Electric Charges and Fields

1

ELECTRIC CHARGE (Q)

It is a scalar quantity. It is of two kinds. Benjamin Franklin suggested the names **positive** and **negative** in place of the old names vitreous and resinous, respectively.

Positive charges develop on a body due to removal of electrons. Negative charges develop due to addition of electrons.

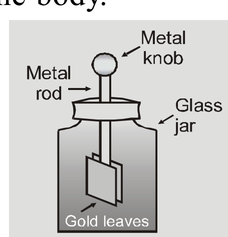
Unit of Q : coulomb   Dimensions of Q : A^1T^1

Think
1. State coulomb 2. Franklin 3. Faraday

Think
It is possible to measure the charge by electroscope

Electroscope

It is a simple apparatus with which the presence of electric charge on a body is detected (see fig.) When metal knob is touched with a charged body, some charge is transferred to the gold leaves, which then diverges due to repulsion. The separation gives a rough idea of the amount of charge on the body.



Basic Properties of Charges

(i) **Charge is a scalar :** represents excess or deficiency of electron -

(ii) **Charge is transferable :** If a charged body is put in contact with an uncharged body, charge gained by the uncharged body is always lesser than initial charge present on charged body. The process of charge transfer is called 'conduction' .

(iii) **Charge is always associated with mass :** i.e. charge can not exist without mass though mass can exist without charge. So the presence of charge itself is a convincing proof of existence of mass.

In charging, the mass of a body changes i.e. (+ve) Charged => Ms $\downarrow$: (–ve) charged = > M $\uparrow$

(iv) **Quantization of charge :** Charge on any body is always an integral multiple of a fundamental unit of electric charge. This unit is equal to the magnitude of charge on an electron ($|-e| = 1.6 \times 10^{-19}$ C)

$$\boxed{Q = ne}$$

where n is an integer ±1, ±2, ±3, ... and $e = 1.60 \times 10^{-19}$ C

Robert A. Millikan performed the 'oil-drop experiment' which led of the discovery of quantization of charge.

(v) Conservation of charge : In an isolated system the total charge (sum of positive and negative) remains constant.

(i) $_{92}U^{238} \rightarrow {}_{90}TH^{234} + {}_{2}He^{4}$

$[Q = 92e] \rightarrow [Q_1 = 90e] + [Q_2 = 2e]$

(ii) $\gamma \rightarrow e^{+} + e^{-}$

$[Q = 0] \rightarrow [Q_1 = +e] + [Q_2 = -e]$

Think
1.Mass without charge
2. Charge without mass which is possible ?

Think
During radioactive reaction Mass is cons. or not

(vi) Charge is invariant : Charge is independent of F.O.R (Frame of ref.) i.e. charge on a body does not charges what ever be its speed.

(vii) Similar charges repel each other while dissimilar attract : True test of electrification is repulsion and not attraction as attraction may also take place between a charged and an uncharged body and also between two similarly charged bodies.

(viii) Methods of charging : Friction, Induction & Conduction.

(ix) Charge differs from mass in following sense:

(a) In SI units, charge is a derived physical quantity while mass is fundamental

(b) Charge is always conserved but mass is not

(c) Though both charge and mass are quantised, the quanta of charge is e^- while that of mass it is yet not clear.

(d) If v ↑ = > M ↑ while charge is invariant.

Introduce $m = \dfrac{m_0}{\sqrt{1-\dfrac{v^2}{c^2}}}$

Solved Examples

Ex.1 A copper sphere of mass 2.0 g contains about 2×10^{22} atoms. The charge on the nucleus of each atom is 29e.

(i) How many electorns must be removed form the sphere to give it a charge of +2μC?

(ii) Determine the fraction of electrons removed.

(iii) Is there any change in mass of sphere when it is given positive charge?

Sol. (i) Number of electrons to be removed

$$n = \frac{Q}{e} = \frac{2 \times 10^{-6}}{1.6 \times 10^{-19}} = 1.25 \times 10^{13}$$

(ii) Total number of electrons in the sphere = $29 \times 2 \times 10^{22} = 5.8 \times 10^{23}$

Fraction of electrons removed

$$= \frac{1.25 \times 10^{13}}{5.8 \times 10^{23}} = 2.16 \times 10^{-11}$$

Think
1. What is Quark
2. Why in Macroscopic studies quantization is neglected

Thus 2.16×10^{-9}% of electrons are to be removed to give the sphere a charge of 2μC.

(iii) Yes mass decreases, when body is given a positive charge.

decrease of mass

$$\Delta m = 9 \times 10^{-31} \times 1.25 \times 10^{13} = 1.125 \times 10^{-17} \text{ kg}$$

ELECTROSTATIC FORCE

Coulomb's law

The electrostatic force of interaction (repulsion or attraction) between two electric charges q_1 and q_2 separated by a distance r, is directly proportional to the product of the charges, inversely proportional to the square of distance between them and acts along the straight line joining two charges.

$$F = \frac{kq_1q_2}{r^2}$$

Think
1. Point charge
2. Stationary charge

If charges are placed in vacuum

$$k = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{ Nm}^2\text{C}^{-2}$$

Permittivity : Permittiveity is a measure of the ability of the medium surrounding electric charges to allow electric lines of force to pass through it. It determines the forces between the charges.

