SET, RELATIONS AND FUNCTIONS

1. SET THEORY

Collection of well defined objects which are distinct and distinguishable. A collection is said to be well defined if each and every element of the collection has some definition.

Notation of a set : Sets are denoted by capital letters like A, B, C or { } and the entries within the bracket are known as elements of set.

Cardinal number of a set : Cardinal number of a set X is the number of elements of a set X and it is denoted by

n(X) e.g. X = $[x_1, x_2, x_3]$ ∴ n(X) = 3

2. REPRESENTATION OF SETS

Set Listing Method (Roster Method) :

In this method a set is described by listing all the elements, separated by commas, within braces { }

Set builder Method (Set Rule Method) :

In this method, a set is described by characterizing property P(x) of its elements x. In such case the set is described by {x : P(x) holds} or {x | P(x) holds}, which is read as the set of all x such that P(x) holds. The symbol '|' or ':' is read as such that.

3. TYPE OF SETS

Finite set :

A set X is called a finite set if its element can be listed by counting or labeling with the help of natural numbers and the process terminates at a certain natural number n. i.e. n(X) = finite no. eg (a) A set of English Alphabets (b) Set of soldiers in Indian Army

Infinite set :

A set whose elements cannot be listed counted by the natural numbers (1, 2, 3.......n) for any number n, is called a infinite set. e.g.

(a) A set of all points in a plane

(b) X = $\{x : x \in R, 0 < x < 0.0001\}$

(c) X = $\{x : x \in Q, 0 \le x \le 0.0001\}$

Singleton set :

A set consisting of a single element is called a singleton set. i.e. n (X) = 1,
e.g. $\{x : x \in N, 1 < x < 3\}$, {{}} : Set of null set, {ϕ} is a set containing alphabet ϕ.

Null set :

A set is said to be empty, void or null set if it has no element in it, and it is denoted by ϕ. i.e. X is a null set if n(X) = 0. e.g. : $\{x : x \in R \text{ and } x^2 + 2 = 0\}$, $\{x : x > 1 \text{ but } x < 1/2\}$,
$\{x : x \in R, x^2 < 0\}$

Equivalent Set :

Two finite sets A and B are equivalent if their cardinal numbers are same i.e. n(A) = n(B).

Equal Set :

Two sets A and B are said to be equal if every element of A is a member of B and every element of B is a member of A. i.e. A = B, if A and B are equal and A ≠ B, if they are not equal.

4. UNIVERSAL SET

It is a set which includes all the sets under considerations i.e. it is a super set of each of the given set. Thus, a set that contains all sets in a given context is called the universal set. It is denoted by U. e.g. If A = {1, 2, 3}, B = {2, 4, 5, 6} and C = {1, 3, 5, 7}, then U = {1, 2, 3, 4, 5, 6, 7} can be taken as the universal set.

5. DISJOINT SET

Sets A and B are said to be disjoint iff A and B have no common element or $A \cap B = \phi$. If $A \cap B \neq \phi$ then A and B are said to be intersecting or overlapping sets.

e.g. : (i) If A = {1, 2, 3}, B = {4, 5, 6} and C = {4, 7, 9} then A and B are disjoint set where B and C are intersecting sets. **(ii)** Set of even natural numbers and odd natural numbers are disjoint sets.

6. COMPLEMENTARY SET

Complementary set of a set A is a set containing all those elements of universal set which are not in A. It is denoted by $\overline{A}$, A^C or A′. So $A^C = \{x : x \in U \text{ but } x \notin A\}$. e.g. If set A = {1, 2, 3, 4, 5} and universal set U = {1, 2, 3, 4,.......50} then

$\overline{A}$ = {6, 7, ...50}

NOTE :

All disjoint sets are not complementary sets but all complementary sets are disjoint.

7. SUBSET

A set A is said to be a subset of B if all the elements of A are present in B and is denoted by $A \subset B$ (read as A is subset of B) and symbolically written as :

$x \in A \Rightarrow x \in B \Leftrightarrow A \subset B$

Number of subsets :

Consider a set X containing n elements as $\{x_1, x_2, \ldots\ldots, x_n\}$ then the total number of subsets of X = 2^n

Proof : Number of subsets of above set is equal to the number of selections of elements taking any number of them at a time out of the total n elements and it is equal to 2^n

$\because \quad {}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots\ldots + {}^nC_n = 2^n$

Types of Subsets :

A set A is said to be a **proper subset** of a set B if every element of A is an element of B and B has at least one element which is not an element of A and is denoted by $A \subset B$.

The set A itself and the empty set is known as **improper subset** and is denoted as

$A \subseteq B$.

e.g. If X = $\{x_1, x_2, \ldots, x_n\}$ then total number of proper sets = $2^n - 2$ (excluding itself and the null set). The statement $A \subset B$ can be written as $B \supset A$, then B is called the **super set** of A and is written as $B \supset A$.

8. POWER SET

The collection of all subsets of set A is called the power set of A and is denoted by P(A)

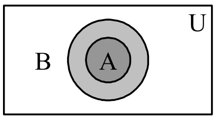
i.e. P(A) = {x : x is a subset of A}. If X = $\{x_1, x_2, x_3, \ldots\ldots\ldots x_n\}$ then

$n(P(X)) = 2^n$; $n(P(P(x))) = 2^{2^n}$.

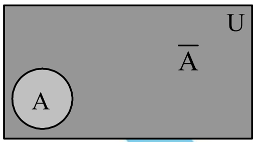
9. VENN (EULER) DIAGRAMS :

The diagrams drawn to represent sets are called Venn diagram or Euler-Venn diagrams. Here we represents the universal U as set of all points within rectangle and the subset A of the set U is represented by the interior of a circle. If a set A is a subset of a set B, then the circle representing A is drawn inside the circle representing B. If A and B are not equal but they have some common elements, then to represent A and B by two intersecting circles.

e.g. If A is subset of B then it is represented diagrammatically in fig.



e.g. If A is a set then the complement of A is represented in fig.



10. OPERATIONS ON SETS :

Union of sets :

If A and B are two sets then union ($\cup$) of A and B is the set of all those elements which belong either to A or to B or to both A and B. It is also defined as $A \cup B = \{x : x \in A \text{ or } x \in B\}$. It is represented through Venn diagram in fig.1 & fig.2

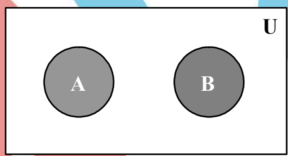


Fig. (1)

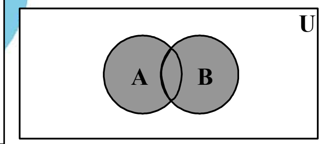
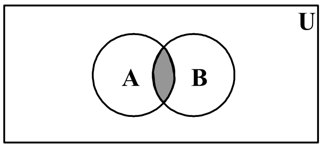


Fig. (2)

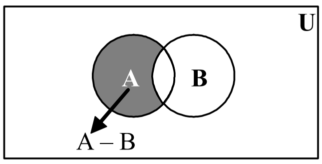
Intersection of sets :

If A and B are two sets then intersection ($\cap$) of A and B is the set of all those elements which belong to both A and B. It is also defined as $A \cap B = \{x : x \in A \text{ and } x \in B\}$ represented in Venn diagram (see fig.)



Difference of two sets :

If A and B are two sets then the difference of A and B, is the set of all those elements of A which do not belong to B.



Thus, $A - B = \{x : x \in A \text{ and } x \notin B\}$

or $A - B = \{x \in A ; x \notin B\}$

Clearly $x \in A - B \Leftrightarrow x \in A$ and $x \notin B$

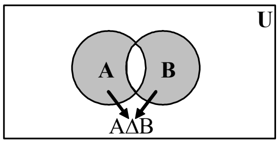
It is represented through the Venn diagrams.

