Q.1 The value of $\lim_{x\to 2}([2-x]+[x-2]-x)$ equals (where ['] denotes greatest integer function)

(A) 0 (B) 3

(C) –3 (D) Does not exist

Q.2 The value of $\lim_{\theta\to 0^+}\frac{\sin\sqrt{\theta}}{\sqrt{\sin\theta}}$ is equal to

(A) 0 (B) 1

(C) –1 (D) Does not exist

Q.3 The value of $\lim_{h\to 0}\{\frac{1}{h\cdot(8+h)^{1/3}}-\frac{1}{2h}\}$ equals

(A) $\frac{1}{12}$ (B) $\frac{-4}{3}$

(C) $\frac{-16}{3}$ (D) $\frac{-1}{48}$

Q.4 The value of $\lim_{x\to\infty}\{\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x}\}$ equals

(A) 0 (B) $\frac{1}{2}$

(C) 2 (D) log2

Q.5 The value of $\lim_{x\to 0}\frac{\sin x+\log(1-x)}{x^2}$ equals

(A) 0 (B) $\frac{1}{2}$

(C) $\frac{-1}{2}$ (D) –1

Q.6 The value of $\lim_{x\to 0}\frac{\tan x-\sin x}{x^3}$ equals

(A) 1 (B) $\frac{1}{2}$

(C) $\frac{-1}{2}$ (D) –1

Q.7 $\lim_{n\to\infty}(3^n+5^n+7^n)^{1/n}$ is equal to

(A) e^3 (B) e^5

(C) 5 (D) 7

Q.8 The value of $\lim_{x\to 0}\frac{\cos(\sin x)-\cos x}{x^4}=$

(A) 1 (B) 6

(C) $-\frac{1}{6}$ (D) $\frac{1}{6}$

Q.9 The value of $\lim_{x\to 0}(\frac{1}{x^2}-\cot x)$ equals

(A) $-\infty$ (B) 0

(C) ∞ (D) Does not exist

Q.10 The value of $\lim_{x\to\infty}\frac{[x]+[2x]+\cdots..+[nx]}{n^2}$, where [.] is greatest integer function, is equal to

(A) x (B) $\frac{x}{2}$

(C) $\frac{x}{3}$ (D) $\frac{x^2}{2}$

Q.11 The function $f(x)=\frac{\cos x-\sin x}{\cos 2x}$ is not defined at $x=\frac{\pi}{4}$. The value of $f\left(\frac{\pi}{4}\right)$ so that f(x) is continuous at $x=\frac{\pi}{4}$ is

(A) 1 (B) –1

(C) $\sqrt{2}$ (D) $\frac{1}{\sqrt{2}}$

Q.12 The function $f(x)=\frac{\log(1+ax)-lo\ (1-bx)}{x}$ is not defined at $x=0$ is not defined at $x=0$. The value which should be assigned to f at $x=0$, so that it is continuous at $x=0$, is

(A) a – b (B) a + b

(C) loga +logb (D) 0

Q.13 The value of f(0), so that the function $f(x)=\frac{2x-s\ ^{-1}x}{2x+\sin^{-1}x}, x\neq 0$, is continuous at each point in its domain, is equal to

(A) 2 (B) $\frac{1}{3}$

(C) $\frac{2}{3}$ (D) $-\frac{1}{3}$

Q.14 The value of f(0) so that the function $f(x)=\frac{\log(1+x^2\tan x)}{\sin x^3}, x\neq 0$, continuous at $x=0$, is given by

(A) 1 (B) 0

(C) 2 (D) –1

Q.15 In order that $f(x)=(x+1)^{\cot x}$ is continuous at $x=0$, f(0) must be equal to

(A) 0 (B) e

(C) $\frac{1}{e}$ (D) e^2

Q.16 The value of f(0), so that the function $f(x)=\frac{1-\cos(1-\cos x)}{x^4}$ is continuous everywhere, is