ε_0 = permittivity of free space (vacuum) = 8.85×10^{-12} C^2/Nm^2 = $\dfrac{1}{36\pi \times 10^9}$ C^2/N-m^2

Dimensions $[\varepsilon_0] = \dfrac{[Q]^2}{[F]\,[\text{length}]^2} = \dfrac{T^2A^2}{MLT^{-2}L^2}$

$$[\varepsilon_0] = M^{-1}L^{-3}T^4A^2$$

Relative Permittivity : The relative permittiveity or the dielectric constant (ε_r or K) of a mecdium is defined as the ratio of the permittivity ε of the medium to the permittivity of Air/FS/vacuum.

$$\varepsilon_r \text{ or } K = \frac{\varepsilon}{\varepsilon_0}$$

If charges are placed in a medium with permittivity e, then the forces caused by induced charges in the medium reduce the force between point charges. The new force is,

$$F = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{q_1q_2}{r^2}$$

$$F_{medium} = \frac{F_{vacuum}}{\varepsilon_r}$$

Think
Why the permittivity of water is so much high as compared to mica

Medium	ε_r
Vaccum	1
Air	1.00059
Water	80
Mica	6
Teflon	2
Glass	5-10
PVC	4.5
Metal/Alloy	∞

Note: Coulomb's law is not valid for distances less than 10^{-15} m. Further if charges are moving then magnetic forces also act.

Vector Form

$\vec{F}_{12}$ = force on charge q_1 due to charge q_2

$\vec{F}_{21}$ = force on charge q_2 due to charge q_1

$$\vec{F}_{21} = k\frac{q_1q_2}{r^2}\hat{r} = -\frac{kq_1q_2}{r^2}\hat{r}, \qquad \vec{F}_{21} = k\frac{q_1q_2}{r^3}\vec{r} = -\frac{kq_1q_2}{r^3}\vec{r} \qquad \vec{F}_{12} = -\vec{F}_{21}$$

Comparative study of fundamental forces :

S.No.	Force	Nature	Range	Relative strength
1	Graviational force	Attractive	Long range	1
2	Electromagnetic force	Attractive or repulsive	Long range	1036
3	Nuclear force	Attractive	Short range	1039
4	Weak force	Unknown	Short range	1014

Principle of superposition (Forces)

When many charges are present say q_1, q_2, q_3 ..... then net force on any charge, say q_3 will be vector sum of the forces due to other charges q_1, q_2 on q_3

$$\vec{F}_3 = \vec{F}_{31} + \vec{F}_{32}$$

The resultant of two forces $\vec{F}_1$ and $\vec{F}_2$ (see fig.) is determined by using.

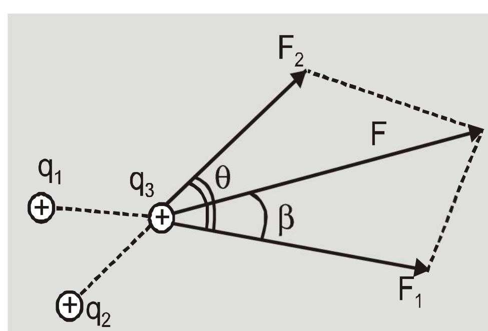
$$F = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

The resultant makes an angle β with the F_1.

$$\tan\beta = \frac{F_2\sin\theta}{F_1 + F_2\cos\theta}$$

Solved Examples

Ex.2 Atomic number of copper is 29, its atomic weight is 63.5 gm/mole. If two pieces of copper having weight 10 gm are taken and on one of the pieces 1 electron per 1000 atom is transferred to the other piece and there after this these pieces are placed 10 cm apart, then what will be the coulomb force between them.



Sol. In 1 mole copper (63.5 gm) there are 6×10^{23} atoms (N_A = Avagedro number = 6×10^{23} atoms)

Number of atoms = $\frac{6 \times 10^{23} \times 10}{63.5} = 9.45 \times 10^{22}$

For every 1000 atoms, 1 electron is transferred therefore total number of transferred electron is

$$= \frac{9.45 \times 10^{22}}{1000} = 9.45 \times 10^{19}$$

Therefore charge on one piece is 9.45×10^{19} e and on the other will be (9.45×10^{19}e)

Force when they are kept 10 cm apart

$$F = \frac{1}{4\pi \in_0} \frac{q_1 q_2}{r^2}$$

$$= \frac{1}{4\pi \in_0} \frac{(9.45 \times 10^{19})^2 e^2}{(10 \times 10^{-2})^2}$$

$$= \frac{9 \times 10^9 \times (9.45 \times 10^{19})^2 \times (1.6 \times 10^{-19})^2}{(10 \times 10^{-2})^2}$$

$= 2 \times 10^{14}$ N

Ex.3 Two similar small balls having mass m and charge q are suspended by silk strings having length ℓ, according to the figure. If in the figure θ is an acute angle then for equilibrium what will the distance between the centre of the two balls.

Sol. The force & acting on the system are as follows.

T is tension in string, F is coulomb force and mg is weight.

For equilibrium,

T cos θ = mg and T sin θ = F

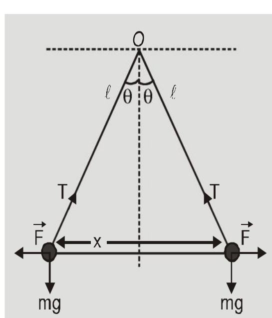
$$\therefore \tan\theta = \frac{F}{mg} = \frac{1}{4\pi \in_0 x^2} \frac{q^2}{mg}$$

If θ is small

$$\therefore \tan\theta \simeq \sin\theta \approx \frac{x}{2\ell}$$

$$\therefore \frac{x}{2\ell} = \frac{1}{4\pi \in_0 x^2} \frac{q^2}{mg}$$

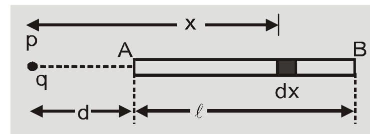
$$x = \left\{ \frac{q^2 \ell}{2\pi \in_0 mg} \right\}^{1/3}$$



Ex.4 On an insulated rod haring length L, Q charge is evenly distributed and a point charge q is placed at a distance d from are of its ends as shown. What will be the total electric force on q.

