KINEMATICS

1. REST AND MOTION :

* An object is said to be in motion wrt a frame of reference S_1, when its location is changing with time in same frame of reference S_1.
* Rest and motion are relative terms.
*Absolute rest and absolute motion have no meaning.

Motion is broadly classified into 3 categories.

1. Rectilinear and translatory motion.
2. Circular and rotatory motion.
3. Oscillatory and vibratory motion.

1.1 *Rectilinear or 1-D Motion*

When a particle is moving along a straight line, then its motion is a rectilinear motion.

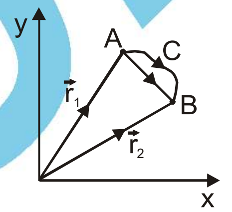
Parameters of rectilinear motion or translatory motion or plane motion :

(A) ***Time :***

* It is a scalar quantity and its SI unit is second(s).
* At a particular instant of time, a physical object can be present at one location only.
* Time can never decrease.

(B) ***Position or location -*** It is defined with respect to some reference point (origin) of given frame of reference.

Consider a particle which moves from location $\vec{r}_1$ (at time t_1) to location $\vec{r}_2$ (at time t_2) as shown in the figure below, following path ACB.



(C) ***Distance :***

The length of the actual path traversed by the particle is termed as its distance.
Distance = length of path ACB.

* Its SI unit is metre and it is a scalar quantity.
* It can never decrease with time.

(D) ***Displacement :***

The change in position vector of the particle for a given time interval is known as its displacement.

$$\overrightarrow{AB} = \vec{r} = \vec{r}_2 - \vec{r}_1$$

* Displacement is a vector quantity and its SI unit is metre.
* It can decrease with time.

For a moving particle in a given interval of time

* Displacement can be +ve, −ve or 0, but distance would be always +ve.
* Distance ≥ Magnitude of displacement.
* Distance is always equal to displacement only and only if particle is moving along a straight line without any change in direction.

(E) ***Average speed and average velocity :***

Average speed and average velocity are always defined for a time interval.

$$\text{Average speed}(v_{av}) = \frac{\text{Total distance travelled}}{\text{Time interval}} = \frac{\Delta s}{\Delta t}$$

$$\text{Average velocity}(\vec{v}_{av}) = \frac{\text{Displacement}}{\text{Time interval}} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1}$$

* Average speed is a scalar quantity, while average velocity is a vector quantity. Both have the same SI units, i.e., m/s.

For a moving particle in a given interval of time

* Average speed can be a many valued function but average velocity would be always a single-valued function.
* Average velocity can be positive, negative or 0 but average speed would be always positive.

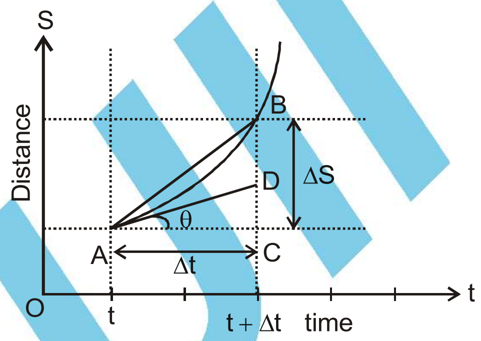
(F) *Instantaneous speed and instantaneous velocity*

Instantaneous speed is also defined exactly like average speed i.e. it is equal to the ratio of total distance and time interval, but with one qualification that time interval is extremely (infinitesimally) small. The instantaneous speed is the speed at a particular instant of time and may have entirly different value than that of average speed. Mathematically.

$$v = \lim_{\Delta s \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt} \qquad ...(4)$$

When Δs is the distance travelled in time Δt.

As Δt tends to zero, the ratio defining speed becomes finite and equals to the first derivative of the distance. The speed at the moment 't' is is called the instantaneous speed at time 't'.



Instantaneous speed is equal to the slope of the tangent at given instant.

On the distance - time plot, the speed is equal to the slope of the tangent to the curve at the time instant 't'. Let A and B point on the plot corresponds to the time t and t + Δt during the motion. As Δt approaches zero, the chord AB becomes the tangent AC at A. The slope of the tangent equal ds/dt, which is equal to the intantaneous speed at 't'.

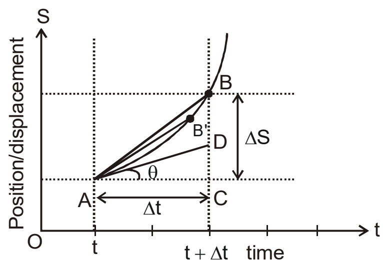
$$v = \tan\theta = \frac{DC}{AC} = \frac{ds}{dt}$$

(G) *Instantaneous velocity :*

Instantaneous velocity is defined exactly like speed. It is equal to the ratio of total displacement and time interval, but with one qualification that time interval is extremely (infinitesimally) small. Thus, instantaneous velocity can be termed as the average velocity at a particular instant of time when Δt tend to zero and may have entirely different value that of average velocity : Mathematically.

$$v = \lim_{\Delta t \to 0} \frac{\Delta r}{\Delta t} = \frac{dr}{dt}$$

As Δt tends to zero, the ratio defining velocity becomes finite and equals to the first derivative of the position vector.
The velocity at the moment 't' is called the instantaneous velocity or simply velocity at time 't'.



Instantaneous velocity is equal to the slope of the tangent at given instant.

The magnitude of average velocity $|v_{avg}|$ and average speed v_{avg} may not be equal, but magnitude of instantaneous velocity $|v|$ is always equal to instantaneous speed v.

$$\frac{dx}{x^2} = dt \quad \Rightarrow \int_1^x \frac{dx}{x^2} = \int_0^t dt \quad \Rightarrow \left[-\frac{1}{x}\right]_1^x = t \quad \Rightarrow -\frac{1}{x} + 1 = t \Rightarrow x = \frac{1}{1-t}$$

Average and instantaneous acceleration.

When the velocity of a moving object/particle changes with time, we can say that it is accelerated.
Average acceleration,

$(\vec{a}_{av}) = \dfrac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \dfrac{\Delta\vec{v}}{\Delta t} = \dfrac{\text{Change in velocity}}{\text{Time interval}}$

Instantaneous acceleration,

$(\vec{a}) = \lim_{\Delta t \to 0} \vec{a}_{av} = \dfrac{d\vec{v}}{dt}$ = Rate of change of velocity

Acceleration is a vector quantity whose direction is same as that of change in velocity vector. Its SI unit is m/s^2.

* When direction of acceleration and velocity are opposite to each other, then acceleration is termed as retardation.

* $\vec{a} = \dfrac{d\vec{v}}{dt} = \dfrac{d^2\vec{r}}{dt^2} = \vec{v}\dfrac{d\vec{v}}{d\vec{r}}$

Deduce the following equations for unifromly accelerated motion by using intergration technique.

(A) $v = u + at$ (B) $s = ut + \frac{1}{2}at^2$ (C) $v^2 - u^2 = 2as$ (D) $s_{nth} = u + \frac{a}{2}(2n - 1)$

First equation of motion. Acceleration is defined as

$$a = \frac{dv}{dt}$$

or $dv = adt$ ...(1)

When time = 0, velocity = u (say)

When time = t, velocity = v (say)

Integrating equation (1) within the above limits of time and velocity, we get

$$\int_u^v dv = \int_0^t a\,dt \quad \text{or} \quad [v]_u^v = a\int_0^t dt = a[t]_0^t$$

or $v - u = a(t - 0)$

or $v = u + at$ ...(2)

Second equation of motion. Velocity is defined as

$$v = \frac{ds}{dt}$$

or $ds = v\,dt = (u + at)\,dt$ ...(iii)

When time = 0, displacement travelled = 0

When time = t, displacement travelled = s (say).