Symmetric difference of two sets :

Set of those elements which are obtained by taking the union of the difference of A & B is (A – B) & the difference of B & A is (B – A), is known as the symmetric difference of two sets A & B and it is denoted by $(A \Delta B)$.

Thus $A \Delta B = (A - B) \cup (B - A)$

Representation through the venn diagram is given in the fig.



11. NUMBER OF ELEMENTS IN DIFFERENT SETS

If A, B & C are finite sets and U be the finite universal set, then

(i) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

(ii) $n(A \cup B) = n(A) + n(B)$ (if A & B are disjoint sets)

(iii) $n(A - B) = n(A) - n(A \cap B)$

(iv) $n(A \Delta B) = n[(A - B) \cup (B - A)]$
$= n(A) + n(B) - 2n(A \cap B)$

(v) $n(A \cup B \cup C)$
$= n(A) + n(B) + n(C) - n(A \cap B)$
$- n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

(vi) $n(A' \cup B') = n(A \cap B)' = n(U) - n(A \cap B)$

(vii) $n(A' \cap B') = n(A \cup B)' = n(U) - n(A \cup B)$

12. CARTESIAN PRODUCT OF TWO SETS

Cartesian product of A to B is a set containing the elements in the form of ordered pair (a, b) such that $a \in A$ and $b \in B$. It is denoted by $A \times B$.

i.e. $A \times B = \{(a, b) : a \in A \text{ and } b \in B\}$
$= \{(a_1, b_1),(a_1, b_2),(a_2, b_1),(a_2, b_2),(a_3, b_1),(a_3, b_2)\}$

If set $A = \{a_1, a_2, a_3\}$ and $B = \{b_1, b_2\}$ then $A \times B$ and $B \times A$ can be written as :

$A \times B = \{(a, b) : a \in A \text{ and } b \in B$ and
$B \times A = \{(b, a) ; b \in B \text{ and } a \in A\}$

Clearly $A \times B \neq B \times A$ until A and B are equal

Note :

1. If number of elements in A : n(A) = m and n(B) = n then number of elements in $(A \times B) = m \times n$

2. Since $A \times B$ contains all such ordered pairs of the type (a, b) such that $a \in A$ & $b \in B$, that means it includes all possibilities in which the elements of set A can be related with the elements of set B. Therefore, $A \times B$ is termed as largest possible relation defined from set A to set B, also known as universal relation from A to B.

13. ALGEBRAIC OPERATIONS ON SETS

Idempotent operation :

For any set A, we have (i) $A \cup A = A$ and (ii) $A \cap A = A$

Proof :

(i) $A \cup A = \{x : x \in A \text{ or } x \in A\}$
$= \{x : x \in A\} = A$

(ii) $A \cap A = \{x : x \in A \text{ \& } x \in A\}$
$= \{x : x \in A\} = A$

Identity operation :

For any set A, we have

(i) $A \cup \phi = A$ and

(ii) $A \cap U = A$ i.e. ϕ and U are identity elements for union and intersection respectively

Proof :

(i) $A \cup \phi = \{x : x \in A \text{ or } x \in \phi\}$
$= \{x : x \in A\} = A$

(ii) $A \cap U = \{x : x \in A \text{ and } x \in U\}$
$= \{x : x \in A\} = A$

Commutative operation :

For any set A and B, we have

(i) $A \cup B = B \cup A$ and (ii) $A \cap B = B \cap A$

i.e. union and intersection are commutative.

Associative operation :

If A, B and C are any three sets then

(i) $(A \cup B) \cup C = A \cup (B \cup C)$

(ii) $(A \cap B) \cap C = A \cap (B \cap C)$

i.e. union and intersection are associative.

Distributive operation :

If A, B and C are any three sets then

(i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

i.e. union and intersection are distributive over intersection and union respectively.

De-Morgan's Principle :

If A and B are any two sets, then

(i) $(A \cup B)' = A' \cap B'$

(ii) $(A \cap B)' = A' \cup B'$

Proof : (i) Let x be an arbitrary element of $(A \cup B)'$. Then $x \in (A \cup B)' \Rightarrow x \notin (A \cup B)$
$\Rightarrow x \notin A$ and $x \notin B$ $\Rightarrow x \in A' \cap B'$

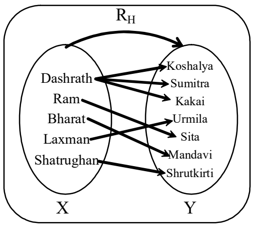
Again let y be an arbitrary element of $A' \cap B'$. Then $y \in A' \cap B'$
$\Rightarrow y \in A'$ and $y \in B'$ $\Rightarrow y \notin A$ and $y \notin B$
$\Rightarrow y \notin (A \cup B)$ $\Rightarrow y \in (A \cup B)'$
$\therefore A' \cap B' \subseteq (A \cup B)'$. Hence $(A \cup B)' = A' \cap B'$

Similarly (ii) can be proved.

14. RELATION

A relation R from set X to Y ($R : X \to Y$) is a correspondence between set X to set Y by which some or more elements of X are associated with some or more elements of Y. Therefore a relation (or binary relation) R, from a non-empty set X to another non-empty set Y, is a subset of X × Y. i.e. $R_H : X \to Y$ is nothing but subset of A × B.

e.g. Consider a set X and Y as set of all males and females members of a royal family of the kingdom Ayodhya X = {Dashrath, Ram, Bharat, Laxman, shatrughan} and Y = {Koshaliya, Kakai, sumitra, Sita, Mandavi, Urmila, Shrutkirti} and a relation R is defined as "was husband of "from set X to set Y.



Then R_H = {(Dashrath, Koshaliya), (Ram, sita), (Bharat, Mandavi), (Laxman, Urmila), (Shatrughan, Shrutkirti), (Dashrath, Kakai), (Dashrath, Sumitra)}

Note :

(i) If a is related to b then symbolically it is written as a R b where a is pre-image and b is image

(ii) If a is not related to b then symbolically it is written as a $\not R$ b.

Domain, Co-domain & Range of Relation

Domain : of relation is collection of elements of the first set which are participating in the correspondence i.e. it is set of all pre-images under the relation R. e.g. Domain of R_H : {Dashrath, Ram, Bharat, Laxman, Shatrughan}

Co-Domain : All elements of set Y irrespective of whether they are related with any element of X or not constitute co-domain. e.g. Y= {Koshaliya, Kakai, Sumitra, Sita, Mandavi, Urmila, Shrutkirti} is co-domain of R_H.

Range : of relation is a set of those elements of set Y which are participating in correspondence i.e. set of all images. Range of R_H : {Koshaliya, Kakai, Sumitra, Sita, Mandavi, Urmila, Shrutkirti}.

15. TYPES OF RELATIONS

Reflexive Relation

R : X → Y is said to be reflexive iff x R x ∀ x ∈ X. i.e. every element in set X, must be a related to itself therefore ∀ x ∈ X; (x, x) ∈ R then relation R is called as reflexive relation.

Identity Relation :

Let X be a set. Then the relation I_x = {(x, x) : x ∈ X} on X is called the identity relation on X. i.e. a relation I_x on X is identity relation if every element of X related to itself only. e.g. y = x

Note : All identity relations are reflexive but all reflexive relations are not identity.

Symmetric Relation

R : X → Y is said to be symmetric iff (x, y) ∈ R ⇒ (y, x) ∈ R for all (x, y) ∈ R i.e. x R y ⇒ y R x for all (x, y) ∈ R. e.g. perpendicularity of lines in a plane is symmetric relation.

Transitive Relation

R : X → Y is transitive iff (x, y) ∈ R and (y, z) ∈ R ⇒ (x, z) ∈ R for all (x, y) and (y, z) ∈ R. i.e. x R y and y R z ⇒ x R z. e.g. The relation "being sister of " among the members of a family is always transitive.