Sol. Considering a small element dx of the rod at a distance x from q, force on q due to this small element

$$dF = \frac{q dQ}{4\pi \in_0 x^2}$$



where dQ is charge in small element dx and dQ

$$= \left(\frac{Q}{L}\right) dx$$

$$\therefore F = dF = \frac{qQdx}{4\pi \in_0 Lx^2} = \frac{1}{4\pi \in_0} \frac{qQ}{L} \int_d^{d+L} \frac{dx}{x^2}$$

$$= \frac{1}{4\pi \in_0} \frac{qQ}{L} \left[\frac{-1}{x}\right]_d^{d+L}$$

$$= \frac{1}{4\pi \in_0} \frac{qQ}{L} \left[\frac{1}{d} - \frac{1}{d+L}\right]$$

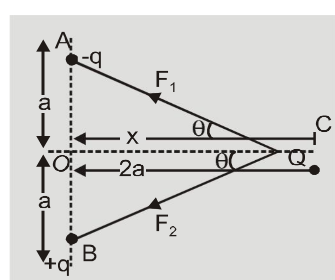
$$= \frac{qQ}{4\pi \in_0 d(d+L)}$$

Ex.5 Two similar negative charges –q are situated at point (0, a) & (0, –a) along Y-axis. A positive charge Q is left from point (2a, 0) Analyse the motion of Q.

Sol. Due to symmetry the y components of forces acting on Q due to charges –q at A and B will balance each other and the x components will add up along direction O.

If at any instant Q is at a distance x from O

$F = F_1 \cos\theta + F_2 \cos\theta = 2F_1 \cos\theta$ [$\because F_1 = F_2$]



$$= 2\frac{1}{4\pi\in_0}\left[\frac{-qQ}{(a^2+x^2)}\right]\left[\frac{x}{(a^2+x^2)^{1/2}}\right]$$

$$F = -\frac{1}{4\pi\in_0}\frac{2qQx}{(a^2+x^2)^{3/2}}$$ i.e. $F \propto -x$

$\because$ $F \propto -x$, the motion is oscillatory having amplitude 2a but it will not be S.H.M.

If $x << a$

$$F = -\frac{1}{4\pi\in_0}\frac{2qQ}{a^3}x = -kx$$

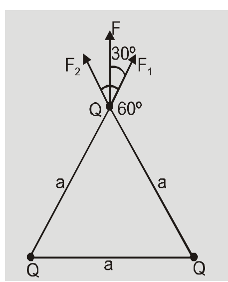
and the motion is S.H.M. with time period T, where

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{4\pi\in_0 ma^3}{2Qq}}$$

Ex.6 Three equal charges (Q, each) are placed on the vertices of an equilateral triangle of side a. What is the resultant force on any one charge due to the other two?

Sol. The charges are shown in fig. The resultant force

$$F = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos 60^o}$$

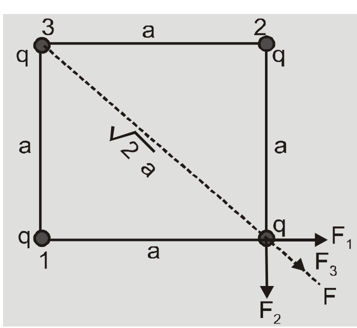


with $F_1 = F_2 = kQ^2/a^2$

$$\boxed{F = \frac{\sqrt{3}kQ^2}{a^2}}$$

From symmetry the direction is as shown along y-axis.

Ex.7 Consider four equal charges (q, each) placed on the corners of a square with side a. Determine the magnitude and direction of the resultant force on the charge on lower right corner.



Sol. The forces on the charge on lower right corner due to charges 1, 2, 3 are $F_1 = kq^2/a^2$, $F_2 = kq^2/a^2$, $F_3 = kq^2/2a^2$

The resultant of F_1 and F_2 is

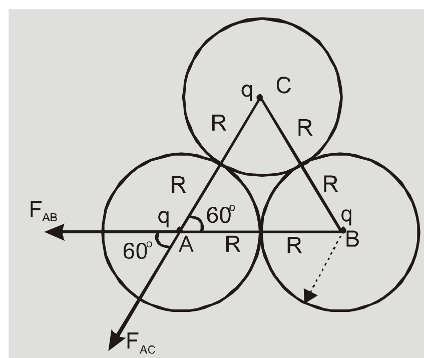
$$F_{12} = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos 90^o} = \sqrt{2}kq^2/a^2$$

This is in the direction parallel to F_3. Therefore the total force on the said charge is $F = F_{12} + F_3$

$$F = \frac{1}{2}\frac{kq^2}{a^2}(1+2\sqrt{2})$$ The direction of F is 45° below the horizontal line

Ex.8 Three identical spheres each having a charge q and radis R, are kept in such a way that each touches the other two. Find the magnitude of the electric force on any sphere due to other two.

Sol. For external points a charged sphere behaves as if the whole of its charge was concentrated at its centre.



Force on A due to B is F_{AB}

$$= \frac{1}{4\pi\varepsilon_0}\frac{q \times q}{(2R)^2} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{4R^2} \text{ along } \overrightarrow{BA}$$

Force on A due to C. F_{AC}

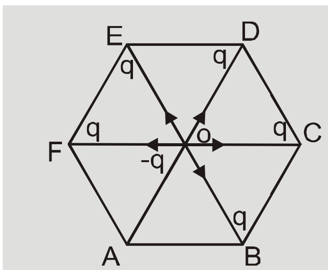
$$= \frac{1}{4\pi\varepsilon_0}\frac{q \times q}{(2R)^2} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{4R^2} \text{ along } \overrightarrow{CA}$$

Now, angle between BA and CA is 60^0 and $F_{AB} = F_{AC} = F$.

$$F_A = \sqrt{F^2 + F^2 + 2FF\cos 60} = \sqrt{3}F = \frac{1}{4\pi\varepsilon_0}\frac{(\sqrt{3})}{4}\left[\frac{q}{R}\right]^2$$

Ex.9 Five point charges, each of value +q are palced on five vertices of a regular hexagon of side L m. What is the magnitude of the force on a point charge of value -q coloumb placed at the centre of the hexagon?