(A) $\frac{1}{8}$ (B) $\frac{1}{2}$

(C) $\frac{1}{4}$ (D) $\frac{1}{6}$

Q.17 Let $f(x)=\begin{cases}ax+1, & x<1\\ 3, & x=1\\ bx^2+1, & x\geq 1\end{cases}$. The value of 'a' and 'b' for which f(x) is continuous at $x=1$ is given by

(A) 2, 2 (B) 1, 2

(C) 3, 2 (D) 1, 1

Q.18 Which of the following functions have finite number of points of discontinuity on real set? (where [.] represents greatest integer function)

(A) tanx (B) x[x]

(C) $\frac{|x|}{x}$ (D) $\sin[m\pi x], n\in N$

Q.19 The values of a and b, so that the function $f(x)=\begin{cases}x^2+3x+a: & \text{when } x\leq 1\\ bx+2 & ;\ \text{when } x>1\end{cases}$ is differentiable for each $x\in$ in R; are respectively

(A) 3, 5 (B) 5, 7

(C) 4, 8 (D) 6, 5

Q.20 Let $f(x) = \max\{x, x^3\}$. Set of points on which f(x) is not differentiable is
(A) $\{-1,1\}$ (B) $\{-1,0\}$
(C) $\{0,1\}$ (D) $\{-1,0,1\}$

Q.21 The number of points where $f(x) = |x^2 - 3x + 2|$ is non-differentiable is
(A) 0 (B) 1
(C) 2 (D) 3

Q.22 If [] represents greatest integer function, then the left hand derivative of $f(x) = [x] \sin \pi x$ at $x = k$, k is an integer, is given by
(A) $(-1)^k(k-1)\pi$ (B) $(-1)^{k-1}(k-1)\pi$
(C) $(-1)^k \cdot k\pi$ (D) $(-1)^{k-1} \cdot k\pi$

Q.23 The largest set of points, where the function $f(x) = x|x|$ is differentiable, is given by
(A) $(-\infty, \infty)$ (B) $(-\infty, 0) \cup (0, \infty)$
(C) $(0, \infty)$ (D) $(0, \infty)$

Q.24 The set of all points, where the function $f(x) = \frac{x}{1+|x|}$ is differentiable, is given by
(A) $(-\infty, \infty)$ (B) $(-\infty, 0) \cup (0, \infty)$
(C) $(0, \infty)$ (D) $(0, \infty)$

Q.25 If $f(x) = |x - 2020|$, then $f'(x)$ at $x = 2019$, is given by
(A) 1 (B) –1
(C) 0 (D) 2019

Q.26 Let $f(2) = 4, f'(2) = 4$. Then $\lim\limits_{x\to 2} \frac{xf(2)-2f(x)}{x-2}$ is
(A) $-\frac{1}{3}$ (B) –2
(C) –4 (D) 3

Q.27 $\lim\limits_{x\to 0}(\frac{4^x+9^x}{2})^{\frac{1}{x}}$ equals
(A) 2 (B) 6
(C) 16 (D) 112

Q.28 The value of $\lim\limits_{x\to 0}(\frac{\sin 2x}{x} + \frac{e^x-1}{x} + \frac{\log(1+x)}{x})$ is equal to
(A) 4 (B) 3
(C) 2 (D) 1

Q.29 If the function $f(x) = (1-x)\tan\frac{\pi x}{2}$ is continuous at $x = 1$, then $f(1) =$
(A) $\frac{2}{\pi}$ (B) $\frac{\pi}{2}$
(C) 0 (D) 2π

Q.30 Let the function f(x) be defined as

$$f(x) = \begin{cases} \frac{\ln x - 1}{x-e}, x \neq e \\ k, x = e \end{cases}$$

The value of k, for which the function is continuous at $x = e$, is equal to
(A) e (B) $\frac{1}{e}$
(C) e^2 (D) -e

Q.31 At $x = 0$, the function $y = e^{-|x|}$ is
(A) Continuous
(B) Continuous and differentiable
(C) Differentiable with derivative $= 1$
(D) Differentiable with derivative $= -1$