Integrating equation (3) within the above limits of time and distance, we get

$$\int_0^s ds = \int_0^t (u + at)\,dt = u\int_0^t dt + a\int_0^t t\,dt \quad \text{or} \quad [s]_0^s = u[t]_0^t + a\left[\frac{t^2}{2}\right]_0^t$$

or $s - 0 = u(t - 0) + a\left[\dfrac{t^2}{2} - 0\right]$

or $s = ut + \dfrac{1}{2}at^2$ ...(4)

Third equation of motion. By the definitions of acceleration and velocity,

$$a = \frac{dv}{dt} = \frac{dv}{ds} \times \frac{ds}{dt} = \frac{dv}{ds} \times v$$

or $ads = vdv$ ...(5)

When time = 0, velocity = u, displacement travelled = 0

When time = t, velocity = v, displacement travelled = s (say)

Integrating equation (5) within the above limits of velocity and displacement, we get

$$\int_0^s a\,ds = \int_u^v v\,dv \quad \text{or} \quad a\int_0^s ds = \int_u^v v\,dv \quad \text{or} \quad a[s]_0^s = \left[\frac{v^2}{2}\right]_u^v$$

or $a[s-0] = \frac{v^2}{2} - \frac{u^2}{2}$ or $2as = v^2 - u^2$

or $v^2 - u^2 = 2as$ ...(6)

Fourth equation of motion. By definition of velocity,

$$v = \frac{ds}{dt}$$

or $ds = vdt = (u + at)\,dt$ ...(7)

When time = (n – 1) second, displacement travelled

= s_{n-1} (say).

When time = n second, displacement travelled = s_n (say)

Integrating equation (7) within the above limits of time and distance, we get

$$\int_{s_{n-1}}^{s_n} ds = \int_{n-1}^{n} (u+at)dt \quad \text{or} \quad [s]_{s_{n-1}}^{s_n} = u\int_{n-1}^{n} dt + a\int_{n-1}^{n} t\,dt \quad \text{or} \quad s_n - s_{n-1} = u[t]_{n-1}^{n} + a\left[\frac{t^2}{2}\right]_{n-1}^{n}$$

$$= u[n-(n-1)] + \frac{a}{2}[n^2 - (n-1)^2] \qquad = u + \frac{a}{2}[n^2 - (n^2 - 2n + 1)]$$

$$s_{nth} = u + \frac{a}{2}(2n-1) \qquad ...(8)$$

where $s_{nth} = s_n - s_{n-1}$ = displacement in n^{th} second.

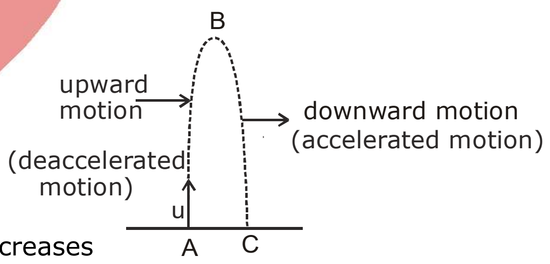
2. MOTION UNDER GRAVITY :

I FORMAT : (When a body is thrown vertically upward)

It includes two types of motion

(i) Deaccelerated motion from A to B because the direction of velocity and acceleration is opposite. So speed decreases

(ii) Accelerated motion from B to C because the direction of velocity and acceleration is same (downward). So speed increases



(a) Time of flight :

It is the time taken by the particle to reach the ground. If the particle is thrown vertically upward with initial velocity u then

$u_i = u$

$a = -g$ (take downward direction negative)

from equation

$S = ut + \frac{1}{2}at^2 \Rightarrow S_{net} = 0$ (when particle again reaches the ground)

$t = T$ (time of flight)

$0 = uT - \frac{1}{2}gT^2 \Rightarrow T = \frac{2u}{g}$

(b) Maximum Height :

from $v^2 = u^2 + 2as$

at maximum height v = 0, $s = H_{max}$

$\Rightarrow 0 = u^2 - 2gH_{max} \Rightarrow H_{max} = \frac{u^2}{2g}$

(c) Final velocity

from v = u + at

$v = v_f \quad a = -g \quad t = T = \frac{2u}{g} \Rightarrow \quad v_f = u - g\left(\frac{2u}{g}\right)$

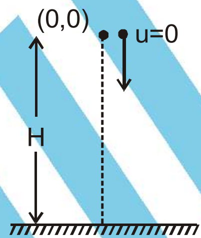
$v_f = -u$

i.e. the body reaches the ground with the same speed with which it was thrown vertically upwards as it thrown vertically upward.

II Format (Free fall) :

A body released near the surface of the earth is accelerated downward under the influence of force of gravity.

(a) Time of Flight :



from equation $S = ut + \frac{1}{2}at^2$

$S = -H, u = 0, a = -g$

t = T (Let assume)

$\Rightarrow \quad -H = (0)T - \frac{1}{2}gt^2 \Rightarrow T = \sqrt{\frac{2H}{g}}$

(b) Final Velocity when body reaches the ground

from $v^2 - u^2 = 2as$

$s = -H \quad v = v_f u = 0 \; a = -g$

$\Rightarrow \quad v_f^2 - 0 = 2(-g)(-H) \Rightarrow v_f = \sqrt{2gH}$

TWO DIMENSIONAL MOTION OR MOTION IN A PLANE

VECTOR

1. SCALAR :

In physics we deal with two type of physical quantity one is scalar and other is vector. Each scalar quantity has a magnitude and a unit.

For example mass = 4kg

Magnitude of mass = 4

and unit of mass = kg

Example of scalar quantities : mass, speed, distance etc.

Scalar quantities can be added, subtracted and multiplied by simple laws of algebra.

2. VECTOR :

Vector are the physical quantites having magnitude as well as specified direction.

For example :

Speed = 4 m/s (is a scalar)

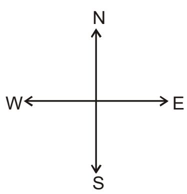
Velocity = 4 m/s toward north (is a vector)

If someone wants to reach some location then it is not sufficient to provide information about the distance of that location it is also essential to tell him about the proper direction from the initial location to the destination.

The magnitude of a vector ($\vec{A}$) is the absolute value of a vector and is indicated by $|\vec{A}|$ or A.

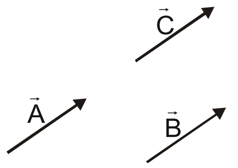
Example of vector quantity : Displacement, velocity, acceleration, force etc.

Knowledge of direction



Equality of Vectors.

Vectors having equal magnitude and same direction are called equal vectors



if $|\vec{A}| = |\vec{B}| = |\vec{C}|$

and $\hat{A} = \hat{B} = \hat{C}$

then $\vec{A} = \vec{B} = \vec{C}$

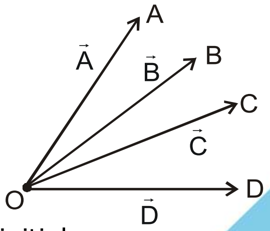
Collinear vectors :

Any two vectors are co-linear then one can be express in the term of other.

$\vec{a} = \lambda\vec{b}$ (where λ is a constant)

Co-initial vector : If two or more vector start from same point then they called co-initial vector.

e.g.



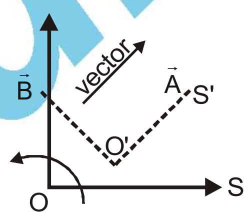
here A, B, C, D are co-initial.

Coplanar vectors :

Three (or more) vectors are called coplanar vectors if they lie in the same plane or are parallel to the same plane. Two (free) vectors are always coplanar.

Important points

☛ If the frame of reference is translated or rotated the vector does not change (though its components may change).



Two vectors are called equal if their magnitudes and directions are same, and they represent values of same physical quantity.

Multiplication and division of a vector by a scalar

Multiplying a vector $\vec{A}$ with a positive number λ gives a vector $(\vec{B} = \lambda\,\vec{A})$ whose magnitude become λ times but the direction is the same as that of $\vec{A}$. Multiplying a vector $\vec{A}$ by a negative number λ gives a vector $\vec{B}$ whose direction is opposite to the direction of $\vec{A}$ and whose magnitude is $-\lambda$ times $|\vec{A}|$.

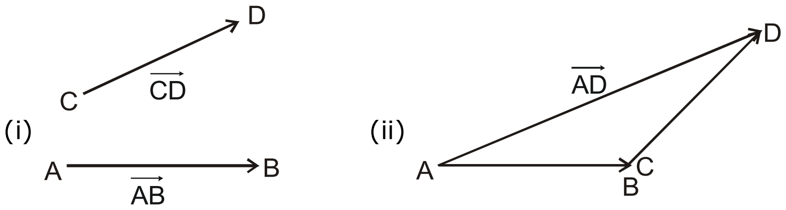
The division of vector $\vec{A}$ by a non-zero scalar m is defined as multiplication of $\vec{A}$ by $\frac{1}{m}$.

