Exercise-1

Marked Questions can be used as Revision Questions.

PART - I : SUBJECTIVE QUESTIONS

Section (A) : Definition, Projectile on a horizontal plane

A-1. Two bodies are projected at angles θ and $(90 - \theta)$ to the horizontal with the same speed. Find the ratio of their times of flight?

A-2. In above question find the ratio of the maximum vertical heights?

A-3. A body is so projected in the air that the horizontal range covered by the body is equal to the maximum vertical height attained by the body during the motion. Find the angle of projection?

A-4. A projectile can have the same range R for two angles of projections at a given speed. If T_1 & T_2 be the times of flight in two cases, then find out relation between T_1, T_2 and R?

A-5. A cricketer can throw a ball to a maximum horizontal distance of 100 m. To what height above the ground can the cricketer throw the same ball with same speed.

A-6. A player kicks a football at an angle of 45º with an initial speed of 20 m/s. A second player on the goal line 60 m away in the direction of kick starts running to receive the ball at that instant. Find the constant speed of the second player with which he should run to catch the ball before it hits the ground [$g = 10\ m/s^2$]

A-7. The direction of motion of a projectile at a certain instant is inclined at an angle α to the horizontal. After t seconds it is inclined an angle β. Find the horizontal component of velocity of projection in terms of g, t, α and β. (α and β are positive in anticlockwise direction)

A-8. A gun kept on a straight horizontal road is used to hit a car, travelling along the same road away from the gun with a uniform speed of $72 \times \sqrt{2}$ km/hour. The car is at a distance of 50 metre from the gun, when the gun is fired at an angle of 45° with the horizontal. Find (i) the distance of the car from the gun when the shell hits it, (ii) the speed of projection of the shell from the gun. [$g = 10\ m/s^2$] **[IIT 1974]**

Section (B) : Projectile from a tower

B-1. A projectile is fired horizontally with a velocity of 98 m/s from the top of a hill 490 m high. Find :
(take $g = 9.8\ m/s^2$)
(i) The time taken to reach the ground
(ii) The distance of the target from the foot of hill
(iii) The velocity with which the particle hits the ground

B-2. From the top of a tower of height 50m a ball is projected upwards with a speed of 30 m/s at an angle of 30° to the horizontal. Then calculate -
(i) Maximum height from the ground
(ii) At what distance from the foot of the tower does the projectile hit the ground.
(iii) Time of flight.

Section (C) : Equation of trajectory

C-1. The equation of a projectile is $y = \sqrt{3}\,x - \frac{gx^2}{2}$, find the angle of projection. Also find the speed of projection. Where at t = 0, x = 0 and y = 0 also $\frac{d^2x}{dt^2} = 0$ & $\frac{d^2y}{dt^2} = -g$.

C-2 A bullet is fired from horizontal ground at some angle passes through the point $\left(\frac{3R}{4}, \frac{R}{4}\right)$, where 'R' is the range of the bullet. Assume point of the fire to be origin and the bullet moves in x-y plane with x-axis horizontal and y-axis vertically upwards. Angle of projection is $\frac{\alpha\pi}{180}$ radian. Find α :

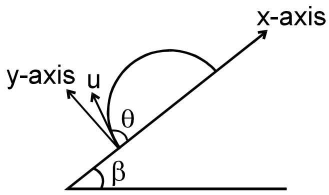
C-3. The radius vector of a point A relative to the origin varies with time t as $\vec{r} = at\hat{i} - bt^2\hat{j}$, where a and b are positive constants and $\hat{i}$ and $\hat{j}$ are the unit vectors of the x and y axes. Find:

(i) The equation of the point's trajectory y (x) ; plot this function

(ii) The time dependence of the velocity **v** and acceleration **a** vectors as well as of the moduli of these quantities.

Section (D) Projectile on an inclined plane

D-1. A particle is projected at an angle θ with an inclined plane making an angle β with the horizontal as shown in figure, speed of the particle is u, after time t find :



(a) x component of acceleration ?
(b) y component of acceleration ?
(c) x component of velocity ?
(d) y component of velocity ?
(e) x component of displacement ?
(f) y component of displacement ?
(g) y component of velocity when particle is at maximum distance from the incline plane ?

PART - II : ONLY ONE OPTION CORRECT TYPE

A-1. A ball is thrown upwards. It returns to ground describing a parabolic path. Which of the following remains constant?

(A) Speed of the ball
(B) Kinetic energy of the ball
(C) Vertical component of velocity
(D) Horizontal component of velocity.

A-2. A bullet is fired horizontally from a rifle at a distant target. Ignoring the effect of air resistance, which of the following is correct?

	Horizontal Acceleration	Vertical Acceleration
(A)	10 ms^{-2}	10 ms^{-2}
(B)	10 ms^{-2}	0 ms^{-2}
(C)	0 ms^{-2}	10 ms^{-2}
(D)	0 ms^{-2}	0 ms^{-2}.

A-3. A point mass is projected, making an acute angle with the horizontal. If angle between velocity and acceleration $\vec{g}$ is θ at any time t during the motion, then θ is given by

(A) $0^\circ < \theta < 90^\circ$ (B) $\theta = 90^\circ$ (C) $\theta < 90^\circ$ (D) $0^\circ < \theta < 180^\circ$

A-4. It was calculated that a shell when fired from a gun with a certain velocity and at an angle of elevation $\frac{5\pi}{36}$ rad should strike a given target in the same horizontal plane. In actual practice, it was found that a hill just prevented the trajectory. At what angle of elevation should the gun be fired to hit the target.

(A) $\frac{5\pi}{36}$ rad (B) $\frac{11\pi}{36}$ rad (C) $\frac{7\pi}{36}$ rad (D) $\frac{13\pi}{36}$ rad.

A-5. A projectile is thrown with a speed v at an angle θ with the upward vertical. Its average velocity between the instants at which it crosses half the maximum height is

(A) v sin θ, horizontal and in the plane of projection

(B) v cos θ, horizontal and in the plane of projection

(C) 2v sin θ, horizontal and perpendicular to the plane of projection

(D) 2v cos θ, vertical and in the plane of projection.