Q.32 Suppose f(x) is differentiable at $x = 1$ and $\text{Lt}_{n\to 0}\frac{1}{n}f(1+n) = 5$, then $f'(1)$ equals
(A) 6
(B) 5
(C) 4
(D) 3

Q.33 If $f(x) = \log|x|$ then for $x \neq 0$, $f'(x)$ is equal to
(A) $\frac{1}{|x|}$
(B) $\frac{1}{x}$
(C) $-\frac{1}{x}$
(D) $\pm\frac{1}{x}$

Q.34 Differential coefficient of $\log_{10} x$ w.r.t. $\log_x 10$ is
(A) $-\frac{(\log x)^2}{(\log 10)^2}$
(B) $\frac{(\log_{10} x)^2}{(\log 10)^2}$
(C) $\frac{(\log_x 10)^2}{(\log 10)^2}$
(D) $-\frac{(\log 10)^2}{(\log x)^2}$

Q.35 In [0,1], Lagrange mean value theorem is NOT applicable to
(A) $f(x) = \begin{cases} \frac{1}{2} - x, x < \frac{1}{2} \\ (\frac{1}{2} - x)^2, x \geq \frac{1}{2} \end{cases}$
(B) $f(x) = \begin{cases} \frac{\sin x}{x}, x \neq 0 \\ 1, x = 0 \end{cases}$
(C) $f(x) = x|x|$
(D) $f(x) = |x|$

Q.36 Let $a = \lim_{x\to 0} x\cot x$ and $b = \lim x\log x$, then
(A) $a = b$
(B) $b > a$
(C) $a = b + 1$
(D) $b = a + 1$

Q.37 $\lim_{x\to\infty}\frac{(1+x+x^2)}{(\ln x)^3}$ is equal to
(A) 2
(B) e^2
(C) e^{-2}
(D) ∞

Q.38 Let $f(x) = \sqrt{1-\sqrt{1-x^2}}$, then f(x) is
(A) Continuous on $[-1,1]$ and differentiable on $(-1,1)$
(B) Continuous on $[-1,1]$ and differentiable on $(-1,0) \cup (0,1)$
(C) Continuous and differentiable on $(-1,1)$
(D) Discontinuous at $x = 0$

Q.39 The value of $\lim_{h\to 0}\frac{f(x+h)-f(x-h)}{h}$ is equal to
(A) $f'(x)$
(B) 0
(C) $2f'(x)$
(D) $-f'(x)$

Q.40 If $f(x) = \begin{cases} \frac{x^2+3x+p}{2(x^2-1)}, x \neq 1 \\ \frac{5}{4}, x = 1 \end{cases}$ is continuous at $x = 1$ then
(A) $p = 2$
(B) $p = 0$
(C) $p = -4$
(D) $p = 5$

Q.41 If $y = \frac{e^x - e^{-x}}{e^x + e^{-x}}$, then $\frac{dy}{dx} =$
(A) $1 + y^2$
(B) $1 - y^2$
(C) $y^2 - 1$
(D) $y^2 + 2$

Q.42 If $y = \cos^{-1}(\cos x)$ then $\frac{dy}{dx}$ at $x = \frac{5\pi}{4}$ is equal to

(A) 1 (B) –1

(C) $\frac{1}{\sqrt{2}}$ (D) $\sqrt{2}$

Q.43 If $f(x) = -\sqrt{25 - x^2}$, then $\lim_{x\to 1} \frac{f(x)-f(1)}{x-1}$ is equal to

(A) $\frac{1}{24}$ (B) $\frac{1}{5}$

(C) $-\sqrt{24}$ (D) $\frac{1}{\sqrt{24}}$

Q.44 If f be a function of ' x ' such that $f(9) = 9, f'(9) = 3$, then $\lim_{x\to 9} \frac{\sqrt{f(x)}-3}{\sqrt{x}-3}$ is

(A) 9 (B) 3

(C) 1 (D) $\frac{1}{3}$

Q.45 The value of $\lim_{x\to 0}(1 - \frac{1}{2^x})(\frac{1}{\sqrt{\tan x+4}-2})$ is

