Exercise-1

✍ **Marked Questions can be used as Revision Questions.**

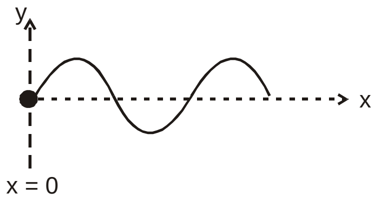
PART - I : SUBJECTIVE QUESTIONS

Section (A) : Equation of travelling wave (Including sine Wave)

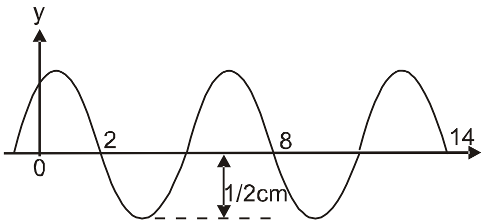
A-1. Consider the wave y = (10 mm) sin [$(5\pi\ cm^{-1})\ x - (60\pi\ s^{-1})\ t + \frac{\pi}{4}$]. Find (a) the amplitude (b) the wave number (c) the wavelength (d) the frequency (e) the time period and (f) the wave velocity (g) phase constant of SHM of praticle at x = 0.

A-2. Two identical traveling waves, moving in the same direction are out of phase by $\pi/2$ rad. What is the amplitude of the resultant wave in terms of the common amplitude y_m of the two combining waves?

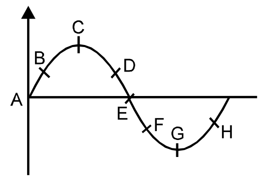
A-3. The string shown in figure is driven at a frequency of 5.00 Hz. The amplitude of the motion is 12.0 cm, and the wave speed is 20.0 m/s. Furthermore, the wave is such that y = 0 at x = 0 and t = 0. Determine (a) the angular frequency and (b) wave number for this wave (c) Write an expression for the wave function. Calculate (d) the maximum transverse speed and (e) the maximum transverse acceleration of a point on the string.



A-4.✍ The sketch in the figure shows displacement time curve of a sinusoidal wave at x = 8 m. Taking velocity of wave v = 6m/s along positive x-axis, write the equation of the wave.



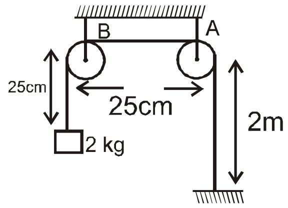
A-5. A transverse wave is travelling along a string from left to right. The fig. represents the shape of the string (snap-shot) at a given instant. At this instant
(a) which points have an upward velocity
(b) which points have downward velocity
(c) which points have zero velocity
(d) which points have maximum magnitude of velocity.



Section (B) : Speed of a wave on a string

B-1. A piano string having a mass per unit length equal to 5.00×10^{-3} kg/m is under a tension of 1350 N. Find the speed with which a wave travels on this string.

B-2.✍ In the arrangement shown in figure, the string has mass of 5g. How much time will it take for a transverse disturbance produced at the floor to reach the pulley A ? Take g = 10 m/s^2.



B-3. A uniform rope of length 20 m and mass 8 kg hangs vertically from a rigid support. A block of mass 2 kg is attached to the free end of the rope. A transverse pulse of wavelength 0.06 m is produced at the lower end of the rope. What is the wavelength of the pulse when it reaches the top of the rope?

B-4. A particle on a stretched string supporting a travelling wave, takes minimum time 5.0 ms to move from its mean position to the mean position. The distance between two consecutive particles, which are at their mean positions, is 2.0 cm. Find the frequency, the wavelength and the wave speed.

B-5. Two wires of different densities but same area of cross-section are soldered together at one end and are stretched to a tension T. The velocity of a transverse wave in the first wire is half of that in the second wire. Find the ratio of the density of the first wire to that of the second wire.

B-6. A 4.0 kg block is suspended from the ceiling of an elevator through a string having a linear mass density of 1.6×10^{-3} kg/m. Find (a) speed (with respect to the string) with which a wave pulse can proceed on the string if the elevator accelerates up at the rate of 6 m/s^2. (b) In part (a) at some instant speed of lift is 40 m/s in upward direction, then speed of the wave pulse with respect to ground at that instant is (Take g = 10 m/s^2)

Section (C) : Power transmitted along the string

C-1. A 6.00 m segment of a long string has a mass of 180 g. A high-speed photograph shows that the segment contains four complete cycles of a wave. The string is vibrating sinusoidally with a frequency of 50.0 Hz and a peak-to-valley displacement of 15.0 cm. (The "peak-to-valley" displacement is the vertical distance from the farthest positive displacement to the farthest negative displacement.) (a) Write the function that describes this wave traveling in the positive x direction. (b) Determine the average power being supplied to the string.

C-2. A transverse wave of amplitude 5 mm and frequency 10 Hz is produced on a wire stretched to a tension of 100 N. If the wave speed is 100 m/s, what average power is the source transmitting to the wire? ($\pi^2 = 10$)

Section (D) : Interference, Reflection, Transmission

D-1. The equation of a plane wave travelling along positive direction of x-axis is $y = a\sin\frac{2\pi}{\lambda}(\nu t - x)$ When this wave is reflected at a rigid surface and its amplitude becomes 80%, then find the equation of the reflected wave

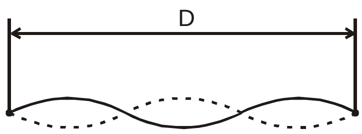
D-2. A series of pulses, each of amplitude 0.150 m, are sent on a string that is attached to a wall at one end. The pulses are reflected at the wall and travel back along the string without loss of amplitude. When two waves are present on the same string. The net displacement of a given point is the sum of the displacements of the individual waves at the point. What is the net displacement at point on the string where two pulses are crossing, (a) if the string is rigidly attached to the post ? (b) If the end at which reflection occurs is free to slide up and down?

D-3. Two waves, each having a frequency of 100 Hz and a wavelength of 2 cm, are travelling in the same direction on a string. What is the phase difference between the waves (a) if the second wave was produced 10 m sec later than the first one at the same place (b) if the two waves were produced at a distance 1 cm behind the second one? (c) If each of the waves has an amplitude of 2.0 mm, what would be the amplitudes of the resultant waves in part (a) and (b)?

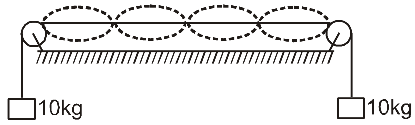
Section (E) : Standing waves and Resonance

E-1. What are (a) the lowest frequency (b) the second lowest frequency and (c) the third lowest frequency for standing waves on a wire that is 10.0 m long has a mass of 100 g and is stretched under a tension of 25 N which is fixed at both ends ?

E-2. A nylon guitar string has a linear density of 7.20 g/m and is under a tension of 150 N. The fixed supports are distance D = 90.0 cm apart. The string is oscillating in the standing wave pattern shown in figure. Calculate the (a) speed. (b) wavelength and (c) frequency of the traveling waves whose superposition gives this standing wave.