Note :

(i) Every null relation is a symmetric and transitive relation.

(ii) Every singleton relation is a transitive relation.

(iii) Universal and identity relations are reflexive, symmetric as well as transitive.

Anti-symmetric Relation

Let A be any set. A relation R on set A is said to be an antisymmetric relation iff (a, b) ∈ R and (b, a) ∈ R ⇒ a = b for all a, b ∈ A e.g. Relations "being subset of "; "is greater than or equal to" and "identity relation on any set A" are antisymmetric relations.

Equivalence Relation

A relation R from a set X to set Y (R : X → Y) is said to be an equivalence relation iff it is reflexive, symmetric as well as transitive. The equivalence relation is denoted by ~ e.g. Relation "is equal to" Equality, Similarity and congruence of triangles, parallelism of lines are equivalence relation.

16. INVERSE OF A RELATION

Let A, B be two sets and let R be a relation from a set A to B. Then the inverse of R, denoted by R^{-1}, is a relation from B to A and is defined by R^{-1} = {(b, a) : (a, b) ∈ R}, Clearly, (a, b) ∈ R ⇔ (b, a) ∈ R^{-1} Also,

Dom of R = Range of R^{-1} and

Range of R = Dom of R^{-1}

1. FUNCTION

If to every value (Considered as real unless other–wise stated) of a variable x, which belongs to some collection (Set) E, there corresponds one and only one finite value of the quantity y, then y is said to be a function (Single valued) of x or a dependent variable defined on the set E ; x is the argument or independent variable .

If to every value of x belonging to some set E there corresponds one or several values of the variable y, then y is called a multiple valued function of x defined on E.Conventionally the word **"FUNCTION"** is used only as the meaning of a single valued function, if not otherwise stated.

Pictorially : $\xrightarrow[\text{input}]{x} \boxed{f} \xrightarrow[\text{output}]{f(x)=y}$, y is called the image of x & x is the pre-image of y under f.

Every function from A → B satisfies the following conditions .

(i) $f \subset A \times B$ **(ii)** $\forall\ a \in A \Rightarrow (a, f(a)) \in f$ and

(iii) $(a, b) \in f$ & $(a, c) \in f \Rightarrow b = c$

2. DOMAIN, CO–DOMAIN & RANGE OF A FUNCTION :

Let $f : A \to B$, then the set A is known as the domain of f & the set B is known as co-domain of f . The set of all f images of elements of A is known as the range of f . Thus :

Domain of $f = \{a \mid a \in A, (a, f(a)) \in f\}$

Range of $f = \{f(a) \mid a \in A, f(a) \in B\}$

It should be noted that range is a subset of co–domain . If only the rule of function is given then the domain of the function is the set of those real numbers, where function is defined. For a continuous function, the interval from minimum to maximum value of a function gives the range.

3. IMPORTANT TYPES OF FUNCTIONS :

(i) Polynomial Function :

If a function f is defined by $f(x) = a_0 x^n + a_1 x^{n-1} + a_2 x^{n-2} + \dots + a_{n-1} x + a_n$ where n is a non negative integer and $a_0, a_1, a_2, \dots, a_n$ are real numbers and $a_0 \neq 0$, then f is called a polynomial function of degree n .

Note : (a) A polynomial of degree one with no constant term is called an odd linear function . i.e. $f(x) = ax$, $a \neq 0$

(b) There are two polynomial functions, satisfying the relation ;

$f(x).f(1/x) = f(x) + f(1/x)$. They are :

(i) $f(x) = x^n + 1$ &

(ii) $f(x) = 1 - x^n$, where n is a positive integer .

(ii) Algebraic Function :

y is an algebraic function of x, if it is a function that satisfies an algebraic equation of the form $P_0(x)\, y^n + P_1(x)\, y^{n-1} + \dots\dots + P_{n-1}(x)\, y + P_n(x) = 0$ Where n is a positive integer and $P_0(x), P_1(x)$ .... are Polynomials in x.

e.g. $y = |x|$ is an algebraic function, since it satisfies the equation $y^2 - x^2 = 0$.

Note that all polynomial functions are Algebraic but not the converse. A function that is not algebraic is called **Transcedental Function** .

(iii) Fractional Rational Function :

A rational function is a function of the form.

$y = f(x) = \dfrac{g(x)}{h(x)}$, where g (x) & h (x) are polynomials & $h(x) \neq 0$.

(iv) Exponential Function :

A function $f(x) = a^x = e^{x \ln a}$ ($a > 0$, $a \neq 1$, $x \in R$) is called an exponential function. The inverse of the exponential function is called the logarithmic function . i.e. $g(x) = \log_a x$.

(v) Absolute Value Function :

A function $y = f(x) = |x|$ is called the absolute value function or Modulus function. It is defined as : $y = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$

(vi) Signum Function :

A function $y = f(x) = \text{Sgn}(x)$ is defined as follows : $y = f(x) = \begin{cases} 1 & \text{for } x > 0 \\ 0 & \text{for } x = 0 \\ -1 & \text{for } x < 0 \end{cases}$

It is also written as $\text{Sgn}\, x = |x|/x$; $x \neq 0$; $f(0) = 0$

(vii) Greatest Integer Or Step Up Function :

The function $y = f(x) = [x]$ is called the greatest integer function where [x] denotes the greatest integer less than or equal to x . Note that for :

$-1 \leq x < 0$; $[x] = -1$ 0

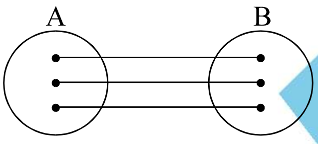
$\leq x < 1$; $[x] = 0$
$1 \leq x < 2$; $[x] = 1$
$2 \leq x < 3$; $[x] = 2$
and so on .

(viii) Fractional Part Function :
It is defined as :
$g(x) = \{x\} = x - [x]$.
e.g. the fractional part of the no. 2.1 is $2.1 - 2 = 0.1$ and the fractional part of -3.7 is 0.3. The period of this function is 1 and graph of this function is as shown .

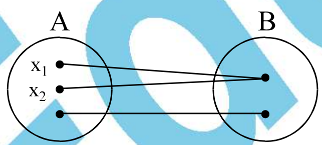
4. CLASSIFICATION OF FUNCTIONS :

One – One Function (Injective mapping) :

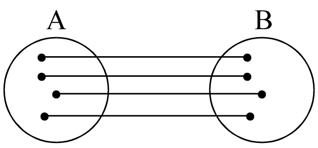
A function $f: A \rightarrow B$ is said to be a one–one function or injective mapping if different elements of A have different f images in B . Thus for $x_1, x_2 \in A$ & $f(x_1)$, $f(x_2) \in B$, $f(x_1) = f(x_2) \Leftrightarrow x_1 = x_2$ or $x_1 \neq x_2 \Leftrightarrow f(x_1) \neq f(x_2)$.



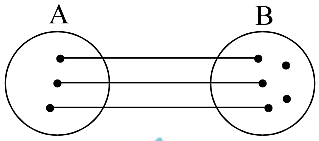
Many–one function :
A function $f: A \rightarrow B$ is said to be a many one function if two or more elements of A have the same f image in B . Thus $f: A \rightarrow B$ is many one if for ; $x_1, x_2 \in A$, $f(x_1) = f(x_2)$ but $x_1 \neq x_2$.



Onto function (Surjective mapping) :
If the function $f: A \rightarrow B$ is such that each element in B (co–domain) is the f image of atleast one element in A, then we say that f is a function of A 'onto' B . Thus $f: A \rightarrow B$ is surjective if $\forall\, b \in B$, $\exists$ some $a \in A$ such that $f(a) = b$.



