UNIT AND DIMENSION

1. PHYSICAL QUANTITY

The quantites which can be measured by an instrument and by means of which we can describe the laws of physics are called physical quantities.

Types of physical quantities :

- Fundamental
- Derived
- Supplementry

1.1 Fundamental

Although the number of physical quantities that we measure is very large, we need only a limited number of units for expressing all the physical quantities since they are interrelated with one another. So, certain physical quantities have been chosen arbitrarily and their units are used for expressing all the physical quantities, such quantities are known as **Fundamental, Absolute** or **Base Quantities** (such as length, time and mass in mechanics)

(i) All other quantites may be expressed in terms of fundamental quantities.

(ii) They are independent of each other and cannot be obtained from one another.

An international body named General Conference on Weights and Measures chose seven physical quantities as fundamental :

(1) length (2) mass (3) time (4) electric current,

(5) thermodynamic temperature (6) amount of substance

(7) luminous intensity.

Note : These are also called as absolute or base quantities.

In mechanics, we treat length, mass and time as the three basic or fundamental quantities.

1.2 Derived : Physical quantities which can be expressed as combination of base quantities are called as derived quantities.

For example : Speed, velocity, acceleration, force, momentum, pressure, energy etc.

Ex.1 $\textbf{\textit{Speed}} = \dfrac{\text{distance}}{\text{time}} = \dfrac{\text{length}}{\text{time}}$

1.3 Supplementary : Beside the seven fundamental physical quantities two supplementary quantities are also defined, they are :

(1) Plane angle (2) Solid angle.

Note : The supplementary quantities have only units but no dimensions.

2. MAGNITUDE :

Magnitude of physical quantity = (numerical value) × (unit)

Magnitude of a physical quantity is always constant. It is independent of the type of unit.

$\Rightarrow$ numerical value $\propto \dfrac{1}{\text{unit}}$

or $n_1u_1 = n_2u_2$ = constant

3. UNIT :

Measurement of any physical quantity is expressed in terms of an internationally accepted certain basic reference standard called unit.

The units for the fundamental or base quantities are called fundamental or base unit. Other physical quantities are expressed as combination of these base units and hence, called derived units.

A complete set of units, both fundamental and derived is called a system of unit.

Supplementary units :

(1) Plane angle : radian (rad)

(2) Solid angle : steradian (sr)

* The SI system is at present widely used throughout the world. In IIT JEE only SI system is followed.

Definitions of some important SI Units

(i) Metre : 1 m = 1,650, 763.73 wavelengths in vaccum, of radiation corresponding to organ-red light of krypton-86.

(ii) Second : 1 s = 9,192, 631,770 time periods of a particular from Ceasium - 133 atom.

(iii) Kilogram : 1kg = mass of 1 litre volume of water at 4°C

(iv) Ampere : It is the current which when flows through two infinitely long straight conductors of negligible cross-section placed at a distance of one metre in vacuum produces a force of 2×10^{-7} N/m between them.

(v) Kelvin : 1 K = 1/273.16 part of the thermodynamic temperature of triple point of water.

(vi) Mole : It is the amount of substance of a system which contains as many elementary particles (atoms, molecules, ions etc.) as there are atoms in 12g of carbon - 12.

(vii) Candela : It is luminous intensity in a perpendicular direction of a surface of $\left(\frac{1}{600000}\right)m^2$ of a black body at the temperature of freezing point under a pressure of 1.013×10^5 N/m^2.

(viii) Radian : It is the plane angle between two radiia of a circle which cut-off on the circumference, an arc equal in length to the radius.

(ix) Steradian : The steradian is the solid angle which having its vertex at the centre of the sphere, cut-off an area of the surface of sphere equal to that of a square with sides of length equal to the radius of the sphere.

4. DIMENSIONS

Dimensions of a physical quantity are the power to which the fundamental quantities must be raised to represent the given physical quantity.

For example, $\quad \text{density} = \frac{\text{mass}}{\text{volume}} = \frac{\text{mass}}{(\text{length})^3}$

or $\quad \text{density} = (\text{mass})\,(\text{length})^{-3} \qquad \ldots(i)$

Thus, the dimensions of density are 1 in mass and −3 in length. The dimensions of all other fundamental quantities are zero.

