EXERCISE LEVEL –I

EL– I

Q.1 $\sum_{i=1}^{4} 2n + 3 = ?$

(a) 5 (b) 12 (c) 21 (d) 32

Q.2 Is the sum of 1, 2, 3, and 4, which equals 10, considered a series?

(a) 1+2+3+4 only (b) 10 only

(c) 1+2+3+4 and 10 (d) neither 1+ 2+ 3+ 4 nor 10

Q.3 The sequence 1, 1, 2, 3, 5 is part of the Fibonacci sequence.

(a) True (b) False

(c) Can't be determine (d) None of these

Q.4 Which of the following relations produces the Fibonacci sequence?

(a) $a_n = a_{n-1} + a_{n-2}$ (b) $a_{n-1} = a_n + a_{n-2}$ (c) $a_{n-2} = a_n + a_{n-1}$ (d) $a_n = a_{n+1} + a_{n-2}$

Q.5 Which of the following sequences is finite?

(a) 48, 24, 12… (b) 1, 2, 3 … (c) 2,4,6,8,10 (d) 2,3,5,7,11,13, …

Q.6 The values of $7\log(\frac{16}{15}) + 5\log(\frac{25}{24}) + 3\log(\frac{81}{80})$ is.

(a) log 2 (b) log 3 (c) 1 (d) 0

Q.7 The value of $\frac{\log_a(\log_b x)}{\log_b(\log_a b)}$ is

(a) $\log_b a$ (b) $\log_a b$ (c) $-\log_a b$ (d) $-\log_b a$

Q.8 The number of solutions of $\log_2(x-1) = 2\log_2(x-3)$ is

(a) 2 (b) 1 (c) 6 (d) 7

Q.9 If $\log_y x = \log_z y = \log_x z$, then

(a) $x < y < z$ (b) $x > y \geq z$ (c) $x < y \leq z$ (d) $x = y = z$

Q.10 If $a_n = 4_n + 6$, find 15^{th} term of the sequence.

(a) 6 (b) 10 (c) 60 (d) 66

Q.11 Which statement among the ones provided above is/are correct?

1. If (a_n) represents the nth term of an arithmetic progression (AP), then $a_n = \frac{a_{n+k}+a_{n-k}}{3}$

2. In an arithmetic progression (AP), if the sum of m terms is equal to the sum of n terms, then the sum of (m + n) terms is always zero.

3. The sum to infinity of the series $\frac{1}{3} + \frac{1}{15} + \frac{1}{35} + \cdots$ is $\frac{1}{2}$.

(a) (1) and (2) (b) (2) and (3)

(b) (3) and (1) (d) All (1), (2) and (3)

Q.12 If x, y, z are in arithmetic progression (AP), then the terms $1\frac{1}{\sqrt{x}+\sqrt{y}}, \frac{1}{\sqrt{z}+\sqrt{x}}, \frac{1}{\sqrt{y}+\sqrt{z}}$ are also in

(a) AP (b) GP (c) HP (d) AP and HP

Q.13 If $1, \log_3\sqrt{(3^{1-x}+2)}, \log_3(4.3^x - 1)$ are in AP, then x equals

(a) $\log_3 4$ (b) $1 - \log_3 4$ (c) $1-\log_4 3$ (d) $\log_4 3$

Q.14 If $a_1, a_2, \ldots, a_n$ are in AP with common difference d, then the sum of the series

Sin d $(\text{cosec}\, a_1 \,\text{cosec}\, a_2 + \text{cosec}\, a_2 \,\text{cosec}\, a_3 + \cdots + \text{cosec}\, a_{n-1} \,\text{cosec}\, a_n)$ is

(a) sec a_1 - sec a_n (b) cot a_1 - cot a_n

(c) tan a_1 –tan a_n (d) cosec a_1 =cosec a_n

Q.15 If (S_1), (S_2), and (S_3) represent the sums of the first (n_1), (n_2), and (n_3) terms, respectively, of an arithmetic progression (A.P.), then $\frac{S_1}{n_1}(n_2 - n_3) + \frac{S_2}{n_2}(n_3 - n_1) + \frac{S_3}{n_3}(n_1 - n_2) =$

(a) 0 (b) 1 (c) $1 - \left(\frac{1}{10}\right)^{106}$ (d) None of these

Q.16 If $a_1, a_2, a_3, \ldots, a_{24}$ are in arithmetic progression and $a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} = 225$, then $a_1 + a_2 + a_3 + \cdots + a_{23} + a_{24}$ is equal to

(a) 909 (b) 75 (c) 750 (d) 900

Q.17 Let the sequence, $a_1, a_2, a_3, \ldots, a_{2n}$ form an AP, then $a_1^2 - a_2^2 + a_3^2 - \cdots + a_{2n-1}^2 - a_{2n}^2$ is equal to

(a) $\frac{n}{2n-1}(a_1^2 - a_{2n}^2)$ (b) $\frac{2n}{n-1}(a_{2n}^2 - a_1^2)$ (c) $\frac{n}{n+1}(a_1^2 + a_{2n}^2)$ (d)None of these

Q.18 How many terms of the A.P. $-6, -\frac{11}{2}, -5 \ldots$ are needed to give the sum -25?