Into function :
If $f: A \rightarrow B$ is such that there exists atleast one element in co–domain which is not the image of any element in domain, then $f(x)$ is into .

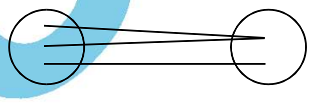


Thus a function can be one of these four types:

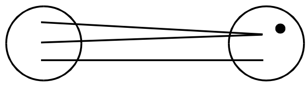
(a) one–one onto (injective & surjective)

(b) one–one into (injective but not surjective)

(c) many–one onto (surjective but not injective)



(d) many–one into (neither surjective nor injective)



Identity function :
The function $f: A \rightarrow A$ defined by $f(x) = x \ \forall\, x \in A$ is called the identity of A and is denoted by I_A.
It is easy to observe that identity function is a bijection .

Constant function :
A function $f: A \rightarrow B$ is said to be a constant function if every element of A has the same f image in B . Thus $f: A \rightarrow B$; $f(x) = c$, $\forall\, x \in A$, $c \in B$ is a constant function. Note that the range of a constant function is a singleton and a constant function may be one-one or many-one, onto or into .

5. ODD & EVEN FUNCTIONS :
If $f(-x) = f(x)$ for all x in the domain of 'f' then f is said to be an even function.
e.g. $f(x) = \cos x$; $g(x) = x^2 + 3$.
If $f(-x) = -f(x)$ for all x in the domain of 'f' then f is said to be an odd function.
e.g. $f(x) = \sin x$; $g(x) = x^3 + x$.

SOLVED PROBLEMS

Ex.1 If a set A = {a, b, c} then find the number of subsets of the set A and also mention the set of all the subsets of A.

Sol. Since n(A) = 3

∴ number of subsets of A is $2^3 = 8$

and set of all those subsets is P(A) named as power set

P(A) : {ϕ, {a}, {b}, {c}, {a,b},{b,c},{a,c},{a,b,c}}

Ex.2 Show that n {P[P(ϕ)]} = 4

Sol. We have P(ϕ) = {ϕ}

∴ P(P(ϕ)) = {ϕ, {ϕ}}

⇒ P[P(P(ϕ))] = { ϕ, {ϕ}, {{ϕ}}, {ϕ, {ϕ}}}.

Hence, n{P[P(ϕ)]} = 4

Ex.3 If A = {x : x = 2n + 1, n ∈ Z and B = {x : x = 2n, n ∈ Z}, then find A ∪ B.

Sol. A ∪ B = {x : x is an odd integer} ∪ {x : x is an even integer} = {x : x is an integer} = Z

Ex.4 If A = {x : x = 3n, n ∈ Z} and B = {x : x = 4n, n ∈ Z} then find A ∩ B.

Sol. We have,

x ∈ A ∩ B ⇔ x = 3n, n ∈ Z and x = 4n, n ∈ Z

⇔ x is a multiple of 3 and x is a multiple of 4

⇔ x is a multiple of 3 and 4 both

⇔ x is a multiple of 12 ⇔ x = 12n, n ∈ Z

Hence A ∩ B = {x : x = 12n, n ∈ Z}

Ex.5 If A and B be two sets containing 3 and 6 elements respectively, what can be the minimum number of elements in A ∪ B? Find also, the maximum number of elements in A ∪ B.

Sol. We have,

n(A ∪ B) = n(A) + n(B) – n(A ∩ B).

This shows that n(A ∪ B) is minimum or maximum according as n(A ∩ B) is maximum or minimum respectively.

Case-I

When n(A ∩ B) is minimum, i.e.,

n(A ∩ B) = 0

This is possible only when A ∩ B = ϕ. In this case,

n(A ∪ B) = n(A) + n(B) – 0 = n(A) + n(B) = 3 + 6 = 9.

So, maximum number of elements in A ∪ B is 9.

Case-II

When n(A ∩ B) is maximum.

This is possible only when A ⊆ B. In this case, n(A ∩ B) = 3

∴ n(A ∪ B) = n(A) + n(B) – n(A ∩ B)

= (3 + 6 – 3) = 6

So, minimum number of elements in A ∪ B is 6.

Ex.6 If A = {2, 3, 4, 5, 6, 7} and B = {3, 5, 7, 9, 11, 13} then find A – B and B – A..

Sol. A – B = {2, 4, 6} & B – A = {9, 11, 13}

Ex.7 If the number of elements in A is m and number of element in B is n then find

(i) The number of elements in the power set of A × B.

(ii) number of relation defined from A to B

Sol. **(i)** Since n(A) = m; n(B) = n then n(A × B)= mn

So number of subsets of A × B = 2^{mn}

⇒ n (P(A × B)) = 2^{mn}

(ii) number of relation defined from A to B = 2^{mn}

Any relation which can be defined from set A to set B will be subset of A × B

∵ A × B is largest possible relation A → B

∴ no. of relation from A → B = no. of subsets of set (A × B)

Ex.8 Let A and B be two non-empty sets having elements in common, then prove that A × B and B × A have n^2 elements in common.

Sol. We have

(A × B) ∩ (C × D) = (A ∩ C)× (B ∩ D)

On replacing C by B and D by A, we get

$\Rightarrow$ $(A \times B) \cap (B \times A) = (A \cap B) \times (B \cap A)$

It is given that AB has n elements so $(A \cap B) \times (B \cap A)$ has n^2 elements

But $(A \times B) \cap (B \times A)$

$= (A \cap B) \times (B \cap A)$

$\therefore$ $(A \times B) \cap (B \times A)$ has n^2 elements

Hence $A \times B$ and $B \times A$ have n^2 elements in common.

Ex.9 Let R be the relation on the set N of natural numbers defined by

$R : \{(x, y)\} : x + 3y = 12 \quad x \in N, y \in N\}$ Find

(i) R **(ii)** Domain of R **(iii)** Range of R

Sol. **(i)** We have, $x + 3y = 12$

$\Rightarrow x = 12 - 3y$

Putting y = 1, 2, 3, we get

x = 9, 6, 3 respectively

For y = 4, we get $x = 0 \notin N$. Also for $y > 4$, $x \notin N$

$\therefore$ $R = \{(9, 1), (6, 2), (3, 3)\}$

(ii) Domain of R = {9, 6, 3}

(iii) Range of R = {1, 2, 3}

Ex.10 If $X = \{x_1, x_2, x_3\}$ and $y = (x_1, x_2, x_3, x_4, x_5\}$ then find which is a reflexive relation of the following :

(a) $R_1 : \{(x_1, x_1), (x_2, x_2)$

(b) $R_1 : \{(x_1, x_1), (x_2, x_2), (x_3, x_3)$

(c) $R_3 : \{(x_1, x_1), (x_2, x_2), (x_3, x_3), (x_1, x_3), (x_2, x_4)$

(d) $R_3 : \{(x_1, x_1), (x_2, x_2), (x_3, x_3), (x_4, x_4)$

Sol. (a) non-reflexive because $(x_3, x_3) \notin R_1$

(b) Reflexive (c) Reflexive

(d) non-reflexive because $x_4 \notin X$

Ex.11 If x = {a, b, c} and y = {a, b, c, d, e, f} then find which of the following relation is symmetric relation :

R_1 : { } i.e. void relation

R_2 : {(a, b)}

R_3 : {(a, b), (b, a)(a, c)(c, a)(a, a)}

Sol. R_1 is symmetric relation because it has no element in it.

R_2 is not symmetric because $(b, a) \in R_2$ & R_3 is symmetric.

Ex.12 If x = {a, b, c} and y = (a, b, c, d, e} then which of the following are transitive relation.

(a) R_1 = { }

(b) R_2 = {(a, a)}

(c) R_3 = {(a, a}.(c, d)}

(d) R_4 = {(a, b), (b, c)(a, c),(a, a), (c, a)}

Sol. **(a)** R_1 is transitive relation because it is null relation.