For convenience, the fundamental quantities are represented by one letter symbols. Generally mass is denoted by M, length by L, time by T and electric current by A.

The thermodynamic temperature, the amount of substance and the luminous intensity are denoted by the symbols of their units K, mol and cd respectively. The physical quantity that is expressed in terms of the base quantities is enclosed in square brackets.

$$[\sin\theta] = [\cos\theta] = [\tan\theta] = [e^x] = [M^0L^0T^0]$$

5. USE OF DIMENSIONS

Theory of dimensions have following main uses :

Conversion of units :

This is based on the fact that the product of the numerical value (n) and its corresponding unit (u) is a constant, i.e.,

$$n[u] = \text{constant}$$

or $\quad n_1[u_1] = n_2\,[u_2]$

Suppose the dimensions of a physical quantity are a in mass, b in length and c in time. If the fundamental units in one system are M_1, L_1 and T_1 and in the other system are M_2, L_2 and T_2 respectively. Then we can write.

$$n_1[M_1^a L_1^b T_1^c] = n_2[M_2^a L_2^b T_2^c] \quad ...(i)$$

Here n_1 and n_2 are the numerical values in two system of units respectively. Using Eq. (i), we can convert the numerical value of a physical quantity from one system of units into the other system.

Ex.1 ***The value of gravitation constant is G = 6.67 × 10^{-11} Nm^2/kg^2 in SI units. Convert it into CGS system of units.***

Sol. The dimensional formula of G is $[M^{-1} L^3 T^{-2}]$.

Using equation number (i), i.e.,

$$n_1[M_1^{-1} L_1^3 T_1^{-2}] = n_2[M_2^{-1} L_2^3 T_2^{-2}]$$

$$n_2 = n_1\left[\frac{M_1}{M_2}\right]^{-1}\left[\frac{L_1}{L_2}\right]^{3}\left[\frac{T_1}{T_2}\right]^{-2}$$

Here, $n_1 = 6.67 \times 10^{-11}$

$M_1 = 1$ kg, $M_2 = 1$ g $= 10^{-3}$ kg $L_1 = 1$ m, $L_2 = 1$cm $= 10^{-2}$ m, $T_1 = T_2 = 1$s

Substituting in the above equation, we get

$$n_2 = 6.67 \times 10^{-11}\left[\frac{1\text{kg}}{10^{-3}\text{kg}}\right]^{-1}\left[\frac{1\text{m}}{10^{-2}\text{m}}\right]^{3}\left[\frac{1\text{s}}{1\text{s}}\right]^{-2}$$

or $n_2 = 6.67 \times 10^{-8}$

Thus, value of G in CGS system of units is 6.67×10^{-8} dyne cm^2/g^2.

To check the dimensional correctness of a given physical equation :

Every physical equation should be dimensionally balanced. This is called the 'Principle of Homogeneity'. The dimensions of each term on both sides of an equation must be the same. On this basis we can judge whether a given equation is correct or not. But a dimensionally correct equation may or may not be physically correct.

Ex.2 ***Show that the expression of the time period T of a simple pendulum of length l given by T = $2\pi\sqrt{\frac{l}{g}}$ is dimensionally correct.***

Sol. $T = 2\pi\sqrt{\frac{l}{g}}$

Dimensionally $[T] = \sqrt{\frac{[L]}{[LT^{-2}]}} = [T]$

As in the above equation, the dimensions of both sides are same. The given formula is dimensionally correct.

Principle of Homogeneity of Dimensions.

This principle states that the dimensions of all the terms in a physical expression should be same. For example, in the physical expression $s = ut + \frac{1}{2}at^2$, the dimensions of s, ut and $\frac{1}{2}at^2$ all are same.

Limitations of Dimensional Analysis

The method of dimensions has the following limitations :

(i) By this method the value of dimensionless constant can not be calculated.

(ii) By this method the equation containing trigonometrical, exponential and logarithmic terms cannot be analysed.

(iii) If a physical quantity depends on more than three factors, then relation among them cannot be established because we can have only three equations by equalising the powers of M, L and T.