At here $\vec{A}$ and $\vec{B}$ are co-linear vector

3. LAWS OF ADDITION AND SUBTRACTION OF VECTORS :

3.1 Triangle rule of addition : Steps for additing two vector representing same physical quantity by triangle law.

(i) Keep vectors s.t. tail of one vector coincides with head of other.
(ii) Join tail of first to head of the other by a line with arrow at head of the second.
(iii) This new vector is the sum of two vectors. (also called reultant)



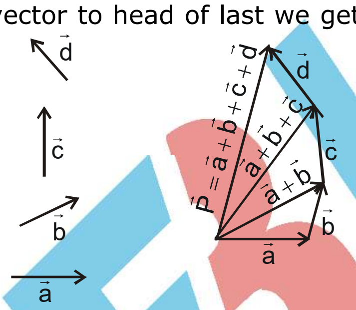
(iii) $\vec{AB} + \vec{CD} = \vec{AD}$

Take example here.

Q. A boy moves 4 m south and then 5 m in direction 37° E of N. Find resultant displacement.

3.2 Polygon Law of addition :

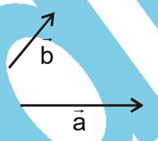
This law is used for adding more than two vectors. This is extension of triangle law of addition. We keep on arranging vectors s.t. tail of next vector lies on head of former.
When we connect the tail of first vector to head of last we get resultant of all the vectors.

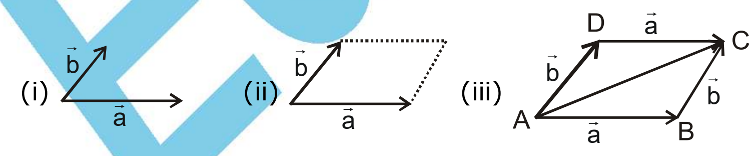


Note : $\vec{P} = ((\vec{a} + \vec{b}) + \vec{c}) + \vec{d} = ((\vec{c} + \vec{a} + \vec{d})) + \vec{d}$ [Associative Law]

3.3 Parallelogram law of addition :

Steps :
(i) Keep two vectors such that there tails coincide.
(ii) Draw parallel vectors to both of them considering both of them as sides of a parallelogram.
(iii) Then the diagonal drawn from the point where tails coincide represents the sum of two vectors, with its tail at point of coincidence of the two vectors.

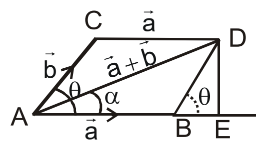




$\vec{AC} = \vec{a} + \vec{b}$

Note : $\vec{AC} = \vec{a} + \vec{b}$ and $\vec{AC} = \vec{b} + \vec{a}$ thus $\vec{a} + \vec{b} = \vec{b} + \vec{a}$ [Cummutative Law]
Note : Angle between 2 vectors is the angle between their positive directions.

Suppose angle between these two vectors is θ, and $|\vec{a}| = a, |\vec{b}| = b$



$$
\begin{aligned}
(AD)^2 &= (AE)^2 + (DE)^2 \\
&= (AB + BE)^2 + (DE)^2 \\
&= (a + b\cos\theta)^2 + (b\sin\theta)^2 \\
&= a^2 + b^2\cos^2\theta + 2ab\cos\theta + b^2\sin^2\theta \\
&= a^2 + b^2 + 2ab\cos\theta
\end{aligned}
$$

Thus, $AD = \sqrt{a^2 + b^2 + 2ab\cos\theta}$

or $\quad |\vec{a}+\vec{b}| = \sqrt{a^2+b^2+2ab\cos\theta}$

angle α with vector a is $\tan\alpha = \frac{DE}{AE} = \frac{b\sin\theta}{(a+b\cos\theta)}$

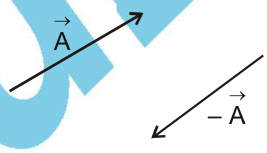
Important points :

- To a vector, only a vector of same type can be added that represents the same physical quantity and the resultant
is also a vector of the same type.
- As $R = [A^2 + B^2 + 2AB\cos\theta]^{1/2}$ so R will be maximum when, $\cos\theta = max = 1$,
i.e., $\theta = 0^o$, i.e. vectors are like or parallel and $R_{max} = A + B$.
- $|\vec{A}| = |\vec{B}|$ and angle between them θ then $R = 2A\cos\theta/2$
- $|\vec{A}| = |\vec{B}|$ and angle between them $\pi - \theta$ then $R = 2A\sin\theta/2$
- The resultant will be minimum if, $\cos\theta = min = -1$, i.e., $\theta = 180^o$, i.e. vectors are antiparallel and $R_{min} = A - B$.
- If the vectors A and B are orthogonal, i.e., $\theta = 90^o$, $R = \sqrt{A^2+B^2}$
- As previously mentioned that the resultant of two vectors can have any value from (A – B) to (A + B) depending on the angle between them and the magnitude of resultant decreases as θ increases 0^o to 180^o.
- Minimum number of unequal coplanar vectors whose sum can be zero is three.
- The resultant of three non-coplanar vectors can never be zero, or minimum number of non coplanar vectors whose sum can be zero is four.

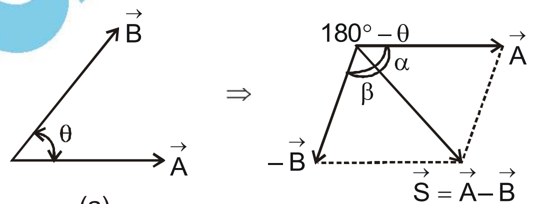
4. SUBTRACTION OF VECTOR :

Negative of a vector say $-\vec{A}$ is a vector of the same magnitude as vector $\vec{A}$ but pointing in a direction opposite to that of $\vec{A}$.

Thus, $\vec{A}-\vec{B}$ can be written as $\vec{A}+(-\vec{B})$ or $\vec{A}-\vec{B}$ is really the vector addition of $\vec{A}$ and $-\vec{B}$.



Suppose angle between two vectors $\vec{A}$ and $\vec{B}$ is θ. Then angle between $\vec{A}$ and $-\vec{B}$ will be $180° - \theta$ as shown in figure.



(a) (b)

Magnitude of $\vec{S} = \vec{A} - \vec{B}$ will be thus given by

$$S = |\vec{A}-\vec{B}| = \sqrt{A^2+B^2+2AB\cos(180°-\theta)}$$

or $\quad S = \sqrt{A^2+B^2-2AB\cos\theta}$ ....(i)

For direction of $\vec{S}$ we will either calculate angle α or β, where,

$$\tan\alpha = \frac{B\sin(180° - \theta)}{A + B\cos(180° - \theta)} = \frac{B\sin\theta}{A - B\cos\theta} \qquad ...(ii)$$

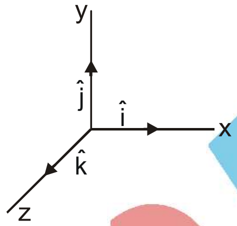
or $$\tan\beta = \frac{A\sin(180° - \theta)}{B + A\cos(180° - \theta)} = \frac{A\sin\theta}{B - A\cos\theta} \qquad ...(iii)$$

5. UNIT VECTOR AND ZERO VECTOR

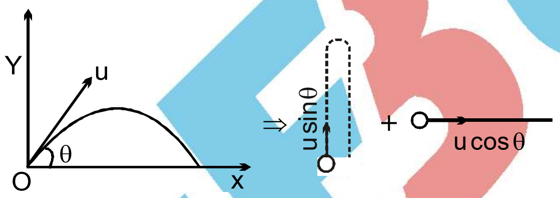
Unit vector is a vector which has a unit magnitude and points in a particular direction. Any vector $(\vec{A})$ can be written as the product of unit vector $(\hat{A})$ in that direction and magnitude of the given vector.

$$\vec{A} = A\,\hat{A} \quad \text{or} \quad \hat{A} = \frac{\vec{A}}{A}$$

A unit vector has no dimensions and unit. Unit vectors along the positive x-, y-and z-axes of a rectangular coordinate system are denoted by $\hat{i}, \hat{j}$ and $\hat{k}$ respectively such that $|\hat{i}| = |\hat{j}| = |\hat{k}| = 1$.