(A) $\log_a 16$ (B) does not exist

(C) $3\ell n\, 2$ (D) $4\ell n2$

Q.46 If $\lim_{x\to 0} \frac{\sin 6x}{\tan 3x} = a$, $\lim_{x\to\infty} \frac{\sin x}{x} = b$, $\lim_{x\to\infty} \frac{\log x}{x} = c$ then value of $a + b + c$ is

(A) 1 (B) 2

(C) 3 (D) 4

Q.47 $\lim_{x\to 0} \frac{x\tan 2x - 2x\tan x}{(1-\cos 2x)^2}$ equals

(A) 1 (B) $\frac{1}{3}$

(C) $\frac{1}{4}$ (D) $\frac{1}{2}$

Q.48 $\lim_{x\to 2^+}(\frac{[x]^3}{3} - [\frac{x}{3}]^3)$ is, (where [ ] is G.I.F.)

(A) 0 (B) $\frac{64}{27}$

(C) $\frac{8}{3}$ (D) does not exist

Q.49 Let α and β be the distinct root of $ax^2 + bx + c = 0$, then $\lim_{x\to\alpha} \frac{1-\cos(ax^2+bx+c)}{(x-\alpha)^2}$ is equal to

(A) $\frac{1}{2}(\alpha - \beta)^2$ (B) $-\frac{a^2}{2}(\alpha - \beta)^2$

(C) 0 (D) $\frac{a^2}{2}(\alpha - \beta)^2$

Q.50 The value of $\lim_{x\to 0} \frac{\sin(\pi\cos^2 x)}{x^2}$ equals

(A) $-\pi$ (B) π

(C) $\frac{\pi}{2}$ (D) e

Q.51 $\lim_{x\to 0}(\frac{1}{x^2} - \frac{1}{\tan^2 x})$ equals

(A) 0 (B) $\frac{1}{3}$

(C) $\frac{2}{3}$ (D) $\frac{1}{2}$

Q.52 If $\lim_{x\to 0}(\cos x + a\sin bx)^{1/x} = e^2$, then the values of a and b can be

(A) $a = 1, b = -2$ (B) $a = 2\sqrt{2}, b = \sqrt{2}$

(C) $a = 2\sqrt{2}, b = \frac{1}{\sqrt{2}}$ (D) $a = -2, b = 1$

Q.53 If {x} denotes the fractional part of x. Then $\lim_{x\to 0}\frac{\{x\}}{\tan\{x\}}$ is equal to

(A) 1 (B) 0

(C) –1 (D) Limit doesn't exist

Q.54 If α and β be the roots of the equation $ax^2+bx+c=0$ then $\lim_{x\to\frac{1}{\alpha}}\sqrt{\frac{1-\cos^2(cx^2+bx+a)}{4(1-\alpha x)^2}}$ equals

(A) Not possible (B) $\left|\frac{c}{2\alpha}\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)\right|$

(C) $\left|\frac{c}{2\alpha}\left(\frac{1}{\alpha}-\frac{1}{\beta}\right)\right|$ (D) $\left|\frac{c}{2}\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)\right|$

Q.55 If $\lim_{x\to\infty}(1+\frac{a}{x}+\frac{b}{x^2})^{2x}=e^2$, then the value of a and b are

(A) $a\in R, b\in R$ (B) $a=1, b\in R$

(C) $a\in R, b=2$ (D) $a=1, b=2$

Q.56 If the value of $\lim_{x\to 0}\frac{(1+\sin x)^{\text{cosec}}-e+(\frac{\sin x}{2})e}{\sin^2 x}$ isk, then $(24k-11e)$ is equal to

(A) 0 (B) 1

(C) 2 (D) 3

Q.57 Let $S_n=\frac{1}{1.4}+\frac{1}{4.7}+\frac{1}{7.10}+\cdots$. to n terms then $\lim_{n\to\infty}S_n$ is equal to