ERROR'S

Systematic errors

The systematic errors are those errors that tend to be in one direction, either positive or negative. Some of the sources of systematic errors are :

(a) Instrumental errors that arise from the errors due to imperfect design or calibration of the measuring instrument, zero error in the instrument, etc.

(c) Personal errors that arise due to an individual's bias, lack of proper setting of the apparatus or individual's carelessness in taking observations without observing proper precautions, etc. Systematic errors can be minimised by improving experimental techniques, selecting better instruments and removing personal bias as far as possible.

Random errors

The random errors are those errors, which occur irregularly and hence are random with respect to sign and size. These can arise due to random and unpredictable fluctuations in experimental conditions

Least count error

The smallest value that can be measured by the measuring instrument is called its least count. All the readings or measured values are good only up to this value. The least count error is the error associated with the resolution of the instrument.

The magnitude of the difference between the true value of the quantity and the individual measurement value is called the absolute error of the measurement. This is denoted by $|\Delta a|$. In absence of any other method of knowing true value, we considered arithmatic mean as the true value. Then the errors in the individual measurement values are

$\Delta a_1 = \text{amean} - a_1,$

$\Delta a_2 = \text{amean} - a_2,$

....

....

$\Delta a_n = a_{mean} - a_n$

The Δa calculated above may be positive in certain cases and negative in some other cases. But absolute error |?a| will always be positive.

(b) The arithmetic mean of all the absolute errors is taken as the final or mean absolute error of the value of the physical quantity a. It is represented by ?amean.

Thus,

Δa_{mean} = ($|\Delta a_1|+|\Delta a_2|+|\Delta a_3|+...+|_? a_n|$)/n $|\Delta a_i|$/n

If we do a single measurement, the value we get may be in the range amean ± ?amean

i.e. a = amean ± ?amean

or,$a_{mean} - \Delta a_{mean} \leq a\Delta \leq a_{mean} + \Delta a_{mean}$

This implies that any measurement of the physical quantity a is likely to lie between ($a_{mean}+ \Delta a_{mean}$) and ($a_{mean} - \Delta a_{mean}$).

(c) Instead of the absolute error, we often use the relative error or the percentage error (δa). The relative error is the ratio of the mean absolute error Δa_{mean} to the mean value amean of the quantity measured. Relative error = $\Delta a_{mean}/a_{mean}$ When the relative error is expressed in percent, it is called the percentage error (δa).

Thus, Percentage error $\delta a = (\Delta a_{mean}/a_{mean}) \times 100\%$ Let us now consider an example.

Combination of Errors

(a) Error of a sum or a difference

Suppose two physical quantities A and B have measured values A ± ΔA, B ± ΔB respectively where ΔA and ΔB are their absolute errors. We wish to find the error ΔZ in the sum Z = A + B. We have by addition, Z ± ΔZ = (A ± ΔA) + (B ± ΔB).

The maximum possible error in Z ΔZ = ΔA + ΔB For the difference Z = A – B,

we have

$Z \pm \Delta Z = (A \pm \Delta A) - (B \pm \Delta B)$

$= (A - B) \pm \Delta A \pm \Delta B$

or, $\pm \Delta Z = \pm \Delta A \pm \Delta B$

The maximum value of the error ΔZ is again ΔA + ΔB.

(b) Error of a product or a quotient

Suppose Z = AB and the measured values of A and B are A ± ΔA and B ± ΔB. Then Z ± ΔZ = (A ± ΔA) (B ± ΔB) = AB ± B ΔA ± A ΔB ± ΔA ΔB.

Dividing LHS by Z and RHS by AB we have, 1±(ΔZ/Z) = 1 ± (ΔA/A) ± (ΔB/B) ± (ΔA/A)(ΔB/B). Since ΔA and ΔB are small, we shall ignore their product.

Hence the maximum relative error

$\Delta Z/ Z = (\Delta A/A) + (\Delta B/B)$.

(c) Error in case of a measured quantity raised to a power

Suppose $Z = A^2$,

Then,

$\Delta Z/Z = (\Delta A/A) + (\Delta A/A) = 2 (\Delta A/A)$. Hence, the relative error in A2 is two times the error in A.