Sol. If there had been a sixth charge +q at the remaining vertex of hexagon force due to all the six charges on -q at O will be zero (as the forces due to individual charges will balance each other), i.e., $\vec{F}_R = 0$.



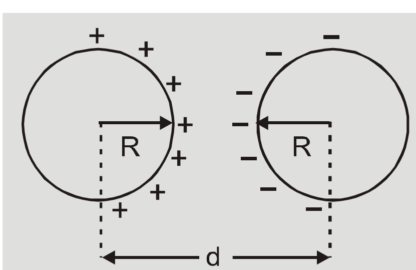
Now If $\vec{f}$ is the force due to sixth charge and $\vec{F}$ due to remaining five charges, $\vec{F} + \vec{f} = 0$ i.e. $\vec{F} = -\vec{f}$

or, $F = f = \frac{1}{4\pi\varepsilon_0}\frac{q \times q}{L^2} = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{L}\right]^2$

Ex.10 Two charged spheres of radius 'R' are kept at a distance 'd' (d > 2R). One has a charge +q and the other -q. The force between them will be-

[1] $\frac{1}{4\pi\varepsilon_0}\frac{q^2}{d^2}$ [2] $> \frac{1}{4\pi\varepsilon_0}\frac{q^2}{d^2}$

[3] $< \frac{1}{4\pi\varepsilon_0}\frac{q^2}{d^2}$ [4] None of these



Sol. [2] Redistribution of charge will take place due to mutual attraction and hence effective distance will be less than d.

ELECTRIC FIELD

The electric field $\vec{E}$ at any point is equal in magnitude to the force experienced per unit (test) charge placed at that point, and is directed along the direction of the force experienced

$$\vec{E} = \frac{\vec{F}}{q_0}, \quad \vec{E} = \lim_{q_0 \to 0}\frac{\vec{F}}{q_0}$$

Electric field (intensity) $\vec{E}$ is a vector quantity.

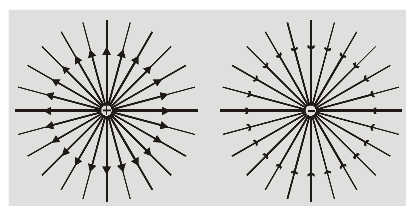
Electric field due to a positive charge is always away from the charge and that due to a negative charge is always towards the charges.

Unit of [E] : newton/coulomb = volt/metre

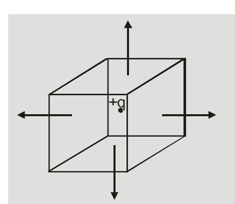
Dimensions of [E] : $\frac{[F]}{[q]} = \frac{MLT^{-2}}{TA} = MLT^{-3}A^{-1}$

Electric lines of force

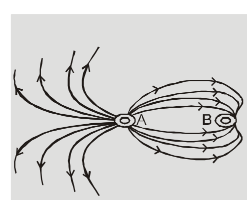
[1] Electric lines of force usually start or diverge out form positive charge and end or converge on negative charge.



[2] The number of lines originating or terminating on a charge is proportinal to the megnitude of charge. In SI units 1% shows electric lines associated with unit (i.e., 1 coulomb) charge. So if a bodty encloses a charge q, total lines of force or flux associated with it is q/ε_0. If the body is cubical and charge is situated at its center the lines of froce through each frace will be q/ε_0.



Total lines of force (q/e_0) while through each face ($q/6e_0$)

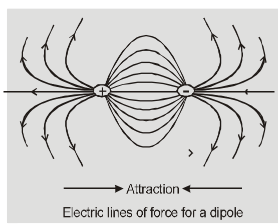


Charge A is positive B is negative and $q_a > q_b$

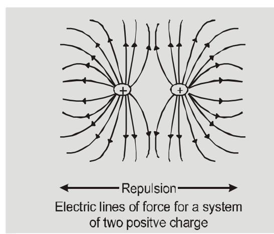
[3] Lines of force never cross each other because if they cross then intensity at that point will have two directions which is not possible.

[4] The electric lines of force can nerver be closed loops, as line can never start and end on the same charge. If a line of force is a closed curve, work done round a closed path will not be zero and electric field will not remain conservative.

[5] Lines of force have tendency to contract longitudinally like a stretched elastic string producing attraction between opposite charges and repel each other laterally resulting in, repulsion between similar charges and 'edge-effect' (curving of lines of froce near the edges of a charged conductor)

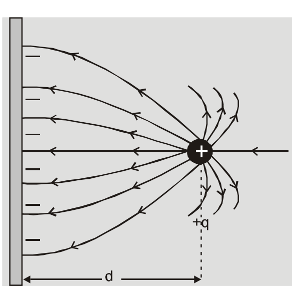


Electric lines of force for a dipole

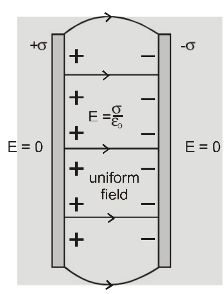


Electric lines of force for a system of two positve charge

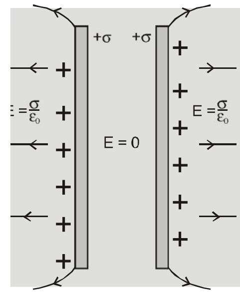
[6] Electric lines of force end or start normally on the surface of a conductor. If a line of force is not normal to the surface of a conductor, electric intensity will have a component along the surface of the conductor and hence conductor will not remain equipotential which is not possible as in electrostatics conductor is an equipotential surface.