E-3. The length of the wire shown in figure between the pulleys is 1.5 m and its mass is 15 g. Find the frequency of vibration with which the wire vibrates in four loops leaving the middle point of the wire between the pulleys at rest. ($g = 10\ m/s^2$)

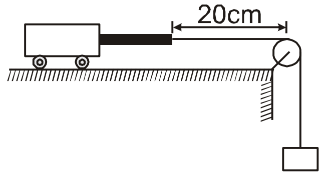


E-4. A string oscillates according to the equation $y' = (0.50\ cm)\ \sin\left[\left(\frac{\pi}{3}cm^{-1}\right)x\right]\ \cos\ [(40\ \pi\ s^{-1})t]$. What are the (a) amplitude and (b) speed of the two waves (identical except for direction of travel) whose superposition gives this oscillation? (c) What is the distance between nodes? (d) What is the transverse speed of a particle of the string at the position x = 1.5 cm when $t = \frac{9}{8}s$?

E-5. A string vibrates in 4 loops with a frequency of 400 Hz.
(a) What is its fundamental frequency ?
(b) What frequency will cause it to vibrate into 7 loops.

E-6. The vibration of a string of length 60 cm is represented by the equation, $y = 3\cos(\pi x/20)\cos(72\pi t)$ where x & y are in cm and t in sec.
(i) Write down the component waves whose superposition gives the above wave.
(ii) Where are the nodes and antinodes located along the string.
(iii) What is the velocity of the particle of the string at the position x = 5 cm & t = 0.25 sec.

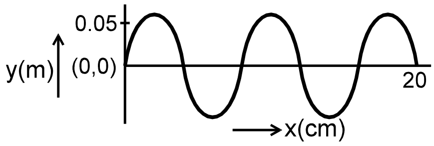
E-7. A heavy string is tied at one end to a movable support and to a light thread at the other end as shown in figure. The thread goes over a fixed pulley and supports a weight to produce a tension. The lowest frequency with which the heavy string resonates is 120 Hz. If the movable support is pushed to the right by 20 cm so that the joint is placed on the pulley, what will be the minimum frequency at which the heavy string can resonate?



PART - II : ONLY ONE OPTION CORRECT TYPE

Section (A) : Equation of travelling wave (Including sine Wave)

A-1. For the wave shown in figure, the equation for the wave, travelling along +x axis with velocity $350\ ms^{-1}$ when its position at t = 0 is as shown



(A) $0.05\sin\left(\frac{314}{4}x - 27475\ t\right)$
(B) $0.05\sin\left(\frac{379}{5}x - 27475\ t\right)$
(C) $1\sin\left(\frac{314}{4}x - 27475\ t\right)$
(D) $0.05\sin\left(\frac{289}{5}x + 27475\ t\right)$

A-2. The displacement of a wave disturbance propagating in the positive x-direction is given by $y = 1/(1 + x^2)$ at time t = 0 and $y = 1/[1 + (x - 1)^2]$ at t = 2 seconds where x and y are in meters. The shape of the wave disturbance does not change during the propagation. The velocity of the wave is: **[JEE - 1990]**

(A) 2.5 m/s (B) 0.25 m/s (C) 0.5 m/s (D) 5 m/s

A-3. A transverse wave is described by the equation $Y = Y_0 \sin 2\pi(ft - x/\lambda)$. The maximum particle velocity is equal to four times the wave velocity if **[JEE - 1984]**

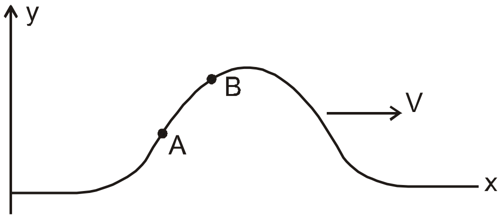
(A) $\lambda = \pi Y_0/4$ (B) $\lambda = \pi Y_0/2$ (C) $\lambda = \pi Y_0$ (D) $\lambda = 2\pi Y_0$

A-4. A travelling wave on a string is given by $y = A \sin [\alpha x + \beta t + \frac{\pi}{6}]$. If $\alpha = 0.56$ /cm, $\beta = 12$/sec, A = 7.5 cm, then position and velocity of particle at x = 1 cm and t = 1s is
(A) 4.6 cm, 46.5 cm s^{-1} (B) 3.75 cm, 77.94 cm s^{-1}
(C) 1.76 cm, 7.5 cm s^{-1} (D) 7.5 cm, 75 cm s^{-1}

A-5. A transverse wave of amplitude 0.50m, wavelength 1m and frequency 2 Hz is propagating on a string in the negative x-direction. The expression of the wave is **[REE - 1989]**
(A) $y(x, t) = 0.5 \sin (2\pi x - 4\pi t)$ (B) $y(x, t) = 0.5 \cos (2\pi x + 4\pi t)$
(C) $y(x, t) = 0.5 \sin (\pi x - 2\pi t)$ (D) $y(x, t) = 0.5 \cos (2\pi x - 2\pi t)$

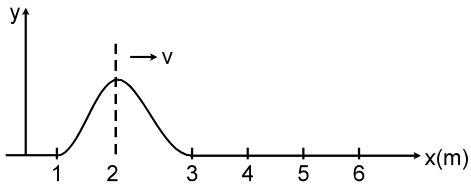
A-6. Two small boats are 10m apart on a lake. Each pops up and down with a period of 4.0 seconds due to wave motion on the surface of water. When one boat is at its highest point, the other boat is at its lowest point. Both boats are always within a single cycle of the waves. The speed of the waves is
(A) 2.5 m/s (B) 5.0 m/s (C) 14 m/s (D) 40 m/s

A-7. A wave pulse is generated in a string that lies along x-axis. At the points A and B, as shown in figure, if R_A and R_B are ratio of wave speed to the particle speed respectively then:



(A) $R_A > R_B$ (B) $R_B > R_A$
(C) $R_A = R_B$
(D) Information is not sufficient to decide.

A-8. Wave pulse on a string shown in figure is moving to the right without changing shape. Consider two particles at positions $x_1 = 1.5$ m and $x_2 = 2.5$ m. Their transverse velocities at the moment shown in figure are along directions:

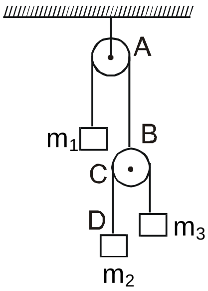


(A) positive y–axis and positive y–axis respectively
(B) negative y–axis and positive y–axis respectively
(C) positive y–axis and negative y–axis respectively
(D) negative y–axis and negative y–axis respectively

A-9. An observer standing at the sea-coast observes 54 waves reaching the coast per minute. If the wavelength of wave is 10m, The velocity of wave is **[REE - 1979]**
(A) 19 m/sec (B) 29 m/sec (C) 9 m/sec (D) 39 m/sec

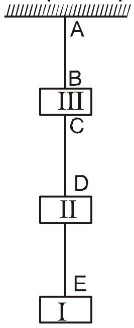
Section (B) : Speed of a wave on a string

B-1. Both the strings, shown in figure, are made of same material and have same cross-section. The pulleys are light. The wave speed of a transverse wave in the string AB is v_1 and in CD it is v_2. The v_1/v_2 is