In general, if $Z = A^p B^q/C^r$ Then,

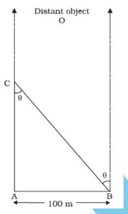
$\Delta Z/Z = p (\Delta A/A) + q (\Delta B/B) + r (\Delta C/C)$.

SOLVED EXAMPLE

Ex.1 Calculate the angle of (a) 1º (degree) (b) 1' (minute of arc or arcmin) and (c) 1" (second of arc or arc second) in radians. Use 360º = 2π rad, 10 = 60' and 1' = 60"

Ans. (a) We have 360º = 2π rad 1º = (π /180) rad = 1.745 × 10^{-2} rad (b) 1º = 60' = 1.745 × 10^{-2} rad 1' = 2.908×10^{-4} rad ; 2.91 × 10^{-4} rad (c) 1' = 60" = 2.908 × 10^{-4} rad 1" = 4.847 × 10^{-4} rad ; 4.85 × 10^{-6} rad

Ex.2 A man wishes to estimate the distance of a nearby tower from him. He stands at a point A in front of the tower C and spots a very distant object O in line with AC. He then walks perpendicular to AC up to B, a distance of 100 m, and looks at O and C again. Since O is very distant, the direction BO is practically the same as AO; but he finds the line of sight of C shifted from the original line of sight by an angle θ = 40º (θ is known as 'parallax') estimate the distance of the tower C from his original position A.



Ans. We have, parallax angle θ = 400 From Fig. 2.3, AB = AC tan θ AC = AB/tanθ = 100 m/tan 40º = 100 m/0.8391 = 119 m

Ex.3 The moon is observed from two diametrically opposite points A and B on Earth. The angle θ subtended at the moon by the two directions of observation is 1º 54'. Given the diameter of the Earth to be about 1.276 × 10^7 m, compute the distance of the moon from the Earth.

Ans. We have ? = 1º 54' = 114'

= (114 × 60)" × (4.85 ×10^{-6}) rad

= 3.32 × 10^{-2} rad,

since 1" = 4.85 ×10^{-6} rad. Also b = AB =1.276 × 10^7m Hence from Eq. (2.1), we have the earth-moon distance, D = b /θ

$$= \frac{1.276 \times 10^{7}}{3.32 \times 10^{-2}} = 3.84 \times 108\text{m}$$

Ex.4 The Sun's angular diameter is measured to be 1920''. The distance D ofthe Sun from the Earth is 1.496 × 10^{11} m. What is the diameter of the Sun ?

Ans. Sun's angular diameter α

= 1920"

= 1920 × 4.85 × 10^{-6} rad

= 9.31 × 10^{-3} rad

Sun's diameter

d = αD

= (9.31 × 10^{-3}) × (1.496 × 10^{11}) m

= 1.39 ×10^9 m

Ex.5 If the size of a nucleus (in the range of 10^{-15} to 10^{-14} m) i s scaled up to the tip of a sharp pin, what roughly is the size of an atom ? Assume tip of the pin to be in the range 10^{-5} m to 10^{-4} m.

Ans. The size of a nucleus is in the range of 10^{-15} m and 10^{-14} m. The tip of a sharp pin is taken to be in the range of 10–5 m and 10–4 m. Thus we are scaling up by a factor of 10^{10}. An atom roughly of size 10^{-10} m will be scaled up to a size of 1 m. Thus a nucleus in an atom is as small in size as the tip of a sharp pin placed at the centre of a sphere of radius about a metre long.

Ex.6 The temperatures of two bodies measured by a thermometer are t_1 = 20 ºC ± 0.5 ºC and t_2 = 50 ºC ± 0.5 ºC. Calculate the temperature difference and the error theirin.

Ans. t' = t_2– t_1 = (50 ºC±0.5 ºC) – (20ºC ± 0.5 ºC)
t' = 30 ºC ± 1 ºC

Ex.7 The resistance R = V/I where V = (100 ± 5)V and I = (10 ± 0.2)A. Find the percentage error in R.

Ans. The psercentage error in V is 5% and in I it is 2%. The total error in R would therefore be
5% + 2% = 7%.

Ex.8 5.74 g of a substance occupies 1.2 cm³. Express its density bykeeping the significant figures in view.

