EXERCISE LEVEL –I

EL– I

Q.1 The eccentricity of the ellipse given by the equation $9x^2 + 5y^2 - 30y = 0$ is...

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{3}{4}$ (d) $\frac{5}{6}$

Q.2 The latus rectum of the conic represented by $3x^2 + 4y^2 - 6x + 8y - 50$ is...

(a) 3 (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{2}{\sqrt{3}}$ (d) $\frac{4}{\sqrt{3}}$

Q.3 If the separation between the foci is 2 and the distance between the directrices is 5, then the equation of the ellipse in standard form is...

(a) $6x^2 + 10y^2 = 5$ (b) $6x^2 + 10y^2 = 15$ (c) $x^2 + 3y^2 = 10$ (d) $10x^2 + 6y^2 = 15$

Q.4 The equation $\frac{x^2}{2-r} + \frac{y^2}{r-5} + 1 = 0$represents an ellipse if and only if.

(a) $r > 2$ (b) $r > 5$ (c) $2 < r < 5$ (d) $r < 2$

Q.5 The total distance from any point on the ellipse $3x^2 + 4y^2 = 24$ to its foci is...

(a) $8\sqrt{2}$ (b) $4\sqrt{2}$ (c) $16\sqrt{2}$ (d) 6

Q.6 The coordinates of one of the foci of the ellipse $4x^2 + 9y^2 = 1$ are.

(a) $(\frac{\sqrt{5}}{6}, 0)$ (b) $(\frac{\sqrt{5}}{3}, 0)$ (c) $(\frac{\sqrt{5}}{4}, 0)$ (d) $(\frac{\sqrt{5}}{5}, 0)$

Q.7 The length of the latus rectum of the ellipse represented by $x = 3(\cos t + \sin t), y = 4(\cos t - \sin t)$ is determined by...

(a) $\frac{5\sqrt{2}}{2}$ (b) $\frac{7\sqrt{2}}{2}$ (c) $\frac{9\sqrt{2}}{2}$ (d) $\sqrt{2}$

Q.8 The count of values for c such that the straight line $y = 4x + c$ touches the curve $\frac{x^2}{4} + \frac{y^2}{1} = 1$, are

(a) 0 (b) 1 (c) 2 (d) Infinite

Q.9 Tangents are extended to the ellipse $\frac{x^2}{9} + \frac{y^2}{5} = 1$ at the extremities of the latus rectum. The area enclosed by the resulting quadrilateral is.

(a) 27 (b) $\frac{27}{2}$ (c) $\frac{27}{4}$ (d) $\frac{27}{55}$

Q.10 If tangents are extended to the ellipse $x^2 + 2y^2 = 2$ then the path of the midpoints of the segments intercepted by the tangents between the coordinate axes is given by the locus.

(a) $x^2 + 2y^2 = 4x^2y^2$ (b) $2x^2 + y^2 = 4x^2y^2$

(c) $2x^2 + y^2 = 4$ (d) $x^2 + 2y^2 = 4$

Q.11 If the line $px + qy = 1$ intersects the ellipse $x^2 + 4y^2 = 4$ at points where the eccentric angles vary by $\frac{\pi}{3}$ then r^2 is equal to

(a) $\frac{3}{4}(4p^2 + q^2)$ (b) $\frac{4}{3}(4p^2 + q^2)$ (c) $\frac{2}{3}(4p^2 + q^2)$ (d) $\frac{3}{4}(p^2 + 4q^2)$

Q.12 The smallest area of the triangle created by any tangent to the ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with coordinate – axes is.

(a) $\frac{a^2+b^2}{2}$ (b) $\frac{(a+b)^2}{2}$ (c) ab (d) $\frac{(a-b)^2}{2}$

Q.13 The product of the perpendiculars drawn from the foci onto any tangent to the ellipse $3x^2 + 4y^2 = 12$ is.