(b) R_2 is transitive relation because all singleton relations are transitive.

(c) R_3 is transitive relation

(d) R_4 is also transitive relation

Ex.13 Let R be a relation on the set N of natural numbers defined by $xRy \Leftrightarrow x$ divides y ' for all $x, y \in N$.

Sol. This relation is an antisymmetric relation on set N. Since for any two numbers $a, b \in N$. $a \mid b$ and $b \mid a \Rightarrow a = b$, i.e., a R b andb R a $\Rightarrow a = b$.

It should be noted that this relation is not antisymmetric on the set Z of integers, because we find that for any non zero integer a a R (a) and (–a) R a, but $a \neq -a$.

Ex.14 Prove that the relation R on the set Z of all integers numbers defined by $(x, y) \in R \Leftrightarrow x - y$ is divisible by n is an equivalence relation on Z.

Sol. We observe the following properties

Reflexivity :

For any $a \in N$, we have

$a - a = 0 \times n \Rightarrow a - a$ is divisible by $n \Rightarrow (a, a) \in R$

Thus (a, a) R for all Z. so, R is reflexive on Z.

Symmetry :

Let $(a, b) \in R$. Then $(a, b) \in R$

$\Rightarrow$ (a – b)is divisible by n

$\Rightarrow$ $(a - b) = np$ for some $p \in Z$

$\Rightarrow$ $b - a = n(-p)$

$\Rightarrow$ b – a is divisble by n

$\Rightarrow$ $(b, a) \in R$

Thus $(a, b) \in R \Rightarrow (b, a) \in R$ for all $a, b, \in Z$.

So R is symmetric on Z.

Transitivity :

Let a, b, c ∈ Z such that (a, b) ∈ R and (b, c) ∈ R. Then

(a, b) ∈ R ⇒ (a – b) is divisible by n ⇒ a – b = np for some p ∈ Z

(b, c) ∈ R ⇒ (b – c) is divisible by n ⇒ b – c = np for some q ∈ Z

∴ (a, b) ∈ R and (b, c) ∈ R ⇒ a – b = np and b – c = nq

⇒ (a – b) + (b – c) = np + nq ⇒ a – c = n(p + q)

⇒ a – c is divisible by n

⇒ (a, c) ∈ R

Thus (a, b) ∈ R and (b, c) ∈ R

⇒ (a, c) ∈ R for all a, b, c ∈ Z. So R is transitive relation on Z.

Thus, R being reflexive, symmetric and transitive is an equivalence relation on Z.

Ex.15 Let a relation R_1 on the set R of real numbers be defined as (a, b) ∈ R_1 ⇔ 1 + ab > 0 for all a, b ∈ R. Show that R_1 is reflexive and symmetric but not transitive.

Sol. We observe the following properties :

Reflexivity :

Let a be an arbitrary element of R. Then

a ∈ R ⇒ 1 + a . a = 1 + a^2 > 0

⇒ (a, a) ∈ R_1

Thus (a, a) ∈ R_1 for all a ∈ R. So R_1 is reflexive on R.

Symmetry :

Let (a, b) ∈ R. Then

(a, b) ∈ R_1 ⇒ 1 + ab > 0

⇒ 1 + ba > 0

⇒ (b, a) ∈ R_1

Thus (a, b) ∈ R_1 ⇒ (b, a) ∈ R_1 for all a, b ∈ R

So R_1 is symmetric on R

Transitive :

We observe that (1, 1/2) ∈ R_1 and (1/2, –1) ∈ R_1 but (1,–1) ∈ R_1 because

1 + 1 × (–1) = 0 $\not>$ 0. So R_1 is not transitive on R.

Ex.16 Let A be the set of first ten natural numbers and let R be a relation on A defined by (x, y) ∈ R ⇔ x + 2y = 10 i.e., R = {(x, y) : x ∈ A, y ∈ A and x + 2y = 10}. Express R and R^{-1} as sets of ordered pairs. Determine also :

(i) Domains of R and R^{-1}

(ii) Range of R and R^{-1}

Sol. We have (x, y) ∈ R ⇔ x + 2y = 10 ⇔ y = $\frac{10-x}{2}$, x, y ∈ A

where A = {1,2,3,4,5,6,7,8,9,10}

Now, x = 1 ⇒ y = $\frac{10-1}{2} = \frac{9}{2} \notin$ A

This shows that 1 is not related to any element in A. Similarly we can observe that 3, 5, 7, 9 and 10 are not related to any element of a under the defined relation. Further we find that

for x = 2, y = $\frac{10-2}{2}$ = 4 ∈ A

∴ (2, 4) ∈ R

for x = 4, y = $\frac{10-4}{2}$ = 3 ∈ A

∴ (4, 3)∈ R

for x = 6, y = $\frac{10-6}{2}$ = 2 ∈ A

∴ (6, 2) ∈ R

for x = 8, y = $\frac{10-8}{2}$ = 1 ∈ A

∴ (8, 1) ∈ R

Thus R = {(2, 4), (4, 3), (6, 2), (8, 1)}

⇒ R^{-1} = {(4, 2), (3, 4), (2, 6), (1, 8)}

Clearly, Dom (R) = {2, 4, 6, 8} = Range (R^{-1})

and Range (R) = {4, 3, 2, 1} = Dom (R^{-1})

EXERCISE

Q.1 Which of the following collections are sets
(i) The collection of all good athletes of India.
(ii) The collection of fat boys of your locality.
(iii) The collection of all those students of your class whose age exceeds 15 years.

Q.2 Rewrite the following statements, using set notations :
(i) A is an empty set and B is a non-empty set.
(ii) number of elements in A is 6.
(iii)0 is a whole number but not a natural number.

Q.3 Let A = [2, 4, 6, 8, 10]. Insert the appropriate symbol $\in$ or $\notin$ in the blank spaces :
(i) 6 ……..A (ii) 1 …….. A (iii) 5 …….. A
(iv) 10 ….. A (v) 2 ……… A (vi) 3 ……… A

Q.4 Write the following sets in the roster form
(i) A = set of all letters in the word 'LETTER'
(ii) B ={x: x is a natural number less then 7}
(iii) C = $\{x : x$ is an integer and $2 \le x < 5\}$

Q.5 List all the elements of each of the following sets :
(i) $A = \left\{x : x \in Z \text{and} -\frac{1}{2} < x < \frac{7}{2}\right\}$
(ii) $B = \{x : x \in Z \text{ and } x^2 \le 4\}$
(iii) $C = \{x : x = 2n, n \in N \text{ and } n \le 5\}$

Q.6 State whether the give set is finite or infinite:
(i) $A = \{x \in R;\ 0 < x < 1\}$
(ii) $B = \{x \in Z;\ x < 1\}$
(iii) $C = \{x \in Z;\ -15 < x < 15\}$

Q.7 Which of the following sets are empty sets
(i) $A = \{x : x \in W, x + 3 < 3\}$
(ii) $B = \{x : x \in Q, 1 < x < 2\}$
(iii) $C = \{0\}$

Q.8 Which of the following in the pairs of equal sets?
(i) A = set of the letters in the word 'ALLOY'
B = set of the letters in the word 'LOYAL'
(ii) $C = \{x : x \in Z, x^2 \le 4\}$ and
$D = \{x : x \in Z, x^2 - 4 = 0\}$
(iii) $E = \{x : x Z, x - 3 = 0\}$ and
$F = (x : x \in Z, x^2 - 9 = 0\}$

Q.9 Which of the following are pairs of equivalent sets ?
(i) A = {1, 2, 3}, B = {2, 4, 6}
(ii) P = {0}, Q = ϕ
(iii) C = {–2, –1, 0}, D = {1, 2, 3}

Q.10 State in each whether $A \subseteq B$ or $A \not\subseteq B$
(i) A = {0, 1, 2, 3}, B = {1, 2, 3, 4, 5}
(ii) A = {x, y, z}, B = {z, y, x}
(iii) $A = \{x : x \in Z, x^2 = 1\}$, $B = \{x : x \in N, x^2 = 1\}$

Q.11 Write down all subsets of each of the following
(i) A = {3} (ii) B = {-2, 5}

Q.12 If A = {1, 2, 3}, find P(A) and n{P(A)}.

