.130 $\left(\tan\frac{A}{2}\right)\left(\tan\frac{B}{2}\right) = \frac{(a+b-c)}{(a+b+c)}$

Q.131 $b\cos^2\frac{C}{2} + c\cos^2\frac{B}{2} = s$

Q.132 $a\left(\cos^2\frac{C}{2} - \cos^2\frac{B}{2}\right) = (b-c)\cos^2\frac{A}{2}$

Q.133 $\frac{1}{a}\cos^2\frac{A}{2} + \frac{1}{b}\cos^2\frac{B}{2} + \frac{1}{c}\cos^2\frac{C}{2} = \frac{s^2}{abc}$

Q.134 If $3\tan\frac{A}{2}\tan\frac{C}{2} = 1$, show that a, b, c are in AP.

Q.135 In a ΔABC, if a, b, c, are in AP, show that

$2\sin\frac{A}{2}\sin\frac{C}{2}=\sin\frac{B}{2}$.

Q.136 In a ΔABC, if a, b, c, are in AP, show that

$\left(\cot\frac{A}{2}\right)\left(\cot\frac{C}{2}\right)$ = 3.

Q.137 Find the principal solution of each of the following equations :

(i) sinx= $\frac{\sqrt{3}}{2}$ (ii) cosx= $\frac{1}{2}$ (iii) tanx = $\sqrt{3}$

(iv) cotx = $\sqrt{3}$ (v) cosecx=2 (vi) secx= $\frac{2}{\sqrt{3}}$

Q.138 Find the principal solution of each of the following equations :

(i) sinx= $\frac{-1}{2}$ (ii) $\sqrt{2}\cos x+1=0$ (iii) tan x = –1

(iv) $\sqrt{3}$ cosecx+2=0 (v) tanx= $-\sqrt{3}$ (vi) $\sqrt{3}$ secx+ 2=0

Find the general solution of each of the following equations : (Q.139 to Q.151)

Q.139 (i) sin3x=0 (ii) sin $\frac{3x}{2}$ =0 (iii) sin $\left(x+\frac{\pi}{5}\right)$ = 0

(iv) cos2x= 0 (v) cos $\frac{5x}{2}$ =0 (vi) cos $\left(x+\frac{\pi}{10}\right)$ = 0

(vii) tan2x=0 (viii) tan $\left(3x+\frac{\pi}{6}\right)$ =0 (ix) tan $\left(2x-\frac{\pi}{4}\right)$ =0

Q.140 (i) sinx= $\frac{\sqrt{3}}{2}$ (ii) cosx=1

Q.141 (i) cosx= $\frac{-1}{2}$ (ii) cosecx= $-\sqrt{2}$ (iii) tanx=–1

Q.142 (i) sin2x= $\frac{1}{2}$ (ii) cos3x= $\frac{1}{\sqrt{2}}$

Q.143 (i) sec3x=–2 (ii) cot4x=–1 (iii) cosec3x = $\frac{-2}{\sqrt{3}}$

Q.144 (i) $4\cos^2x=1$ (ii) $4\sin^2x-3=0$ (iii) $\tan^2x=1$

Q.145 (i) cos3x=cos2x (ii) cos5x=sin3x (iii) cosmx= sinnx

Q.146 sin x = tan x

Q.147 4 sin x cos x + 2 sin x + 2cos x + 1 = 0

Q.148 $\sec^2x$ = 1 – tan 2x

Q.149 $\tan^3x$ – 3tan x = 0

Q.150 sin x + sin 3x + sin 5x = 0

Q.151 sin x tan x – 1 = tan x – sin x

ANSWER

1. $\sin\theta=-\frac{1}{2}$, $\tan\theta=\frac{1}{\sqrt{3}}$, $\cot\theta=\sqrt{3}$, cosec θ = –2, secθ= $\frac{-2}{\sqrt{3}}$ **2.** $\cos\theta=\frac{\sqrt{3}}{2}$, tanθ = $\frac{-1}{\sqrt{3}}$, cotθ = $-\sqrt{3}$, sec θ = $\frac{2}{\sqrt{3}}$, cosec θ = –2 **3.** $\sin\theta=\frac{3}{5}$, $\cos\theta=-\frac{4}{5}$, $\tan\theta=\frac{-3}{4}$, $\cot\theta=\frac{-4}{3}$, $\sec\theta=\frac{-5}{4}$

4. $\sin\theta=\frac{-1}{\sqrt{2}}$, $\cos\theta=\frac{1}{\sqrt{2}}$, $\tan\theta=-1$ cot θ = –1, cosec θ = $-\sqrt{2}$ **5.** cos x = $-\frac{1}{5}$, cot x = $\frac{1}{2\sqrt{6}}$