(a) 4 (b) 3 (c) $2\sqrt{3}$ (d) 6

Q.14 If the line $lx + my + n = 0$ is a normal to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, then

(a) $\frac{a^2}{l^2} + \frac{b^2}{m^2} = \frac{(a^2+b^2)^2}{n^2}$ (b) $\frac{a^2}{l^2} - \frac{b^2}{m^2} = \frac{(a^2-b^2)^2}{n^2}$

(c) $\frac{a^2}{l^2} + \frac{b^2}{m^2} = \frac{(a^2-b^2)^2}{n^2}$ (d) $\frac{a^2}{l^2} - \frac{b^2}{m^2} = \frac{(a^2+b^2)^2}{n^2}$

Q.15 If the normal to the ellipse $\frac{x^2}{14}+\frac{y^2}{5}=1$ at the point θ intersect curve at the again at the point 2θ then $\cos\theta$ is equal to.

(a) $\frac{2}{3}$ (b) $\frac{-2}{3}$ (c) $\frac{3}{4}$ (d) $\frac{-3}{4}$

Q.16 The equation describing the director circle of the ellipse $4x^2+9y^2=36$ is given by...

(a) $x^2+y^2=4$ (b) $x^2+y^2=9$ (c) $x^2+y^2=13$ (d) $x^2+y^2=5$

Q.17 The eccentricity of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ that goes through the points (2, 2) and (-3, 1) is...

(a) $\frac{1}{\sqrt{5}}$ (b) $\sqrt{\frac{2}{5}}$ (c) $\sqrt{\frac{3}{5}}$ (d) $\sqrt{\frac{4}{5}}$

Q.18 In an ellipse, where the distance between the foci is 8 and the distance between the directrices is 25, the ratio of the length of the major axis to the minor axis is...

(a) $\frac{5}{\sqrt{17}}$ (b) $\frac{3}{\sqrt{17}}$ (c) $\frac{4}{\sqrt{17}}$ (d) $\frac{6}{\sqrt{17}}$

Q.19 The radius of the circle that passes through the point where the ellipse intersects $\frac{x^2}{16}+\frac{y^2}{9}=1$ and $x^2-y^2=0$ is

(a) $\frac{12}{5}$ (b) $\frac{12\sqrt{2}}{5}$ (c) $\frac{6\sqrt{2}}{5}$ (d) $\frac{9\sqrt{2}}{5}$

Q.20 If the distance between the directrices is four times the distance between the foci, then the eccentricity of the ellipse is...

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{1}{4}$ (d) $\frac{1}{6}$

Q.21 Consider the ellipse,$5x^2+9y^2=45$, let F_1 and F_2, where F_1 and F_2 are its foci and P is a point on the ellipse. The sum of the distances PF_1+PF_2 is...

(a) 3 (b) $2\sqrt{3}$ (c) 6 (d) $2\sqrt{5}$

Q.22 If the focal distance of one end of the minor axis of an ellipse (with axes along the x and y axes, respectively, denoted as k) and the distance between the foci is 2h, then the equation of the ellipse is.

(a) $\frac{x^2}{h^2}+\frac{y^2}{k^2}=1$ (b) $\frac{x^2}{k^2}+\frac{y^2}{k^2-h^2}=1$ (c) $x^2+\frac{y^2}{k^2}=1$ (d) $\frac{x^2}{h^2}+y^2=1$

Q.23 The sum of the squares of the perpendiculars from two points on $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ the minor axis, each at a distance $\sqrt{a^2-b^2}$ from the center is.

(a) a^2 (b) b^2 (c) $2a^2$ (d) $2b^2$

Q.24 The line $2x+y=3$ intersects the ellipse $4x^2+y^2=5$ at points P and Q. If θ is the angle between the normals at these points, then $\tan\theta$ is equal to.

(a) $\frac{1}{2}$ (b) $\frac{3}{4}$ (c) $\frac{3}{5}$ (d) 5

Q.25 The number of points on the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ from which a pair of perpendicular tangents can be drawn to the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=$ is.