A-6. A particle moves along the parabolic path $y = ax^2$ in such a way that the x component of the velocity remains constant, say c. The acceleration of the particle is

(A) ac $\hat{k}$ (B) $2ac^2\ \hat{j}$ (C) $ac^2\ \hat{j}$ (D) $a^2c\ \hat{j}$

A-7. During projectile motion, acceleration of a particle at the highest point of its trajectory is

(A) g (B) zero

(C) less than g (D) dependent upon projection velocity

A-8. The speed at the maximum height of a projectile is half of its initial speed u. Its range on the horizontal plane is :

(A) $\frac{2u^2}{3g}$ (B) $\frac{\sqrt{3}u^2}{2g}$ (C) $\frac{u^2}{3g}$ (D) $\frac{u^2}{2g}$

A-9. The velocity of projection of a projectile is $(6\hat{i} + 8\hat{j})$ ms^{-1}. The horizontal range of the projectile is ($g = 10$ m/sec^2)

(A) 4.9 m (B) 9.6 m (C) 19.6 m (D) 14 m

Section (B) : Projectile from a tower

B-1. One stone is projected horizontally from a 20 m high cliff with an initial speed of 10 ms^{-1}. A second stone is simultaneously dropped from that cliff. Which of the following is true?

(A) Both strike the ground with the same speed.

(B) The stone with initial speed 10 ms^{-1} reaches the ground first.

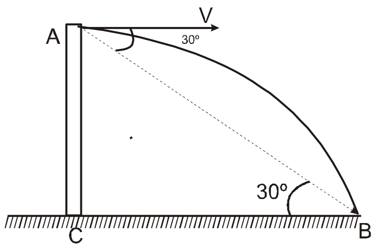
(C) Both the stones hit the ground at the same time.

(D) The stone which is dropped from the cliff reaches the ground first.

B-2. An object is thrown horizontally from a point 'A' from a tower and hits the ground 3s later at B. The line from ‘A’ to ‘B’ makes an angle of 30º with the horizontal. The initial velocity of the object is : (take g = 10 m/s^2)

(A) $15\sqrt{3}$ m/s (B) 15 m/s

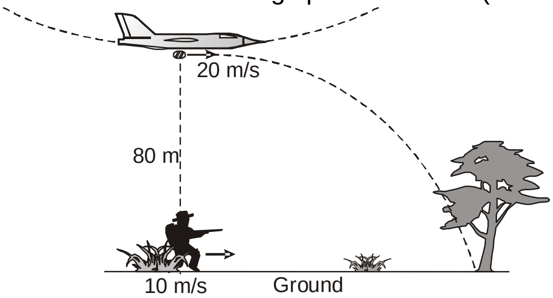
(C) $10\sqrt{3}$ m/s (D) $25/\sqrt{3}$ m/s



B-3. A body is projected horizontally from the top of a tower with initial velocity 18 ms^{-1}. It hits the ground at angle 45º. What is the vertical component of velocity when it strikes the ground?

(A) $18\sqrt{3}$ ms^{-1} (B) 18 ms^{-1} (C) $9\sqrt{2}$ ms^{-1} (D) 9 ms^{-1}

B-4. A bomber plane moving at a horizontal speed of 20 m/s releases a bomb at a height of 80 m above ground as shown. At the same instant a Hunter of negligible height starts running from a point below it, to catch the bomb with speed 10 m/s. After two seconds he realized that he cannot make it, he stops running and immediately holds his gun and fires in such direction so that just before bomb hits the ground, bullet will hit it. What should be the firing speed of bullet. (Take $g = 10\ m/s^2$)



(A) 10 m/s
(B) $20\sqrt{10}$ m/s
(C) $10\sqrt{10}$ m/s
(D) None of these

Section (C) : Equation of trajectory

C-1. A ball is projected from a certain point on the surface of a planet at a certain angle with the horizontal surface. The horizontal and vertical displacement x and y varies with time t in second as :
$x = 10\sqrt{3}t$ and $y = 10t - t^2$
The maximum height attained by the ball is
(A) 100 m (B) 75 m (C) 50 m (D) 25 m.

C-2. A ball is thrown upward at an angle of 30° with the horizontal and lands on the top edge of a building that is 20 m away. The top edge is 5m above the throwing point. The initial speed of the ball in metre/second is (take $g = 10\ m/s^2$) :

(A) $u = 40\sqrt{\dfrac{(4+\sqrt{3})}{13\sqrt{3}}}$ m/s
(B) $u = 40\sqrt{\dfrac{4-\sqrt{3}}{13\sqrt{3}}}$ m/s
(C) $u = 40\sqrt{\dfrac{4+\sqrt{3}}{13}}$ m/s
(D) $u = 40\dfrac{40}{\sqrt{\sqrt{3}\ (4+\sqrt{3})}}$ m/s

Section (D) : Projectile on an inclined plane

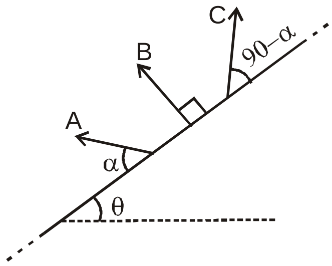
D-1. A plane surface is inclined making an angle θ with the horizontal. From the bottom of this inclined plane, a bullet is fired with velocity v. The maximum possible range of the bullet on the inclined plane is
(A) $\dfrac{v^2}{g}$ (B) $\dfrac{v^2}{g(1+\sin\theta)}$ (C) $\dfrac{v^2}{g(1-\sin\theta)}$ (D) $\dfrac{v^2}{g(1+\cos\theta)}$

D-2. A ball is horizontally projected with a speed v from the top of a plane inclined at an angle 45° with the horizontal. How far from the point of projection will the ball strike the plane?
(A) $\dfrac{v^2}{g}$ (B) $\dfrac{\sqrt{2}v^2}{g}$ (C) $\dfrac{2v^2}{g}$ (D) $\left[\dfrac{2\sqrt{2}v^2}{g}\right]$

D-3. A particle is projected at angle 37° with the incline plane in upward direction with speed 10 m/s. The angle of incline plane is given 53°. Then the maximum distance from the incline plane attained by the particle will be
(A) 3m (B) 4 m (C) 5 m (D) zero

D-4. On an inclined plane of inclination 30°, a ball is thrown at an angle of 60° with the horizontal from the foot of the incline with a velocity of $10\sqrt{3}\ ms^{-1}$. If $g = 10\ ms^{-2}$, then the time in which ball will hit the inclined plane is -
(A) 1 sec. (B) 6 sec. (C) 2 sec. (D) 4 sec.