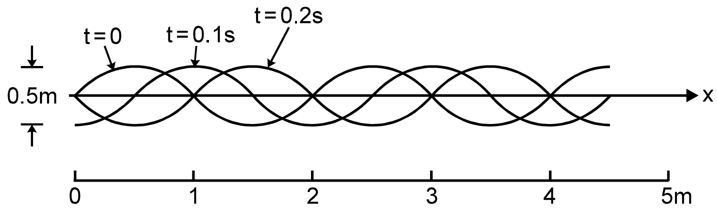
(A) 1 (B) 2 (C) $\sqrt{2}$ (D) $\frac{1}{\sqrt{2}}$

B-2. Three blocks I, II, & III having mass of 1.6 kg, 1.6 kg and 3.2 kg respectively are connected as shown in the figure. The linear mass density of the wire AB, CD and DE are 10 g/m, 8 g/m and 10 g/m respectively. The speed of a transverse wave pulse produced in AB, CD and DE are : (g = 10 m/sec^2)



(A) 80 m/s, $20\sqrt{10}$ m/s, 40 m/s
(B) $20\sqrt{10}$ m/s, 40 m/s, 80 m/s
(C) $20\sqrt{10}$ m/s in all
(D) 80 m/s in all

B-3. Three consecutive flash photographs of a travelling wave on a string are reproduced in the figure here. The following observations are made. Mark the one which is correct. (Mass per unit length of the string = 3 g/cm.)



(A) displacement amplitude of the wave is 0.25 m, wavelength is 1 m, wave speed is 2.5 m/s and the frequency of the driving force is 0.2/s.
(B) displacement amplitude of the wave is 2.0 m, wavelength is 2 m, wave speed is 0.4 m/s and the frequency of the driving force is 0.7/s.
(C) displacement amplitude of the wave is 0.25 m, wavelength is 2 m, wave speed is 5 m/s and the frequency of the driving force is 2.5 /s.
(D) displacement amplitude of the wave is 0.5 m, wavelength is 2 m, wave speed is 2.5 m/s and the frequency of the driving force is 0.2/s.

B-4. A heavy ball is suspended from the ceiling of a motor car through a light string. A transverse pulse travels at a speed of 50 cm/s on the string when the car is at rest and 52 cm/s when the car accelerates on a horizontal road. Then acceleration of the car is : (Take g = 10 m/s^2.)
(A) 2.7 m/s^2
(B) 3.7 m/s^2
(C) 2.4 m/s^2
(D) 4.1 m/s^2

Section (C) : Power transmitted along the string

C-1. A wave moving with constant speed on a uniform string passes the point x = 0 with amplitude A_0, angular frequency ω_0 and average rate of energy transfer P_0. As the wave travels down the string it gradually loses energy and at the point x = □, the average rate of energy transfer becomes $\frac{P_0}{2}$. At the point x = □, angular frequency and amplitude are respectively :
(A) ω_0 and $A_0/\sqrt{2}$
(B) $\omega_0/\sqrt{2}$ and A_0
(C) less than ω_0 and A_0
(D) $\omega_0/\sqrt{2}$ and $A_0/\sqrt{2}$

C-2. A sinusoidal wave with amplitude y_m is travelling with speed V on a string with linear density ρ. The angular frequency of the wave is ω. The following conclusions are drawn. Mark the one which is correct.
(A) doubling the frequency doubles the rate at which energy is carried along the string
(B) if the amplitude were doubled, the rate at which energy is carried would be halved
(C) if the amplitude were doubled, the rate at which energy is carried would be doubled
(D) the rate at which energy is carried is directly proportional to the velocity of the wave.

C-3. Sinusoidal waves 5.00 cm in amplitude are to be transmitted along a string having a linear mass density equal to 4.00×10^{-2} kg/m. If the source can deliver a average power of 90 W and the string is under a tension of 100 N, then the frequency at which the source can operate is (take $\pi^2 = 10$):
(A) 45 Hz (B) 50 Hz (C) 30 Hz (D) 62 Hz

C-4. The average power transmitted through a given point on a string supporting a sine wave is. 0.40 watt when the amplitude of wave is 2 mm. What average power will transmitted through this point its amplitude is increased to 4 mm.
(A) 0.40 watt (B) 0.80 watt (C) 1.2 watt (D) 1.6 watt

Section (D) : Interference, Reflection, Transmission

D-1. When two waves of the same amplitude and frequency but having a phase difference of ϕ, travelling with the same speed in the same direction (positive x), meets at a point then.
(A) their resultant amplitude will be twice that of a single wave but the frequency will be same
(B) their resultant amplitude and frequency will both be twice that of a single wave
(C) their resultant amplitude will depend on the phase angle while the frequency will be the same
(D) the frequency and amplitude of the resultant wave will depend upon the phase angle.

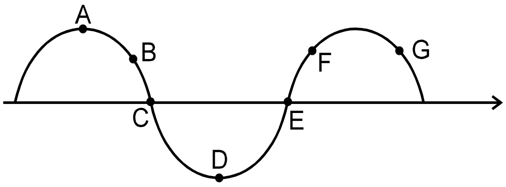
D-2. The rate of transfer of energy in a wave depends
(A) directly on the square of the wave amplitude and square of the wave frequency
(B) directly on the square of the wave amplitude and square root of the wave frequency
(C) directly on the wave frequency and square of the wave amplitude
(D) directly on the wave amplitude and square of the wave frequency

D-3. Two waves of amplitude A_1, and A_2 respectively and equal frequency travels towards same point. The amplitude of the resultant wave is
(A) $A_1 + A_2$ (B) $A_1 - A_2$
(C) between $A_1 + A_2$ and $A_1 - A_2$ (D) Can not say

D-4. A wave pulse, travelling on a two piece string, gets partially reflected and partially transmitted at the junction. The reflected wave is inverted in shape as compared to the incident one. If the incident wave has speed v and the transmitted wave v′,
(A) $v' > v$ (B) $v' = v$ (C) $v' < v$
(D) nothing can be said about the relation of v and v′.

D-5. The effects are produced at a given point in space by two waves described by the equations, $y_1 = y_m \sin \omega t$ and $y_2 = y_m \sin (\omega t + \phi)$ where y_m is the same for both the waves and ϕ is a phase angle. Tick the incorrect statement among the following.
(A) the maximum intensity that can be achieved at a point is twice the intensity of either wave and occurs if $\phi = 0$
(B) the maximum intensity that can be achieved at a point is four times the intensity of either wave and occurs if $\phi = 0$
(C) the maximum amplitude that can be achieved at the point is twice the amplitude of either wave and occurs at $\phi = 0$
(D) When the intensity is zero, the net amplitude is zero, and at this point $\phi = \pi$.

D-6. The following figure depicts a wave travelling in a medium. Which pair of particles are in phase.