(a) 2 (b) 3 (c) 4 (d) 1

Q.26 A common tangent is $9x^2+16y^2=144.\ y^2=x-4$ and $x^2+y^2-12x+32=0$ is

(a) $y=3$ (b) $y=-3$ (c) $x=4$ (d) $x=-4$

Q.27 The equation of the chord of the ellipse $x^2+4y^2=4$ with the midpoint at $(-1,\frac{1}{2})$ is

(a) $x-2y+2=0$ (b) $x+2y=0$ (c) $3x-2y+4=0$ (d) $2x-2y+3=0$

Q.28 The pole of the line $x+4y=5$ with respect to the ellipse $5x^2+4y^2=10$ is

(a) $(2,2)$ (b) $(\frac{2}{5},\frac{3}{5})$ (c) $(\frac{2}{5},2)$ (d) $(2,\frac{3}{5})$

Q.29 The locus of the midpoints of the portions of the tangents to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ intercepted between the axes is.

(a) $\frac{a^2}{x^2}+\frac{b^2}{y^2}=4$ (b) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=4$ (c) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=16$ (d) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=25$

Q.30 A rod of given length moves in such a way that its endpoints lie on two fixed perpendicular lines. Any point on the rod, excluding the midpoint, traces. A rod of given length moves in such a way that its endpoints lie on two fixed perpendicular lines. Any point on the rod, excluding the midpoint, traces

(a) A circle (b) A parabola (c) An ellipse (d) A straight line

Q.31 The locus of the feet of the perpendiculars drawn from the center to the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ on any tangents on it is.

(a) $(x^2+y^2)^2=9x^2+4y^2$ (b) $(x^2+y^2)^2=4x^2+9y^2$

(c) $(x^2+y^2)^2=3x^2+2y^2$ (d) $(x^2+y^2)^2=2x^2+3y^2$

Q.32 The locus of the foot of the perpendicular from any focus onto any tangent to $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$is.

(a) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ (b) $x^2+y^2=a^2+b^2$ (c) $x^2+y^2=a^2$ (d) $x^2+y^2=a^2-b^2$

Q.33 The locus of the foot of the perpendicular from the center to any tangent of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is

(a) A circle (b) A pair of straight lines

(c) Another ellipse (d) Curve of degree 4

Q.34 If tangents are drawn to the ellipse $x^2+2y^2=2$ at all points on the ellipse, excluding its four vertices, then the midpoints of the tangents intercepted between the coordinate axes lie on the curve.

(a) $\frac{1}{4x^2}+\frac{1}{2y^2}=1$ (b) $\frac{x^2}{2}+\frac{y^2}{4}=1$ (c) $\frac{x^2}{4}+\frac{y^2}{2}=1$ (d) $\frac{1}{2x^2}+\frac{1}{4y^2}=1$

Q.35 Given that the length of the latus rectum of an ellipse, with its major axis along the x-axis and center at the origin, is 8, and the distance between the foci is equal to the length of its minor axis. Determine which of the following points lies on it.

(a) $(4\sqrt{3},2\sqrt{3})$ (b) $(4\sqrt{3},2\sqrt{2})$ (c) $(4\sqrt{2},2\sqrt{2})$ (d) $(4\sqrt{2},2\sqrt{3})$

Q.36 If the tangents to the ellipse $4x^2+y^2=8$ at the points (1, 2) and (a, b) are perpendicular to each other, then the value of a^2 is equal to.

(a) $\frac{64}{17}$ (b) $\frac{2}{17}$ (c) $\frac{4}{17}$ (d) $\frac{128}{17}$

Q.37 In an ellipse centered at the origin, if the difference between the lengths of the major and minor axes is 10, and one of the foci is located at $(0,5\sqrt{3})$, then the length of its latus rectum is.

