EXERCISE LEVEL –I

EL–I

Q.1 If α and β denote the eccentric angles corresponding to the endpoints of a focal chord of an ellipse, then the eccentricity of the ellipse can be expressed as:

(a) $\frac{\cos\alpha+\cos\beta}{\cos(\alpha-\beta)}$ (b) $\frac{\sin(\alpha-\beta)}{\sin\alpha-\sin\beta}$ (c) $\frac{\cos\alpha-\cos\beta}{\cos(\alpha-\beta)}$ (d) $\frac{\sin\alpha+\sin\beta}{\sin(\alpha+\beta)}$

Q.2 Midpoints of chords drawn through the positive end of the minor axis of an ellipse lie on.

(a) A circle (b) A parabola (c) An ellipse (d) A hyperbola

Q.3 If $P(\theta)$ and $Q(\frac{\pi}{2}+\theta)$ are two points on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ the focus of midpoint of PQ is.

(a) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{1}{2}$ (b) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=4$ (c) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=2$ (d) $\frac{x^2}{b^2}+\frac{y^2}{a^2}=4$

Q.4 AB is diameter of, $x^2+9y^2=25$. The eccentric angel of A is $\frac{\pi}{6}$ then the acentric angel of B is

(a) $\frac{5\pi}{6}$ (b) $\frac{-5\pi}{6}$ (c) $\frac{-2\pi}{3}$ (d) $\frac{\pi}{3}$

Q.5 In the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$; $y=x$ and $3y+2x=0$ are a pair of conjugate diameter. Then the acentric of the ellipse is.

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\frac{2}{\sqrt{3}}$

Q.6 If the polar with respect to $y^2=4ax$ touches the ellipse $\frac{x^2}{\alpha^2}+\frac{y^2}{\beta^2}=1$ the locus of its pole is.

(a) $\frac{x^2}{\alpha^2}-\frac{y^2}{(\frac{4a^2\alpha^2}{\beta^2})}=1$ (b) $\frac{x^2}{\alpha^2}+\frac{\beta^2y^2}{4a^2}=1$ (c) $\alpha^2x^2+\beta^2y^2=1$ (d) $\alpha^2x^2-\beta^2y^2=1$

Q.7 The equation of the hyperbola with directrix, $2x+y=1$), corresponding focus at (1, 2), and eccentricity $\sqrt{3}$ is given by.

(a) $7x^2+12xy-2y^2-2x+14y-22=0$ (b) $2x^2+12xy-7y^2-2x+14y-22=0$
(c) $7x^2-12xy+2y^2-2x+14y-22=0$ (d) $7x^2+12xy-2y^2-2x-14y-22=0$

Q.8 In the foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{b^2}=1$ and the hyperbola $\frac{x^2}{144}-\frac{y^2}{81}=\frac{1}{25}$ coincide then the value b^2 equals

(a) 9 (b) 6 (c) 8 (d) 7

Q.9 For the $x^2\sec^2\alpha-y^2\operatorname{cosec}^2\alpha=1$ which of the following remain constant with the change of α?

(a) Abscissae of vertices (b) Abscissae of foci
(c) Eccentricity (d) Directrices

Q.10 The locus of a point of the from (e^t+e^{-t}, e^t-e^{-t}), is

(a) $x^2-y^2=2$ (b) $x^2-y^2=1$ (c) $x^2+y^2=1$ (d) $x^2-y^2=4$

Q.11 The equation $2x^2+3y^2-8x-18y+35=k$ represents.

(a) No locus if $k=0$ (b) An ellipse if $k<0$
(c) A point if $k=0$ (d) A hyperbola if $k>0$

Q.12 If e and e' are the eccentricities of a hyperbola and its conjugate, then the expression $e^2+e'^2$ is equal to.

(a) $4e^2e'^2$ (b) $2ee'$ (c) e'^2e^2 (d) ee'

Q.13 If the eccentricity of a hyperbola is $\sqrt{3}$, then the eccentricity of its conjugate hyperbola is.

(a) $\sqrt{2}$ (b) $\sqrt{3}$ (c) $\sqrt{\frac{3}{2}}$ (d) $2\sqrt{3}$

Q.14 If the separation between the foci and the separation between the directrices of the hyperbola are $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ are in the ration 3: 2, then a: b is

(a) $\sqrt{2}:1$ (b) $\sqrt{3}:\sqrt{2}$ (c) 1: 2 (d) 2: 1

Q.15 Let PQ be a latus rectum of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ and O be the center. If $\triangle$ OPQ is an equilateral triangle and e eccentric of the hyperbola. Then.

(a) $e=\frac{3}{2}$ (b) $e=2$ (c) $e=\frac{1+\sqrt{13}}{2\sqrt{3}}$ (d) $e=3$

Q.16 The line $5x+12y+9=0$ is tangent to the hyperbola $x^2-9y^2=9$ at the point.

