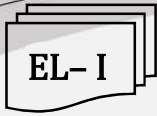


EXERCISE LEVEL - I


 EL- I

- Q.1** $\lim_{x \rightarrow k} (x - [x])$ Where k is an integer, is equivalent to (where $[\cdot]$ signifies the greatest integer function).
- (a) 1 (b) 0 (c) -1 (d) Does not exist
- Q.2** The value of $\lim_{x \rightarrow 1} \frac{x^{P+1} - Px - x + P}{(x-1)^2}$; ($P \in \mathbb{N}$) equal.
- (a) $\frac{P(P-1)}{2}$ (b) P (c) $\frac{P(P+1)}{2}$ (d) $\frac{P^2-2P}{2}$
- Q.3** The value of $\lim_{x \rightarrow 0} \frac{\tan x - \sin x}{x^3}$ equals
- (a) 1 (b) $\frac{1}{2}$ (c) $\frac{-1}{2}$ (d) -1
- Q.4** The value of $\lim_{x \rightarrow \frac{\pi}{4}} \frac{\sin x - \cos x}{(x - \frac{\pi}{4})^2}$ equals
- (a) $\sqrt{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) 2 (d) $\frac{1}{2}$
- Q.5** The value of $\lim_{x \rightarrow 0} \frac{x \tan 2x - 2x \tan x}{(1 - \cos 2x)^2}$ is equal to
- (a) 2 (b) -2 (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$
- Q.6** The value of $\lim_{x \rightarrow 0} \frac{\sin(\pi \cos^2 x)}{x^2}$ equals
- (a) $-\pi$ (b) π (c) $\frac{\pi}{2}$ (d) 2π
- Q.7** The value of $\lim_{x \rightarrow 0} \frac{\sqrt{1 - \cos 2x}}{x}$ equals
- (a) 0 (b) 1 (c) $\sqrt{2}$ (d) Does not exist
- Q.8** The value of $\lim_{\theta \rightarrow \frac{\pi}{2}} (\sec \theta - \tan \theta)$ equals
- (a) 0 (b) 1 (c) 2 (d) ∞
- Q.9** The value of $\lim_{x \rightarrow 0} (\frac{1}{x^2} - \cot x)$ is
- (a) 1 (b) 0 (c) ∞ (d) Limit does not exist
- Q.10** The value of $\lim_{x \rightarrow \infty} \left\{ \sqrt{x + \sqrt{x + \sqrt{x}}} - \sqrt{x} \right\}$ equals
- (a) 0 (b) $\frac{1}{2}$ (c) 2 (d) $\log 2$
- Q.11** The value of $\lim_{x \rightarrow 0} \frac{\sin \alpha x - \sin \beta x}{e^{\alpha x} - e^{\beta x}}$ equals
- (a) 0 (b) 1 (c) -1 (d) $\alpha - \beta$
- Q.12** The value of $\lim_{x \rightarrow 0} \left\{ \tan \left(\frac{\pi}{4} + x \right) \right\}^{\frac{1}{x}}$ equals
- (a) e^2 (b) e^3 (c) e^{-2} (d) 1
- Q.13** The value of $\lim_{x \rightarrow \infty} \left(\frac{x^2 + 5x + 3}{x^2 + x + 2} \right)^x$ equals
- (a) e^4 (b) e^2 (c) e^3 (d) e
- Q.14** The value of $\lim_{x \rightarrow \infty} \left(\frac{x+6}{x+1} \right)^{x+4}$ equals
- (a) 0 (b) 1 (c) e^4 (d) e^5