A vector of zero magnitude is called a **zero or a null vector**. Its direction is arbitrary.



Assume that effect of air friction and wind resistance are negligible and value of 'acceleration due to gravity $\vec{g}$ is constant.

Take point of projection as origin and horizontal and vertical direction as +ve X and Y-axes, respectively.

For X-axis	**For Y - axis**
$u_x = u\cos\theta$,	$u_y = u\sin\theta$
$a_x = 0$,	$a_y = -g$,
$v_x = u\cos\theta$, and	$v_y = u\sin\theta - gt$, and
$x = u\cos\theta \times t$	$y = u\sin\theta\, t - \frac{1}{2}gt^2$

It is clear from above equations that horizontal component of velocity of the particle remains constant while vertical component of velocity is first decreasing, gets zero at the highest point of trajectory and then increases in the opposite direction. At the highest point, speed of the particle is minimum.

The time, which projectile takes to come back to same (initial) level is called the time of flight (T).

At initial and final points, y = 0,

So $u\sin\theta\, t - \frac{1}{2}gt^2 = 0$

$\Rightarrow \quad t = 0$ and $t = \frac{2u\sin\theta}{g}$ So, $T = \frac{2u\sin\theta}{g}$

Range (R) The horizontal distance covered by the projectile during its motion is said to be range of the projectile

$$R = u\cos\theta \times T = \frac{u^2\sin 2\theta}{g}$$

For a given projection speed, the range would be maximum for $\theta = 45°$.

Maximum height attained by the projectile is

$$H = \frac{u^2 \sin^2\theta}{2g}$$

at maximum height the vertical component of velocity is 0.

Time of ascent = Time of descent = $\frac{u\sin\theta}{g} = \frac{T}{2}$

- Speed, kinetic energy, momentum of the particle initialy decreases in a projectile motion and attains a minimum value (not equal to zero) and then again increases.

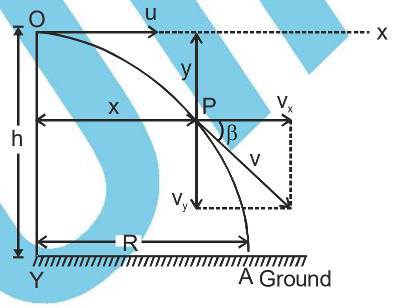
- θ is the angle between $\vec{v}$ and horizontal which decreases to zero. (at top most point) and again increases in the negative direction

Projectile fired parallel to horizontal. As shown in shown figure suppose a body is projected horizontally with velocity u from a point O at a certain height h above the ground level. The body is under the influence of two simultaneous independent motions:

(i) Uniform horizontal velocity u.

(ii) Vertically downward accelerated motion with constant acceleration g.

Under the combined effect of the above two motions, the body moves along the path OPA.



Trajectory of the projectile. After the time t, suppose the body reaches the point P(x, y).

The horizontal distance covered by the body in time t is

$$x = ut \qquad \therefore\ t = \frac{x}{u}$$

The vertical distance travelled by the body in time t is given by

$$s = ut + \frac{1}{2}at^2$$

or $$y = 0 \times 1 + \frac{1}{2}gt^2 = \frac{1}{2}gt^2$$

[For vertical motion, u = 0]

or $$y = \frac{1}{2}g\left(\frac{x}{u}\right)^2 = \left(\frac{g}{2u^2}\right)x^2 \qquad \left[\because t = \frac{x}{u}\right]$$

or $$y = kx^2 \qquad \left[\text{Here } k = \frac{g}{2u^2} = \text{a constant}\right]$$

As y is a quadratic function of x, so the trajectory of the projectile is a parabola.

Time of flight. It is the total time for which the projectile remains in its flight (from 0 to A). Let T be its time of flight.

For the vertical downward motion of the body, we use

$$s = ut + \frac{1}{2}at^2$$

or $h = 0 \times T + \frac{1}{2}gT^2$ or $T = \sqrt{\frac{2h}{g}}$

Horizontal range. It is the horizontal distance covered by the projectile during its time of flight. It is equal to OA = R. Thus R = Horizontal velocity × time of flight = u × T

or $$R = u\sqrt{\frac{2h}{g}}$$

Velocity of the projectile at any instant. At the instant t (when the body is at point P), let the velocity of the projectile be v. The velocity v has two rectangular components:

Horizontal component of velocity, $v_x = u$
Vertical component of velocity, $v_y = 0 + gt = gt$
$\therefore$ The resultant velocity at point P is

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2 + g^2t^2}$$

If the velocity v makes an angle β with the horizontal, then

$$\tan\beta = \frac{v_y}{v_x} = \frac{gt}{u} \quad \text{or} \quad \beta = \tan^{-1}\left(\frac{gt}{u}\right)$$

6. RELATIVE MOTION

The word 'relative' is a very general term, which can be applied to physical, nonphysical, scalar or vector quantities. For example, my height is five feet and six inches while my wife's height is five feet and four inches. If I ask you how high I am relative to my wife, your answer will be two inches. What you did? You simply subtracted my wife's height from my height. The same concept is applied everywhere, whether it is a relative velocity, relative acceleration or anything else. So, from the above discussion we may now conclude that relative velocity of A with respect of B (written as $\vec{v}_{AB}$) is

$$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$$

Similarly, relative acceleration of A with respect of B is

$$\vec{a}_{AB} = \vec{a}_A - \vec{a}_B$$

If it is a one dimensional motion we can treat the vectors as scalars just by assigning the positive sign to one direction and negative to the other. So, in case of a one dimensional motion the above equations can be written as

$$v_{AB} = v_A - v_B$$

and $$a_{AB} = a_A - a_B$$

Further, we can see that

$$\vec{v}_{AB} = -\vec{v}_{BA} \quad \text{or} \quad \vec{a}_{BA} = -\vec{a}_{AB}$$

IMPORTANT NOTE :

PROCEDURE TO SOLVE THE VECTOR EQUATION.

$$\vec{A} = \vec{B} + \vec{C} \quad ...(1)$$

(a) Their are 6 variables in this equation which are following :

(1) Magnitude of $\vec{A}$ and its direction

(2) Magnitude of $\vec{B}$ and its direction

(3) Magnitude of $\vec{C}$ and its direction.

(b) We can solve this equation if we know the value of 4 varibales [Note : two of them must be directions]

(c) If we know the two direction of any two vectors then we will put them on the same side and other on the different side.

For example

If we know the directions of $\vec{A}$ and $\vec{B}$ and $\vec{C}$'s direction is unknown then we make equation as follows : -

$$\vec{C} = \vec{A} - \vec{B}$$

(d) Then we make vector diagram according to the equation and resolve the vectors to know the unknown values.

SOLVED EXAMPLE

Ex.1 A car is moving along a straight line, say OP in Fig. 3.1. It moves from O to P in 18 s and returns from P to Q in 6.0 s. What are the average velocity and average speed of the car in going (a) from O to P ? and (b) from O to P and back to Q ?

Ans. (a) Average velocity $= \dfrac{\text{Displacement}}{\text{Time interval}}$

$$\vec{\upsilon} = \frac{+360\,\text{m}}{18\ \text{s}} = +20\ \text{m s}^{-1}$$

$$\text{Average speed} = \frac{\text{Path length}}{\text{Time interval}} = \frac{360\ \text{m}}{18\ \text{s}} = 20\,\text{m s}^{-1}$$

Thus, in this case the average speed is equal to the magnitude of the average velocity.

(b) In this case,

$$\text{Average velocity} = \frac{\text{Displacement}}{\text{Time interval}} = \frac{+240\ \text{m}}{(19+6.0)\ \text{s}}$$

$$= +10\ \text{ms}^{-1}$$

$$\text{Average speed} = \frac{\text{Path length}}{\text{Time interval}} = \frac{OP+PQ}{\Delta t}$$

$$= \frac{(360+120)}{24\ \text{s}} = 20\ \text{m s}^{-1}$$

Thus, in this case the average speed is not equal to the magnitude of the average velocity. This happens because the motion here involves change in direction so that the path length is greater than the magnitude of displacement. This shows that **speed is, in general, greater than the magnitude of the velocity. ??**

If the car in Example 3.1 moves from O to P and comes back to O in the same time interval, average speed is 20 m/s but the average velocity is zero !