Ans. There are 3 significant figures in the measured mass whereas there are only 2 significant figures in the measured volume. Hence the density should be expressed to only 2 significant figures.

$$\text{Density} = \frac{5.74}{1.2}\ \text{g cm}^{-3} = 4.8\ \text{g cm}^{-3}$$

Exercise - I — UNSOLVED PROBLEMS

Q.1 Fill in the blanks
(a) The volume of a cube of side 1 cm is equal to ... m^3
(b) The surface area of a solid cylinder of radius 2.0cm, and height 10.0 cm is equal to .... $(mm)^2$
(c) A vehicle moving with a speed of 18 km, h^{-1} covers ....m in 1s
(d) The relative density of lead is 11.3. Its density is ..... g cm^{-3} or ..... kg m^{-3}.

Q.2 Fill in blanks by suitable conversion of units.
(a) 1 kg m^2 s^{-2} = ... g cm^2 s^{-2}
(b) 1 m = ... 1 y (c) 3 m s^{-2} = ... km h^{-2}
(d) G = 6.67 × 10^{-11} N m^2 $(kg)^{-2}$ = ... $(cm)^3$ s^{-2} g^{-1}.

Q.3 A calorie is a unit of heat or energy and it equals about 4.2 J where 1 J = 1 kg m^2 s^{-2} . Suppose we employ a system of units in which the unit of mass equals α kg, the unit of length equals β m, the unit of time is γ s. Show that a calorie has a magnitude 4.2 α^{-1} β^{-2} γ^2 in terms of the new units.

Q.4 Explain this statement clearly :
"To call a dimensional quantity ''large' or 'small' is meaningless without specifying a standard for comparison ". In view of this, reframe the following statements wherever necessary :
(a) atoms are very small objects
(b) a jet plane moves with great speed
(c) the mass of Jupiter is very large
(d) the air inside this room contains a large number of molecules
(e) a proton is much more massive than an electron
(f) the sped of sound is much smaller than the speed of light.

Q.5 A new unit of length is chosen such that the speed of light in vacuum is unity. What is the distance between the Sun and the Earth in terms of the new unit if light takes 8 min and 20 s to cover this distance ?

Q.6 which of the following is the most precise device for measuring length :
(a) a vernier callipers with 20 divisions on the sliding scale
(b) a screw gauge of pitch 1 mm and 100 divisions on the circular scale
(c) an optical instrument that can measure length to with within a wavelength of light ?

Q.7 A student measures the thickness of a human hair by looking at it through a microscope of magnification 100. He makes 20 observations and finds that the average width of the hair in the field of view of the microscope is 3.5 mm. What is the estimate on the thickness of hair ?

Q.8 Answer the following :
(A) You are given a thread and a metre scale. How will you estimate the diameter of the thread ?
(b) A screw gauge has a pitch of 1.0 mm and 200 divisions of the circular scale. Do you think it is possible to increase the accuracy of the screw gauge arbitrarily by increasing the number of divisions on the circular scale ?
(c) The mean daimeter of a thin brass rod is to be measured by vernier callipers. Why is a set of 100 measurements of the diameter expected yield a more reliable estimate than a set of 5 measurements only

Q.9 The photograph of a house occupation area of 1.75 cm^2 on a 35 mm slide. The slide is projected onto a screen, and the area of the house on the screen is 1.55 m^2. What is the linear magnification of the projector-screen arrangement.

Q.10 State the number of significant figures in the following :
(a) 0.007 m^2 (b) 2.64 × 10^{24} kg
(c) 0.2370 g cm^{-3} (d) 6.320 J
(e) 6.032 N m^{-2} (f) 0.0006032 m^2

Q.11 The length, breadth and thickness of a rectangular sheet of metal are 4.234 m, 1.005 m, and 2.01 cm respectively. Given the area and volume of the sheet to correct significant figures.

Q.12 The mass of a box measured by a grocer's balance is 2.3 kg. Two gold pieces of masses 20.15 g and 20.17 g are added to the box. What is (1) the total mass of the box, (b) the difference in the masses of the pieces to correct significant figures

Q.13 A physical quantity related to four observables a, b, c and d as follows :

$$P = a^3 b^2 / (\sqrt{cd})$$

The percentage errors of measurement in a, b, c and d are 1%, 3%, 4% and 2% respectively. What is the percentage error in the quantity P ?