D-5. Three stones A, B, C are projected from surface of very long inclined plane with equal speeds and different angles of projection as shown in figure. The incline makes an angle θ with horizontal. If H_A, H_B and H_C are maximum height attained by A, B and C respectively above inclined plane then : (Neglect air friction)

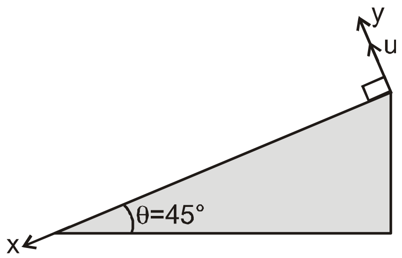


(A) $H_A + H_C = H_B$ (B) $H_A^2 + H_C^2 = H_B^2$

(C) $H_A + H_C = 2H_B$ (D) $H_A^2 + H_C^2 = 2H_B^2$

PART - III : MATCH THE COLUMN

1. An inclined plane makes an angle θ = 45° with horizontal. A stone is projected normally from the inclined plane, with speed u m/s at t = 0 sec. x and y axis are drawn from point of projection along and normal to inclined plane as shown. The length of incline is sufficient for stone to land on it and neglect air friction. Match the statements given in column I with the results in column II. (g in column II is acceleration due to gravity.)



Column I	Column II
(A) The instant of time at which velocity of stone is parallel to x-axis	(p) $\frac{2\sqrt{2}u}{g}$
(B) The instant of time at which velocity of stone makes an angle θ = 45° with positive x-axis. in clockwise direction	(q) $\frac{2u}{g}$
(C) The instant of time till which (starting from t = 0) component of displacement along x-axis become half the range on inclined plane is	(r) $\frac{\sqrt{2}u}{g}$
(D) Time of flight on inclined plane is	(s) $\frac{u}{\sqrt{2}g}$

2. A particle is projected from level ground. Assuming projection point as origin, x-axis along horizontal and y-axis along vertically upwards. If particle moves in x-y plane and its path is given by $y = ax - bx^2$ where a, b are positive constants. Then match the physical quantities given in column-I with the values given in column-II. (g in column II is acceleration due to gravity.)

Column I	Column II
(A) Horizontal component of velocity	(p) $\frac{a}{b}$
(B) Time of flight	(q) $\frac{a^2}{4b}$
(C) Maximum height	(r) $\sqrt{\frac{g}{2b}}$
(D) Horizontal range	(s) $\sqrt{\frac{2a^2}{bg}}$

Exercise-2

✍ Marked Questions can be uasd as Revision Questions.

PART - I : ONLY ONE OPTION CORRECT TYPE

1. A particle moves in the xy plane with only an x-component of acceleration of 2 ms^{-2}. The particle starts from the origin at t = 0 with an initial velocity having an x-component of 8 ms^{-1} and y-component of –15 ms^{-1}. Velocity of particle after time t is :

(A) $[(8 + 2t)\,\hat{i} - 15\,\hat{j}]$ m s^{-1}
(B) zero
(C) $2t\,\hat{i} + 15\,\hat{j}$
(D) directed along z-axis.

2.✍ A plane flying horizontally at a height of 1500 m with a velocity of 200 ms^{-1} passes directly overhead an antiaircraft gun. Then the angle with the horizontal at which the gun should be fired for the shell with a muzzle velocity of 400 m s^{-1} to hit the plane, is -

(A) 90º (B) 60º (C) 30º (D) 45º

3.✍ If R and h represent the horizontal range and maximum height respectively of an oblique projection whose start point (i.e. point of projecteion) & end point are in same horizontal level. Then $\frac{R^2}{8h} + 2h$ represents

(A) maximum horizontal range
(B) maximum vertical range
(C) time of flight
(D) velocity of projectile at highest point

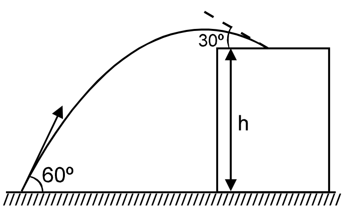
4.✍ A projectile is thrown with velocity v making an angle θ with the horizontal. It just crosses the top of two poles, each of height h, after 1 second and 3 second respectively. The time of flight of the projectile is

(A) 1 s (B) 3 s (C) 4 s (D) 7.8 s

5. A body has an initial velocity of 3 ms^{-1} and has a constant acceleration of 1 ms^{-2} normal to the direction of the initial velocity. Then its velocity, 4 second after the start is

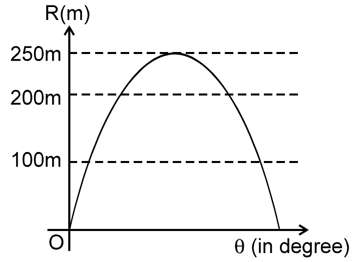
(A) 7 ms^{-1} along the direction of initial velocity
(B) 7 ms^{-1} along the normal to the direction of the initial velocity
(C) 7 ms^{-1} mid-way between the two directions
(D) 5 ms^{-1} at an angle of $\tan^{-1}\frac{4}{3}$ with the direction of the initial velocity

6. A stone projected at an angle of 60º from the ground level strikes at an angle of 30º on the roof of a building of height 'h'. Then the speed of projection of the stone is :



(A) $\sqrt{2gh}$
(B) $\sqrt{6gh}$
(C) $\sqrt{3gh}$
(D) $\sqrt{gh}$

7. From the ground level, a ball is to be shot with a certain speed. Graph shows the range R it will have versus the launch angle θ. The least speed the ball will have during its flight if θ is chosen such that the flight time is half of its maximum possible value, is equal to (take g = 10 m/s^2)

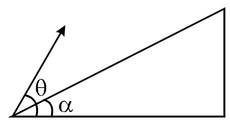


(A) 250 m/s (B) $50\sqrt{3}$ m/s (C) 50 m/s (D) $25\sqrt{3}$ m/s

8. A particle at a height 'h' from the ground is projected with an angle 30° from the horizontal, it strikes the ground making angle 45° with horizontal. It is again projected from the same point at height h with the same speed but with an angle of 60° with horizontal. Find the angle it makes with the horizontal when it strikes the ground :

(A) $\tan^{-1}(4)$ (B) $\tan^{-1}(5)$ (C) $\tan^{-1}(\sqrt{5})$ (D) $\tan^{-1}(\sqrt{3})$

9. A projectile is fired at an angle θ with the horizontal. Find the condition under which it lands perpendicular on an inclined plane of inclination α as shown in figure.