- Q.15** The value of $\lim_{x \rightarrow 0} \left(\frac{1+\tan x}{1+\sin x} \right)^{\operatorname{cosec} x}$ equals
 (a) e (b) $\frac{1}{e}$ (c) 1 (d) 0
- Q.16** The value of $\lim_{x \rightarrow 0} \left(\frac{a^x + b^x + c^x}{3} \right)^{\frac{1}{x}}$ equals
 (a) $(abc)^{\frac{1}{3}}$ (b) $\frac{1}{3}(abc)$ (c) $\log(abc)^{\frac{1}{3}}$ (d) 1
- Q.17** The value of $\lim_{x \rightarrow 0^+} ((\sin x)^{\frac{1}{x}} + (\frac{1}{x})^{\sin x})$ equals
 (a) 0 (b) 1 (c) ∞ (d) -1
- Q.18** The value of $\lim_{x \rightarrow 0} |x|^{\sin x}$ equals
 (a) 0 (b) -1 (c) 1 (d) Does not exist
- Q.19** If $y = \log(\sec x + \tan x)$, then $\frac{dy}{dx} =$
 (a) $\tan x$ (b) $\sec x$ (c) $\tan^2 x$ (d) $\sec^2 x$
- Q.20** Let $f(x) = \log(x + \sqrt{x^2 + 1})$, then $f'(x)$ equals
 (a) $\sqrt{x^2 + 1}$ (b) $\frac{x}{\sqrt{x^2 + 1}}$ (c) $1 + \frac{x}{\sqrt{x^2 + 1}}$ (d) $\frac{1}{\sqrt{x^2 + 1}}$
- Q.21** Let $f(x) = e^x \cdot g(x)$, $g(0) = 4$, $g'(0) = 2$, then $f'(0)$ equals
 (a) 1 (b) 2 (c) 3 (d) 6
- Q.22** The value of $\frac{dy}{dx}$ is, when $y = f\left(\frac{1}{x}\right)$ and $f'(x) = \sin(x^2)$
 (a) $-x^2 \sin\left(\frac{1}{x^2}\right)$ (b) $\frac{-1}{x^2} \sin(x^2)$ (c) $\frac{-1}{x^2} \sin\left(\frac{1}{x^2}\right)$ (d) $-x^2 \sin(x^2)$
- Q.23** If $\sqrt{x} + \sqrt{y} = 4$, then $\frac{dy}{dx}$ at $x = 1$ is given by
 (a) 3 (b) -1 (c) -3 (d) 1
- Q.24** If $y = |\cos x| + |\sin x|$, then $\frac{dy}{dx}$ at $x = \frac{2\pi}{3}$ is
 (a) $\frac{1-\sqrt{3}}{2}$ (b) 0 (c) $\frac{1}{2}(\sqrt{3} - 1)$ (d) $\frac{\sqrt{3}+1}{2}$
- Q.25** The value of $\lim_{x \rightarrow 0} \frac{\log \cos x}{x}$ is equal to
 (a) 0 (b) ∞ (c) 1 (d) Does not exist
- Q.26** If $\lim_{x \rightarrow a} \frac{a^x - x^a}{x^x - a^a} = -1$, then
 (a) $a = 1$ (b) $a = 0$ (c) $a = e$ (d) $a = \frac{1}{e}$
- Q.27** The value of $\lim_{x \rightarrow 0} \frac{x \cos x - \log(1+x)}{x^2}$ is equal to
 (a) $\frac{1}{2}$ (b) 0 (c) 1 (d) Does not exist
- Q.28** The value of $\lim_{x \rightarrow 0} \frac{(1-x)^n - 1}{x}$ equals
 (a) $n!$ (b) $(n-1)!$ (c) $-n$ (d) n
- Q.29** If $G(x) = -\sqrt{25 - x^2}$, then $\lim_{x \rightarrow 1} \frac{G(x) - G(1)}{x - 1}$ has the value
 (a) $\frac{1}{24}$ (b) $\frac{1}{5}$ (c) $-\sqrt{24}$ (d) $\frac{1}{\sqrt{24}}$
- Q.30** The value of $\lim_{x \rightarrow \frac{\pi}{3}} \frac{\sin(\frac{\pi}{3} - x)}{2 \cos x - 1}$ equals
 (a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\sqrt{3}$ (d) $\frac{2}{\sqrt{3}}$
- Q.31** The value of $\lim_{x \rightarrow 0} \frac{e^{x^2} - \cos x}{x^2}$ equals
 (a) $\frac{3}{2}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) 2
- Q.32** The value of $\lim_{x \rightarrow y} \frac{\sin^2 x - \sin^2 y}{x^2 - y^2}$ equals
 (a) 0 (b) 1 (c) $\frac{\sin y}{y}$ (d) $\frac{\sin 2y}{2y}$