Q.13 Express each of the following sets as an interval
(i) $A = \{x : x \in R, -4 < x < 0\}$
(ii) $B = \{x : x \in R, 0 \le x < 3\}$
(iii) $C = \{x : x \in R, 2 < x \le 6\}$

Q.14 Write each of the following intervals in the set-builder from:
(i) A =]-2, 2[(ii) B = [5, 9] (iii) C = [8, 11]

Q.15 If A = {3, {4, 5}, 6}, find which of the following statement are true:
(i) $\{4, 5\} \subseteq A$ (ii) $\{4, 5\} \in A$ (iii) $\{\{4, 5\}\} \subseteq A$

Q.16 If A = {a, b, c, d, e, f} B = {c, e, g, h} and C = {a, e, m, n}, find :
(i) $A \cup B$ (ii) $B \cup C$ (iii) $A \cup C$

Q.17 If A = (1, 2, 3, 4, 5}, B = (4, 5, 6, 7, 8},
C = {7,8,9,10,11} and D = {10,11,12,13,14},
find : (i) $A \cup B$ (ii) $B \cup C$ (iii) $A \cup C$ (iv) $B \cup D$

Q.18 If A = {3, 5, 7, 9, 11}, B = {7, 9, 11, 13},
C = {11, 13, 15} and D = {15, 17}, find :
(i) $A \cap B$ (ii) $A \cap C$ (iii) $B \cap C$

Q.19 If $A = \{x : x \in N\}$, $B = \{x : x \in N$ and x is even},
$C = \{x : x \in N$ and x is odd} and
$D = \{x : x \in N$ and x is prime} then find:
(i) $A \cap B$ (ii) $A \cap C$ (iii) $A \cap D$

Q.20 If $A = \{2x : x \in N \text{ and } 1 \le x < 4\}$,
$B = \{(x + 2) : x \in N \text{ and } 2 \le x < 5\}$ and
$C = \{x : x \in N \text{ and } 4 < x < 8\}$, find:
(i) $A \cap B$ (ii) $A \cup B$ (iii) $(A \cup B) \cap C$

Q.21 If A = {2, 4, 6, 8, 10, 12} and B = {3, 4, 5, 6, 7, 8, 10}, find:
(i) (A–B) (ii) (B–A) (iii) $(A-B) \cup (B-A)$

Q.22 If A = {a, b, c, d, e}, B = {a, c, e, g} and C = {b, e, f, g}, find:
(i) $A \cap (B-C)$ (ii) $A-(B \cup C)$ (iii) $A-(B \cap C)$

Q.23 If A = $\left\{\frac{1}{x} : x \in N \text{ and } x < 8\right\}$ and

B = $\left\{\frac{1}{2x} : x \in N \text{ and } x \leq 4\right\}$, find:

(i) A ∪ B (ii) A ∩ B (iii) A–B (iv) B–A

Q.24 If R is the set of all real numbers and Q is the set of all rational numbers then what is the set (R – Q) ?

Q.25 If A = {2, 3, 5, 7, 11} and B = ϕ, find :

(i) A ∪ B (ii) A ∩ B

Q.26 If A and B are two sets such that A ⊆ B, then find: (i) A ∪ B (ii) A ∩ B (iii) A – B

Q.27 Which of the following sets are pairs of disjoint sets: Justify your answer.

(i) A = {3, 4, 5, 6} and B = {2, 5, 7, 9}

(ii) C = {1, 2, 3, 4, 5} and D = (6, 7, 9, 11}

(iii) E = {x : x ∈ N, x is even and x < 8}

F = {x : x = 3n, n ∈ N and n < 4}

Q.28 If U = {1, 2, 3, 4, 5, 6, 7, 8, 9},
S = (1, 2, 3, 4}, B = {2, 4, 6, 8} and
C = {1, 4, 5, 6}, find:

(i) A′ (ii) B′ (iii) C′ (iv) (B′)′

(v) (A ∪ B)′ (vi) (A ∩ C)′

Q.29 If U = {a, b, c, d, e}, A = {a, b, c} and B = {b, c, d, e}, then verify that :

(i) (A ∪ B)′ = (A′ ∩ B′) (ii) (A ∩ B)′ = (A′ ∪ B′)

Q.30 If U is the universal set and A ⊂ U, then fill in the blanks:

(i) A ∪ A′ = …. (ii) A ∩ A′ = …..

(iii) ϕ′ ∩ A = ….. (iv) U′ ∩ A = …..

Q.31 If A = {a, b, c, d, e}, B = {a, c, e, g} and C = {b, e, f, g}, verify that:

(i) A ∪ B = B ∪ A (ii) A ∪ C = C ∪ A

(iii) B ∪ C = C ∪ B (iv) A ∩ B = B ∩ A

Q.32 If A = {a, b, c, d, e}, B = {a, c, e, g} and C = {b, e, f, g}, verify that:

(i) A ∩ (B – C) = (A ∩ B) – (A ∩ C)

(ii) A –(B ∩ C) = (A – B) ∪ (A – C)

Q.33 If A = {x : x∈N, x ≤ 7}, B = {x: x is prime, x < 8} and C = {x : x ∈ N, x is odd and x < 10}, verify that:

(i) A ∪ (B ∩ C) = (A ∪ B) ∩ (A ∪ C)

(ii) A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ B)

Q.34 If A = {1, 2, 3, 4, 5, 6, 7, 8, 9},
A ={2,4,6,8} and B ={2, 3, 5, 7}, verify that:

(i) (A ∪ B)′ = (A′ ∩ B′)

(ii) (A ∩ B)′ = (A′ ∪ B′)

Q.35 Let A = {a, b, c}, B = {b, c, d, e} and C = {c, d, e, f} be subsets of U = {a, b, c, d, e, f}. Then, verify that:

(i) (A′)′ = A (ii) (A ∪ B)′ = (A′ ∩ B′)

(iii) (A ∩ B)′ = (A′ ∪ B′)

Q.36 Given an example of three sets A, B, C such that A ∩ B ≠ ϕ, B ∩ C ≠ ϕ, A ∩ C ≠ ϕ and A ∩ B ∩ C = ϕ.

Q.37 Let A = {a, b, c, e, f}, B = {c, d, e, g} and C = {b, c, f, g} be subsets of the set U = {a, b, c, d, e, f, g, h}.
Draw Venn diagrams to represent the following sets: (i) A ∩ B (ii) A ∪ (B ∩ C) (iii) A–B

Q.38 Let A = {2, 4, 6, 8, 10}, B = {4, 8, 12, 16} and C = {6, 12, 18, 24}. Using Venn diagrams, verify that: A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C)

Q.39 Let A = {a, e, I, o, u}, B = {a, d, e, o, v} and C = {e, o, t, m}. Using Venn diagrams, verify that: A ∪ (B ∩ C) = (A ∪ B) ∩ (A ∪ C)

Q.40 Let A⊂ B ⊂U. Exhibit it in a Venn diagram.

Q.41 Let A = {2, 3, 5, 7, 11, 13} and
B = {5, 7, 9, 11, 15} be subsets of
U = {2, 3, 5, 7, 9, 11, 13, 15}.
Using Venn diagrams, verify that:

(i) (A ∪ B)′ = (A′ ∩ B′) (ii) (A ∩ B)′ = (A′ ∪ B′)

Q.42 Using Venn diagrams, show that (A–B),(A∩B) and (B – A) are disjoint sets, taking A = {2,4,6,8,10,12} and B = {3,6,9,12,15}.