(A) $\sin\alpha = \cos(\theta - \alpha)$ (B) $\cos\alpha = \sin(\theta - \alpha)$ (C) $\tan\theta = \cot(\theta - \alpha)$ (D) $\cot(\theta - \alpha) = 2\tan\alpha$

10. A ball is thrown eastward across level ground. A wind blows horizontally to the east, and assume that the effect of wind is to provide a constant force towards the east, equal in magnitude to the weight of the ball. The angle θ (with horizontal east) at which the ball should be projected so that it travels maximum horizontal distance is

(A) 45° (B) 37° (C) 53° (4) 67.5°

PART - II : SINGLE AND DOUBLE VALUE INTEGER TYPE

1. A hunter in a valley is trying to shoot a deer on a hill. The distance of the deer along his line of sight is $10\sqrt{181}$ meters and the height of the hill is 90 meters. His gun has a muzzle velocity of 100 m/sec. Minimum how many meters above the deer should he aim his rifle in order to hit it? [g = 10 m/s^2]

2. A stone is thrown in such a manner that it would just hit a bird at the top of a tree and afterwards reach a maximum height double that of the tree. If at the moment of throwing the stone the bird flies away horizontally with constant velocity and the stone hits the bird after some time.The ratio of horizontal velocity of stone to that of the bird is $\frac{1}{n} + \frac{1}{\sqrt{n}}$. Find 2n.

3. If 4 seconds be the time in which a projectile reaches a point P of its path and 5 seconds the time from P till it reaches the horizontal plane passing through the point of projection. The height of P above the horizontal plane (in m) will be - [g = 9.8 m/sec^2]

4. A particle moves along the parabolic path $x = y^2 + 2y + 2$ in such a way that the y-component of velocity vector remains 5m/s during the motion. The magnitude of the acceleration of the particle (in m/s^2) is :

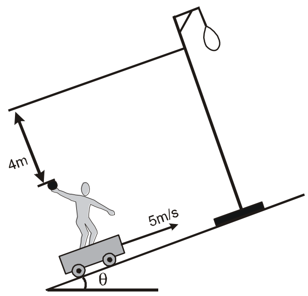
5. A person standing on the top of a cliff 30 m high has to throw a packet to his friend standing on the ground 40 m horizontally away. If he throws the packet directly aiming at the friend with a speed of $\frac{125}{3}$ m/s. Packet falls at a distance $\frac{20}{\alpha}$ m from the friend. Here α is an integer. Find α. [Use g = 10 m/s^2].

6. From the top of a tower of height 40 m, a ball is projected upwards with a speed of 20 m/sec at an angle of 30º to the horizontal. Distance from the foot of the tower where the ball hit the ground is $40\sqrt{\beta}$ m. Here β is an integer. Find β

7. A particle is projected from a point (0, 1) on Y-axis (assume + Y direction vertically upwards) aiming towards a point (4, 9). It falls on ground on x axis in 1 sec. If the speed of projection is $\sqrt{\beta}$ m/s, where β is an integer. Find β. Taking g = 10 m/s^2 and all coordinate in metres.

8. A Bomber flying upward at an angle of 53º with the vertical releases a bomb at an altitude of 800 m. The bomb strikes the ground 20 sec after its release. Velocity of the bomber at the time of release of the bomb is V m/s. Find $\frac{V}{4}$. [Given sin 53º = 0.8 ; g = 10 ms^{-2}]

9. A man is travelling on a flat car which is moving up a plane inclined at $\cos\theta = 4/5$ to the horizontal with a speed 5 m/s. He throws a ball towards a stationary hoop located perpendicular to the incline in such a way that the ball moves parallel to the slope of the incline while going through the centre of the hoop. The centre of the hoop is 4 m high from the man's hand calculate the time taken by the ball to reach the hoop in second.



10. A ball starts falling with zero initial velocity on a smooth inclined plane forming an angle α with the horizontal. Having fallen the distance h, the ball rebounds elastically off the inclined plane. At distance nhsinα from the impact point the ball rebounds for the second time. Here n is an integer. Find n.

11. A stone is projected horizontally with speed v from a height h above ground. A horizontal wind is blowing in direction opposite to velocity of projection and gives the stone a constant horizontal acceleration f (in direction opposite to initial velocity). As a result the stone falls on ground at a point vertically below the point of projection. Then find the value of $\frac{f^2h}{gv^2}$ (g is acceleration due to gravity)

12. A small ball rolls of the top of a stairway horizontally with a velocity of 4.5 m s^{-1}. Each step is 0.2 m high and 0.3 m wide. If g is 10 ms^{-2}, then the ball will strike the nth step where n is equal to (assume ball strike at the edge of the step).

13. If at an instant the velocity of a projectile be 60 m/s and its inclination to the horizontal be 30°, at what time interval (in sec) after that instant will the particle be moving at right angles to its former direction. (g = 10 m/s^2)

PART - III : ONE OR MORE THAN ONE OPTIONS CORRECT TYPE

1. A projectile is projected at an angle α (> 45°) with an initial velocity u. The time t at which its horizontal component will equal the vertical component in magnitude:

(A) $t = \frac{u}{g}(\cos\alpha - \sin\alpha)$ (B) $t = \frac{u}{g}(\cos\alpha + \sin\alpha)$

(C) $t = \frac{u}{g}(\sin\alpha - \cos\alpha)$ (D) $t = \frac{u}{g}(\sin^2\alpha - \cos^2\alpha)$

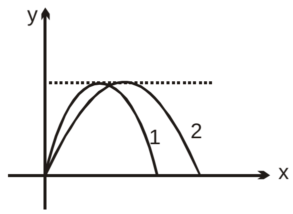
2. At what angle should a body be projected with a velocity 24 ms^{-1} just to pass over the obstacle 14 m high at a distance of 24 m. [Take g = 10 ms^{-2}]

(A) $\tan\theta = 19/5$ (B) $\tan\theta = 1$ (C) $\tan\theta = 3$ (D) $\tan\theta = 2$

3. A particle is projected from a point on the ground with an initial velocity of u = 50 m/s at an angle of 53° with the horizontal (tan 53° = 4/3, g = 10 m/s^2 = acceleration due to gravity).

(A) The velocity of the particle will make angle 45° with the horizontal after time 1 s.
(B) The velocity of the particle will make angle 45° with the horizontal after time 7 s.
(C) The average velocity between the point of projection and the highest point on its path is horizontal.
(D) The average velocity between two points on same height will be horizontal.

4. Two stones are projected from level ground. Trajectories of two stones are shown in figure. Both stones have same maximum heights above level ground as shown. Let T_1 and T_2 be their time of flights and u_1 and u_2 be their speeds of projection respectively (neglect air resistance). Then

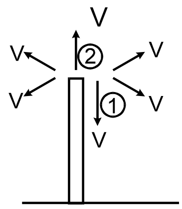


(A) $T_2 > T_1$ (B) $T_1 = T_2$
(C) $u_1 > u_2$ (D) $u_1 < u_2$

5. A projectile of mass 1 kg is projected with a velocity of $\sqrt{20}$ m/s such that it strikes on the same level as the point of projection at a distance of $\sqrt{3}$ m. Which of the following options are correct?