(A) A and D (B) B and F
(C) C and E (D) B and G

D-7. Three waves of equal frequency having amplitudes 10 μm, 4 μm and 7 μm arrive at a given point with a successive phase difference of $\pi/2$. The amplitude of the resulting wave in μm is given by
(A) 7 (B) 6 (C) 5 (D) 4

Section (E) : Standing waves and Resonance

E-1. A wave represented by the equation $y = a \cos(kx - \omega t)$ is superposed with another wave to form a stationary wave such that the point $x = 0$ is a node. The equation for other wave is : **[JEE - 1988]**
(A) $a \sin(kx + \omega t)$ (B) $-a \cos(kx + \omega t)$ (C) $-a \cos(kx - \omega t)$ (D) $-a \sin(kx - \omega t)$

E-2. A stretched sonometer wire resonates at a frequency of 350 Hz and at the next higher frequency of 420 Hz. The fundamental frequency of this wire is

(A) 350 Hz (B) 5 Hz (C) 70 Hz (D) 170 Hz

E-3. Equations of a stationary wave and a travelling wave are y_1 = a sinkx cos ωt and y_2 = a sin (ωt – kx). The phase difference between two points $x_1 = \frac{\pi}{3k}$ and $x_2 = \frac{3\pi}{2k}$ is ϕ_1 for the first wave and ϕ_2 for the second wave. The ratio $\frac{\phi_1}{\phi_2}$ is :

(A) 1 (B) 5/6 (C) 3/4 (D) 6/7

E-4. Two stretched wires A and B of the same lengths vibrate independently. If the radius, density and tension of wire A are respectively twice those of wire B, then the fundamental frequency of vibration of A relative to that of B is **[REE - 1990]**

(A) 1 : 1 (B) 1 : 2 (C) 1 : 4 (D) 1 : 8

E-5. A steel wire of mass 4.0 g and length 80 cm is fixed at the two ends. The tension in the wire is 50 N. The wavelength of the fourth harmonic of the fundamental will be :

(A) 80 cm (B) 60 cm (C) 40 cm (D) 20 cm

E-6. One end of two wires of the same metal and of same length (with radius, r and 2r) are joined together. The wire is used as sonometer wire and the junction is placed in between two bridges. The tension T is applied to the wire. If at a junction a node is formed then the ratio of number of loops formed in the wires will be: **[JEE - 1985]**

(A) 1 : 2 (B) 2 : 3 (C) 3 : 4 (D) 4 : 5

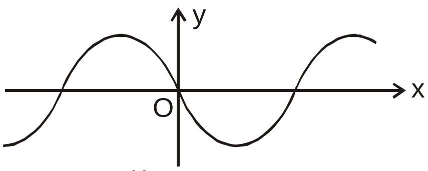
E-7. In a stationary wave represented by y = a sin ωt cos kx, amplitude of the component progressive wave is

(A) $\frac{a}{2}$ (B) a (C) 2a (D) None of these

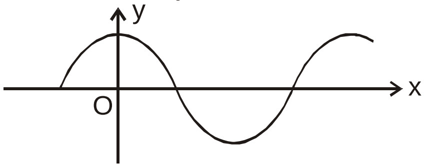
PART - III : MATCH THE COLUMN

1. For four sinusoidal waves, moving on a string along positive x direction, displacement-distance curves (y-x curves) are shown at time t = 0. In the right column, expressions for y as function of distance x and time t for sinusoidal waves are given. All terms in the equations have general meaning. Correctly match y-x curves with corresponding equations.

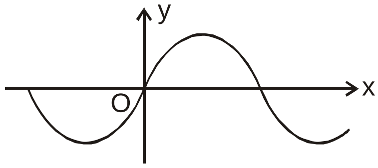
Column-I **Column-II**

(A)


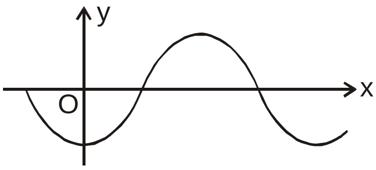
(p) y = A cos (ωt – kx)

(B)


(q) y = – A cos (kx – ωt)

(C)


(r) y = A sin (ωt – kx)

(D)


(s) y = A sin (kx – ωt)

(t) y = A cos (kx – ωt)

2. In case of string waves, match the statements in column-I with the statements in column-II.

Column-I	**Column-II**
(A) A tight string is fixed at both ends and	(p) At the middle, antinode is formed

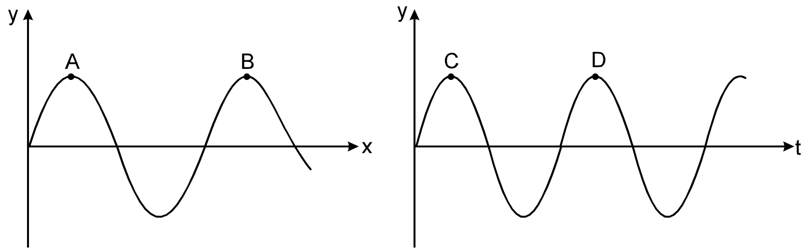
	sustaining standing wave		in odd harmonic
(B)	A tight string is fixed at one end and free at the other end	(q)	At the middle, node is formed in even harmonic
(C)	A tight string is fixed at both ends and vibrating in four loops	(r)	the frequency of vibration is 300% more than its fundamental frequency
(D)	A tight string is fixed at one end and free at the other end, vibrating in 2nd overtone	(s)	Phase difference between SHMs of any two particles will be either π or zero.
		(t)	The frequency of vibration is 400% more than fundamental frequency.

Exercise-2

✍ **Marked Questions can be used as Revision Questions.**

PART - I : ONLY ONE OPTION CORRECT TYPE

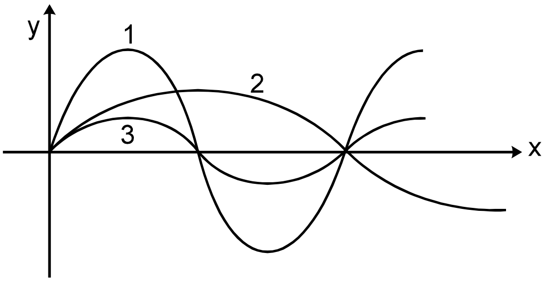
1.✍ The same progressive wave is represented by two graphs I and II. Graph I shows how the displacement 'y' varies with the distance x along the wave at a given time. Graph II shows how y varies with time t at a given point on the wave. The ratio of measurements AB to CD, marked on the curves, represents :



(I) (II)

(A) wave number k
(B) wave speed V.
(C) frequency ν.
(D) angular frequency ω.

2. Graph shows three waves that are separately sent along a string that is stretched under a certain tension along x–axis. If ω_1, ω_2 and ω_3 are their angular frequencies respectively then :



(A) $\omega_1 = \omega_3 > \omega_2$
(B) $\omega_1 > \omega_2 > \omega_3$
(C) $\omega_2 > \omega_1 = \omega_3$
(D) $\omega_1 = \omega_2 = \omega_3$

3.✍ A transverse periodic wave on a string with a linear mass density of 0.200 kg/m is described by the following equation $y = 0.05 \sin(420t - 21.0\, x)$, where x and y are in meters and t is in seconds. The tension in the string is equal to :

(A) 32 N
(B) 42 N
(C) 66 N
(D) 80 N

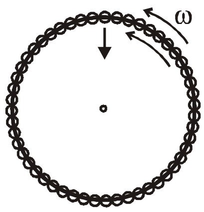
4. A heavy but uniform rope of length L is suspended from a ceiling. A particle is dropped from the ceiling at the instant when the bottom end is given the jerk. Where will the particle meet the pulse :