Ex.2 The position of an object moving along x-axis is given by x = a + bt² where a = 8.5 m, b = 2.5 m s⁻² and t is measured in seconds. What is its velocity at t = 0 s and t = 2.0 s. What is the average velocity between t = 2.0 s and t = 4.0 s ?

Ans. In notation of differential calculus, the velocity is $\upsilon = \dfrac{dx}{dt}(a+bt^2) = 2b\,t = 5.0\,\text{m s}^{-1}$

At t = 0 s, υ = 0 m s^{-1} and at t = 2.0 s.

υ = 10 m s^{-1},

$$\text{Average velocity} = \frac{x(4.0)-x(2.0)}{4.0-2.0}$$

$$\frac{a+16b-a-4b}{2.0} = 6.0 \times b = 6.0 \times 2.5 = 15\ \text{m s}^{-1}$$

From Fig. 3.7, we note that during the period t =10 s to 18 s the velocity is constant. Between period t =18 s to t = 20 s, it is uniformly decreasing and during the period t = 0 s to t = 10 s, it is increasing. Note that for uniform motion, velocity is the same as the average velocity at all instants.

Ex.3 Obtain equations of motion for constant acceleration using method of calculus.

Ans. By definition $a = \dfrac{d\upsilon}{dt}$

$d\upsilon = a\,d\,t$

Integrating both sides

$$\int_{\upsilon_0}^{\upsilon} d\upsilon = \int_0^t a\,dt = a\int_0^t dt \qquad \text{(a is constant)}$$

$$\upsilon - \upsilon_0 = at,\ \ \upsilon = \upsilon_0 + at$$

Further, $\upsilon = \dfrac{dx}{dt}$, $dx = \upsilon\,dt$

Integrating both sides

$$\int_{x_0}^{x} dx = \int_0^t \upsilon\,dt = \int_0^t (\upsilon_0 + at)\,dt$$

$$x - x_0 = \upsilon_0 t + \frac{1}{2}a\,t^2$$

We can write

$$a = \frac{d\upsilon}{dt} = \frac{d\upsilon}{dx}\frac{dx}{dt} = \upsilon\frac{d\upsilon}{dx} \quad \text{or, } \upsilon\,d\upsilon = a\,dx$$

Integrating both sides,

$$\int_{\upsilon_0}^{\upsilon} \upsilon\,d\upsilon = \int_{x_0}^{x} a\,dx, \quad \frac{\upsilon^2 - \upsilon_0^2}{2} = a(x-x_0)$$

$$\upsilon^2 = \upsilon_0^2 + 2a(x-x_0)$$

The advantage of this method is that it can be used for motion with non-uniform acceleration also. Now, we shall use these equations to some important cases.

Ex.4 A ball is thrown vertically upwards with a velocity of 20 m s^{-1} from the top of a multistory building. The height of the point from where the ball is thrown is 25.0 m from the ground. (a) How high will the ball rise ? and (b) how long will it be before the ball hits the ground? Take g = 10 ms^{-2}.

Ans. (a) Let us take the y-axis in the vertically upward direction with zero at the ground, as shown in Fig. 3.13. Now υ_0 = + 20 m s^{-1},

a = – g = –10 m s^{-2},

υ = 0 m s^{-1}

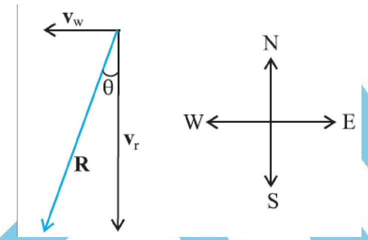
If the ball rises to height y from the point of launch, then using the equation $\upsilon^2 = \upsilon_0^2 + 2a(y - y_0)$

we get

$0 = (20)^2 + 2(-10)(y - y_0)$

Solving, we get, $(y - y_0)$ = 20 m. (b) We can solve this part of the problem in two ways. **Note carefully the methods used**.

Ex.5 Rain is falling vertically with a speed of 35 m s^{-1}. Winds starts blowing after sometime with a speed of 12 m s^{-1} in east to west direction. In which direction should a boy waiting at a bus stop hold his umbrella ?

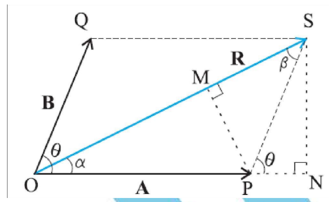


Ans. The velocity y of the rain and the wind are represented by the vectors $\mathbf{v_r}$ and $\mathbf{v_w}$ in Fig. 4.7 and are in the direction specified by the problem. Using the rule of vector addition, wesee that the resultant of $\mathbf{v_r}$ and $\mathbf{v_w}$ is **R** as shown in the figure. The magnitude of **R** is $R = \sqrt{\upsilon_r^2 + \upsilon_w^2} = \sqrt{35^2 + 12^2}$ = m s^{-1} = 37 m s^{-1}

The direction θ that R makes with the vertical is given by $\tan\theta = \frac{\upsilon_w}{\upsilon_r} = \frac{12}{35} = 0.343$ or, θ=tan^{-1} (0.343) = 19º

Therefore, the boy should hold his umbrella in the vertical plane at an angle of about 19º with the vertical towards the east.

Ex.6 Find the magnitude and direction of the resultant of two vectors A and B in terms of their magnitudes and angle ƒ∅between them. Fig. 4.9 (d) A vector A resolved into components along x, y-, and z-axes



Ans. Let OP and OQ represent the two vectors A and B making an angle θ. Then, using the parallelogram method of vector addition, OS represents the resultant vector **R : R = A + B** SN is normal to OP and PM is normal to OS. From the geometry of the figure,

$OS^2 = ON^2 + SN^2$

but ON = OP + PN = A + B cos θ SN = B sin θ

$OS^2 = (A + B\cos\theta)^2 + (B\sin\theta)^2$

or, $R^2 = A^2 + B^2 + 2AB\cos\theta$

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta} \quad (4.24a)$$

In Δ OSN, SN = OS sinθ = R sinθ, and

in ΔPSN, SN = PS sin θ = B sin θ

Therefore, R sin θ = B sin θ

$$\text{or,} \quad \frac{R}{\sin\theta} = \frac{B}{\sin\alpha} \quad (4.24b)$$

Similarly, PM = A sin α = B sin β

$$\text{or,} \quad \frac{A}{\sin\beta} = \frac{B}{\sin\alpha} \quad (4.24c)$$

Combining Eqs. (4.24b) and (4.24c), we get

$$\frac{R}{\sin\theta} = \frac{A}{\sin\beta} = \frac{B}{\sin\alpha} \quad (4.24d)$$

Using Eq. (4.24d), we get:

$$\sin\alpha = \frac{B}{R}\sin\theta \quad (4.24e)$$

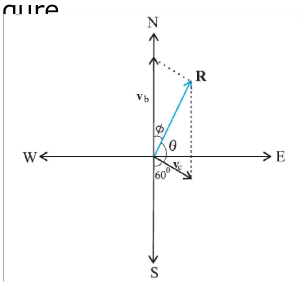
where R is given by Eq. (4.24a).

$$\text{or,} \quad \tan\alpha = \frac{SN}{OP + PN} = \frac{B\sin\theta}{A + B\cos\theta} \quad (4.24f)$$

Equation (4.24a) gives the magnitude of the resultant and Eqs. (4.24e) and (4.24f) its direction. Equation (4.24a) is known as the **Law of cosines** and Eq. (4.24d) as the **Law of sines.**

Ex.7 A motorboat is racing towards north at 25 km/h and the water current in that region is 10 km/h in the direction of 60° east of south. Find the resultant velocity of the boat.