(a) $n = 5$ (b) $n = 20$ (c) Both (a) and (b) (d)None of these

Q.19 In an A.P., if p^{th} term is $\frac{1}{q}$ and q^{th} term is $\frac{1}{p}$, the sum of first pq terms is

(a) $\frac{1}{2}(pq + 1)$ (b) $\frac{1}{3}(pq + 1)$ (c) $\frac{2}{3}(pq + 1)$ (d) $\frac{2}{3}(pq + 1)$

Q.20 If $\frac{a^n + b^n}{a^{n-1} + b^{n-1}}$ is the A.M. between a and b, then find the value of n.

(a)$n = 1$ (b)$n = 5$ (c)$n = 6$ (d)$n = 5$

Q.21 Between 1 and 31, (m) numbers have been inserted in such a manner that the resulting sequence forms an arithmetic progression (A.P.), and the ratio of the 7^{th} and $(m-1)^{th}$ numbers is 5:9. Determine the value of (m).

(a) $m = 14$ (b) $m = 21$ (c) $m = 10$ (d) $m = 29$

Q.22 Determine the sum of all natural numbers between 100 and 1000 that are multiples of 5.

(a) 98450 (b) 10454 (c) 95412 (d) 56523

Q.23 If p^{th}, q^{th} and r^{th} terms of G.P. are x, y, z respectively, the $x^{q-r}y^{r-p}z^{p-q}$ is equal to

(a) 0 (b)1 (c)-1 (d)None of these

Q.24 Find the value of n so that $\frac{a^{n+1} + b^{n+1}}{a^n + b^n}$ may be the geometric mean between a & b.

(a) $n = \frac{-4}{5}$ (b) $n = \frac{-1}{2}$ (c) $n = \frac{-1}{5}$ (d) $n = \frac{-2}{3}$

Q.25 The determinant $\Delta = \begin{vmatrix} a & b & a\alpha + b \\ b & c & b\alpha + c \\ a\alpha + b & b\alpha + c & 0 \end{vmatrix}$ is equal to zero, if

(a) a, b, c are in A.P. (b) a, b, c are in G.P.

(c) a, b, c are in H.P. (d) a is a root of $ax^2 + bx + c = 0$

Q.26 Consider an infinite geometric series with the first term a and common ratio r. If its sum is 4 and the second term is $\frac{3}{4}$, then

(a) $a = \frac{4}{7}, r = \frac{3}{7}$ (b) $a = 2, r = \frac{3}{8}$ (c) $a = \frac{3}{2}, r = \frac{1}{2}$ (d) $a = 3, r = \frac{1}{4}$

Q.27 If the fifth term of a geometric progression (G.P.) is 2, then the product of its first 9 terms is

(a) 256 (b) 512 (c)1024 (d)None of these

Q.28 Find a G.P. for which sum of the first two terms is – 4 and the fifth term is 4 times the third term

(a) $\frac{-4}{3}, \frac{-8}{3}, \frac{-16}{3}, \ldots$ (b) $4, -8, 16, -32$ (c) Both (a) & (b) (d) None of these

Q.29 Find the sum of the sequence 7, 77, 777, 7777 ... to n terms.

(a) $\frac{7}{9}\left[\frac{10(10^n - 1)}{9} - n\right]$ (b) $\frac{2}{3}\left[\frac{10(10^n - 1)}{9} - n\right]$ (c) $\frac{2}{3}\left[\frac{4(4^n - 1)}{9} - n\right]$ (d) $\frac{2}{3}\left[\frac{1(1^{n-1})}{9} - n\right]$

Q.30 How many terms of G.P. 2, 4, 8, 16 ... are required to give sum 254?