(A) at a distance lf $\frac{2L}{3}$ from the bottom
(B) at a distance $\frac{L}{3}$ from the bottom
(C) at a distance $\frac{3L}{4}$ from the bottom
(D) None of these

5. An object of specific gravity ρ is hung from a thin steel wire. The fundamental frequency for transverse standing waves in the wire is 300 Hz. The object is immersed in water so that one half of its volume is submerged. The new fundamental frequency in Hz is **[JEE - 1995]**

(A) $300\left(\frac{2\rho-1}{2\rho}\right)^{1/2}$
(B) $300\left(\frac{2\rho}{2\rho-1}\right)^{1/2}$
(C) $300\left(\frac{2\rho}{2\rho-1}\right)$
(D) $300\left(\frac{2\rho-1}{2\rho}\right)$

6. (a) A circular loop of rope of length L rotates with uniform angular velocity ω about an axis through its centre on a horizontal smooth platform. Velocity of pulse (with respect to rope) produced due to slight radial displacement is given by



(A) ωL
(B) $\frac{\omega L}{2\pi}$
(C) $\frac{\omega L}{\pi}$
(D) $\frac{\omega L}{4\pi^2}$

(b) In the above question if the motion of the pulse and rotation of the loop, both are in same direction then the velocity of the pulse w.r.t. to ground will be :

(A) ωL
(B) $\frac{\omega L}{2\pi}$
(C) $\frac{\omega L}{\pi}$
(D) $\frac{\omega L}{4\pi^2}$

(c) In the above question if both are in opposite direction then the velocity of the pulse w.r.t. to ground will be:

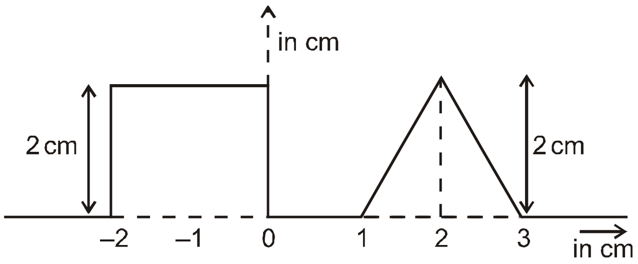
(A) ωL
(B) $\frac{\omega L}{2\pi}$
(C) $\frac{\omega L}{\pi}$
(D) 0

7. Which of the following function correctly represents the travelling wave equation for finite positive values of x and t :
(A) $y = x^2 - t^2$
(B) $y = \cos x^2 \sin t$
(C) $y = \log (x^2 - t^2) - \log(x - t)$
(D) $y = e^{2x} \sin t$

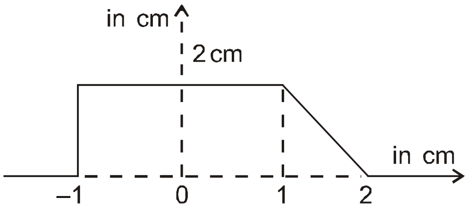
8. A 75 cm string fixed at both ends produces resonant frequencies 384 Hz and 288 Hz without there being any other resonant frequency between these two. Wave speed for the string is :
(A) 144 m/s
(B) 216 m/s
(C) 108 m/s
(D) 72 m/s

9. Two wave pulses travel in opposite directions on a string and approach each other. The shape of the one pulse is same with respect to the other.
(A) The pulses will collide with each other and vanish after collision
(B) The pulses will reflect from each other i.e., the pulse going towards right will finally move towards left and vice versa.
(C) The pulses will pass through each other but their shapes will be modified
(D) The pulses will pass through each other without any change in their shape

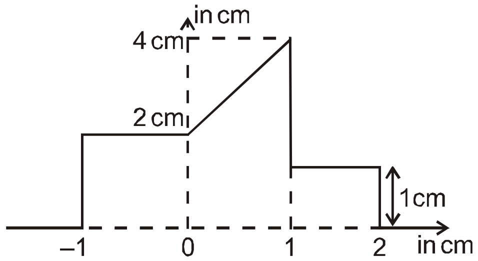
10. The figure shows at time t = 0 second, a rectangular and triangular pulse on a uniform wire are approaching each other. The pulse speed is 0.5 cm/s. The resultant pulse at t = 2 second is



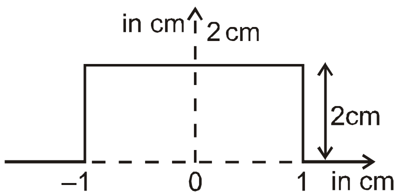
(A)



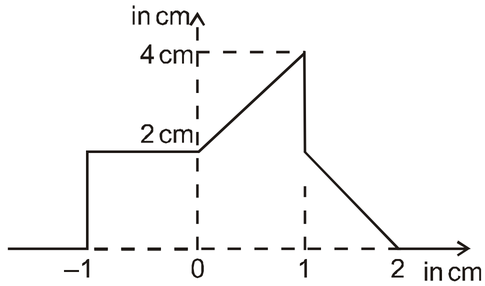
(B)



(C)



(D)



11. When a wave pulse travelling in a string is reflected from a rigid wall to which string is tied as shown in figure. For this situation two statements are given below.

(1) The reflected pulse will be in same orientation of incident pulse due to a phase change of π radians.
(2) During reflection the wall exerts a force on string in upward direction.
For the above given two statements choose the correct option given below.
(A) Only (1) is true (B) Only (2) is true (C) Both are true (D) Both are wrong

12. A wave travels on a light string. The equation of the wave is $Y = A \sin (kx - \omega t + 30°)$. It is reflected from a heavy string tied to an end of the light string at x = 0. If 64% of the incident energy is reflected the equation of the reflected wave is
(A) $Y = 0.8 A \sin (kx - \omega t + 30° + 180°)$ (B) $Y = 0.8 A \sin (kx + \omega t + 30° + 180°)$
(C) $Y = 0.8 A \sin (kx + \omega t - 30°)$ (D) $Y = 0.8 A \sin (kx + \omega t + 30°)$

13. The wave–function for a certain standing wave on a string fixed at both ends is $y(x, t) = 0.5 \sin (0.025\pi x) \cos 500 t$ where x and y are in centimeters and t is in seconds. The shortest possible length of the string is :
(A) 126 cm (B) 160 cm (C) 40 cm (D) 80 cm

14. Equation of a standing wave is expressed as $y = 2A \sin\omega t \cos kx$. In the equation, quantity ω/k represents
(A) the transverse speed of the particles of the string.
(B) the speed of the component waves.
(C) the speed of the standing wave.
(D) a quantity that is independent of the properties of the string.

15. A sonometer wire is divided in many segments using bridges. If fundamental natural frequencies of the segments are n_1, n_2, n_3.... then the fundamental natural frequency of entire sonometer wire will be (If the divisions were not made) :

(A) $n = n_1 + n_2 + n_3 + ...$ (B) $n = \sqrt{n_1 \times n_2 \times n_3 \times ....}$ (C) $\frac{1}{n} = \frac{1}{n_1} + \frac{1}{n_2} + \frac{1}{n_3} + ...$ (D) none of the above

16. A string of length L, fixed at its both ends is vibrating in its first overtone mode. Consider two elements of the string of same small length at positions $\ell_1 = 0.2$ L and $\ell_2 = 0.45$ L from one end. If K_1 and K_2 are their respective maximum kinetic energies then

(A) $K_1 = K_2$ (B) $K_1 > K_2$ (C) $K_1 < K_2$

(D) it is not possible to decide the relation.