Q.43 If A and B are two sets such that n(A)=37, n(B)=26 and n(A ∪ B)= 51, find n(A ∩ B).

Q.44 If P and Q are two sets such that n(P ∪ Q) = 75, n(P ∩ Q) = 17 and n(P) = 49, find n(Q).

Q.45 If A and B are two sets such that n(A) = 24, n(B) = 22 and n(A ∩ B) = 8, find:

(i) n(A ∪ B) (ii) n(A – B) (iii) n(B – A)

Q.46 If A and B are two sets such that n(A–B)=24, n(B–A)=19 and n(A ∩ B) = 11, find:

(i) n(A) (ii) n(B) (iii) n(A ∪ B)

Q.47 In a committee, 50 people speak Hindi, 20 speak English and 10 speak both Hindi and English. How many speak at least one of these two languages ?

Q.48 In a group of 50 persons, 30 like tea, 25 like coffee and 16 like both. How many like
(i) either tea or coffee?
(ii) neither tea nor coffee?

Q.49 In a class of a certain school 50 students offered mathematics, 42 offered biology and 24 offered both the subjects. Find the number of students.
(i) offering mathematics only,
(ii) offering biology only and
(iii) offering any of the two subjects.

Q.50 In an examination, 56% of the candidates failed in English and 48% failed in science. If 18% failed in both English and Sicience, find the percentage of those who passed in both the subjects.

Q.51 In a group of 65 people, 40 like cricket, 10 like both cricket and tennis. How many like tennis only and not cricket ? How many like tennis ?

Q.52 A school awarded 42 medals in hockey, 18 in basketball and 23 in cricket. If these medals were bagged by a total of 65 students and only 4 students got exactly two of the three sports, how many students recieved medals in exactly two of the three sports?

Q.53 In a survey of 60 people, it was found that 25 people read newspaper H, 26 read newspaper T, 26 read newspaper I, 9 read both H and I, 11 read both H and T, 8 read both T and I, and 3 read all the three newspaper. Find
(i) the number of people who read at least one of the newspapers,
(ii) the number of people who read exactly one newspaper.

Q.54 In a class, 18 students offered physics, 23 offered chemistry and 24 offered mathematics. Of these, 13 are in both chemistry and mathematics; 12 in physics and chemistry; 11 in mathematics and physics, and 6 in all the three subjects. Find
(i) how many students are there in the class;
(ii) how many offered mathematics but not chemistry;
(iii) how many are taking exactly one of the three subjects.

Q.55 In a survey of 100 students, the number of students studying the various languages is found as: English only 18; English but not Hindi 23; English and Sanskrit 8; Sanskrit and Hindi 8; English 26; Sanskrit 48 and no language 24. Find
(i) how many students are studying Hindi;
(ii) how many students are studying English and Hindi both.

Q.56 Find the values of a and b, when:
(i) $(a + 3, b - 2) = (5, 1)$
(ii) $(a + b, 2b - 3) = (4, -5)$

Q.57 If $A=\{0,1\}$ and $B=\{1,2,3\}$, show that $A\times B \neq B\times A$.

Q.58 If $P=\{a,b\}$ and $Q=\{x,y,z\}$, show that $P\times Q\neq Q\times P$.

Q.59 If $A = \{2, 3, 5\}$ and $B = \{5, 7\}$, find
(i) $A \times B$ (ii) $B \times A$ (iii) $A \times A$ (iv) $B \times B$

Q.60 If $A = \{x \in N : x \leq 3\}$ and $B = \{x \in W, x < 2\}$, find $(A \times B)$ and $(B \times A)$. Is $(A \times B) = (B \times A)$?

Q.61 If $A=\{1,3,5\}$, $B=\{3,4\}$ and $C=\{2,3\}$, verify that:
(i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$
(ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Q.62 Let $A = \{x \in W : x < 2\}$, $B = \{x \in N : 1 < x \leq 4\}$ and $C = \{3, 5\}$. Verify that:
(i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$
(ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Q.63 If $A\times B=\{(-2,3),(-2,4),(0,3),(0,4),(3,3),(3,4)\}$ find A and B.

Q.64 Let $A = \{2, 3\}$ and $B = \{4, 5\}$. Find $(A \times B)$. How many subsets will $(A \times B)$ have ?

Q.65 Let A and B be two sets such that $n(A) = 3$ and $n(B) = 2$. If $a \neq b \neq c$ and $(a, 0), (b, 1), (c, 0)$ are in $A \times B$, find A and B.

Q.66 If $A = \{-1, 1\}$, find $(A \times A \times A)$.

Q.67 If $A \subseteq B$, prove that $(A \times C) \subseteq (B \times C)$.

Q.68 Define a relation on a set A. What do you mean by the domain and range of a relation ?

Q.69 If A and B are two sets contain m and n elements respectively, how many different relations can be defined from A to B ?

Q.70 Find the domain and range of each of the following relations:

(i) R = {(–1, 1), (1, 1), (2, 4), (–2, 4),(3, 9)}

(ii) $R = \left\{\left(x, \frac{1}{x}\right) : x \text{ is an integer}, 0 < x < 6\right\}$

(iii) R = {(x,y):x and y are integers and xy = 4}

(iv) R = {(x, y) : x, y ∈ Z and $x^2 + y^2 = 25$}

Q.71 Let A = {1,2,3, 4, 5, 6}. Define a relation R on A by R = {(x, y) : y = (x + 1)}.

(i) Depict R, using arrow diagram.

(ii) Write down the domain, co-domain and range of R.

Q.72 Let A = {1, 2, 3, 5} and B(4, 6, 9}.

Let R = {(x, y) : |x – y| is odd, x ∈ A and y ∈ B}. Write R in the roster form.

Q.73 Let A = {1, 2, 3, 4, 6} and let

R = {(a, b) : a, b ∈ A and a divides b}.

(i) Write R in the roster form.

(ii) Find dom (R) and range (R).

Q.74 Let A = {1, 2, 3, 4, 5} and B = {1, 2, 3, 4}. Let R be the relation, 'is greater than' from A to B, Write R as a set of ordered pairs.

Q.75 What is an equivalence relation ?

Show that the relation of 'similarity' on the set S of all triangles in a plane is an equivalence relation.

Q.76 Let R = {(a, b) : a, b ∈ N and a = b^2}.

Show that R satisfies none of reflexivity, symmetry and transitivity.

Q.77 Let A = {–1, 1, 2, 3,) and B = {1, 4, 9, 16}

Let f = {(x, y) : x ∈ A, y ∈ B and y = x^2}.

Show that f is a function from A to B. Find its domain and range.

Q.78 Let A={2,3,5,7} and B={3,5,9,13,15}.Let f={(x,y):x ∈ A, y ∈ B and y=2x–1}.Write f in the roster form. Show that f is a function from A to B. Find the domain and range of f.

Q.79 Let f = {(2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (3, 4)}. Is f a function? Give reason.

Q.80 Let g={(1,2),(2,5),(3,8),(4,10),(5,12),(6,12)}. Is g a functions ? If yes, find its domain and range. If no, give reason.

Q.81 Let f = {(0, –5), (1, –2), (2, 1), (3, 4), (4, 7)} be a linear function from Z into Z. Find f.

Q.82 Let $f(x) = \begin{cases} x^2, \text{when } 0 \le x \le 2 \\ 2x, \text{when } 2 \le x \le 5 \end{cases}$ and let

$g(x) = \begin{cases} x^2, \text{when } 0 \le x \le 3 \\ 2x, \text{when } 3 \le x \le 5 \end{cases}$.