Fixed point charge near infinite metal plate



Parallel metal plates havin dissimilar charges

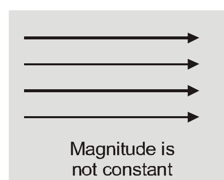


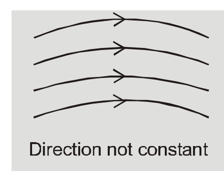
Parallel metal plates havin similar charges

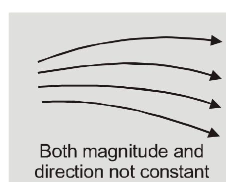
[7] If in aregion of space, there is no electric field will be no lines of force. This is why inside a conductor or at a neutral point where resultant intensity is zero there is no line of force.

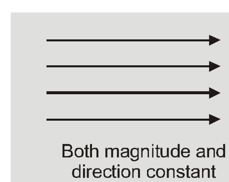
[8] The number of lines of force per unit normal area at a point represents magnitude of electric filed intensity. The crowded lines represent strong field while distant lines shows a weak field.

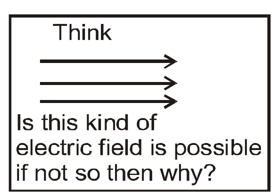
[9] If the lines of force are equidistant straight lines the field is uniform and if lines of force are not equidistant or straight lines or both, the field will be non-uniform. The first three represent nonuniform field while last shows uniform field.


Magnitude is not constant


Direction not constant


Both magnitude and direction not constant

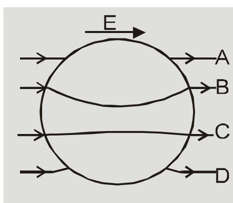

Both magnitude and direction constant



Solved Examples

Ex.11 A Solid metallic sphere is placed in a uniform electric field. Which of the lines A, B, C and D shows the correct path and why ?

Sol. Path (A) is wrong as lines of force do not start or end normally on the surface of a conductor. Path (B) and (C) are wrong as lines of force should not exist inside a conductor. Also lines of froce are not normal to the surface of conductor. Path (D) represents the correct situation as here line of force does not exist inside the conductor and starts and ends normally on its surface.



Ex.12 A metallic slab is introduced between the two plates of a charged parallel plate capacitor. Sketch the electric lines of force between the plates.

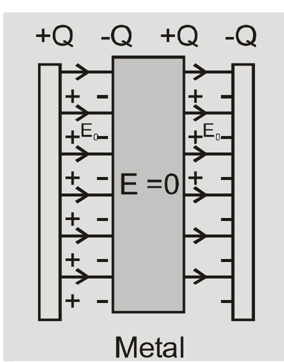
Sol. Keeping in mind the following properties :

[1] Lines of force start from positive charge and end on negative charge.

[2] Lines of force start and end normally on the surface of a conductor.

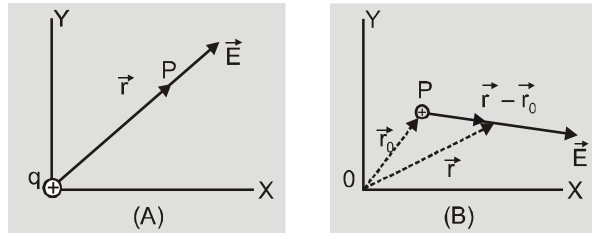
[3] Lines of force do not exist inside a conductor (as field inside a conductor is zero)

The field between the plates is shown as



Electric field due to a point charge

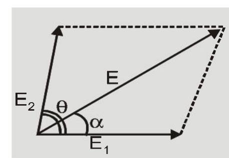
$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r} \quad \Rightarrow \quad \vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^3}\vec{r}$$



If the position vector of the charge is $\vec{r_0}$, then electric field at position $\vec{r}$ is $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q(\vec{r}-\vec{r_0})}{|\vec{r}-\vec{r_0}|^3}$

Superposition of electric fields

The resultant electric field at any point is equal to the vector sum of electric fields at that point due to various charges.



$$\vec{E} = \vec{E_1} + \vec{E_2} + \vec{E_3} + ...$$

The magnitude of the resultant of two electric fields is given by

$$E = \sqrt{E_1^2 + E_2^2 + 2E_1E_2\cos\theta}$$

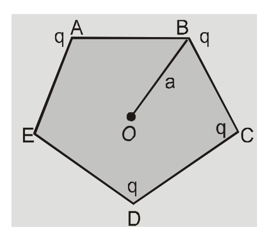
The direction is given by $\tan\alpha = \frac{E_2\sin\theta}{E_1 + E_2\cos\theta}$

Solved Examples

Ex.13 Charges with magnitude q are placed at 4 corners of a regular pentagon. These charges are at distance 'a' from the centre of the pentagon. Find electric field intensity at the centre of the pentagon.

Sol. Charges are placed at corners A, B, C and D of the pentagon. If charge q is placed at the fifth corner also then by symmetry the intensity $\vec{E}$ at centre O is zero.

Electric field at O due to new charge placed at



$E = \frac{q}{4\pi\epsilon_0 a^2}$ along $\vec{EO}$

$\therefore$ Electric field due to charges at A, B, C and D.

$E = \frac{q}{4\pi\epsilon_0 a^2}$ along $\vec{OE}$

Ex.14 Along x-axis at positions x = 1, x = 2, x = 4 and x = 8 charges q is placed. What will be electric field at x = 0 due to these charges. What will be the value of electric field if the charges are alternately positive and negative.