(a) 5 (b) 6 (c) 8 (d) 10

Q.38 If the tangent at a point (α, β) (where $\beta>0$) to the parabola $y^2=x$ is also a tangent to the ellipse $x^2+2y^2=1$, then the value of a is equal to

(a) $\sqrt{2}-1$ (b) $\sqrt{2}+1$ (c) $2\sqrt{2}+1$ (d) $2\sqrt{2}-1$

Q.39 If the line $x-2y=12$ is tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at the point $(3,\frac{-9}{2})$ then the length of the latus rectum of the ellipse is.

(a) 5 (b) $8\sqrt{3}$ (c) $12\sqrt{2}$ (d) 9

Q.40 The tangent and normal to the ellipse $3x^2+5y^2=32$ at the point P(2, 2) intersect the x-axis at Q and R, respectively. The area (in square units) of the triangle PQR is.

(a) $\frac{16}{3}$ (b) $\frac{14}{3}$ (c) $\frac{34}{15}$ (d) $\frac{68}{15}$

Q.41 If the normal to the ellipse $3x^2+4y^2=12$ at a point P on it is parallel to the line $2x+y=4$, and the tangent to the ellipse at P passes through Q(4, 4), then the length of PQ is equal to

(a) $\frac{\sqrt{61}}{2}$ (b) $\frac{5\sqrt{5}}{2}$ (c) $\frac{\sqrt{157}}{2}$ (d) $\frac{\sqrt{221}}{2}$

Q.42 Through which of the following points does an ellipse, with foci at (0, 2) and (0, -2) and a minor axis of length 4, pass?

(a) $(\sqrt{2},2)$ (b) $(2,2\sqrt{2})$ (c) $(1,2\sqrt{2})$ (d) $(2,\sqrt{2})$

Q.43 If the distance between the foci of an ellipse is 6 and the distance between its directrices is 12, then the length of its latus rectum is?

(a) $\frac{3}{\sqrt{2}}$ (b) $\sqrt{3}$ (c) $3\sqrt{2}$ (d) $2\sqrt{3}$

Q.44 If e_1 and e_2 are the eccentricities of the ellipse, $\frac{x^2}{18}+\frac{y^2}{4}=1$ and the hyperbola $\frac{x^2}{9}-\frac{y^2}{4}=1$ respectively and (e_1, e_2) is a point on the ellipse $15x^2+3y^2=k$ then k is equal to.

(a) 14 (b) 15 (c) 17 (d) 16

Q.45 Let $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1(a>b)$ be a given ellipse, length of whose latus rectum is 10. If its eccentricity is the maximum value of the function, $\phi(t)=\frac{5}{12}+t-t^2$ then a^2+b^2 is equal to

(a) 135 (b) 116 (c) 126 (d) 145

Q.46 Consider x = 4 as one of the directrices to an ellipse with its center at the origin and find its eccentricity. $\frac{1}{2}$. If P(1, β), β > 0 is a point on this ellipse, then the equation of the normal to it at P is

(a) $7x - 4y = 1$ (b) $4x - 2y = 1$ (c) $4x - 3y = 2$ (d) $8x - 2y = 5$

Q.47 Which of the following points lies on the locus of the foot of the perpendicular drawn upon any tangent to the ellipse, $\frac{x^2}{4} + \frac{y^2}{2} = 1$ from any of its foci?

(a) $(1,2)$ (b) $(-2, \sqrt{3})$ (c) $(-1, \sqrt{3})$ (d) $(-1, \sqrt{2})$

Q.48 If the normal at one end of a latus rectum of an ellipse passes through an extremity of the minor axis, then the eccentricity 'e' of the ellipse satisfies.

(a) $e^2 + 2e - 1 = 0$ (b) $e^2 + e - 1 = 0$ (c) $e^4 + 2e^2 - 1 = 0$ (d) $e^4 + e^2 - 1 = 0$

Q.49 If the points of intersection of the ellipse $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$ and the circle $x^2 + y^2 = 4b, b > 4$ lie on the curve$y^2 = 3x^2$, then b is equal to

(a) 5 (b) 6 (c) 12 (d) 10

Q.50 Let's consider a tangent drawn to the ellipse $\frac{x^2}{27} + y^2 = 1$ at $(3\sqrt{3}\cos\theta, \sin\theta)$ where $\theta \in (0, \frac{\pi}{2})$ The value of θ , such that the sum of intercepts on the axes made by this tangent is minimized, is equal to.