17. A stone is hung in air from a wire which is stretched over a sonometer. The bridges of the sonometer are 40 cm apart when the wire is in unison with a tuning fork of frequency 256. When the stone is completely immersed in water, the length between the bridges is 22 cm for re-establishing (same mode) unison with the same tuning fork. The specific gravity of the material of the stone is:

(A) $\frac{(40)^2}{(40)^2 + (22)^2}$ (B) $\frac{(40)^2}{(40)^2 - (22)^2}$ (C) $256 \times \frac{22}{40}$ (D) $256 \times \frac{40}{22}$

18. A wire of density 9 gm/cm^3 is stretched between two clamps 1.00 m apart while subjected to an extension of 0.05 cm. The lowest frequency of transverse vibrations in the wire is **[JEE - 1975]**
(Assume Young's modulus $Y = 9 \times 10^{10}$ N/m^2)

(A) 35 Hz (B) 45 Hz (C) 75 Hz (D) 90 Hz

19. A 20 cm long rubber string fixed at both ends obeys Hook's law. Initially when it is stretched to make its total length of 24 cm, the lowest frequency of resonance is υ_0. It is further stretched to make its total length of 26 cm. The lowest frequency of resonance will now be :

(A) the same as υ_0 (B) greater than υ_0 (C) lower than υ_0 (D) None of these

20. Two wires made of the same material, one thick and the other thin, are connected to form a composite wire. The composite wire is subjected to some tension. A wave travelling along the wire crosses the junction point. The characteristic that undergoes a change at the junction point is

(A) Frequency only (B) Speed of propagation only

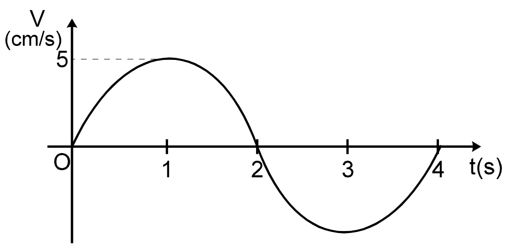
(C) Wavelength only (D) The speed of propagation as well as the wavelength

21. Standing waves are generated on string loaded with a cylindrical body. If the cylinder is immersed in water, the length of the loops changes by a factor of 2.2. The specific gravity of the material of the cylinder is **[Olympiad 2014, Stage-I]**

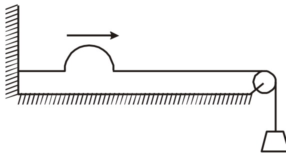
(A) 1.11 (B) 2.15 (C) 2.50 (D) 1.26

PART - II : SINGLE AND DOUBLE VALUE INTEGER TYPE

1. A certain transverse sinusoidal wave of wavelength 20 cm is moving in the positive x direction. The transverse velocity of the particle at x = 0 as a function of time is shown. The amplitude of the motion is equal to $\frac{x}{\pi}$ (in cm). Find x :

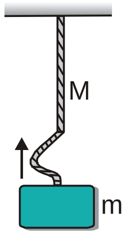


2. Figure shows a string of linear mass density 1.0 g/cm on which a wave pulse is travelling. Find the time taken (in milli second) by the pulse in travelling through a distance of 60 cm on the string. (Take g = 10 m/s^2).



3. A wire of 9.8×10^{-3} kg per meter mass passes over a frictionless pulley fixed on the top of an inclined frictionless plane which makes an angle of 30° with the horizontal. Masses M_1 & M_2 are tied at the two ends of the wire. The mass M_1 rests on the plane and the mass M_2 hangs freely vertically downwards. The whole system is in equilibrium. Now a transverse wave propagates along the wire with a velocity of 100 m/sec. Find the ratio of masses M_1 to M_2. **[REE - 1993, 4]**

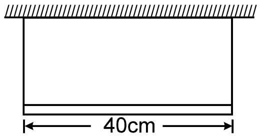
4. A uniform rope of length ℓ and mass M hangs vertically from a rigid support. A block of mass m (m = M) is attached to the free end of the rope. A transverse pulse of wavelength λ is produced at the lower end of the rope. The wavelength of the pulse, when it reaches the top of the rope, is $a\lambda$. Find a^2



5. A non–uniform rope of mass M and length L has a variable linear mass density given by $\mu = kx$, where x is the distance from one end of the wire and k is a constant. The time required for a pulse generated at one end of the wire to travel to the other end is given by $t = \sqrt{(pML/9F)}$ where F (constant) is the tension in the wire then find p

6. A man generates a symmetrical pulse in a string by moving his hand up and down. At t = 0 the point in his hand moves downward from mean position. The pulse travels with speed 3 m/s on the string & his hand passes 6 times in each second from the mean position. Then the point on the string at a distance 3m will reach its upper extreme first time at time t = n/12. Find n

7. A uniform horizontal rod of length 40 cm and mass 1.2 kg is supported by two identical wires as shown in figure. Where should a mass of 4.8 kg be placed on the rod from left end (in cm) so that the same tuning fork may excite the wire on left into its fundamental vibrations and that on right into its first overtone? Take $g = 10\ m/s^2$.



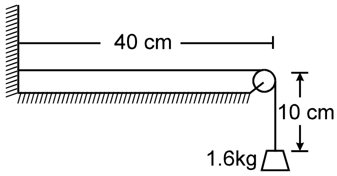
8. A string of length 'ℓ' is fixed at both ends. It is vibrating in its 3rd overtone with maximum amplitude 'a'. The amplitude at a distance $\frac{\ell}{3}$ from one end is $= \sqrt{p}\frac{a}{2}$. Find p

9. A string 120 cm in length and fixed at both ends sustains a standing wave, with the consecutive points of the string at which the displacement amplitude is equal to 3.5 mm (not maximum) being separated by 15.0 cm. The maximum displacement amplitude is $3.5\ x^{1/2}$ in mm. Then find x. To which overtone do these oscillations correspond?

10. A string of mass ' m ' and length ℓ, fixed at both ends is vibrating in its fundamental mode.The maximum amplitude is ' a ' and the tension in the string is ' T '. If the energy of vibrations of the string is $\frac{\pi^2 a^2 T}{\eta L}$. Find η **[JEE (Main) 2003; 4/60]**

11. A travelling wave of amplitude 5A is partially reflected from a boundary with the amplitude 3 A. Due to superposition of two waves with different amplitudes in opposite directions a wave pattern is formed. Determine the ratio of amplitude at antinode to node.

12. A 50 cm long wire of mass 20 g supports a mass of 1.6 kg as shown in figure. Find the first overtone frequency of the portion of the string between the wall and the pulley. Take g = 10 m/s^2.



13. A 1 m long rope, having a mass of 40 g, is fixed at one end and is tied to a light string at the other end. The tension in the string is 400 N. Find the wavelength in second overtone (in cm).

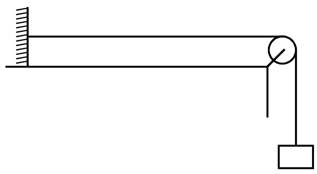
14. In an experiment of standing waves, a string 90 cm long is attached to the prong of an electrically driven tuning fork that oscillates perpendicular to the length of the string at a frequency of 60 Hz. The mass of the string is 0.044 kg. What tension (in newton) must the string be under (weights are attached to the other end) if it is to oscillate in four loops?