Sol. By superposition theory

......|............|...........|...........|.............∞

x = 1   x = 2   x = 4   x = 8

$$E = \frac{q}{4\pi \in_0}\left[\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{4^2}+\frac{1}{8^2}+.....\right]$$

$$= \frac{q}{4\pi \in_0}\left[\frac{1}{1}+\frac{1}{4}+\frac{1}{16}+\frac{1}{64}+.....\right]$$

Terms in the braket are G.P. with first term a = 1 and common ratio $r = \frac{1}{4}$ .Its sum $S = \frac{a}{1-r}$

$$\therefore\ E = \frac{q}{4\pi \in_0}\left[\frac{1}{1-1/4}\right] = \left[\frac{1}{4\pi \in_0}\right]\frac{4}{3}q$$

If the charges are alternately positive and negative

$$E = \frac{q}{4\pi \in_0}\left[1-\frac{1}{4}+\frac{1}{16}-\frac{1}{64}+....\right]$$

where a = 1, r = –1/4

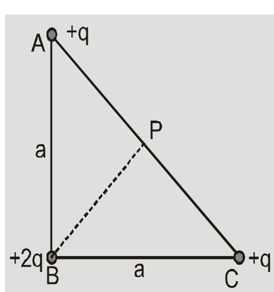
$$E = \frac{q}{4\pi \in_0}\left[\frac{1}{1-(-1/4)}\right]$$

$$= \left[\frac{q}{4\pi \in_0}\right]\frac{4q}{5}$$

Ex.15 Three charges +q, +q, +2q are arranged as shown in figure. What is the field at point P (center of side AC)

Sol. The sum of fields at P due to charges at A and C add up to zero (because of equal magnitude and opposite direction) . Thus the net field at P is that due to +2q charge. Its direction is along the line BP and its magnitude is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{2q}{(BP)^2}$$

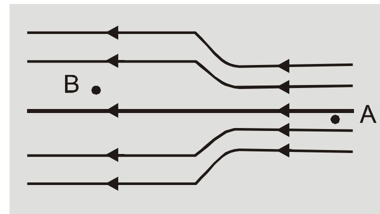


$$BP = \sqrt{BC^2 - PC^2}$$

$$= \sqrt{a^2 - a^2/2} = a/\sqrt{2}$$

Thus $\boxed{E = \frac{q}{\pi\varepsilon_0 a^2}}$

Ex.16 Fig. shows field lines of an electric field, the line spacing perpendicular to the page is the same every where. If the magnitude of the field at A is 40 N/C, then the magnitude of the field at B is approximately.



Sol. From the diagram we notice that density of lines at B is approximately half of that at A. Since the density of field lines is proportional to the strengths of field, we expect B = 20 N/C.

Ex.17 An infinite plane of positive charge has a surface charge density σ. A metal ball B of mass m and charge q is attached to a thread and tied to a point A on the sheet PQ. Find the angle θ which AB makes with the plane PQ.

Sol. Due to positive charge the ball will experience electrical force $F_e = qE$ horizontally away from the sheet while the weight of the ball will act vertically downwards and hence if T is the tension in the string, for equilibrium of ball :

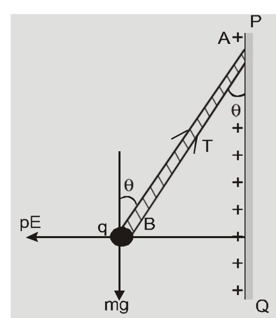
Along horizontal, $T\sin\theta = qE$

And along vertical, $T\cos\theta = mg$ So $\tan\theta = \frac{qE}{mg}$

and $T = [(mg)^2 + (qE)^2]^{1/2}$

The field E produced by the sheet of charge PQ having charge density

s is $E = \frac{\sigma}{2\varepsilon_0}$



So, $\tan\theta = \frac{q\sigma}{2\varepsilon_0 mg}$

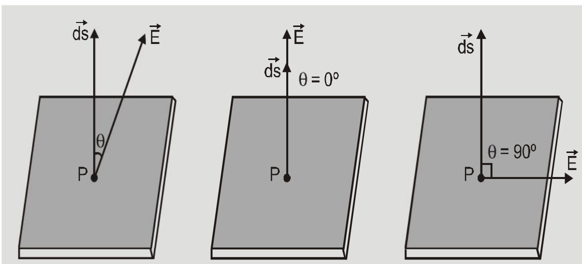
i.e., $\theta = \tan^{-1}\left[\frac{q\sigma}{2\varepsilon_0 mg}\right]$

GAUSS'S LAW AND APPLICATIONS

Electric flux

[1] It is a real scalar physical quantity with units volt x m and dimensions

$$\phi_E = Eds = \frac{F}{q}ds = \frac{MLT^{-2}}{AT}L^2 = ML^3T^{-3}A^{-1}$$

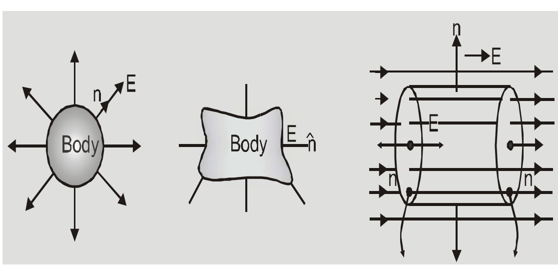


$d\phi_E = Eds\cos\theta$ $(d\phi_E)_{max} = Eds$ $(d\phi_E)_{min} = 0$

[2] It will be maximum when cos θ is max = 1, i.e., $\theta = 0^0$, i.e., electric field is normal to the area with $(d\phi_E)_{max} = E\ ds$

[3] It will be minimum when cos θ is min = 0, i.e., $\theta = 90^0$, i.e.field is parallel to the area with $(d\phi_E)_{min} = 0$

[4] For a closed body outward flux is taken as positive while inward flux is taken as negative.