- Q.33** The value of $\lim_{x \rightarrow 0} \frac{(4^x - 1)^3}{\sin \frac{x}{4} \log(1 + \frac{x^2}{3})}$ equals
 (a) $3(\log 4)^3$ (b) $4(\log 4)^3$ (c) $12(\log 4)^3$ (d) $15(\log 4)^3$
- Q.34** The integer n for which $\lim_{x \rightarrow 0} \frac{(\cos x - 1)(\cos x - e^x)}{x^n}$ is a finite non-zero number, is
 (a) 1 (b) 2 (c) 3 (d) 4
- Q.35** $\lim_{x \rightarrow 0} \frac{\sin(nx)((a-n)nx - \tan x)}{x^2} = 0$, when n is a non-zero positive integer, then a is equal to
 (a) $\frac{n+1}{2}$ (b) $n^2 + 1$ (c) $\frac{1}{n+1}$ (d) $n + \frac{1}{n}$
- Q.36** If $f(x) = \frac{x-3}{x+4}$, $g(x) = -\frac{2(2x+1)}{x^2+x-12}$ and $r(x) = \frac{2}{x-3}$, then value of $\lim_{x \rightarrow 3} \{f(x) + g(x) + r(x)\}$ is
 (a) Zero (b) $\frac{2}{7}$ (c) $-\frac{2}{7}$ (d) -1
- Q.37** Value of $\lim_{x \rightarrow \infty} (\frac{1}{x} - \frac{1}{x^2} + \frac{1}{x^3} - \dots \dots \infty)^{\frac{1}{x}}$ is equal to
 (a) Zero (b) $\frac{1}{e}$ (c) e (d) 1
- Q.38** Value of $\lim_{x \rightarrow 0} \frac{1 - \cos(1 - \cos x)}{x^4}$ equals
 (a) $\frac{1}{4}$ (b) $\frac{1}{8}$ (c) $\frac{1}{16}$ (d) $-\frac{1}{16}$
- Q.39** If $\lim_{x \rightarrow 0} \frac{x(1 + p \cos x) - q \sin x}{x^3} = 2$, then the value of $p + 2q$ equals
 (a) $-\frac{9}{2}$ (b) $-\frac{11}{2}$ (c) $-\frac{29}{2}$ (d) $-\frac{31}{2}$
- Q.40** The value of $\lim_{x \rightarrow \infty} \frac{(x+2)(4x+5)}{x^2(x+6)}$ is equal to
 (a) Zero (b) 1 (c) 4 (d) $\frac{5}{3}$
- Q.41** If $f(x) = x^2 \ln x$, then $f'(e)$ equals
 (a) e (b) $2e$ (c) $4e$ (d) $3e$
- Q.42** If $y = |\sin 2x| + |\tan x|$, then value of $\frac{dy}{dx}$ at $x = \frac{3\pi}{4}$ is
 (a) $-\frac{1}{2}$ (b) $\frac{1}{2}$ (c) -2 (d) 2
- Q.43** If $y = |\tan x| + |\sec x|$, then $\frac{dy}{dx}$ at $x = \frac{5\pi}{6}$ is
 (a) -2 (b) 2 (c) $-\frac{2}{3}$ (d) $\frac{2}{3}$
- Q.44** If $y = |\sin x|$, then value of $\frac{dy}{dx}$ at $x = \frac{3\pi}{4}$ is
 (a) $-\frac{1}{2}$ (b) $\frac{1}{2}$ (c) $-\frac{1}{\sqrt{2}}$ (d) $\frac{1}{\sqrt{2}}$
- Q.45** The value of $\lim_{x \rightarrow 0} \frac{\sin(\pi \cos^2 x)}{x^2}$ is
 (a) π (b) 0 (c) $\frac{\pi}{2}$ (d) 2
- Q.46** Let $\lim_{x \rightarrow 0} \frac{\sin 2x}{\tan(\frac{x}{2})} = L_1$ and $\lim_{x \rightarrow 0} \frac{e^{2x} - 1}{x} = L_2$, then the value of $L_1 L_2$ is
 (a) 4 (b) 8 (c) 6 (d) 2
- Q.47** $\lim_{x \rightarrow 0} \frac{2^x - 1}{x} + \lim_{x \rightarrow 0} \frac{3^x - 1}{x} - \lim_{x \rightarrow \infty} (\frac{6^x - 1}{x})$ equals
 (a) 1 (b) 2 (c) 0 (d) 4
- Q.48** $\lim_{x \rightarrow 0} \frac{\log(1+2x)}{x} + \lim_{x \rightarrow 2} \frac{x^4 - 2^4}{x - 2}$ equals
 (a) 30 (b) 32 (c) 35 (d) 34
- Q.49** $\lim_{x \rightarrow -\infty} \frac{\sin x}{x} + \lim_{x \rightarrow -\infty} \frac{\log x}{x}$ equals
 (a) 0 (b) 1 (c) 3 (d) ∞
- Q.50** $\lim_{x \rightarrow 0} \frac{2^x - 1}{\sqrt{1+x} - 1} =$
 (a) 2 (b) $\log_6 2$ (c) $\frac{\log_e 2}{2}$ (d) $2 \log_6 2$
- Q.51** $\lim_{x \rightarrow 1} (\sec \frac{\pi}{2x}) \cdot \ln x$ is equal to
 (a) $\frac{\pi}{2}$ (b) $-\frac{2}{\pi \ln 2}$ (c) $\frac{2}{\pi \ln 2}$ (d) $\frac{\pi}{\ln 2}$