(a) 4 (b) 5 (c) 6 (d) 7

Q.31 $i^2 + i^4 + i^6 + \cdots$ upto $(2k + 1)$ terms, $k \in N$ is

(a) 0 (b) 1 (c) -1 (d) k

Q.32 If $1 + \cos a + \cos^2 a + \cdots \infty = 2 - \sqrt{2}$, then a, $(0 < a < \pi)$ is

(a)$\frac{\pi}{8}$ (b)$\frac{\pi}{6}$ (c)$\frac{\pi}{4}$ (d)$\frac{3\pi}{4}$

Q.33 If a, b, c are in G.P. and $\log_c a, \log_b c, \log_a b$ are in A.P. then the common difference of the A.P. is

(a) 3 (b) $\frac{3}{2}$ (c) $\frac{1}{2}$ (d) $\frac{2}{3}$

Q.34 If a and b are two different positive real numbers, then which of the following statements is true?
(a) $2\sqrt{ab} > a + b$ (b) $2\sqrt{ab} < a + b$ (c) $2\sqrt{ab} = a + b$ (d)None of these

Q.35 If a is positive and if A and G are the arithmetic mean and the geometric mean of the roots of $x^2 - 2ax + a^2 = 0$ respectively, then
(a) A=G (b) $A = 2G$ (c) 2A=G (d) A^2=G

Q.36 If A.M. and G.M. of roots of a quadratic equation are 8 and 5, respectively, then obtain the quadratic equation.
(a) x^2-16 x + 15 = 0 (b) x^2-16 x + 25 = 0 (c) x^2 -16 x + 5 = 0 (d) x^2-6 x + 5 = 0

Q.37 The sum of two numbers is 6 times their geometric mean, the ratio of numbers is
(a) $(3 + 2\sqrt{2}):(3 - 2\sqrt{2})$ (b) $(2 + 5\sqrt{2}):(2 - 5\sqrt{2})$
(c) $(3 + 5\sqrt{2}):(3 - 5\sqrt{2})$ (d) $(3 + 5\sqrt{2}):(3 - 5\sqrt{2})$

Q.38 Which of the following is true if A means arithmetic mean and G means geometric mean of two numbers?
(a) $A > G$ (b) $A \geq G$ (c) $G < A$ (d) $G \leq A$

Q.39 The ratio of the A.M. and G.M. of two positive numbers a and b is 5: 3. Find the ratio of a to b.
(a) 9:1 (b) 3:5 (c) 1:9 (d) 3:1

Q.40 The harmonic mean between two numbers is $14\frac{2}{5}$ and the geometric mean is 24. The greater number between them is
(a) 72 (b) 54 (c) 36 (d) None of these

Q.41 Suppose a, b, c are in AP and a^2, b^2, c^2 are in GP. If $a < b < c$ and $a + b + c = \frac{3}{2}$, then the value of a is.
(a)$\frac{1}{2\sqrt{2}}$ (b) $\frac{1}{2\sqrt{3}}$ (c) $\frac{1}{2} - \frac{1}{\sqrt{3}}$ (d) $\frac{1}{2} - \frac{1}{\sqrt{2}}$

Q.42 Let the harmonic mean and the geometric mean of two numbers be in the ration $4 : 5$. The two numbers are in the ratio
(a) $1 : 1$ (b) $2 : 1$ (c) $3 : 1$ (d) $4 : 1$

Q.43 If a, b, c, d, e, f are in A.P., then $e - c$ is equal to
(a) $d - c$ (b) $2(d - c)$ (c) $2(c - a)$ (d) $c - b$

Q.44 The m^{th} term of an A.P. is n and n^{th} term is m. Its p^{th} term is
(a) $m - n + p$ (b) $n + p - m$ (c) $m + n - p$ (d) $m + n + p$

Q.45 If 8^{th} term of an A.P. is 15 , then the sum of first 15 terms is
(a) 180 (b) 210 (c) 225 (d) 240

Q.46 The first and last term of an A.P. are 1 and 7 at the sum of its term is 36 then the number of terms will be.
(a) 6 (b) 7 (c) 8 (d) 9

Q.47 If the sum of first n terms of an A.P. is $2n^2 + 5n$, then its n^{th} term is
(a) $3n - 5$ (b) $4n - 3$ (c) $4n + 3$ (d) $3n + 5$

Q.48 If $\frac{3+5+7+\cdots \text{ to n terms}}{5+8+11+\cdots \text{ to 10 terms}} = 7$, then n is equal to
(a) 35 (b) 36 (c) 37 (d) 38

Q.49 If n A.M.'s are inserted between 3 and 17 such that the ratio of the last mean to the first mean is 3:1 then the value of n is.
(a) 4 (b) 6 (c) 8 (d) 9

Q.50 The maximum sum of the series $100 + 98 + 96 + \cdots$ is
(a) 2500 (b) 2550 (c) 2050 (d) 2555

Q.1 List the initial four terms of the following sequences, given their nth terms:
(a) 2^n (b) $n^2 - 16$ (c) $\frac{n+4}{n+1}$

Q.2 A sequence of numbers (a_0, a_1, a_2, a_3,) satisfies the relation $a_n^2 - a_{n-1}a_{n+1} = (-1)^n$. Find a_3, given $a_0 = 1$ and $a_1 = 3$

Q.3 Find the 21^{st}, and 42^{nd} terms of the sequence defined by: $t_n \begin{cases} 0, \text{If n is odd} \\ 1, \text{If n is even} \end{cases}$

Q.4 The sequence (a_n) is defined by a (n) = (n–1)(n–2)(n–3). Show that the first three terms of the sequence are zero, but the rest of the terms are positive.