15. Three consecutive resonant frequencies of a string are 90, 150 and 210 Hz. If the length of the string is 80 cm, what would be the speed (in m/s) of a transverse wave on this string ?

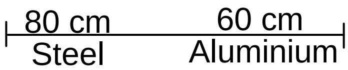
16. A steel wire of length 1 meter, mass 0.1 kg and uniform cross sectional area 10^{-6} m^2 is rigidly fixed at both ends without any stress. The temperature of the wire is lowered by 20ºC. If transverse waves are setup by plucking the string in the middle, calculate the frequency of the fundamental mode of vibration. Young's modulus of steel = 2×10^{11} N/m^2, coefficient of linear expansion of steel = 1.21×10^{-5}/ºC. **[JEE - 1984]**

17. A wire having some linear density is stretched between two rigid supports with tension. It is observed that the wire resonates at a frequency of 420 cycles/sec. The next higher frequency at which the same wire resonates is 490 cycles/sec. Find the possible mode of vibration of string initially: **[JEE - 1971]**

18. Figure shows a string stretched by a block going over a pulley. The string vibrates in its tenth harmonic in resonance with a particular tuning fork. When a beaker containing water is brought under the block so that the block is completely dipped into the beaker, the string vibrates in its eleventh harmonic. If the density of the material of the block is d gram/cm^3 then find greatest integer of d. (node is formed at the pulley)



19. Figure shows an aluminium wire of length 60 cm joined to a steel wire of length 80 cm and stretched between two fixed supports. The tension produced is 40 N. The cross-sectional area of the steel wire is 1.0 mm^2 and that of the aluminium wire is 3.0 mm^2. The minimum frequency of a tuning fork which can produce standing waves in the system with the joint as a node is 10P (in Hz) then find P. Given density of aluminium is 2.6 g/cm^3 and that of steel is 7.8 g/cm^3.



20. A metallic wire with tension T and at temperature 30°C vibrates with its fundamental frequency of 1 kHz. The same wire with the same tension but at 10°C temperature vibrates with a fundamental frequency of 1.001 kHz. The coefficient of linear expansion of the wire is equal to 10^{-K} ºC. Find 2K

PART - III : ONE OR MORE THAN ONE OPTIONS CORRECT TYPE

1. The plane wave represented by an equation of the form y = f(x – v t) implies the propagation along the positive x-axis without change of shape with constant velocity v :

(A) $\frac{\partial y}{\partial t} = -v\left(\frac{\partial y}{\partial x}\right)$ (B) $\frac{\partial y}{\partial t} = -v\left(\frac{\partial^2 y}{\partial x^2}\right)$ (C) $\frac{\partial^2 y}{\partial t^2} = -v^2\left(\frac{\partial^2 y}{\partial x^2}\right)$ (D) $\frac{\partial^2 y}{\partial t^2} = v^2\left(\frac{\partial^2 y}{\partial x^2}\right)$

2. A wave equation which gives the displacement along the Y direction is given by $y = 10^{-4} \sin(60t + 2x)$ where x and y are in meters and t is time in seconds. This represents a wave **[JEE - 1982]**
(A) travelling with a velocity of 30 m/s in the negative x direction
(B) of wavelength π meter
(C) of frequency $30/\pi$ hertz
(D) of amplitude 10^{-4} meter travelling along the negative x direction.

3. The displacement of a particle in a medium due to a wave travelling in the x-direction through the medium is given by $y = A \sin(\alpha t - \beta x)$, where t = time, and α and β are constants :
(A) the frequency of the wave is α
(B) the frequency of the wave is $\alpha/2\pi$
(C) the wavelength is $2\pi/\beta$
(D) the velocity of the wave is α/β

4. The displacement of particles in a string stretched in x-direction is represented by y. Among the following expressions for y, those describing wave motion are : **[JEE - 1987, 2]**
(A) $\cos(kx)\sin(\omega t)$ (B) $k^2x^2 - \omega^2t^2$ (C) $\cos^2(kx + \omega t)$ (D) $\cos(k^2x^2 - \omega^2t^2)$

5. The particle displacement in a wave is given by $y = 0.2 \times 10^{-5} \cos(500t - 0.025x)$, where the distances are measured in meters and time in seconds. Now **[REE - 1994]**
(A) wave velocity is 2×10^4 ms^{-1}
(B) particle velocity is 2×10^4 ms^{-1}
(C) initial phase difference is $\frac{\pi}{2}$
(D) wavelength of the wave is (80π) m

6. A transverse sinusoidal wave of amplitude a, wavelength λ and frequency f is travelling on a stretched string. The maximum speed of any point on the string is v /10, where v is the speed of propagation of the wave. if $a = 10^{-3}$ m and v = 10 m/s, then λ and f are given by
(A) $\lambda = 2\pi \times 10^{-2}$ m (B) $\lambda = 10^{-3}$ m (C) $f = 10^3 / (2\pi)$ Hz (D) $f = 10^4$ Hz

7. If the tension in a string is increased by 21 percent, the fundamental frequency of the string changes by 15 Hz. Which of the following statements will also be correct?
(A) The original fundamental frequency is nearly 150 Hz
(B) The velocity of propagation changes nearly by 4.5%
(C) The velocity of propagation changes nearly by 10%
(D) The fundamental wavelength changes nearly by 10%

8. Two waves of equal frequency f and speed v travel in opposite directions along the same path. The waves have amplitudes A and 3A. Then :
(A) the amplitude of the resulting wave varies with position between maxima of amplitude 4A and minima of zero amplitude
(B) the distance between a maxima and adjacent minima of amplitude is $\frac{\nu}{2f}$
(C) at a point on the path the average displacement is zero $\Rightarrow$ Average displacement of medium particle at any point is zero.
(D) the position of a maxima or minima of amplitude does not change with time

9. Two particles A and B have a phase difference of π when a sine wave passes through the region
(A) A and B oscillates with same frequency
(B) A and B move in opposite directions
(C) A and B are separated by odd multiple of half of the wavelength
(D) the displacements at A and B have equal magnitudes

10. In a stationary wave,
(A) all the particles of the medium vibrate either in phase or in opposite phase
(B) all the antinodes vibrate in opposite phase

(C) the alternate antinodes vibrate in phase
(D) all the particles between consecutive nodes vibrate in phase

11. Following are equations of four waves :

(i) $y_1 = a \sin \omega \left(t - \frac{x}{\nu}\right)$ (ii) $y_2 = a \cos \omega \left(t + \frac{x}{\nu}\right)$

(iii) $z_1 = a \sin \omega \left(t - \frac{x}{\nu}\right)$ (iv) $z_2 = a \cos \omega \left(t + \frac{x}{\nu}\right)$

Which of the following statements is/are correct?
(A) On superposition of waves (i) and (iii), a travelling wave having amplitude $a\sqrt{2}$ will be formed
(B) Superposition of waves (ii) and (iii) is not possible
(C) On superposition of (i) and (ii), a stationary wave having amplitude $a\sqrt{2}$ will be formed
(D) On superposition of (iii) and (iv), a transverse stationary wave will be formed

12. A wave disturbance in a medium is described by $y(x, t) = 0.02 \cos (50\pi t + \pi/2) \cos 10(\pi x)$, where x and y are in meter and t in second. **[JEE - 1995]**
(A) A node occurs at x = 0.15 m (B) An antinode occurs at x = 0.3 m
(C) The speed of wave is 5 ms^{-1} (D) The wavelength is 0.2m.