(a) $(-5,\frac{4}{3})$ (b) $(-5,\frac{-4}{3})$ (c) $(-3,\frac{1}{2})$ (d) $(5,-\frac{17}{6})$

Q.17 If the tangent at the point $(2\sec\theta, 3\tan\theta)$ of the hyperbola $\frac{x^2}{4}-\frac{y^2}{9}=1$ is parallel to $3x-y+4=0$, then the value of 0 is

(a) 30° (b) 60° (c) 90° (d) 45°

Q.18 The equation of a tangent to the hyperbola,$16x^2-25y^2-96x+100y-356=0$,, forming an angle $\frac{\pi}{4}$ with the transverse axis, is given by:

(a) $y=x+2$ (b) $y=x+4$ (c) $x=y+3$ (d) $x+y+2=0$

Q.19 The path followed by a point P (α, β) as it moves, subject to the condition that the line $y=ax+\beta$ is a tangent to the hyperbola, is called the locus, $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, is

(a) A hyperbola (b) A parabola (c) A circle (d) An ellipse

Q.20 If $x=9$ represents the chord of contact for the hyperbola $x^2-y^2=9$, then the equation of the corresponding pair of tangents is.

(a) $9x^2-8y^2+18x-9=0$ (b) $9x^2-8y^2-18x+9=0$

(c) $9x^2-8y^2-18x-9=0$ (d) $8x^2-9y^2+18x+9=0$

Q.21 The equation of the hyperbola with asymptotes given by the straight lines $3x-4y+7=0$ and $4x+3y+1=0$, and passing through the origin, is:

(a) $12x^2-7xy-12y^2+31x+17y=0$ (b) $12x^2-7xy+2y^2=0$

(c) $12x^2+7xy+12y^2-31x+17y=0$ (d) $12x^2+7xy-2y^2=0$

Q.22 If the line $ax+by+c=0$ serves as a normal to the curve $xy=2$, then...

(a) $a>0, b>0$ (b) $a<0, b<0$

(c) $a>0, b<0$ or $a<0, b>0$ (d) Can't be determined

Q.23 If the normal to the curve at a certain point is $(ct,\frac{c}{t})$ If the normal at a point on the curve $xy=c^2$ intersects the curve again at t', then...

(a) $t'=-\frac{1}{t^3}$ (b) $t'=\frac{-1}{t}$ (c) $t'=\frac{1}{t^2}$ (d) $t'^2=\frac{-1}{t^2}$

Q.24 The asymptotes of the curve $xy=hx+ky$ are...

(a) $x=h, y=k$ (b) $x=-h, y=-k$ (c) $x=k, y=h$ (d) $x=-k, y=-h$

Q.25 The result of multiplying the perpendicular distances drawn from any point on a hyperbola to its asymptotes is.

(a) $\frac{ab}{\sqrt{a}+\sqrt{b}}$ (b) $\frac{ab}{a^2+b^2}$ (c) $\frac{a^2b^2}{a^2+b^2}$ (d) $\frac{a^2+b^2}{a^2b^2}$

Q.26 If PN represents the perpendicular from a point on a rectangular hyperbola to its asymptotes, the locus of the mid-point of PN is...

(a) Circle (b) Parabola (c) Ellipse (d) Hyperbola

Q.27 The point where the tangents at t_1 and t_2 intersect on the hyperbola $xy=c^2$ is.

(a) $(\frac{ct_1t_2}{t_1+t_2},\frac{c}{t_1+t_2})$ (b) $(\frac{2ct_1t_2}{t_1+t_2},\frac{2c}{t_1+t_2})$ (c) $(\frac{t_1t_2}{c(t_1+t_2)},\frac{t_1+t_2}{c})$ (d) (ct_1, ct_2)

Q.28 For a rectangular hyperbola with center C intersected by any circle of radius r at four points P, Q, R, S, and the sum $CP^2+CQ^2+CR^2+CS^2$is equivalent to...

(a) r^2 (b) $2r^2$ (c) $3r^2$ (d) $4r^2$

Q.29 The equation of the chord connecting two points (x, y_1) and $(x_2, y_2$ on the rectangular hyperbola $xy=c^2$ is.

(a) $\frac{x}{x_1+x_2}+\frac{y}{y_1+y_2}=1$ (b) $\frac{x}{x_1-x_2}+\frac{y}{y_1-y_2}=1$

(c) $\frac{x}{y_1+y_2}+\frac{y}{x_1+x_2}=1$ (d) $\frac{x}{y_1-y_2}+\frac{y}{x_1-x_2}=1$

Q.30 The equation for the common tangent to the curves s $y^2=8x$ and $xy=-1$is...