- Q.52** $\lim_{x \rightarrow 0} \frac{\sin 2x}{2 - \sqrt{4 - x}}$ is
 (a) 2 (b) 4 (c) 8 (d) 0
- Q.53** $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x\sqrt{x^2}}$ equals
 (a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) 0 (d) Does not exist
- Q.54** $\lim_{x \rightarrow 0^+} ((x \cot x) + (x \log x))$ equals
 (a) 1 (b) 2 (c) 3 (d) 0
- Q.55** $m, n \in \mathbb{I}^+$, then $\lim_{x \rightarrow 0} \frac{\sin x^n}{(\sin x)^m}$ equals
 (a) 1, if $n < m$ (b) 0, if $n = m$ (c) $\frac{n}{m}$ (d) 0, if $n > m$
- Q.56** $\lim_{x \rightarrow \infty} \frac{(1+x+x^2)}{(\ln x)^3}$ is equal to
 (a) 2 (b) e^2 (c) e^{-2} (d) ∞
- Q.57** $\lim_{x \rightarrow d} \left(\frac{4^x + 9^x}{2}\right)^{\frac{1}{x}}$ equals
 (a) 2 (b) 6 (c) 16 (d) 112
- Q.58** If $\lim_{x \rightarrow \infty} \left(1 + \frac{a}{x} + \frac{b}{x^2}\right)^{2x} = e^2$, then the values of a and b are
 (a) $a \in \mathbb{R}, b \in \mathbb{R}$ (b) $a = 1, b \in \mathbb{R}$ (c) $a \in \mathbb{R}, b = 2$ (d) $a = 1, b = 2$
- Q.59** If α, β are roots of $ax^2 + bx + c = 0$, then $\lim_{x \rightarrow \alpha} (1 + ax^2 + bx + c)^{\frac{1}{x-\alpha}}$ is
 (a) $a(\alpha - \beta)$ (b) $\log e^1(\alpha - \beta)$ (c) $e^{a(\alpha-\beta)}$ (d) $e^{a(\alpha+\beta)}$
- Q.60** If $\lim_{x \rightarrow 0} (\cos x + a \sin bx)^{1/x} = e^2$, then the values of a and b can be
 (a) $a = 1, b = -2$ (b) $a = 2\sqrt{2}, b = \sqrt{2}$ (c) $a = 2\sqrt{2}, b = \frac{1}{\sqrt{2}}$ (d) $a = -2, b = 1$

EXERCISE LEVEL – II


 EL- II

Determine the limits for the following in Questions 1 to 4.