(A) $\frac{1}{3}$ (B) 3

(C) $\frac{1}{4}$ (D) ∞

Q.58 Let $t_r=\frac{r}{1+r^2+r^4}$, then $\lim_{n\to\infty}\sum_{r=1}^{n} t_r$ equals

(A) $\frac{1}{4}$ (B) 1

(C) $\frac{1}{2}$ (D) $\frac{1}{3}$

Q.59 If f(x) and g(x) be differentiable functions and $f(1)=g(1)=2$ then $\lim_{x\to 1}\frac{f(1)\cdot g(x)-f(x)\cdot g(1)-f(1)+g(1)}{g(x)-f(x)}$ is equal to

(A) 0 (B) 1

(C) 2 (D) –2

Q.60 $f(x)=\begin{cases}\frac{\sqrt{1+px}-\sqrt{1-px}}{x} & -1\le x<0\\ \frac{2x+1}{x-2} & 0\le x\le 1\end{cases}$; is continuous in the interval $[-1,1]$, then p equals

(A) –1 (B) $-\frac{1}{2}$

(C) $\frac{1}{2}$ (D) 1

Q.61 If $f(x)=\begin{cases}\frac{a|x^2-x-2|}{2+x-x^2}, & x<2\\ b, & x=2\\ \frac{x-[x]}{x-2}, & x>2\end{cases}$; is continuous at $x=2$ and [ ] is G.I.F., then values of a and h are

(A) $a=1, b=1$ (B) $a=1, b=2$

(C) $a=2, b=1$ (D) $a=2, b=2$

Q.62 The function $f(x) = \sin^{-1}(\cos x)$ is

(A) Continuous at $x = 0$ (B) Discontinuous at $x = 0$

(C) Differentiable at $x = 0$ (D) Differentiable at $x = \pi$

Q.63 If $f(x) = \begin{cases} xe^{-\left(\frac{1}{|x|}+\frac{1}{x}\right)}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

(A) Continuous for all x but not differentiable everywhere

(B) Neither differentiable nor continuous

(C) Discontinuous everywhere

(D) Continuous as well as differentiable for all x

Q.64 The number of points at which the function$f(x) = \frac{1}{\log|x|}$ is discontinuous is

(A) 1 (B) 2

(C) 3 (D) 4

Q.65 The value of f(0) so that $f(x) = \frac{(4^x-1)^3}{\sin\frac{x}{4}.\log\left(1+\frac{x^2}{3}\right)}$; is continuous everywhere is

(A) $3(\log 4)^3$ (B) $4(\log 4)^3$

(C) $12(\log 4)^3$ (D) $15(\log 4)^3$

Q.66 Let $f(x) = \begin{cases} \frac{3|x|+4\tan x}{x}, & x \neq 0 \\ k, & x = 0 \end{cases}$; then f(x) is continuous at $x = 0$ for

(A) $k = 7$ (B) $k = 1$

(C) No values of k (D) $k = 2$

Q.67 If the function $f(x) = \frac{x(e^{\sin x}-1)}{1-\cos x}$ is continuous at$x = 0$ then $f(0) =$

(A) 1 (B) 0

(C) 2 (D) $\frac{1}{2}$

Q.68 If $f(x) = \begin{cases} [x] + [-x], & x \neq 0 \\ \lambda, & x = 0 \end{cases}$, where [.] denotes the

(A) –1 (B) 0

(C) 1 (D) No value is possible

Q.69 Let $f(x) = \begin{cases} \frac{\sin[x]}{[x]}, [x] \neq 0 \\ 0, [x] = 0 \end{cases}$, then f(x) is ([.] denotes the greatest integer function)

(A) Continuous at $x = 0$ (B) Left continuous at $x = 0$

(C) Differentiable at $x = 0$ (D) Right continuous at $x = 0$

Q.70 If $f(x) = \begin{cases} px^2 - q, x \in [0,1) \\ x + 1, \quad x \in (1,2] \end{cases}$; and $f(1) = 2$, then the value of the pair (p, q) for which f(x) cannot be continuous at $x = 1$ is