6. $\frac{1}{4}$ **7.** sin x = $\frac{-\sqrt{3}}{2}$, cos x = $-\frac{1}{2}$, tan x = $\sqrt{3}$, cot x = $\frac{1}{\sqrt{3}}$, cosec = $\frac{-2}{\sqrt{3}}$

8. (i) $\frac{\sqrt{3}}{2}$ (ii) 0 (iii) $-\sqrt{3}$ (iv) 1 (v) 2 (vi) $-\sqrt{2}$

9. (i) $\frac{1}{\sqrt{2}}$ (ii) $\frac{2}{\sqrt{3}}$ (iii) $\sqrt{3}$ (iv) 1 (v) –2 (vi) $\frac{1}{2}$

11. (i) $-\frac{1}{2}$ (ii) $\frac{1}{2}$ (iii) $\sqrt{3}$ (iv) 2 (v) 2 (vi) 1 (vii) 1 (viii) $-\frac{1}{2}$ (ix) $-\frac{1}{\sqrt{2}}$

12. sin135° = $\frac{1}{\sqrt{2}}$, cos 135° $-\frac{1}{\sqrt{2}}$, tan 135° = –1, cot 135° = –1, sec135° = $-\sqrt{2}$, cosec135° = $\sqrt{2}$

24. (i) $\frac{21}{221}$ (ii) $\frac{140}{221}$ (iii) $\frac{21}{220}$

27. (i) $\frac{16}{65}$ (ii) $\frac{-63}{65}$ (iii) $\frac{56}{33}$ **28.** (i) $\frac{3}{5}$ (ii) $\frac{-44}{125}$ (iii) $\frac{-3}{4}$

36. (i) 2sin 8x cos2x (ii) 2cos 5x sin 2x
(iii) 2 cos6x cosx (iv) 2sin3x sinx

37. (i) sin10x + sin2x (ii) sin8x – sin2x
(iii) cos 10x + cos 4x (iv) cos 6x – cos 10x

58. (i) $\frac{4\sqrt{5}}{9}$ (ii) $\frac{-1}{9}$ (iii) $-4\sqrt{5}$ **59.** (i) $\frac{24}{25}$ (ii) $\frac{-7}{25}$ (iii) $\frac{-24}{7}$

60. (i) $\frac{-120}{169}$ (ii) $\frac{119}{169}$ (iii) $\frac{-120}{119}$ **61.** (i) $\frac{13}{27}$ (ii) 1

79. (i) $\frac{\sqrt{30}}{6}$ (ii) $\frac{\sqrt{6}}{6}$ (iii) $\sqrt{5}$ **80.** (i) $\frac{2\sqrt{5}}{5}$ (ii) $\frac{1}{\sqrt{5}}$ (iii) 2

81. (i) $\sin\frac{x}{2} = -\frac{\sqrt{(2-\sqrt{3})}}{2}$ (ii) $\cos\frac{x}{2} = -\frac{\sqrt{(2+\sqrt{3})}}{2}$

(iii) $\tan\frac{x}{2} = (2-\sqrt{3})$ **82.** (i) $\frac{120}{169}$ (ii) $\frac{119}{169}$ (iii) $\frac{119}{120}$

83. $\frac{1}{3}$ **113.** $\angle C = 70^\circ$, x = 1.53 cm and b = 3.6 cm

114. $\angle A = 90^\circ$, $\angle B = 30^\circ$ and $\angle C = 60^\circ$

115. c = 124.6 cm, b = 162 cm and $\angle C = 47^\circ$

116. $c = \sqrt{6}$ cm, $\angle A = 105^\circ$, $\angle B = 15^\circ$

117. (i) $\frac{3}{5}$ (ii) $\frac{1}{\sqrt{5}}$ (iii) $\frac{2}{\sqrt{5}}$ (iv) $\frac{1}{2}$ (v) 84 (vi) $\frac{4}{5}$ (vii) $\frac{4}{3}$

118. (i) $\frac{5}{9}$ (ii) $\frac{\sqrt{2}}{3}$ (iii) $\frac{\sqrt{7}}{3}$ (iv) $\sqrt{\frac{2}{7}}$ (v) $2\sqrt{14}$ (vi) $\frac{2\sqrt{14}}{9}$

(vii) $\frac{2\sqrt{14}}{5}$ **119.** (i) $-\frac{16}{65}$ (ii) $\frac{9}{\sqrt{130}}$ (iii) $\frac{7}{\sqrt{130}}$