Q.1 $\lim_{z \rightarrow 1} \frac{\frac{1}{z^3} - 1}{\frac{1}{z^6} - 1}$

Q.2 $\lim_{x \rightarrow 1} \frac{ax^2 + bx + c}{cx^2 + bx + a}, a + b + c \neq 0$

Q.3 $\lim_{x \rightarrow \pi} \frac{\sin(\pi - x)}{\pi(\pi - x)}$

Q.4 $\lim_{r \rightarrow 1} \pi r^2$

Q.5 Find $\lim_{x \rightarrow 1} f(x)$, where $f(x) = \begin{cases} 2x + 3 & x \leq 0 \\ 3(x + 1) & x > 0 \end{cases}$

Q.6 If the function $f(x)$ satisfies, $\lim_{x \rightarrow 1} \frac{f(x) - 2}{x^2 - 1} = \pi$, evaluate $\lim_{x \rightarrow 1} f(x)$

Q.7 Find the value $\lim_{x \rightarrow 0} \frac{\sqrt{x+1} - 1}{x}$

Q.8 Find the value $\lim_{x \rightarrow 0} \frac{\sin 5x}{\tan 3x}$

Q.9 Find the value $\lim_{x \rightarrow 0} \frac{1 - \cos 5x}{1 - \cos 6x}$

Q.10 Find the value $\lim_{x \rightarrow 0} \frac{(x+1)^5 - 1}{x}$

Q.11 Find the value $\lim_{x \rightarrow 0} \frac{1 - \cos x}{\sin^2 x}$

Q.12 Determine the derivative of the function $\sec x$.

Q.13 Determine the derivative of the function $\frac{1}{\sqrt{x}}$.

Q.14 Determine the derivative of the function $x^2 \cos x$.

Q.15 Determine the derivative of the function $e^{\sqrt{\tan x}}$

Q.16 Calculate the derivative of the function $f(x) = (3x + 2)^{3/2}$ with respect to x .

ANSWER KEY – LEVEL – I

Q.	1	2	3	4	5	6	7	8	9	10
Ans.	d	c	b	a	c	b	d	a	d	b
Q.	11	12	13	14	15	16	17	18	19	20
Ans.	B	a	a	d	c	a	b	c	b	d
Q.	21	22	23	24	25	26	27	28	29	30
Ans.	D	c	c	c	a	a	a	c	d	b
Q.	31	32	33	34	35	36	37	38	39	40
Ans.	a	d	c	c	d	c	d	b	c	a
Q.	41	42	43	44	45	46	47	48	49	50
Ans.	d	c	a	c	a	b	c	d	a	d
Q.	51	52	53	54	55	56	57	58	59	60
Ans.	c	c	d	a	d	d	b	b	c	c

ANSWER KEY – LEVEL – II

- 2
- 1
- $\frac{1}{\pi}$
- π
- $\lim_{x \rightarrow 1} = 6$
- $\lim_{x \rightarrow 1} f(x) = 2$
- $\frac{1}{2}$
- $\frac{5}{3}$
- $\frac{25}{36}$
- 5
- $\frac{1}{2}$
- $\sec x \tan x$
- $-\frac{1}{2\sqrt{x^3}}$
- $2x \cos x - x^2 \sin x$
- $e^{\sqrt{\tan x}} \frac{\sec^2 x}{2\sqrt{\tan x}}$
- $\frac{9}{2} (3x + 2)^{\frac{1}{2}}$