Ans. The vector $\mathbf{v_b}$ representing the velocity of the motorboat and the vector $\mathbf{v_c}$ representing the water current are shown in Fig. 4.11 in directions specified by the problem. Using the parallelogram method of addition, the resultant **R** is obtained in the direction shown in the figure



We can obtain the magnitude of R using the Law of cosine :

$$R = \sqrt{\upsilon_b^2 + \upsilon_c^2 + 2\upsilon_b\upsilon_c \cos 120^o}$$

$$= \sqrt{25^2 + 10^2 + 2 \times 25 \times 10(-1/2)} \cong 22 km/h$$

To obtain the direction, we apply the Law of sines

$$\frac{R}{\sin\theta} = \frac{\upsilon_c}{\sin\phi}, \text{ or, } \sin\phi = \frac{\upsilon_c}{R} \sin\phi$$

$$\frac{10 \times \sin 120^o}{21.8} = \frac{10\sqrt{3}}{2 \times 21.8} \cong 0.397 \quad \phi \cong 23.4$$

Ex.8 The position of a particle is given by

$$r = 3.0t\,\hat{i} + 2.0t^2\,\hat{j} + 5.0\hat{k}$$

where t is in seconds and the coefficients have the proper units for r to be in metres. (a) Find v(t) and a(t) of the particle. (b) Find the magnitude and direction of v(t) at t = 1.0 s.

Ans.

$$v(t) = \frac{dr}{dt} = \frac{d}{dt}\left(3.0t\,\hat{i} + 2.0t^2\,\hat{j} + 5.0\hat{k}\right)$$

$$= 3.0\,\hat{i} + 4.0\,\hat{j}, \; a(t) = \frac{dv}{dt} = +4.0\,\hat{j}$$

a = 4.0 m s^{-2} along y- direction

At t = 1.0 s, $v = 3.0\,\hat{i} + 4.0\,\hat{j}$

It's magnitude is $\upsilon = \sqrt{3^2 + 4^2}$ = 5.0 m s^{-1} and direction is

$$= v_0 t + \frac{1}{2}at^2 \quad \text{Or} \quad r = r_0 + v_0 t + \frac{1}{2}at^2$$

It can be easily verified that the derivative of Eq. (4.34a), i.e. $\frac{dr}{dt}$ gives Eq.(4.33a) and it also satisfies the condition that at t=0, **r** = $\mathbf{r_0}$. Equation (4.34a) can be written in component form as

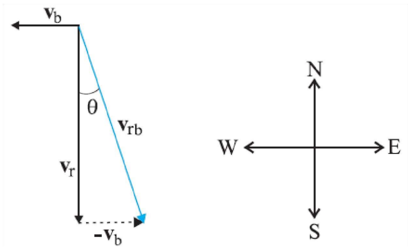
$$x = x_0 + \upsilon_{ax}t + \frac{1}{2}a_x t^2$$

$$y = y_0 + \upsilon_{oy}t + \frac{1}{2}a_y t^2$$

One immediate interpretation of Eq.(4.34b) is that the motions in x- and y-directions can be treated independently of each other. That is, **motion in a plane (two-dimensions) can be treated as two separate simultaneous one-dimensional motions with constant acceleration along two perpendicular directions**. This is an important result and is useful in analysing motion of objects in two dimensions. A similar result holds for three dimensions. The choice of perpendicular directions is convenient in many physical situations, as we shall see in section 4.10 for projectile motion.

Ex.9 Rain is falling vertically with a speed of 35 m s^{-1}. A woman rides a bicycle with a speed of 12 m s^{-1} in east to west direction. What is the direction in which she should hold her umbrella ?

Ans. In Fig. 4.16 $\mathbf{v_r}$ represents the velocity of rain and v_b , the velocity of the bicycle, the woman is riding. Both these velocities are with respect to the ground. Since the woman is ridinga bicycle, the velocity of rain as experienced by



her is the velocity of rain relative to the velocity of the bicycle she is riding. That is vrb = v_r . v_b This relative velocity vector as shown in Fig. 4.16 makes an angle ƒ" with the vertical. It is given by

$$\tan\theta = \frac{\upsilon_b}{\upsilon_r} = \frac{12}{35} = 0.343$$

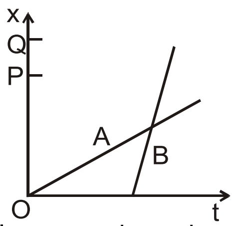
Or $\theta \cong 19^o$

Therefore, the woman should hold her umbrella at an angle of about 19º with the vertical towards the west.

Exercise - I UNSOLVED PROBLEMS

Q.1 In which of the following examples of motion, can the body be considered approximately a point object :
(a) a railway carriage moving without jerks between two stations.
(b) a monkey sitting on top of a man cycling smoothly on a circular track.
(c) a spinning cricket ball that turns sharply on hitting the ground.
(D) a tumbling beaker that has slipped off the edge of a table.

Q.2 The position-time (x-t) graphs for two children A and B returning from their school O to their homes P and Q respectively are shown in figure. Choose the correct entries in the brackets below :



(a) (A/B) lives closer to the school than (B/A)
(b) (A/B) starts from the school earlier than (B/A)
(c) (A/B) walks faster than (B/A).
(d) A and B reach home at the (same/different) time
(e) (A/B) overtakes (B/A) on the road (once/twice).

Q.3 A woman starts from her home at 9.00 am, walks with a speed of 5 km h^{-1} on a straight road up to her office 2.5 km away, stays at the office up to 5.00 pm, and returns home by an auto with a speed of 25 km h^{-1}. Choose suitable scales and plot the x-t graph of her motion.

Q.4 A drunkard walking in a narrow lane takes 5 steps forward and 3 steps backward, followed again by 5 steps forward and 3 steps backward, and son on. Each step is 1 m long and requires 1 s Plot the x-t graph of his motion. Determine graphically and otherwise how long the drunkard takes to fall in a pit 13 m away from the start.

Q.5 A jet airplane travelling at the speed of 500 km h^{-1} ejects its products of combustion at the speed of 1500 km h^{-1} relative to the jet plane. What is the speed of the latter with respect to an observer on the ground ?

Q.6 A car moving along a straight highway with speed of 126 km h^{-1} is brought to a stop within a distance of 200 m. What is the retardation of the car (assumed uniform), and how long does it take for the car to stop ?

Q.7 Two trains A and B of length 400 m each are moving on two parallel tracks with a uniform speed of 72 km h^{-1} in the same direction, with A ahead of B. The driver of B decides to overtake A and accelerates by 1 ms^{-2}. If after 50 s, the guard of B just brushes past the driver of A, what was the original distance between them (guard of B and driver of A) ?

Q.8 On a two-lane road, car A is travelling with a constant speed of 36 km h^{-1}. Two cars B and C approach car A in opposite directions with a constant speed of 54 km h^{-1} each. At a certain instant, when the distance AB is equal to AC, both being 1 km, B decides to overtake A before C does. What minimum acceleration of car B is required to avoid an accident

Q.9 Two towns A and B are connected by a regular bus service with a bus leaving in either direction every T minutes. A man cycling with a speed of 20 km h^{-1} in the direction A to B notices that a bus goes past him every 18 min in the direction of his motion, and every 6 min in the opposite direction. What is the period T of the bus service and with what speed (assumed constant) do the buses ply on the road ?

Q.10 A player throws a ball upwards with an initial speed of 29.4 m s^{-1}.
(a) What is the direction of acceleration during the upward motion of the ball ?
(b) What are the velocity and acceleration of the ball at the highest point of its motion ?
(c) Choose the x = 0 m and t = 0 s to be the location and time of the ball at its highest point, vertically downward direction to be the positive direction of x-axis, and give the signs of position, velocity and acceleration of the ball during its upward, and downward motion.
(d) To what height does the ball rise and after how long does the ball return to the player's hand ? (Take g = 9.8 m s^{-2} and neglect air resistance).

Q.11 Read each statement below carefully and state with reasons and examples (if possible) if it is true or false; A particle in one-dimensional motion
(a) with zero speed at an instant may have non-zero acceleration at that instant .
(b) with zero speed may have non-zero velocity.
(c) with constant speed must have zero acceleration
(d) with positive value of acceleration must be speeding up.

Q.12 A ball is dropped from a height of 90 m on a floor. At each collision with the floor, the ball loses one tenth of its speed. Plot the speed-time graph of its motion between t = 0 to 12 s.