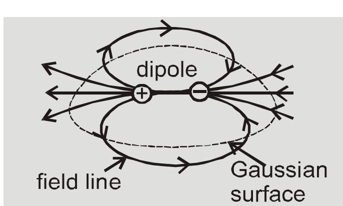


Positive-flux Negative-flux $\phi_E = -\pi R^2 E$ $\phi_E = 0$ $\phi_E = +\pi R^2 E$
Cylinder in a uniform field

Gauss's law

The total (net outward) electric flux through a closed surface is equal to the total charge enclosed by the surface divided by ε_0, the permittivity of free space $\phi = \oint \vec{E}.d\vec{S} = \frac{q}{\varepsilon_0}$

(for area element both symbols dA or dS are used)



Gaussian surface in an arbitary, surface around a charge or charge distribution, imagined for calculation of electric flux through it.

The shape of the Gaussian surface could by any, spherical, cylindrical, cubical, any arbitary shape.

Electric field at Gaussian surface is the resultant $\vec{E}$ at the point. This $\vec{E}$ may arise from charges inside as will as outside the closed Gaussian surface.

Even if total net flux through a closed surface is zero, the electric field E, may be non-zero at the gaussian surface.

In fig.A, the total charge enclosed = 0. Thus the net outward flux should be zero, even though $\vec{E}$ at gaussian surface is not zero.

Gauss's law and Coulomb's law are equivalent. One can derive Coulomb's law by using Gauss's law and vice-versa.

Flux is a measure of number of lines of electric field from a charge.

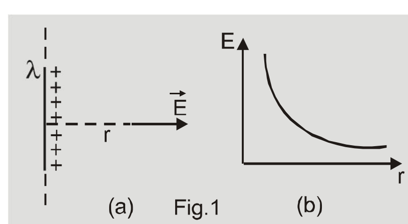
Gauss law can be used for determining E only if charge distribution has some symmetry, so that

$\oint \vec{E}.d\vec{A} = E(\cos\theta)\oint dA$ and flux can be calculated. For example, Gauss's law cannot be used to find electric field due to a dipole.

(i) Electric field due to line charge (infinite length)

The electric field at a distance r from a line charge (density λ) is

$$E = \frac{\lambda}{2\pi\varepsilon_0 r} = k.\frac{2\lambda}{r}$$



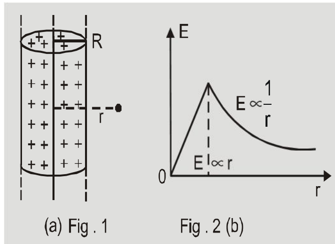
The direction is outward perpendicular to the line charge. The $E \propto (1/r)$ dependence is shown in fig.

(ii) Electric field due to cylinder

If the line charge is a cylinder of radius R, then

(a) Electric field outside

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$



(b) Electric field on the surface

$$E = \frac{\lambda}{2\pi\varepsilon_0 R}$$

(c) Electric field inside (at a distance r from the axis)

$$E = \frac{\lambda r}{2\pi\varepsilon_0 R^2}$$

The direction of the field is outwards (normal to the axis)

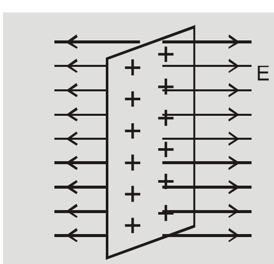
The dependence of the field is shown in figure.2 (b) Inside the charged cylinder,

$$E \propto r$$

outside $\quad E \propto \frac{1}{r}$

(iii) Electric field due to a plane sheet (infinite dimensions)

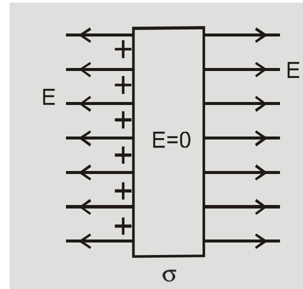
(a) Single sheet of charge



For the surface charge density σ (coulomb/metre^2) the field at a distance r from the sheet is $E = \frac{\sigma}{2\varepsilon_0}$ directed towards outward normal (from a positively charged sheet)

The electric field does not depend on distance.

(b) Charged metal plate

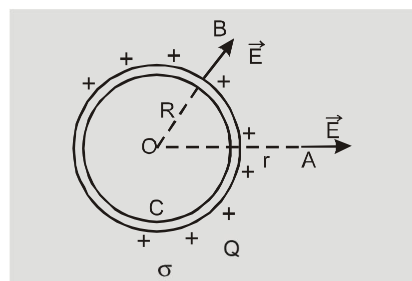


Inside charged metal plate $\quad E = 0$

Outside charged metal plate $\quad E = \frac{\sigma}{\varepsilon_0}$

The field is normally outwards for positively charged plate and inwards for negatively charged plates.

(iv) Electric field due to a charged spherical shell



The charge Q resides on the surface of the spherical shell (radius R).

(a) Field at outside point A

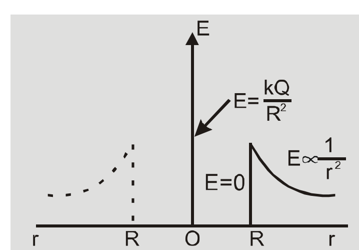
$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}$$

$$\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}}$$

(b) Field at surface point B

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}\hat{r}$$

$$\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}}$$

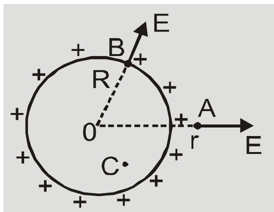


(c) Field at inside point C

$$\boxed{E = 0}$$

The variation of field with distance r from the centre O of the shell in shown in fig. The field at the surface is maximum. And outside the shell field varies as $E \propto 1/r^2$.