(A) The maximum height reached by the projectile can be 0.25 m.
(B) The minimum velocity during its motion can be $\sqrt{15}$ m/s.
(C) The time taken for the flight can be $\sqrt{\frac{3}{5}}$ s.
(D) Maximum angle of projection can be 60°.

6. Particles are projected from the top of a tower with same speed at different angles as shown. Which of the following are True ?

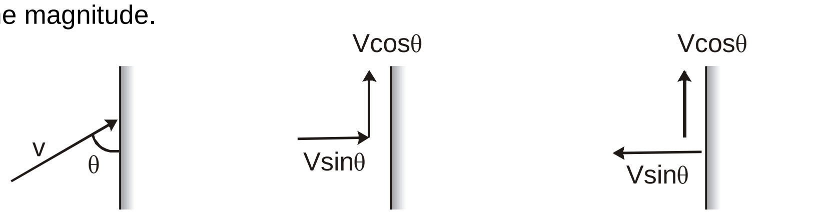


(A) All the particles would strike the ground with (same) speed.
(B) All the particles would strike the ground with (same) speed simultaneously.
(C) Particle 1 will be the first to strike the ground.
(D) Particle 1 strikes the ground with maximum speed.

PART - IV : COMPREHENSION

Comprehension 1

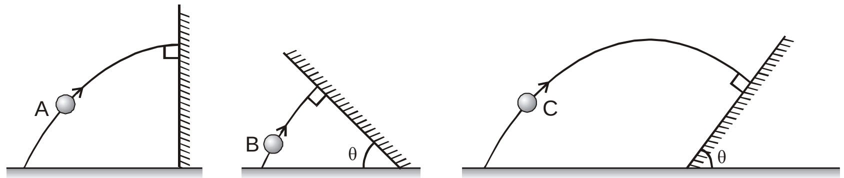
We know how by neglecting the air resistance, the problems of projectile motion can be easily solved and analysed. Now we consider the case of the collision of a ball with a wall. In this case the problem of collision can be simplified by considering the case of elastic collision only. When a ball collides with a wall we can divide its velocity into two components, one perpendicular to the wall and other parallel to the wall. If the collision is elastic then the perpendicular component of velocity of the ball gets reversed with the same magnitude.



Velocity just before collision | Components of velocity just before collision | Components of velocity just after collision

The other parallel component of velocity will remain constant if given wall is smooth.

Now let us take a problem. Three balls 'A' and 'B' & 'C' are projected from ground with same speed at same angle with the horizontal. The balls A, B and C collide with the wall during their flight in air and all three collide perpendicularly with the wall as shown in figure.



1. Which of the following relation about the maximum height H of the three balls from the ground during their motion in air is correct :

(A) $H_A = H_C > H_B$ (B) $H_A > H_B = H_C$ (C) $H_A > H_C > H_B$ (D) $H_A = H_B = H_C$

2. If the time taken by the ball A to fall back on ground is 4 seconds and that by ball B is 2 seconds. Then the time taken by the ball C to reach the inclined plane after projection will be :

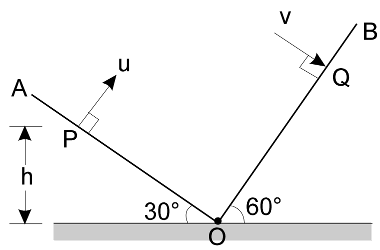
(A) 6 sec. (B) 4 sec. (C) 3 sec. (D) 5 sec.

3. In previous question the maximum height attained by ball 'A' from the ground is :

(A) 10 m (B) 15 m (C) 20 m (D) Insufficient information

Comprehension - 2

Two inclined planes OA and OB having inclinations 30° and 60° with the horizontal respectively intersect each other at O, as shown in figure. A particle is projected from point P with velocity $u = 10\sqrt{3}\ \frac{m}{s}$ along a direction perpendicular to plane OA. If the particle strikes plane OB perpendicular at Q (Take g = 10 m/s²). Then



4. The time of flight from P to Q is :

(A) 5 sec. (B) 2 sec (C) 1 sec (D) None of these

5. The speed with which the particle strikes the plane OB is :

(A) 10 m/s (B) 20 m/s (C) 30 m/s (D) 40 m/s

6. The height h of point P from the ground is :

(A) $10\sqrt{3}$ m (B) 10 m (C) 5 m (D) 20 m

7. The distance PQ is :

(A) 20 m (B) $10\sqrt{3}$ m (C) 10 m (D) 5 m

Exercise-3

✍ **Marked Questions may have for Revision Questions.**

*** Marked Questions may have more than one correct option.**

PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

1. A building 4.8 m high 2b meters wide has a flat roof. A ball is projected from a point on the horizontal ground 14.4 m away from the building along its width. If projected with velocity 16 m/s at an angle of 45º with the ground, the ball hits the roof in the middle, find the width 2b. Also find the angle of projection so that the ball just crosses the roof if projected with velocity $10\sqrt{3}$ m/s. ($g = 10 m/s^2$) **[REE 1995, 6]**

2.✍ A vertical pole has a red mark at some height. A stone is projected from a fixed point on the ground. When projected at an angle of 45º it hits the pole orthogonally 1 m above the mark. When projected with a different speed at an angle of $\tan^{-1}(3/4)$, it hits the pole orthogonally 1.5 m below the mark. Find the speed and angle of projection so that it hits the mark orthogonally to the pole. [$g = 10 m/sec^2$] **[REE 1996, 6]**

3.*✍ The coordinates of a particle moving in a plane are given by x(t) = a cos (pt) and y (t) = b sin (pt), where a, b (< a) and p are positive constants of appropriate dimensions then **[JEE 1999, 3/200]**
(A) the path of the particle is an ellipse
(B) the velocity and acceleration of the particle are normal to each other at $t = \pi/2p$
(C) the acceleration of the particle is always directed towards a focus
(D) the distance travelled by the particle in time interval t = 0 to $= \pi/2p$ is a.