(a) $3y=9x+2$ (b) $y=2x+1$ (c) $2y=x+8$ (d) $y=x+2$

Q.31 If t is considered as a parameter, then $x = a(t + \frac{1}{t})$ and $y = b(t - \frac{1}{t})$ represent.
(a) An ellipse (b) A parabola (c) A hyperbola (d) A circle

Q.32 The equation for the common tangent to the curves is...
(a) $y = x + 2$ (b) $y = 2x + 1$ (c) $y = \frac{x}{2} + 4$ (d) $y = 3x + \frac{2}{3}$

Q.33 The equation of the tangent to the hyperbola $3x^2 - 4y^2 = 12$, which intersects the axes with equal lengths of intercepts, is...
(a) $x - y + 1 = 0$ (b) $x + y + 1 = 0$ (c) $x + y - 1 = 0$ (d) All the correct

Q.34 If the line $y = mx + \sqrt{a^2m^2 - b^2}, m = \frac{1}{2}$ touches the hyperbola $\frac{x^2}{16} - \frac{y^2}{3} = 1$ at the point $(4\sec\theta, \sqrt{3}\tan\theta)$ then θ is.
(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) $\frac{2\pi}{3}$ (d) $\frac{\pi}{6}$

Q.35 A common tangent to $9x^2 - 16y^2 = 144$ and$x^2 + y^2 = 9$ is
(a) $y\sqrt{7} = \sqrt{2}x + 15$ (b) $y\sqrt{7} = 3\sqrt{2}x + 15$
(c) $y = 3\sqrt{2}x + 15$ (d) $y\sqrt{7} = 3x + 15$

Q.36 The locus of the midpoint of a chord of the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$, which pass through the fixed point (1, 2) is a hyperbola whose eccentricity is.
(a) $\frac{3}{2}$ (b) $\frac{\sqrt{7}}{2}$ (c) $\frac{\sqrt{13}}{2}$ (d) $\frac{\sqrt{15}}{2}$

Q.37 The center of a rectangular hyperbola lies on the line $y = 3x$. If one of the asymptotes is $x + y + 2 = 0$, then the other asymptote is.
(a) $x + y + 1 = 0$ (b) $x - y - 1 = 0$ (c) $x - y + 2 = 0$ (d) $x + y + 3 = 0$

Q.38 If the angle between the asymptotes of the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$ is
(a) $\tan^{-1}(\frac{3}{4})$ (b) $2\tan^{-1}(\frac{3}{4})$ (c) $\tan^{-1}(\frac{4}{3})$ (d) $2\tan^{-1}(\frac{4}{3})$

Q.39 If the normal to the rectangular hyperbola $xy = 4$ at the point $(2t_1, \frac{2}{t_1})$ meet the curve again at $(2t_2, \frac{2}{t_2})$, then
(a) $t_1^3t_2 = 1$ (b) $t_1^3t_2 = -1$ (c) $t_2^3t_1 = 1$ (d) $t_1t_2^3 = -1$

Q.40 A hyperbola, with its center at the origin, passing through the point (4, 2), and having a transverse axis of length 4 along the x-axis, has an eccentricity of.
(a) $\frac{3}{2}$ (b) $\sqrt{3}$ (c) $\frac{2}{\sqrt{3}}$ (d) 2

Q.41 The equation for a tangent to the hyperbola, $4x^2 - 5y^2 = 20$, parallel to the line $x - y = 2$ is
(a) $x - y + 7 = 0$ (b) $x - y + 1 = 0$ (c) $x - y - 3 = 0$ (d) $x - y + 9 = 0$

Q.42 If a hyperbola has a conjugate axis length equal to 5, and the distance between its foci is 13, then the eccentricity of the hyperbola is.
(a) 2 (b) $\frac{13}{8}$ (c) $\frac{13}{6}$ (d) $\frac{13}{12}$

Q.43 If the vertices of a hyperbola are at (–2, 0) and (2, 0), and one of its foci is at (–3, 0), which of the following points does not belong to this hyperbola?
(a) $(4, \sqrt{15})$ (b) $(6, 5\sqrt{2})$ (c) $(2\sqrt{6}, 5)$ (d) $(-6, 2\sqrt{10})$

Q.44 If the standard hyperbola passing through the point (4, 6) has an eccentricity of 2, then the equation of the tangent to the hyperbola at (4, 6) is.
(a) $2x - 3y + 10 = 0$ (b) $x - 2y + 8 = 0$ (c) $3x - 2y = 0$ (d) $2x - y - 2 = 0$

Q.45 If one of the directrices of a hyperbola, centered at the origin and passing through the point $(4, -2\sqrt{3})$ is given by $5x = 4\sqrt{5}$, and its eccentricity is \(e\), then.
(a) $4e^4 + 8e^2 - 35 = 0$ (b) $4e^4 - 24e^2 + 35 = 0$
(c) $4e^4 - 12e^2 - 27 = 0$ (d) $4e^4 - 24e^2 + 27 = 0$