(A) (2,0) (B) $(1, -1)$

(C) (4,2) (D) (1,1)

Q.71 If $f(x) = e^{\frac{-1}{x^2}}, x \neq 0$ and $f(0) = 0$ then $f'(0)$ is

(A) 0 (B) 1

(C) e (D) 2

Q.72 If $f(x)=\begin{cases}\frac{1}{1+e^{1/x}}, x\neq 0\\ 0, x=0\end{cases}$; then f(x) is

(A) Continuous at $x=0$

(B) Continuous and differentiable at $x=0$

(C) Continuous but not differentiable at $x=0$

(D) Continuous but not differentiable at $x=0$

Q.73 Domain of differentiability of the function$f(x)=|x-2|\cos x$ is

(A) R (B) $R-\{2\}$

(C) $(0,\infty)$ (D) $\{2\}$

Q.74 Let $f(x)=\frac{\sin(\pi[x+\pi])}{1+[x]^2}$, where [ ] denotes the greatest integer function, then f(x) is

(A) Continuous and differentiable at all $x\in R$

(B) Continuous but not differentiable at some x

(C) Differentiable but not continuous at $x=0$

(D) Neither continuous nor differentiable at $x=0$

Q.75 Let $f(x)=\begin{cases}\frac{|x+1|}{\tan^{-1}(x+1)}, x\neq -1\\ 1, \quad x=-1\end{cases}$. Then f(x) is

(A) Continuous at $x=-1$

(B) Differentiable at $x=-1$

(C) Discontinuous at $x=-1$

(D) Continuous but not derivable at $x=-1$

Q.76 The function $|x^2-3x+2|+\cos|x|$ is not differentiable at x =

(A) $-1,2$ (B) $-1,-2$

(C) $1,2$ (D) $-2,1$

Q.77 Let $f(x)=[x], g(x)=|x|$ and $f\{g(x)\}=h(x)$, where $[\cdot]$ is the greatest integer function. Then $h'(-1)$ is

(A) 0 (B) $-\infty$

(C) Does not exist (D) 5

Q.78 Let $f(x)=\max\{4,1+x^2,x^2-1\}\forall x\in$ R. Total number of points, where f(x) is non-differentiable, is equal to

(A) 2 (B) 4

(C) 6 (D) 0

Q.79 Let $f(x)=\begin{Bmatrix}\max\{|x|,x^2\}, & |x|\leq 2\\ 8-2|x|, & 2<|x|\leq 4\end{Bmatrix}$

Let S be the set of points in the interval $(-4,4)$ at which f is not differentiable. Then S

(A) Equals $\{-2,-1,0,1,2\}$

(B) Equals $\{-2,2\}$

(C) Is an empty set

(D) Equals $\{-2,-1,1,2\}$

Q.80 If $x\log_\theta(\log_e x)-x^2+y^2=4(y>0)$, then $\frac{dy}{dx}$ at x = e is equal to

(A) $\frac{(2e-1)}{2\sqrt{4+e^2}}$ (B) $\frac{(1+2e)}{2\sqrt{4+e^2}}$

(C) $\frac{(1+2e)}{\sqrt{4+e^2}}$ (D) $\frac{e}{\sqrt{4+e^2}}$

EXERCISE LEVEL –II

EL– II

Q.1 Show that the function $\begin{cases} x\sin\left(\frac{1}{x}\right), & \text{when } x \neq 0 \\ 0, & \text{when } x = 0 \end{cases}$, is continuous but not differentiable at $x = 0$

Q.2 If $y = \sin(\cos x^2)$, find $\frac{dy}{dx}$.

Q.3 If $y = \frac{(\sin x + x^2)}{\cot 2x}$, find $\frac{dy}{dx}$.

Q.4 If $y = \frac{\cos x + \sin x}{\cos x - \sin x}$, prove that $\frac{dy}{dx} = \sec^2(x + \frac{\pi}{4})$.

Q.5 If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$.

Q.6 If $\sin y = x\sin(a+y)$, prove that $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$.