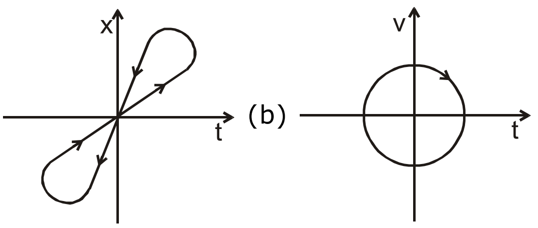
Q.13 Explain clearly, with examples, the distinction between :
(a) magnitude of displacement (sometimes called distance) over an interval of time, and the total length of path covered by a particle over the same interval :
(b) magnitude of average velocity over an interval of time and the average speed over the same interval. [Average speed of a particle over an interval of time is defined as the total path length divided by the time interval.] Show in both (a) and (b) that the second quantity is either greater than or equal to the first. When is the equality sign true ? [For simplicity, consider one-dimensional motion only].

Q.14 A man walks on a straight road from his home to a market 2.5 km away with a speed of 5 km h^{-1}. finding the market closed, he instantly turns and walks back home with a speed of 7.5 km h^{-1} . What is the
(a) magnitude of average velocity, and
(b) average speed of the man over the interval of time (i) 0 to 30 min, (ii) o to 50 min, (iii) 0 to 40 min ? [Note : You will appreciate from this exercise why it is better to define average speed as total path length divided by time, and not as magnitude of average velocity. You would not like to tell the tired man on his return home that his average speed was zero !].

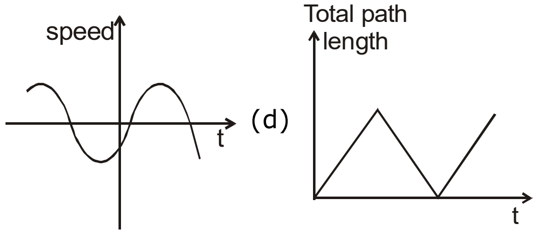
Q.35 in 3.13 and 3.14, we have carefully distinguished between average speed and magnitude of average velocity. No such distinction necessary when we consider instantaneous speed and magnitude of velocity. The instantaneous speed is always equal to the magnitude of instantaneous velocity. Why ?

Q.16 Look at the graphs (a) to (d) Fig. 3.20) carefully and state, with reasons , which of these cannot possibly represent one-dimensional motion of a particle.

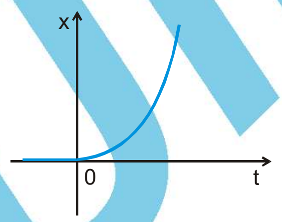
(a) (b)



(c) (d)

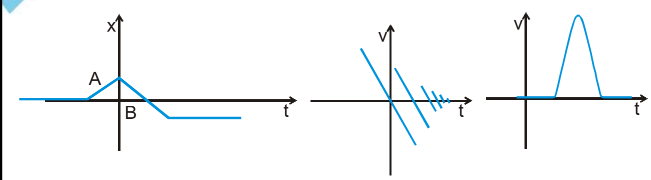


Q.17 Figures 3.21 shows the x-t plot of one-dimensional motion of a particle. Is it correct to say from the graph that the particle moves in a straight line for t < 0 and on a parabolic path for t > 0 ? If not, suggest a suitable physical context for this graph.

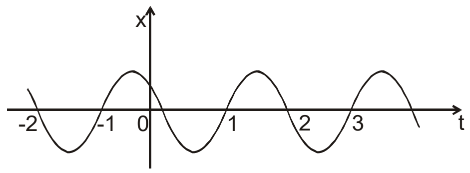


Q.18 A police van moving on a highway with a speed of 30 km h^{-1} fires a bullet at a thief's car speeding away in the same direction with a speed of 192 km h^{-1} . If the muzzle speed of the bullet is 150 m s^{-1}, with what speed does the bullet hit the thief's car ? (Note : Obtain that speed which is relevant for damaging the thief's car).

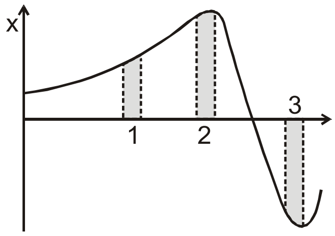
Q.19 Suggest a suitable physical situation for each of the following graphs (fig. 3.22)_ :



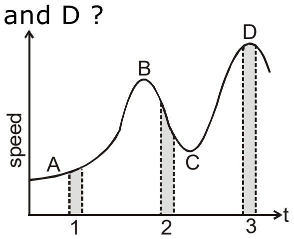
Q.20 Figure 3.23 gives the x-t plot of a particle executing one-dimensional simple harmonic motion. (You will learn about this motion in more detail in Chapter 14). Give the signs of position, velocity and acceleration variables of the particle at t = 0.3 s, 1.2 s, −1.2 s.



Q.21 Figure 3.24 gives the x-t plot of a particle in one-dimensional motion. Three different equal intervals of time are shown. In which interval is the average speed greatest, and in which is it the least ? Give the sign of average velocity for each interval.



Q.22 Figure 3.25 gives a speed-time graph of a particle in motion along a constant direction. Three equal intervals of time are shown. In which interval is the average acceleration greatest in magnitude ? In which interval is the average speed greatest " Choosing the positive direction as the constant direction of motion, give the signs of v and a in the three intervals. What are the accelerations at the points A, B, C and D ?



Q.23 A three-wheeler starts from rest, accelerates uniformly with 1 m s^{-2} on a straight road for 10 s, and then moves with uniform velocity. Plot the distance covered by the vehicle during the n^{th} second (n = 1, 2, 3, ...) versus n. What do you expect this plot to be during accelerated motion : a straight line or a parabola ?

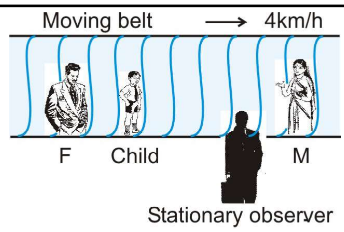
Q.24 A boy standing on a stationary lift (open from above) throws a ball upwards with the maximum initial speed he can, equal to 49 ms^{-1}. How much time does the ball take to return to his hands ? If the lift starts moving up with a uniform speed of 5 ms^{-1} and the boy again throws the ball up with the maximum speed he can, how long does the ball take to return to his hands ? If the lift starts moving up with a uniform accleration of 5 ms^{-2} and the boy again throws the ball up with the maximum speed he can, how long does the ball take to return to his hands ? (g = 9.8 m/s^2)

Q.25 On a long horizontally moving belt (figure 3.26), a child runs to and fro with a speed 9 km h^{-1} (with respect to the belt) between his father and mother located 50 m apart on the moving belt. The belt moves with a speed of 4 km h^{-1}. For an observer on a stationary platform outside, what is the

(a) speed of the child running in the direction of motion of the belt ?

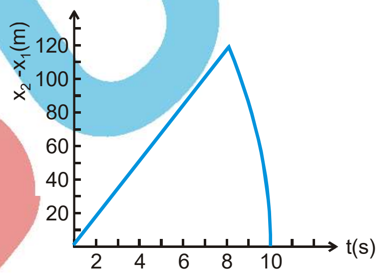
(b) speed of the child running opposite to the direction of motion of the belt ?

(c) time taken by the child in (a) and (b) ?

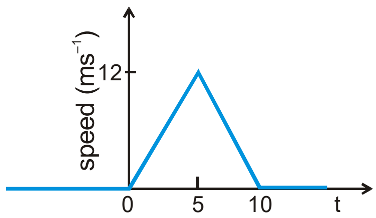


Which of the answers alter if motion is viewed by one of the parents ?

Q.26 Two stones are thrown up simultaneously from the edge of a cliff 200 m high with initial speeds of 15 m s^{-1} and 30 m s^{-1}. Verify that the graph shown in figure 3.27 correctly represents the time variation of the relative position of the second stone with respect to the first. Neglect air resistance and assume that the stones do not rebound after hitting the ground. Take g = 10 m s^{-2}. give the equations for the linear and curved parts of the plot.

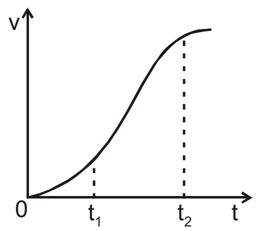


Q.27 The speed-time graph of a particle moving along a fixed direction is shown in figure 3.28. Obtain the distance traversed by the particle between (a) t = 0 s to 10 s.