Q.14 A book with many printing errors contains four different formulas for the displacement y of a particle undergoing a certain periodic motion :
(a) $y = a \sin 2\pi t/T$
(b) $y = a \sin vt$
(c) $y = (a/T) \sin t/a$
(d) $y = (a\sqrt{2})(\sin 2\pi t/T + \cos 2\pi t/T)$

(a) a = maximum displacement of the particle, v = speed of the particle. T = time-period of motion). Rule out the wrong formulas on dimensional grounds.

Q.15 A famous relation in physics relates 'moving mass' m to the 'rest mass' m_0 of a particle in terms of its speed v and the speed of light, c. (This relation first arose as a consequence of special relativity due to Albert Einstein). A boy recalls the relation almost correctly but forget where to put the constant c. He writes : $m = \frac{m_0}{(1-v^2)^{1/2}}$ Guess where to put the missing c.

Q.16 A man walking briskly in rain with speed v must slant his umbrella forward making an angle (with the vertical. A student derives the following relation between θ and v and checks that the relation has a correct limit : as $v \to 0$,($\theta \to 0$, as expected. (We are assuming there is not strong wind and that the rain falls vertically for a stationary man). Do you think this relation can be correct ? If not, guess the correct relation.

Q.17 It is claimed that two cesium clocks, if allowed to run for 100 years, free from any disturbance, may differ by only about 0.02 s. What does this imply for the accuracy of the standard cesium clock in measuring a time-interval of 1 s ?

Q.18 The unit of length convenient on the atomic scale is known as an angstrom and is denoted by Å : 1 Å = 10^{-10} m. The size of a hydrogen atom is about 0.5 Å. What is the total atomic volume in m^3 of a mole of hydrogen atoms ?

Q.19 One mole of an ideal gas at standard temperature and pressure occupies 22.4 L (molar volume). What is the ratio of molar volume to the atomic volume of a mole of hydrogen ? (Take the size of hydrogen molecule to be about 1 Å). Why is this ratio so large ?

Q.20 estimate the average mass density of a sodium atom assuming its size to be about 2.5 Å. (Use the known values of Avogadro's number and the atomic mass of sodium). Compare it with the density of sodium in its crystalline phase : 970 kg m^{-3}. Are the two densities of the same order of magnitude ? If so, why ?

Q.21 The unit of length convenient on the nuclear scale is a fermi : 1 f = 10^{-15} m. Nuclear sizes obey roughly the following empirical relation :

$$r = r_0 A^{1/3}$$

where r is the radius of the nucleus, A its mass number, and r_0 is a constant equal to about, 1.2 f. Show that the rule implies that nuclear mass density is nearly constant for different nuclei. Estimate the mass density of sodium nucleus. Compare it with the average mass density of a sodium atom obtained in 2.20.

Q.22 Explain this common observation clearly : If you look out of the window of a fast moving train, the nearby trees, houses etc. Seem to move rapidly in a direction opposite to the train's motion, but the distant objects (hill tops, the Moon, the stars etc.) seem to the train's motion, but the distant objects (hill tops, the Moon, the stars etc.) seem to be stationary. In fact, since you are aware that you are moving, these distance object seem to move with you.).

Q.23 The principle of 'parallax' in section 2.3.1 is used in the determination of distances of very distant stars. The base line AB is the line joining the Earth's two locations six months apart in its orbit around the Sun. That is, the base line is about the diameter of the Earth's orbit $\approx 3 \times 10^{11}$ m. However, even the nearest stars are so distant that with such a long base line, they show parallax only of the order of 1 " (second) of arc or so. A parsec is a convenient unit of length on the astronomical scale. It is the distance of an object that will show a parallax of 1" (second) of arc from opposite ends of a baseline equal to the distance from the Earth to the Sun. How much is a parsec in terms of metres

Q.24 The nearest star tour solar system is 4.29 light years away. How much is this distance in terms of parsecs ? How much parallax would this star (named Alpha Centauri) show when viewed from two locations of the Earth six months apart in its orbit around the Sun ?