Q.7 If $y = \sec^{-1}(\frac{x+1}{x-1}) + \sin^{-1}(\frac{x-1}{x+1})$, show that $\frac{dy}{dx} = 0$.

Q.8 If $y = \tan^{-1}(\frac{ax-b}{bx+a})$, prove that $\frac{dy}{dx} = \frac{1}{1+x^2}$.

Q.9 If $y = \sin^{-1}(\frac{2x}{1+x^2}) + \sec^{-1}(\frac{1+x^2}{1-x^2})$, show that $\frac{dy}{dx} = \frac{4}{1+x^2}$.

Q.10 Differentiate $(\sin x)^x$ w.r.t. x.

Q.11 If $y = (\cos x)^{\sin x}$, find $\frac{dy}{dx}$.

Q.12 If $x^y = e^{x-y}$, prove that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$.

Q.13 If $x^m \cdot y^n = (x+y)^{m+n}$, then prove that $\frac{dy}{dx} = \frac{y}{x}$.

Q.14 If $x = a\cos^3 t, y = a\sin^3 t$, then find $\frac{dy}{dx}$.

Q.15 Find $\frac{d^2y}{dx^2}$ when $y = x^x$. $(x > 0)$

Q.16 If $y = a\cos(\log x) + b\sin(\log x) x > 0$, prove that $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

Q.17 Verify Rolle's theorem for the function $f(x) = x(x-1)^2$ in the interval $[0,1]$.

Q.18 Verify Rolle's theorem of the function $f(x) = e^x \cdot \cos x$ in the interval $[-\frac{\pi}{2}, \frac{\pi}{2}]$.

Q.19 Verify Rolle's theorem for the function $f(x) = \{\log(x^2+2) - \log 3\}$ on $[-1,1]$.

Q.20 If Rolle's theorem holds for the function $f(x) = x^3 + bx^2 + ax + 5$ on $[1,3]$ with $c = 2 + \frac{1}{\sqrt{3}}$, find the values of a and b.

Q.21 If $p(x) = 51x^{101} - 2323x^{100} - 45x + 1035$.

Using Rolle's theorem prove that at least one root lies between $(45^{\frac{1}{100}}, 46)$.

Q.22 Verify Lagrange's mean value theorem for the function $f(x) = (x-1)(x-2)(x-3)$ in $[0,4]$.

Q.23 Verify the hypothesis and conclusion of Lagrange's mean value theorem for the function $f(x) = \frac{1}{4x-1}, 1 \leq x \leq 4$.

Q.24 Find c of the mean value theorem for the function $f(x) = \frac{1}{4x-1}, 1 \leq x \leq 4$.

Q.25 Find a point on the curve $y = x^3$ where the tangent to the curve is parallel to the chord joining the points (1, 1) and (3, 27).

Q.26 For a function $f(x)$, $f(0) = 0$ and $f'(x) = \frac{1}{1+x^2}$, prove that $f(x) + f(y) = f(\frac{x+y}{1-xy})$.

ANSWER KEY – LEVEL – I

Q.	1	2	3	4	5	6	7	8	9	10
Ans.	C	B	D	B	C	B	D	D	D	B
Q.	11	12	13	14	15	16	17	18	19	20
Ans.	D	B	B	A	B	A	A	C	A	D
Q.	21	22	23	24	25	26	27	28	29	30
Ans.	C	A	A	A	B	C	B	A	A	B
Q.	31	32	33	34	35	36	37	38	39	40
Ans.	A	B	B	A	A	C	D	B	C	C
Q.	41	42	43	44	45	46	47	48	49	50
Ans.	B	B	D	B	D	B	D	C	D	B
Q.	51	52	53	54	55	56	57	58	59	60
Ans.	C	C	D	C	B	A	A	C	C	B
Q.	61	62	63	64	65	66	67	68	69	70
Ans.	A	A	A	C	C	C	C	A	D	D
Q.	71	72	73	74	75	76	77	78	79	80
Ans.	A	D	B	A	C	C	C	A	A	A