What is the average speed of the particle over the intervals in (a) and (b) ?

Q.28 The velocity-time graph of a particle in one-dimensional motion is shown in figure.



Which of the following formulae are correct for describing the motion of the particle over the time-interval t_1 to t_2 :

(a) $x(t_2) = x(t_1) + v(t_1)(t_2 - t_1) + (½) a (t_2 - t_1)^2$

(b) $x(t_2) = v(t_1) + a(t_2 - t_1)$

(c) $v_{arverage} = (x(t_2) - x(t_1))/(t_2 - t_1)$

(d) $a_{arverage} = (v(t_2) - v(t_1))/(t_2 - t_1)$

(e) $x(t_2) = x(t_1) + v_{arverage}(t_2 - t_1) + (½) a_{average}(t_2 - t_1)^2$

(f) $x(t_2) - x(t_1)$ = area under the v-t curve bounded by the t-axis and the dotted line shown.

Q.29 State, for each of the following physical quantities, if it is a scalar or a vector : volume, mass, speed, acceleration, density, number of moles, velocity, angular frequency, displacement, angular velocity.

Q.30 Pick out the two scalar quantities in the following list : force, angular momentum, work, current, linear momentum, electric field, average velocity, magnetic moment, reaction as per Newton's third law, relative velocity.

Q.31 Pick out the only vector quantity in the following list :
Temperature, pressure, impulse, time, power , total path lengthy, energy, gravitational potential, coefficient of friction, charge.

Q.32 State with reasons, whether the following algebraic operations with scalar and vector physical quantities are meaningful :
(a) adding any two scalars,
(b) adding a scalar to a vector of the same dimensions,
(c) multiplying any vector by any scalar, (d) multiplying any two scalars, (e) adding any two vectors, (f) adding a component of a vector to the same vector.

Q.33 Read each statement below carefully and state with reasons, if it is true or false :
(a) The magnitude of a vector is always a scalar,
(b) each component of a vector is always a scalar,
(c) the total path length is always equal to the magnitude of the displacement vector of a particle.
(d) the average speed of a particle (defined as total path length divided by the time taken to cover the path) is either greater or equal to the magnitude of average velocity of the particle over the same interval of time,
(e) Three vectors not lying in a plane can never add up to give a null vector.

Q.34 Establish the following vector inequalities geometrically or otherwise :

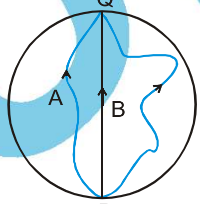
(a) $|a + b| \le |a| + |b|$ (b) $|a + b| \ge |a| - |b|$

(c) $|a - b| \le |a| + |b|$ (d) $|a - b| \ge |a| - |b|$

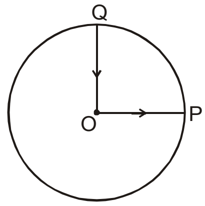
When does the equality sign above apply ?

Q.35 Given a + b + c + d = 0, which of the following statements are correct :
(a) a, b, c and d must each be a null vector,
(b) The magnitude of (a + c) equals the magnitude of (b + d),
(c) The magnitude of a can never be greater than the sum of the magnitudes of b, c and d,
(d) b + c must lie in the plane of a and d if a and d are not collinear, and in the line of a and d, if they are collinear ?

Q.36 The girls skating on a circular ice ground of radius 200 m start from a point P on the edge of the ground and reach a point Q diametrically opposite to P following different paths as shown in figure 4.23. What is the magnitude of the displacement vector for each ? For which girls is this equal to the actual length of path skate ?



Q.37 A cyclist starts from the centre O of a circular park of radius 1 km, reaches the edge P of the park, then cycles along the circumference, and returns to the centre along OQ as shown in figure 4.24. If the round trip takes 10 min, what is the (a) net displacement, (b) average velocity, and (c) average speed of the cyclist ?



Q.38 On an open ground, a motorist follows a track that turns to his left by an angle of 60° after every 500 m. Starting from a given turn, specify the displacement of the motorist at the third, sixth and eight turn. Compare the magnitude of the displacement with the total path length covered by the motorist in each case.

Q.39 A passenger arriving in a new town wishes to go from the station to a hotel located 10 km away on a straight road from the station. A dishonest cabman takes him along a circuitous path 23 km long and reaches the hotel in 28 min. What is (a) the average speed of the taxi, (b) the magnitude of average velocity ? Are the two equal ?

Q.40 Rain is falling vertically with a sped of 30 m s^{-1}. A woman rides a bicycle with a speed of 10 m s^{-1} in the north to south direction. What is the direction in which she should hold her umbrella ?

Q.41 A man can swim with a sped of 4 km/h in still water. How long does he take to cross a river 1 km wide if the river flows steadily at 3 km/h and he makes his strokes normal to the river current ? How far down the river does he go when he reaches the other bank

Q.42 In a harbour, wind is blowing at he speed of 72 km/h and the flag on the mast of a boat anchored in the harbour flutters along the N-E direction. If the boat starts moving at a sped of 51 km/h to the north, what is the direction of the flag on the mast of the boat ?

Q.43 The ceiling of a long hall is 25 m high. What is the maximum horizontal distance that a ball thrown with a speed of 40 m s^{-1} can go without hitting the ceiling of the hall ?

Q.44 A cricketer can throw a ball to a maximum horizontal distance of 100 m. How much high above the ground can the cricketer throw the same ball ?

Q.45 A stone tied to the end of a string 80 cm long is whirled in a horizontal circle with a constant speed. If the stone makes 14 revolutions in 25 s, what is the magnitude and direction of acceleration of the stone

Q.46 An aircraft executes a horizontal loop of radius 1 km with a steady speed of 900km/h. Compare its centripetal acceleration with the acceleration due to gravity.

Q.47 Read each statement below carefully and state, with reasons, if it is rue or false :
(a) The net acceleration of a particle in circular motion is always along the radius of the circle towards the centre
(b) The velocity vector of a particle at a point is always along the tangent to the path of the particle at that point
(c) The acceleration vector of a particle in uniform circular motion averaged over one cycle is a null vector.

Q.48 The position of a particle is given by

$$\vec{r} = 3.0t\hat{i} - 2.0t^2\hat{j} + 4.0\hat{k}\,m$$

where t is in seconds and the coefficients have the proper units for r to be in metres.
(a) Find the v and a of the particle ? (b) What is the magnitude and direction of velocity of the particle at t = 2 s ?

Q.49 A particle starts from the origin at t = 0 s with a velocity of 10.0 $\hat{j}$ m/s and moves in the x-y plane with a constant acceleration of $(8.0\hat{i} + 2.0\hat{j})$ m s^{-2}.
(a) At what time is the x-coordinate of the particle 16 m ? What is the y-coordinate of the particle at that time ? (b) What is the speed of the particle at the time ?

Q.50 $\hat{i}$ and $\hat{j}$ are unit vector along x- and y-axis respectively. What is the magnitude and direction of the vectors $\hat{i} + \hat{j}$, and $\hat{i} - \hat{j}$? What are the components of a vector A = $2\hat{i} + 3\hat{j}$ along the directions of $\hat{i} + \hat{j}$ and $\hat{i} - \hat{j}$?

Q.51 For any arbitrary motion in space, which of the following relations are true :
(a) $V_{average} = (1/2)\,(v(t_1) + v(t_2))$
(b) $V_{average} = [\,r(t_2) - r(t_1)] / (t_2 - t_1)$
(c) V (t) = v (0) at
(d) $r(t) = r(0) + v(0)\,t + (1/2)\,a\,t^2$
(e) $a_{average} = [v(t_2) - v(t_1)] / (t_2 - t_1)$
(The 'average' stands for average of the quantity over the time interval t_1 to t_2)

Q.52 Which of the following quantities are independent of the choice of orientation of the coordinate axes ?
a + b, $3a_x + 2b_y$, |a + b – c|, angle between b and c, a where is a scalar ?

Q.53 Read each statement below carefully and state, with reasons and examples, if it is true or false : A scalar quantity is one that
(a) is conserved in a process
(b) can never take negative values
(c) must be dimensionless
(d) does not vary from one point to another in space
(e) has the same value for observers with different orientations of axes.