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{8}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{3}$

Determine the coordinates of the foci, vertices, major axis length, minor axis length, eccentricity, and the length of the latus rectum for the ellipse.

Q.1 $\frac{x^2}{100}+\frac{y^2}{400}=1$

Q.2 $\frac{x^2}{49}+\frac{y^2}{36}=1$

Q.3 $36x^2+4y^2=144$

Q.4 $4x^2+9y^2=36$

Determine the equation representing the ellipse that meets the specified criteria:

Q.5 Vertices $(0, \pm 13)$, foci $(0, \pm 5)$

Q.6 Ends of major axis $(\pm 3, 0)$, ends of minor axis $(0, \pm 2)$

Q.7 $b = 3$, $c = 4$, center at the origin; foci on the x axis

Q.8 Major axis on the x-axis and passes through the points (4,3) and (6,2)

Q.9 Find the co-ordinates of the foci and the vertices of the ellipse $x^2+4v^2=16$.

Q.10 If $\frac{x^2}{64}+\frac{y^2}{36}=1$ represents the equation of an ellipse, then find the distance between its foci.

Q.11 If the eccentricity of an ellipse is $\frac{\sqrt{13}}{7}$ and distance between its foci is,$2\sqrt{13}$, then find the length of the latus rectum of the ellipse.

Q.12 The minimum and maximum distance of a point, $(\frac{9}{5},\frac{12}{5})$ from the ellipse, $4(3x+4y)^2+9(4x-3y)^2=900$ are m and M then the value of $m + M$ is equal to.

Q.13 The minimum distance of the center of the ellipse $9x^2+16y^2=144$ from the chord of contact of mutually perpendicular tangents of the ellipse is ℓ then 5ℓ is equal to.

ANSWER KEY – LEVEL – I

Q.	1	2	3	4	5	6	7	8	9	10
Ans.	b	a	b	c	b	a	c	c	a	a
Q.	11	12	13	14	15	16	17	18	19	20
Ans.	a	c	b	c	b	c	b	a	b	a
Q.	21	22	23	24	25	26	27	28	29	30
Ans.	c	b	c	c	a	c	a	c	a	c
Q.	31	32	33	34	35	36	37	38	39	40
Ans.	a	c	d	d	b	b	a	b	d	d
Q.	41	42	43	44	45	46	47	48	49	50
Ans.	b	a	c	d	c	b	c	d	c	c

ANSWER KEY – LEVEL – II

1. $F(0, \pm 10\sqrt{3})$; $V(0_L \pm 20)$; Major axis $=40$; Minor axis $= 20$, e $=\frac{\sqrt{3}}{2}$
Latus rectum $= 10$

2. $F(\pm\sqrt{13}, 0)$; $V(\pm 7,0)$ Major axis $=14$; Minor axis $= 12$, e $=\frac{\sqrt{13}}{7}$, Latus rectum $= \frac{72}{7}$

3. $F\left(0, \pm 4\sqrt{2}\right)$; $V(0 \pm 6)$: Major axis $=12$; Minor axis $= 4$, e $= \frac{2\sqrt{2}}{3}$, Latus rectum$= \frac{4}{3}$

4. $F(\pm\sqrt{5}, 0)$; $V(\pm 3,0)$ Major axis $= 6$; Minor axis $= 4$, e $= \frac{\sqrt{5}}{3}$, Latus rectum $= \frac{8}{3}$

5. $\frac{x^2}{144} + \frac{y^2}{169} = 1$

6. $\frac{x^2}{9} + \frac{y^2}{4} = 1$

7. $\frac{x^2}{25} + \frac{y^2}{9} = 1$

8. $x^2 + 4y^2 = 52$ or $\frac{x^2}{52} + \frac{y^2}{13} = 1$