(iv) $\frac{9}{7}$ (v) 630 sq units (vi) $\frac{63}{65}$ (vii) $-\frac{63}{16}$

137. (i) $\frac{\pi}{3}, \frac{2\pi}{3}$ (ii) $\frac{\pi}{3}, \frac{5\pi}{3}$ (iii) $\frac{\pi}{3}, \frac{4\pi}{3}$ (iv) $\frac{\pi}{6}, \frac{7\pi}{6}$

(v) $\frac{\pi}{6}, \frac{5\pi}{6}$ (vi) $\frac{\pi}{6}, \frac{11\pi}{6}$

138. (i) $\frac{7\pi}{6}, \frac{11\pi}{6}$ (ii) $\frac{3\pi}{4}, \frac{5\pi}{4}$ (iii) $\frac{3\pi}{4}, \frac{7\pi}{4}$

(iv) $\frac{4\pi}{3}, \frac{5\pi}{3}$ (v) $\frac{2\pi}{3}, \frac{5\pi}{3}$ (vi) $\frac{5\pi}{6}, \frac{7\pi}{6}$

139. (i) $\frac{n\pi}{3}, n \in I$ (ii) $\frac{2n\pi}{3}, n \in I$ (iii) $\left(n\pi - \frac{\pi}{5}\right), n \in I$

(iv) $(2n+1)\frac{\pi}{4}, n \in I$ (v) $(2n+1)\frac{\pi}{5}, n \in I$

(vi) $\left(n\pi + \frac{2\pi}{5}\right), n \in I$ (vii) $\frac{n\pi}{2}, n \in I$

(viii) $\left(\frac{n\pi}{3} - \frac{\pi}{18}\right), n \in I$ (ix) $\left(\frac{n\pi}{2} + \frac{\pi}{8}\right), n \in I$

140. (i) $x = n\pi + (-1)^n \cdot \frac{\pi}{3}, n \in I$ (ii) $x = 2n\pi$, $n \in I$

141. (i) $x = \left(2\pi \pm \frac{3\pi}{4}\right), n \in I$ (ii) $x = n\pi + (-1)^n \cdot \frac{5\pi}{4}, n \in I$

(iii) $x = \left(n\pi + \frac{3\pi}{4}\right), n \in I$

142. (i) $x = \frac{n\pi}{2} + (-1)^n \cdot \frac{\pi}{12}, n \in I$

(ii) $x = \left(\frac{2n\pi}{3} \pm \frac{\pi}{4}\right), n \in I$

143. (i) $x = \left(\frac{2n\pi}{3} \pm \frac{2\pi}{9}\right), n \in I$ (ii) $x = (4n+3)\frac{\pi}{16}, n \in I$

(iii) $x = \frac{n\pi}{3} + (-1)^n \cdot \frac{4\pi}{9}, n \in I$

144. (i) $x = \left(n\pi \pm \frac{\pi}{3}\right), n \in I$ (ii) $x = \left(n\pi \pm \frac{\pi}{3}\right), n \in I$

(iii) $x = \left(n\pi \pm \frac{\pi}{4}\right), n \in I$

145. (i) $x = 2n\pi$ or $x = \frac{2n\pi}{5}$, where $n \in I$

(ii) $x = \left(\frac{n\pi}{4} + \frac{\pi}{16}\right)$ or $x = \left(n\pi - \frac{\pi}{4}\right)$, where $n \in I$

(iii) $x = \frac{(4k+1)\pi}{2(m+n)}$, where $k \in I$

146. $x = n\pi$ or $x = 2m\pi$, where $m, n \in I$

147. $x = \left(2n\pi \pm \frac{2\pi}{3}\right)$ or $x = m\pi + (-1)^m \cdot \frac{7\pi}{6}$, where $m, n \in I$

148. $x = \frac{n\pi}{2}$ or $x = \left(\frac{m\pi}{2} + \frac{3\pi}{8}\right)$, where $m, n \in I$

149. $x = n\pi$ or $x = \left(m\pi + \frac{\pi}{3}\right)$ or $x = \left(p\pi + \frac{2\pi}{3}\right)$, where $m, n \in I$

150. $x = \frac{n\pi}{3}$ or $x = \left(m\pi \pm \frac{\pi}{3}\right)$, where $m, n \in I$

151. $x = n\pi + (-1)^n \cdot \frac{\pi}{2}$ or $x = \left(m\pi + \frac{3\pi}{4}\right)$.

Where $m, n \in I$