Q.5 Write the next terms of the sequence $\frac{1}{6}, \frac{1}{3}, \frac{1}{2}, \ldots$

Q.6 Find: $\sum_{n=1}^{5}(15 - 9n)$

Q.7 Find the indicated partial sum.
(a) 3,5,9,17,33 ... S_4 (b) $a_n = 2 - 7n; S_5$ (c) $a_n = (-1)^n 2^{2n-3}; S_5$

Q.8 Evaluate: $\sum_{n=1}^{5}(-1)^{n+1}2^n$

Q.9 Express in expanded form. $\sum_{n=1}^{\infty} \frac{n-1}{n}$

Q.10 Express in expanded form. $\sum_{k=0}^{\infty}(-1)^k x^{k+1}$

Q.11 Express in expanded form. $\sum_{i=0}^{\infty}(-2)^{i+1}x^i$

Q.12 Represent the given series using sigma notation. $2 + 2^2x + 2^3x^2 + 2^4x^3 + 2^5x^4$

Q.13 Write the given series using sigma notation. $\frac{3}{4} + \frac{3}{8} + \frac{3}{16} + \cdots + 3(\frac{1}{2})^n$

Q.14 Find the sum of 10 A.M.'s inserted between a and b.

Q.15 For the sequence given by $a_n = 4n^2 - n + 1$, find first three terms.

Q.16 Find the sum of first 32 even natural numbers.

Q.17 Find the n^{th} term of the series $1 + 2 + 4 + 7 + \cdots$

Q.18 Find the sum of infinity of the series whose n^{th} term is $\frac{1}{n(n+1)}$

Q.19 Find the 18th and 25th terms of the sequence defined by

$$a_n = \begin{cases} n(n+2), \text{if n is even natural number} \\ \frac{4n}{n^2+1}, \text{if n is odd natural number} \end{cases}$$

Q.20 Find the sum of all positive integers less than 500 and divisible by both 3 and 7.

Q.21 If the sum of n terms of an A.P. is $5n^2 + 3n$, find the series.

Q.22 The digits of a positive integer, having three digits, are in A.P. and their sum is 15. The number obtained by reversing the digits is 594 less than the original number. Find the number.

Q.23 If a, b, c, d are in G.P., show that a + b, b + c, c + d are in G.P.

Q.24 A square is drawn by joining the midpoints of sides of a given square. A third square is drawn inside the second square in the same way and this process continues indefinitely. If the side of the first square is 16 cm, determine the sum of the areas of all the squares.

Q.25 The sum of integers from 1 to 100 that are divisible by 2 or 5 is equal to.

Q.26 The third term of a G.P. is 4. The product of the first five terms is equal to

Q.27 Consider an infinite G.P. with first term a and common ratio r. If its sum is 4 and the second term is $\frac{3}{4}$ then value of r is equal to

ANSWER KEY – LEVEL – I

Q.	1	2	3	4	5	6	7	8	9	10
Ans.	d	a	a	a	c	a	c	b	d	d
Q.	11	12	13	14	15	16	17	18	19	20
Ans.	d	a	b	b	a	d	a	c	a	a
Q.	21	22	23	24	25	26	27	28	29	30
Ans.	a	a	b	b	b	d	b	c	a	d
Q.	31	32	33	34	35	36	37	38	39	40
Ans.	c	d	b	b	a	b	a	b	a	a
Q.	41	42	43	44	45	46	47	48	49	50
Ans.	d	d	b	c	c	d	c	a	b	b

ANSWER KEY – LEVEL – II

1. (a) 2,4,8,16 (b) $-15,-12,-7,0$ (c) $\frac{5}{2},2,\frac{7}{4},\frac{8}{5}$

2. 3

3. 21^{st} term of the sequence is 0, and 42^{nd} term of the sequence 1.

5. $\frac{2}{3}$

6. -60

7. (a) 34 (b) -95 (c) -100

8. 22

9. $0+\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\cdots$

10. $x-x^2+x^3-x^4+\cdots$

11. $-2+4x-8x^2+16x^3-\cdots$

12. $\sum_{k=1}^{5}2^k x^{k-1}$

13. $\sum_{k=2}^{n}3(\frac{1}{2})^k$