(v) Electric field due to charged conducting sphere



The entire charge Q resides on the surface of a charged conductor. Any charge given to the interior, flows to the surface in less than a nanosecond. So a charged

conducting sphere behaves like spherical shell. Thus,

(a) Field outside $E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$

(b) Field on surface $E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}$

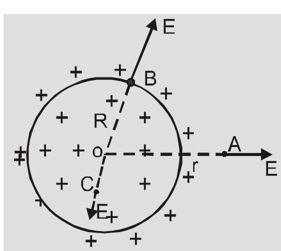
(c) Field inside $E = 0$

Special note : The surface charge density in the above case is $\sigma = Q/4\pi R^2$. In terms of σ the fields are

outside $E = \frac{\sigma}{\varepsilon_0}\left(\frac{R^2}{r^2}\right)$ on surface $E = \frac{\sigma}{\varepsilon_0}$

inside $E = 0$

(vi) Electric field due to a charged sphere (non conducting)



In case of a charged non conducting (plastic etc.) sphere, charges do not flow. As a result, charges exist inside the sphere as well as on the surface.

Assuming uniform charge distribution, the electric field, outside (point A), on the surfac (point B) and inside (point C) are as follows.

(a) Field outside

$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$ directed radially outwards (for positive Q)

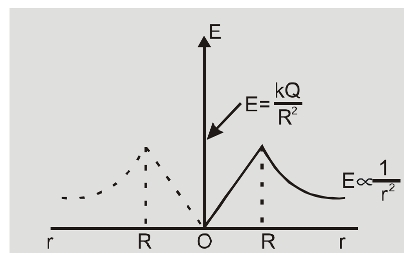
(b) Field on surface

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}$$

(c) Field inside

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^3}r$$

directed radially outwards for positive Q.



The variation of the electric field with distance r from the centre of the charged nonconducting sphere is as shown in fig. The field outside varies inversely as square of the distance. The field at the surface is maximum.

The field inside is directly proportional to the distance.

Special note : The volume charge density in above case, is $\rho = Q/(\frac{4}{3}\pi R^3)$. In terms of ρ, the field are

outside $E = \frac{\rho}{3\varepsilon_0}\left(\frac{R^3}{r^2}\right)$

on surface $E = \frac{\rho}{3\varepsilon_0}R$

inside $E = \frac{\rho}{3\varepsilon_0}r$

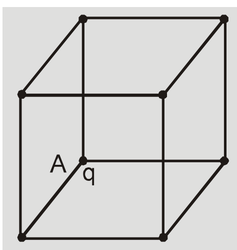
Solved Examples

Ex.18 If a point charge Q is located at the centre of a cube then find (i) flux through the total surface, (ii) flux through one surface.

Sol. (i) According to Gauss's law for closed surface $\phi = Q/\varepsilon_0$, (ii) Since cube is a symmetrical figure thus by symmetry the flux through each surface is $\phi' = Q/6\epsilon_0$

Ex.19 A point charge q is placed at a corner of a cube with side L. Find flux through entire surface and flux through each face.

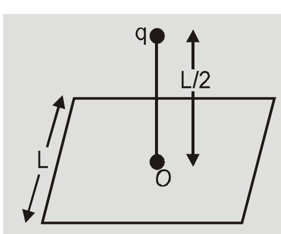
Sol. A corner of a cube can be supposed to be the centre of a big cube made up of 8 such cubes, therefore flux through it is $q/8\epsilon_0$. The direction of E is parallel to the three faces that pass through this face, thus flux through these is zero.



Flux through the other three faces $= \frac{1}{3}\left(\frac{q}{8\epsilon_0}\right) = \frac{q}{24\epsilon_0}$

Ex.20 A point charge +q is located L/2 above the centre of a square having side L. Find the flux through this square.

Sol. The charge q can be supposed to be situated at the centre of a cube having side L with outward flux ϕ. In this core the square is one of its face having flux $\phi/6$



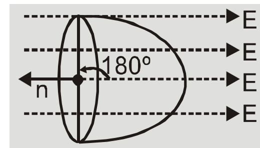
$\therefore$ flux through the square $= \frac{q}{6\varepsilon_0}$

Ex.21 According to the figure, a hemi-spherical object is located in an electric field. Find the outward flux through its curved surface.

Sol. Total outward flux $\phi = \phi_{CS} = \phi_n$

where ϕ_{CS} = flux through curved surface and

ϕ_n = flux through circular base



$\because$ No charge is associated with this surface

$\therefore \; \phi_{CS} = -\phi_n$

$\because$ curved surface perpendicular electric field

$\phi_n = E \times \pi R^2 \cos 180^\circ$

$= -E\,\pi R^2$

$\therefore \; \phi_{CS} = -\phi_n = E\pi R^2$

Ex.22 What is the value of electric flux in SI unit in Y-Z plane of area $2m^2$, if intensity of electric field is $\vec{E} = (5\hat{i} + 2\hat{j})$ N/C.

Sol. $\phi = \vec{E}.\vec{dA}$

$= (5\hat{i} + 2\hat{j}).2\hat{i}$

$= 10$

Ex.23 $2\mu C$ charge is in some Gaussion surface, given outwar flux ϕ, what addition charge is needed if we want that 6ϕ flux enters into the Gaussian surface.

Sol. According to question

$$\frac{(2+Q)\mu C}{\epsilon_0} = -6\phi$$

or $\frac{(2+Q)}{\epsilon_0} = -6\left(\frac{2}{\epsilon_0}\right)$

$Q = -14\mu C$