4. Shots fired simultaneously from the top and foot of a vertical cliff at elevations of 30° and 60° respectively, strike an object simultaneously which is at a height of 100 meters from the ground and at a horizontal distance of $200\sqrt{3}$ meters from the cliff. Find the height of the cliff, the velocities of projection of the shots and the time taken by the shots to hit the object. ($g = 10 m/sec^2$.) **[REE-2000, 5/100]**

5. A ball is projected from the ground at an angle of 45° with the horizontal surface. It reaches a maximum height of 120m and returns to the ground. Upon hitting the ground for the first time, it loses half of its kinetic energy. Immediately after the bounce, the velocity of the ball makes an angle of 30° with the horizontal surface. The maximum height it reaches after the bounce, in metres, is ___________.
[JEE (Advanced) 2018, 3/60]

PART - II : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1.✍ A particle is projected at 60º to the horizontal with a kinetic energy K. The kinetic energy at the highest point is **[AIEEE-2007, 3/120]**
(1) K (2) Zero (3) K/4 (4) K/2

2.✍ A particle has an initial velocity of $3\hat{i} + 4\hat{j}$ and an acceleration of $0.4\hat{i} + 0.3\hat{j}$. Its speed after 10 s is : **[AIEEE-2009, 4/144]**
(1) $7\sqrt{2}$ units (2) 7 units (3) 8.5 units (4) 10 units

3.✍ A particle is moving with velocity $\vec{v} = K(y\hat{i} + x\hat{j})$, where K is a constant. The general equation for its path is : **[AIEEE 2010, 4/144]**
(1) $y = x^2$ + constant (2) $y^2 = x$ + constant (3) xy = constant (4) $y^2 = x^2$ + constant

4. A water fountain on the ground sprinkles water all around it. If the speed of water coming out of the fountain is v, the total area around the fountain that gets wet is : **[AIEEE - 2011, 4/120, –1]**
(1) $\pi\frac{v^2}{g}$ (2) $\pi\frac{v^4}{g^2}$ (3) $\frac{\pi}{2}\frac{v^4}{g^2}$ (4) $\pi\frac{v^2}{g^2}$

5. A boy can throw a stone up to a maximum height of 10m. The maximum horizontal distance that the boy can throw the same stone up to will be : **[AIEEE 2012 ; 4/120, –1]**
(1) $20\sqrt{2}$m (2) 10 m (3) $10\sqrt{2}$m (4) 20m

6. A projectile is given an initial velocity of $(\hat{i} + 2\hat{j})$ m/s, where $\hat{i}$ is along the ground and $\hat{j}$ is along the vertical. If $g = 10 m/s^2$, the equation of its trajectory is : **[JEE (Main) 2013, 4/120]**
(1) $y = x - 5x^2$ (2) $y = 2x - 5x^2$ (3) $4y = 2x - 5x^2$ (4) $4y = 2x - 25x^2$

Answers

EXERCISE-1

PART - I

Section (A) :

A-1. $\tan\theta : 1$ **A-2.** $\tan^2\theta : 1$

A-3. $\tan\theta = 4$ or $\theta = \tan^{-1}(4)$ **A-4.** $T_1T_2 = \frac{2R}{g}$

A-5. 50 m **A-6.** $5\sqrt{2}$ m/s

A-7. $\frac{gt}{\tan\alpha - \tan\beta}$

A-8. (i) 250 m (ii) 50 m/sec.

Section (B) :

B-1. (i) 10 sec. (ii) 980 m (iii) $98\sqrt{2}$ m/s

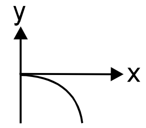
B-2. (i) 61.25 m (ii) $75\sqrt{3}$ m $\approx$ 130 m

(iii) 5 sec.

Section (C) :

C-1. $\theta = 60°$, 2 m/s **C-2.** 53

C-3. (i) $y = -\frac{bx^2}{a^2}$



(ii) $\vec{v} = a\hat{i} - 2bt\hat{j}$,

acceleration $= -2b\hat{j}$,

$|\vec{v}| = \sqrt{a^2 + 4b^2t^2}$,

|acceleration| $= 2b$

Section D :

D-1. (a) $-g\sin\beta$, (b) $-g\cos\beta$,

(c) $u\cos\theta - g\sin\beta \times t$,

(d) $u\sin\theta - g\cos\beta \times t$,

(e) $u\cos\theta \times t - \frac{1}{2}g\sin\beta \times t^2$,

(f) $u\sin\theta \times t - \frac{1}{2}g\cos\beta \times t^2$, (g) zero

PART - II

Section (A) :

A-1.	(D)	**A-2.**	(C)	**A-3.**	(D)
A-4.	(D)	**A-5.**	(A)	**A-6.**	(B)
A-7.	(A)	**A-8.**	(B)	**A-9.**	(B)

Section (B) :

B-1.	(C)	**B-2.**	(A)	**B-3.**	(B)
B-4.	(C)				

Section (C)

C-1.	(D)	**C-2.**	(A)

Section (D)

D-1.	(B)	**D-2.**	(D)	**D-3.**	(A)
D-4.	(C)	**D-5.**	(A)		

PART - III

1. (A) $\to$ r ; (B) $\to$ s ; (C) $\to$ q ; (D) $\to$ p

2. (A) $\to$ r ; (B) $\to$ s ; (C) $\to$ q ; (D) $\to$ p

EXERCISE-2

PART - I

1.	(A)	**2.**	(B)	**3.**	(A)
4.	(C)	**5.**	(D)	**6.**	(C)
7.	(D)	**8.**	(C)	**9.**	(D)
10.	(D)				

PART - II

1.	10	**2.**	4	**3.**	98
4.	50	**5.**	3	**6.**	3
7.	20	**8.**	25	**9.**	1s
10.	8	**11.**	2	**12.**	9
13.	12				

PART - III

1.	(BC)	**2.**	(AB)	**3.**	(ABD)
4.	(BD)	**5.**	(ABCD)	**6.**	(AC)

PART - IV

1.	(A)	**2.**	(C)	**3.**	(C)
4.	(B)	**5.**	(A)	**6.**	(C)
7.	(A)				

EXERCISE-3

PART - I

1. Width of the roof is 9.6 m; $\theta = \tan^{-1}\frac{3}{2}$

or $\theta = 45°$

2. $\frac{\sqrt{3620}}{3}$ m/s , $\tan^{-1}\left(\frac{9}{10}\right)$

3. (AB)

4. 400 m, $V_T = 40$ m/s, $V_F = 40\sqrt{3}$ m/s, T = 10 s.

5. 30 m

PART - II

1.	(3)	**2.**	(1)	**3.**	(4)
4.	(2)	**5.**	(4)	**6.**	(2)

Advance Level Problems (ALP)

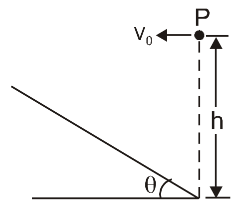
SUBJECTIVE QUESTIONS

1. A projectile aimed at a mark which is in the horizontal plane through the point of projection falls a cm short of it when the elevation is α and goes b cm too far when the elevation is β. Show that if the velocity of projection is same in all the case, the proper elevation is $\frac{1}{2}\sin^{-1}\left[\frac{b\sin 2\alpha + a\sin 2\beta}{a+b}\right]$

2. A particle is thrown over a triangle from one end of a horizontal base and grazing the vertex falls on the other end of the base. If α and β be the base angles and θ the angle of projection prove that $\tan\theta = \tan\alpha + \tan\beta$.