Show that f is a function, while g is not a function.

Q.83 If f(x) = x^3, find the value of $\frac{\{f(5) - f(1)\}}{(5-1)}$.

Q.84 If f(x) = x – 1 and g(x) = x^2 + 1, find:

(i) (f + g) (ii) (f – g) (iii) $\frac{f}{g}$

Q.85 Let $f = \left\{\left(x, \frac{x^2}{1+x^2}\right) : x \in R\right\}$ be a function from R into R. Find range (f).

Q.86 Find the domain of the real-valued function

$f(x) = \frac{x^2 + 2x + 3}{x^2 - 5x + 6}$.

Q.87 Let A = {10, 11, 12, 14, 26}, and

Let f : A → N ; f(n) = highest prime factor of n.

Find the range of f.

Q.88 Find the domain of the real function

$f(x) = \sqrt{x^2 - 4}$.

ANSWER

1. (iii) **2.** (i)A=ϕ and B$\neq\phi$(ii)n(A)=6(iii)0$\in$W but 0$\notin$N

3. (i) $\in$ (ii) $\notin$ (iii) $\notin$ (iv) $\in$ (v) $\in$ (vi) $\notin$

4. (i) A={L,E,T,R} (ii) B={1,2,3,4,5,6}
(iii) C={–2,–1,0,1,2,3,4}

5. (i) A = {0, 1, 2, 3} (ii) B = {–2, –1, 0, 1, 2}
(iii) C = {2, 4, 6, 8, 10}

6. (i) infinite (ii) infinite (iii) finite **7.** A **8.** (i) A = B

9. (i) A and B are equivalent (iii) C and D are equivalent

10. (i) A $\not\subset$ B (ii) A $\subseteq$ B (iii) A $\not\subset$ B

11. (i) P(A)={ϕ,{3}} (ii) P(B)={ϕ,{–2},{5},{–2,5}}

12. P(A) = {ϕ, {1}, {2}, {3}, {1,2}, {2,3}, {1,3}, {1,2, 3}} n{P(A)} = 8

13. (i) A =]–4, 0[(ii) B = [0, 3[(iii) C =]2, 6]

14. (i) A = {x $\in$ R : –2 < x < 2}
(ii) B = {x : x $\in$ R, 5 $\le$ x $\le$ 9}
(iii) C = {x : x $\in$ R, 8 $\le$ x $\le$ 11}

15. (i) False (ii) True (iii) True

16. (i) {a, b, c, d, e, f, g, h}
(ii) {a, c, e, g, h, m, n}
(iii) {a, b, c, d, e, f, m, n}

17. (i) {1, 2, 3, 4, 5, 6, 7, 8}
(ii) {4, 5, 6, 7, 8, 9, 10, 11}
(iii) {1, 2, 3, 4, 5, 7, 8, 9, 10, 11}

18. (i){7,9}(ii){11}(iii)11,13} **19.**(i) B(ii) C(iii) D

20. (i) {4, 6} (ii) {2, 4, 5, 6} (iii) {5, 6}

21. (i) {2, 12} (ii) {3, 5, 7} (iii) {2, 3, 5, 7, 12}

22. (i) {a, c} (ii) {d} (iii) {a, b, c, d}

23. (i) $\left\{1,\frac{1}{2},\frac{1}{3},\frac{1}{4},\frac{1}{5},\frac{1}{6},\frac{1}{7},\frac{1}{8}\right\}$ (ii) $\left\{\frac{1}{2},\frac{1}{4},\frac{1}{6}\right\}$
(iii) $\left\{1,\frac{1}{3},\frac{1}{5},\frac{1}{7}\right\}$ (iv) $\left\{\frac{1}{8}\right\}$

24. (R – Q) = {x : x $\in$ R, x is irrational}

25. (i) {2,3,5,7,11}(ii) ϕ **26.** (i) B (ii) A (iii) ϕ

27. (ii) C and D, since C $\cap$ D = ϕ

28. (i) {5,6,7,8,9} (ii){1,3,5,7,9}(iii) {2,3,7,8,9}
(iv) {2,4,6,8} (v) {5,7,9} (vi) {2,3,5,6,7,8,9}

30. (i) U (ii) ϕ (iii) A (iv) ϕ **43.** 12 **44.** 43 **45.** (i) 38 (ii) 16 (iii) 14 **46.** (i) 35 (ii) 30 (iii) 54

47. 60 **48.** (i) 39 (ii) 11 **49.** (i) 26 (ii) 18 (iii) 68

50. 14 **51.** 25, 35 **52.** 22 **53.** (i) 52 (ii) 30

54. (i) 35 (ii) 11 (iii) 11 **55.** (i) 18 (ii) 3

56. (i) a = 2, b = 3 (ii) a = 5, b = –1

59. (i) A×B={(2,5),(2,7),(3,5),(3,7),(5,5),(5, 7)}
(ii) B×A={(5,2),(5,3),(5,5),(7,2)(7,3)(7,5)}
(iii) A×A={(2,2),(2,3),(2,5),(3,2),(3,3),(3,5), (5,2), (5, 3), (5, 5)}
(iv) B × B ={(5, 5) , (5, 7), (7, 5), (7, 7)}

60. A × B = {(1, 0), (1, 1) (2, 0), (2, 1), (3, 0), (3, 1)}
B × A = {(0, 1), (0, 2) (0, 3), (1, 1) (1, 2) (1, 3)}; no

63. A = {–2, 0, 3} and B = {3, 4}

64. A × B = {(2, 4), (2, 5), (3, 4), (3, 5); number of subsets of (A × B) = 2^4 = 16

65. A = {a, b, c} and B = {0, 1}

66. (A×A×A) = {(–1,1,1), (1,–1,1), (1,1,–1), (–1,–1,1),(–1,1,–1),(1,–1,–1),(1,1,1),(–1,–1,–1)}

69. $2^{m\times n}$

70. (i) dom (R) = {–2, –1, 1, 2, 3}, range (r) = {1, 4, 9}
(ii) dom (R) = {1, 2, 3, 4, 5},
range (R) = $\left\{1,\frac{1}{2},\frac{1}{3},\frac{1}{4},\frac{1}{5}\right\}$
(iii) dom (R) = {–4,–2,–1,1,2,4} = range(R)
(iv)dom(R) = {–5,–4,–3,0,3,4,5}= range(R)

71. (ii) dom (R) = {1, 2, 3, 4, 5},
co-domain = {1, 2, 3, 4, 5, 6},
range (R) = {2, 3, 4, 5, 6}

72. R ={(1, 4),(1, 6),(2, 9),(3, 4),(3, 6)(5, 4),(5, 6)}

73. (i) R ={(1,1),(1,2),(1,3),(1,4),(1,6),(2,2),(2,4), (2,6),(3,3),(3,6),(4,4),(6,6)}
(ii) dom(R)={1,2,3,4,6} and range(R)={1,2,3,4,6}

74. R = {(2, 1), (3, 1), (3, 2), (4, 1), (4, 2), (4, 3), (5, 1), (5, 2), (5, 3), (5, 4)}

77. Dom (f) = {–1, 1, 2, 3} and range (f) = {1, 4, 9}

78. f = {(2, 3), (3, 5), (5, 9), (7, 13)}, dom (f) = {2, 3, 5, 7}, range (f) = {3, 5, 9, 13}.

79. No, since one element, namely2, has more than one images **80.** Yes, dom (g)={1,2,3,4,5,6}, range (g)={2,5,8,10,12} **81.** f(x) = 3x – 5

83. 31 **84.** (i) (f+g)(x)=(x+x^2) (ii) (f–g)(x) = x – x^2 – 2

85. [0, 1] **86.** R–{2,3} **87.** range(f)={3, 5, 7, 11, 13}

88.]–∞, –2] ∪ [2, ∞[