Q.25 A LASER is a source of very intense, monochromatic, and unidirectional beam of light. These properties of a laser light can be exploited to measure long distances. The distance of the Moon from the Earth has been already determined very precisely using a laser as a source of light. A laser light beamed at the Moon takes 2.46 s to return after reflection at the Moon's surface. How much is the radius of the lunar orbit around the Earth ?

Q.26 A SONAR (sound navigation and ranging) used ultrasonic waves to detect and locate objects under water. In a submarine equipped with a SONAR the time delay between generation of a probe wave and the reception of its echo after reflection from an enemy submarine is found to be 77.0 s. What is the distance of the enemy submarine ? (Speed of sound in water = 1450 m s^{-1}).

Q.27 The farthest objects in our Universe discovered by modern astronomers are so distant that light emitted by them takes billions of years to reach the Earth. These objects (known as quasars) have many puzzling features, which have not yet been satisfactorily explained. What is the distance in km of quasar from which light takes 3.0 billion years to reach us ?

Q.28 Precise measurements of physical quantities are a need of modern times. For example, to ascertain the speed of an enemy fighter plane, one must have an accurate method to find its positions at closely separated instants of time. Only then we can hope to shell it by an antiaircraft gun. This was the actual motivation behind the discovery of radar in World War II. Think of different examples in modern science where precision measurements of length, time, mass etc. are needed Also, wherever you can, give a quantitative idea of the precision needed.

Q.29 Just as precise measurements are necessary in science, it is equally important to be able to make rough estimates of quantities using rudimentary ideas and common observations. Think of ways by which you can estimate the following (where an estimate is difficult to obtain, try to get an upper bound on the quantity) :
(a) the total mass of rain-bearing clouds over India during the Monsoon
(b) the mass of an elephant
(c) the wind speed during a storm
(d) the number of stands of hair on your head
(e) the number of air molecules in your classroom.

Q.30 The Sun is a hot plasma (ionized matter) with its inner core at a temperature exceeding 10^7 K, and its outer surface at a temperature, of about 6000 K. At these high temperatures, no substance remains in a solid or liquid phase. In what range do you expect the mass density of the Sun to be In the range of densities of solids and liquids or gases ? Check if your guess is correct from the following data : mass of the Sun = 2.0×10^{30} kg, radius of the sun = 7.0×10^8 m.

Q.31 When the planet Jupiter is at a distance of 824.7 million kilometers from the Earth, its angular diameter is measured to be 35.72" of arc. Calculate the diameter of Jupiter.

Q.32 Assuming that the orbit of the planet Mercury around the Sun to be a circle, copernicus determined the orbital radius to be 0.38 AU. From this, determine the angle of maximum elongation for Mercury and its distance from the Earth when the elongation is maximum.

Q.33 Suppose there existed a planet that went around the Sun twice as fast as the Earth. What would be its orbital size as compared to that of the Earth ?

Q.34 Io, one of the satellites of Jupiter has an orbital period of 1.769 days and the radius of the orbit is 4.22×10^8 m. Show that the mass of Jupiter is about one-thousandth that the Sun.

Q.35 It is a well known fact that during a total solar eclipse the disk of the moon almost completely covers the disk of the Sun. From the fact and from the information you can gather from examples 2.1 and 2.2, determine the approximate diameter of the moon.

Q.36 Suppose the Sunsharnk from its present size so that its radius is halved. What would be the change in tis gravitational potential energy (Calculate the actual number in joules).

Q.37 Let us assume that our galaxy consists of 2.5 $\times 10^{11}$ stars each of one solar mass. How long will a star at a distance of 50,000 ly from the galactic centre take to complete one revolution ? Take the diameter of the Milky Way to be 10^5 ly.

Q.38 A great physicist of this century (P. A.M. Dirac) loved playing with numerical values of Fundamental constants of nature. This led him to an interesting observation. Dirac found that from basic constants of atomic physics (c,e, mass of electron. mass of proton) and the gravitational constant G, he could arrive at a number with the dimension of time. Further, it was a very large number, its magnitude being close to the present estimate on the age of the universe (~ 15 billion years). From the table of fundamental constants in this book, try to see if you too can construct this number (or any other interesting number you can think of). If its coincidence with the age of the universe were significant, what would this imply for the constancy of fundamental constants ?