3. A shell bursts on contact with the ground and pieces from it fly in all directions with all velocities upto 80 feet per second. Show that a man 100 feet away is in danger for $\frac{5}{\sqrt{2}}$ seconds. [Use g = 32 ft/s^2].

4. A stone is projected horizontally from a point P, so that it hits the inclined plane perpendicularly. The inclination of the plane with the horizontal is θ and the point P is at a height h above the foot of the incline, as shown in the figure. Determine the velocity of projection.



5. Two parallel straight lines are inclined to the horizontal at an angle α. A particle is projected from a point mid way between them so as to graze one of the lines and strikes the other at right angles. Show that if θ is the angle between the direction of projection and either of lines, then

$$\tan\theta = \left(\sqrt{2}-1\right)\cot\alpha$$

6. The benches of a gallery in a cricket stadium are 1 m high and 1 m wide. A batsman strikes the ball at a level 1 m above the ground and hits a ball. The ball starts at 35 m/s at an angle of 53º with the horizontal. The benches are perpendicular to the plane of motion and the first bench is 110 m from the batsman. On which bench will the ball hit.

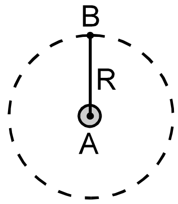
7. A ship is approaching a cliff of height 105 m above sea level. A gun fitted on the ship can fire shots with a speed of 110 ms^{-1}. Find the maximum distance from the foot of the cliff from where the gun can hit an object on the top of the cliff. [g = 10 m/s^2] **[REE 1994, 6]**

8. Shots fired simultaneously from the top and bottom of a vertical cliff with the elevation α and β respectively, strike an object simultaneously at the same point on the ground. If s is the horizontal distance of the object from the cliff, then what is the height of the cliff. ($\beta > \alpha$)

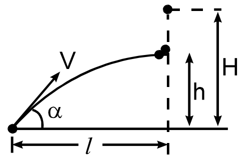
9. Some students are playing cricket on the roof of a building of height 20 m. While playing, ball falls on the ground. A person on the ground returns their ball with the minimum possible speed at angle of projection 45° with the horizontal. The speed of projection is $20\sqrt{\alpha}$ m/s. Here α is an integer. Find α

10. A cannon fires successively two shells with velocity v_0= 250 m/s; the first at the angle θ_1 = 53° and the second at the angle θ_2 = 37° to the horizontal, in the same vertical plane, neglecting the air drag, find the time interval (in sec) between firings leading to the collision of the shells. (g = 10 m/s^2).

11. A small cannon 'A' is mounted on a platform (which can be rotated so that the cannon can aim at any point) and is adjusted for maximum range R of shells. It can throw shells on any point on the shown circle (dotted) on ground. Suddenly a windstorm starts blowing in horizontal direction normal to AB with a speed $\sqrt{2}$ times the velocity of shell. At what least distance can the shell land from point B. (Assume that the velocity of the windstorm is imparted to the shell in addition to its velocity of projection. Also assume that the platform is kept stationary while projecting the shell.)

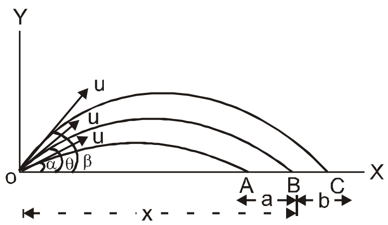


12. A body A falls freely from some altitude H (<< Re). At the moment the first body starts falling another body B is thrown from the earth's surface which collides with the first at an altitude h = H/2 . The horizontal distance of that point of collision is l from the starting point of B. Find the initial velocity and the angle at which it was thrown ?



Answers

1.



$$OA = x - a = \frac{u^2 \sin 2\alpha}{g} \qquad ...(1)$$

$$OC = x + b = \frac{u^2 \sin 2\beta}{g} \qquad ...(2)$$

$$OB = x = \frac{u^2 \sin 2\theta}{g} \qquad ...(3)$$

From eqs. (1) and (2)

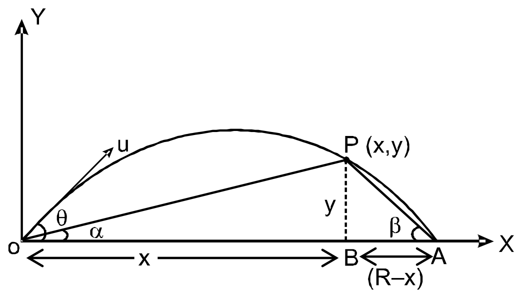
$$x\,(b + a) = \left(\frac{b \sin 2\alpha + a \sin 2\beta}{g}\right)u^2$$

Substituting the value of x from eq. (3), we get

$$\frac{u^2 \sin 2\theta}{g}\,(b + a) = \left(\frac{b \sin 2\alpha + a \sin 2\beta}{g}\right)u^2$$

Solving this equation, we will get θ.

2. The situation is shown in the fig.



From fig $\tan\alpha + \tan\beta = \dfrac{y}{x} + \dfrac{y}{(R - x)}$

where R is the range.

$\therefore \qquad \tan\alpha + \tan\beta = \dfrac{y(R - x) + xy}{x(R - x)}$

or $\qquad \tan\alpha + \tan\beta = \dfrac{y}{x} \times \dfrac{R}{(R - x)} \qquad ....(1)$

but $\quad y = x \tan\theta \left(1 - \dfrac{x}{R}\right)$

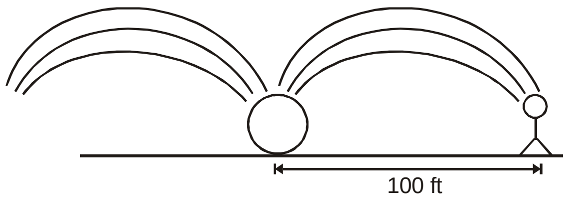
or $\qquad \tan\theta = \dfrac{y}{x} \times \dfrac{R}{(R - x)} \qquad ....(2)$

From equations (1) and (2), we have
$\tan\theta = \tan\alpha + \tan\beta$.