Q.46 If $5x + 9 = 0$, serves as the directrix of the hyperbola $16x^2 - 9y^2 = 144$, then its corresponding focus is located at...
(a) $(\frac{5}{3}, 0)$ (b) $(-\frac{5}{3}, 0)$ (c) $(-5, 0)$ (d) $(5, 0)$

Q.47 A line parallel to the straight line $2x - y = 0$ is tangent to the hyperbola $\frac{x^2}{4} - \frac{y^2}{2} = 1$ at the point(x_1, y_1). Then $x_1^2 + 5y_1^2$ is equal to

(a) 8 (b) 6 (c) 10 (d) 5

Q.48 Consider the point P (3, 3) lying on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$If the normal at point P intersects the x-axis at (9, 0) and e is its eccentricity, then the ordered pair (a^2, e^2) is equivalent to.

(a) (9,3) (b) $(\frac{9}{2}, 3)$ (c) $(\frac{3}{2}, 2)$ (d) $(\frac{9}{2}, 2)$

Q.49 A hyperbola traverses the foci of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ and its transverse and conjugate axes align with the major and minor axes of the ellipse, respectively. If the product of their eccentricities is one, then the equation of the hyperbola is.

(a) $\frac{x^2}{9} - \frac{y^2}{16} = 1$ (b) $x^2 - y^2 = 9$ (c) $\frac{x^2}{9} - \frac{y^2}{4} = 1$ (d) $\frac{x^2}{9} - \frac{y^2}{25} = 1$

Q.50 Suppose a line, $L: 2x + y = k, k > 0$, is a tangent to the hyperbola $x^2 - y^2 = 3$ If \(L\) is also a tangent to the parabola $y^2 = \alpha x$, then the value of α is.

(a) –24 (b) 24 (c) 12 (d) –12

Determine the coordinates of the foci and the vertices, the eccentricity, and the length of the latus rectum for the hyperbolas.

Q.1 $\frac{x^2}{16} - \frac{y^2}{9} = 1$

Q.2 $\frac{y^2}{9} - \frac{x^2}{27} = 1$

Q.3 $9y^2 - 4x^2 = 36$

Derive the equations for the hyperbolas that meet the specified criteria.

Q.4 Vertices $(\pm 2, 0)$, foci $(\pm 3, 0)$

Q.5 Foci $(\pm 3\ 5, 0)$, the latus rectum is of length 8.

Q.6 Vertices $(0, \pm 3)$, foci $(0, \pm 5)$

Q.7 If $20x^2 - 4y^2 = 80$ represent the equation of a hyperbola, then find the length of its latus rectum.

Q.8 For the hyperbola $25y^2 - 100x^2 = 100$, find the eccentricity and the co – ordinates of the vertices and the foci.

Q.9 The tangent at any point of a hyperbola $16x^2 - 25y^2 = 400$ cut off a triangle from the asymptotes and that the portion of it intercepted between the asymptotes, then area of this triangle (in square units), is.

Q.10 Let $16x^2 - 3y^2 - 32x - 12y = 44$ represent a hyperbola product of its square of eccentricity and length of latus rectum is λ then $3\sqrt{3}\lambda$ is.

ANSWER KEY – LEVEL – I

Q.	1	2	3	4	5	6	7	8	9	10
Ans.	b	c	a	b	c	a	a	d	b	d
Q.	11	12	13	14	15	16	17	18	19	20
Ans.	c	c	c	a	c	a	a	a	a	b
Q.	21	22	23	24	25	26	27	28	29	30
Ans.	a	c	a	c	c	d	b	d	a	d
Q.	31	32	33	34	35	36	37	38	39	40
Ans.	c	a	d	c	b	c	b	b	b	c
Q.	41	42	43	44	45	46	47	48	49	50
Ans.	b	d	b	d	b	c	b	b	a	a

ANSWER KEY – LEVEL – II

1. Foci $(\pm 5,0)$, Vertices $(\pm 4,0)$; e $= \frac{5}{4}$, Latus rectum $= \frac{9}{2}$

2. Foci (0 ± 6), Vertices $(0, \pm 3)$; $e = 2$; Latus rectum $= 18$

3. Foci $\left(0, \pm\sqrt{13}\right)$, Vertices $(0, \pm 2)$; e $= \frac{\sqrt{13}}{2}$, Latus rectum $= 9$

4. $\frac{x^2}{4} - \frac{y^2}{5} = 1$

5. $\frac{x^2}{25} - \frac{y^2}{20} = 1$

6. $\frac{y^2}{9} - \frac{x^2}{16} = 1$