3. According to given problem u = 80 f / s

Range = $\dfrac{u^2 \sin 2\theta}{g}$

$\sin 2\theta = \dfrac{100 \times 32}{(80)^2} = 1/2$



$\theta = 15°$ For same Range $\theta = 15°, 75°$
Thus there will be two time of flight

$T_1 = \dfrac{2u\sin 15°}{g} = \dfrac{2 \times 80 \times \sin 15°}{32}$ (minimum time)

$\sin 15° = \dfrac{\sqrt{3}-1}{2\sqrt{2}}$

$T_2 = \dfrac{2u\sin 75°}{g} = \dfrac{2 \times 8 \times \sin 75°}{32}$ (maximum time)

$\sin 75° = \dfrac{\sqrt{3}+1}{2\sqrt{2}}$

Danger time = Maximum time – Minimum time = $(T_2 - T_1)$

$= \dfrac{2 \times 80}{32}$ [sin75° –sin 15°]

$= \dfrac{2 \times 80}{32}\left[\dfrac{\sqrt{3}+1}{2\sqrt{2}} - \dfrac{\sqrt{3}-1}{2\sqrt{2}}\right] = \dfrac{5}{\sqrt{2}}$ sec.

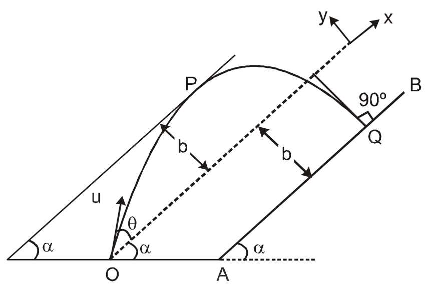
4. $v_0 = \sqrt{\dfrac{2gh}{2+\cot^2\theta}}$

5. Consider the motion of the particle from O to P. The velocity v_y at P is zero.

$v_y^2 = u_y^2 + 2a_y s_y$

$\therefore$ $0 = (u \sin\theta)^2 - 2 (g\cos\alpha)\, b$

or $b = \dfrac{u^2 \sin^2\theta}{2g\cos\alpha}$ ....(i)



Now, consider the motion of the particle from O to Q.
The particle strikes the point Q at 90º to AB, i.e., its velocity along x-direction is zero.

Using $v_x = u_x + a_x t$, we have

$0 = u\cos\theta - (g\sin\alpha)t$

or $t = \dfrac{u\cos\theta}{g\sin\alpha}$ ...(ii)

For motion in y-direction,

$s_y = u_y t + \dfrac{1}{2}a_y t^2$ or $-b = u\sin\theta$

$\left(\dfrac{u\cos\theta}{g\sin\alpha}\right) + \dfrac{1}{2}(-g\cos\alpha)\left(\dfrac{u\cos\theta}{g\sin\alpha}\right)^2$ ...(iii)

From Eqs. (i) and (iii)
or

$-\dfrac{u^2\sin^2\theta}{2g\cos\alpha} = \dfrac{u^2\sin\theta\cos\theta}{g\sin\alpha} - \dfrac{gu^2\cos\alpha\cos^2\theta}{2g^2\sin^2\alpha}$

or $-\dfrac{\sin^2\theta}{2\cos\alpha} = \dfrac{\sin\theta\cos\theta}{\sin\alpha} - \dfrac{\cos\alpha\cos^2\theta}{2\sin^2\alpha}$

Solving, we get $\tan\theta = \left(\sqrt{2}-1\right)\cot\alpha$

6. 6th step

7. 1100 m

8. $h = s(\tan\beta - \tan\alpha)$.

9. 2

10. 10sec

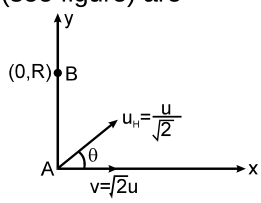
11. Let the speed of shell be u and the speed of wind be v.
The time of flight T remains unchanged due to windstorm

$T = \dfrac{\sqrt{2}\,u}{g}$ ...(1)

Horizontal component of velocity of shell in absence of air

$u_H = \dfrac{u}{\sqrt{2}}$ ...(2)

Hence the net x and y component of velocity of shell (see figure) are



$u_x = \sqrt{2}\,u + \dfrac{u}{\sqrt{2}}\cos\theta$ ...(3a)

$u_y = \dfrac{u}{\sqrt{2}}\sin\theta$ ...(3b)

$\therefore$ The x and y coordinate of point P where shell lands is

$x = u_x T = \left(\sqrt{2}\,u + \dfrac{u}{\sqrt{2}}\cos\theta\right)$

$\dfrac{\sqrt{2}u}{g} = 2R + R\cos\theta$ ...(4a)

$y = u_y T = \left(\frac{u}{\sqrt{2}}\sin\theta\right)\frac{\sqrt{2}u}{g} = R\sin\theta$.(4b)

$\therefore$ The distance S between B and P is given by

$S^2 = (x - 0)^2 + (y - R)^2$

$= (2R + R\cos\theta)^2 + (R\sin\theta - R)^2$

$= R^2 [6 + 4\cos\theta - 2\sin\theta]$

$= R^2 \left[6 + \sqrt{20}\left(\frac{4\cos\theta}{\sqrt{20}} - \frac{2\sin\theta}{\sqrt{20}}\right)\right]$

$\therefore \quad S_{minimum} = R\sqrt{6-\sqrt{20}}$

$= R\sqrt{6-2\sqrt{5}}$ or $R(\sqrt{5}-1)$ **Ans.**

Alternate :

Circle in fig. (1) represents locus of all points where shell lands on the ground in absence of windstorm.

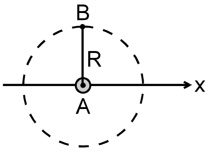


fig.1

Let the speed of shell be 'u' and the speed of wind be $v = \sqrt{2}u$. Let T be the time of flight, which remains unaltered even when the windstorm blows. Since R is the maximum range angle of projection is 45º with the horizontal.

Then $R = \frac{u}{\sqrt{2}} T$ ... (1)

As a result of flow of wind along x-axis, there is an additional shift (Δx) of the shell along x-axis in time of flight.

$\Delta x = vT = \sqrt{2}\,uT = 2R.$

Hence locus of all points where shell lands on ground shifts along x-axis by 2R as shown in fig. (2).

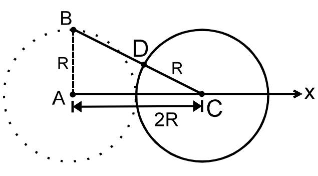


fig.2

From the fig (2).

$BC = \sqrt{R^2 + (2R)^2} = \sqrt{5R^2} = \sqrt{5}\,R$

Hence the minimum required distance is

$BD = BC - DC = \sqrt{5}\,R - R = (\sqrt{5} - 1)\,R$

12. $\nu_0 = \sqrt{gH\left(1+\frac{\ell^2}{H^2}\right)}$, $\tan\alpha = \frac{H}{\ell}$