13. The vibrations of a string of length 600 cm fixed at both ends are represented by the equation $y = 4 \sin\left(\pi \frac{x}{15}\right) \cos (96\pi t)$, where x and y are in cm and t in seconds. **[JEE - 1985]**
(A) The maximum displacement of a point x = 5 cm is $2\sqrt{3}$cm .
(B) The nodes located along the string are at a distance of 15n where integer n varies from 0 to 40.
(C) The velocity of the particle at x = 7.5 cm at t = 0.25 sec is 0
(D) The equations of the component waves whose superposition gives the above wave are $2 \sin 2\pi \left(\frac{x}{30} + 48t\right)$, $2 \sin 2\pi \left(\frac{x}{30} - 48t\right)$.

14. A wave given by $\xi = 10 \sin [80\pi t - 4\pi x]$ propagates in a wire of length 1m fixed at both ends. If another wave of similar amplitude is superimposed on this wave to produce a stationary wave then **[REE - 1998]**
(A) the superimposed wave is $\xi = -10 \sin [80\pi t + 4\pi x]$
(B) the maximum amplitude of the stationary wave is 20 m.
(C) the wave length of the wave is 0.5 m.
(D) the number of total nodes produced in the wire are 3.

15. Consider an element of a stretched string along which a wave travels. During its transverse oscillatory motion, the element passes through a point at y = 0 and reaches its maximum at $y = y_m$. Then, the string element has its maximum **[Olympiad 2014; Stage–I]**
(A) kinetic energy at $y = y_m$. (B) elastic potential energy at $y = y_m$.
(C) kinetic energy at y = 0. (D) elastic potential energy at y = 0.

PART - IV : COMPREHENSION

Comprehension-1

A pulse is started at a time t = 0 along the +x direction on a long, taut string. The shape of the pulse at t = 0 is given by function y with

$$y = \begin{cases} \frac{x}{4} + 1 & \text{for } -4 < x \le 0 \\ -x + 1 & \text{for } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

here y and x are in centimeters. The linear mass density of the string is 50 g/m and it is under a tension of 5N,

1. ✍ The shape of the string is drawn at t = 0 and the area of the pulse enclosed by the string and the x-axis is measured. It will be equal to
(A) 2 cm^2 (B) 2.5 cm^2 (C) 4 cm^2 (D) 5 cm^2

2. ✍ The vertical displacement of the particle of the string at x = 7 cm and t = 0.01 s will be
(A) 0.75 cm (B) 0.5 cm (C) 0.25 cm (D) zero

3. ✍ The transverse velocity of the particle at x = 13 cm and t = 0.015 s will be
(A) –250 cm/s (B) –500 cm/s (C) 500 cm/s (D) –1000 cm/s

Comprehension-2

A transverse sinusoidal wave is generated at one end of a long, horizontal string by a bar that moves up and down through an amplitude of 1/2 cm. The motion is continuous and is repeated regularly 120 times per second. The string has linear density 90 gm/m and is kept under a tension of 900 N. Find:

4. ✍ The maximum value of the transverse component of the tension (in newton)
(A) 1.8 π (B) 10.8 π (C) 9 (D) 18 π

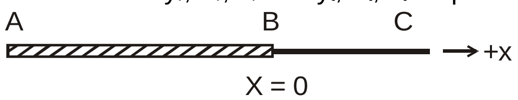
5. ✍ What is the maximum power (in watt) transferred along the string.
(A) 3.24 π^2 (B) 6.48 π^2 (C) 12.96 π^2 (D) 25.92 π^2

6. ✍ What is the transverse displacement y (in cm) when the minimum power transfer occurs
(A) 0 (B) $\frac{1}{2}$ (C) $\frac{1}{4}$ (D) 1

[Leave the answer in terms of π wherever it occurs]

Comprehension-3

In the figure shown a sinusoidal wave is generated at the end 'A'. The wave travels along positive x-axis and during its motion it encounter another string BC at the junction 'B' at x = 0. The density of strings AB and BC are ρ and 9ρ respectively and their radii of cross sections are 2r and r. The wave function, amplitude and wavelength of incident wave are respectively y_i, A_i and λ_i. Similarly for reflected and transmitted wave these parameters are y_r, A_r, λ_r and y_t, A_t, λ_t respectively.



7. Which of the following statement regarding phase difference, $\Delta\phi$ between waves at x = 0 is true ?
(A) $\Delta\phi$ = 0, between y_i and y_r (B) $\Delta\phi$ = 0, between y_r and y_t
(C) $\Delta\phi = \pi$, between y_i and y_t (D) $\Delta\phi = \pi$, between y_r and y_t

8. The ratio of wavelengths λ_r to λ_t (i.e. $\lambda_r : \lambda_t$) will be
(A) 1 : 1 (B) 3 : 2 (C) 2 : 3 (D) None of these

9. The ratio of amplitudes A_r to A_t is (i.e. $A_r : A_t$) will be
(A) 1 : 1 (B) 1 : 4 (C) 4 : 1 (D) None of these

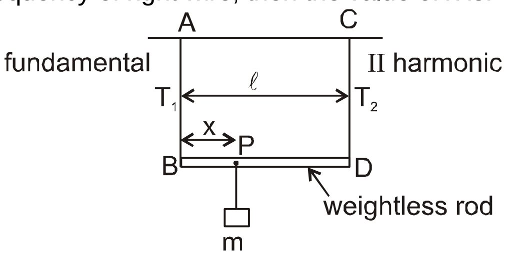
Exercise-3

*** Marked Questions may have more than one correct option.**
✍ Marked Questions can be used as Revision Questions.

PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

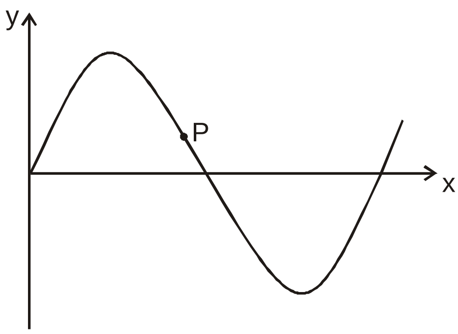
1. A transverse wave travelling in a string produces maximum transverse velocity of 3 m/s and maximum transverse acceleration 90 m/s^2 in a particle. If the velocity of wave in the string is 20 m/s. Determine the equation of the wave ? **[JEE (Mains) 2005 ; 4/60]**

2.✍ A massless rod BD is suspended by two identical massless strings AB and CD of equal lengths. A block of mass ‘m’ is suspended point P such that BP is equal to ‘x’, if the fundamental frequency of the left wire is twice the fundamental frequency of right wire, then the value of x is: **[JEE (Mains) 2006; 3/184]**



(A) l/5 (B) l/4 (C) 4l/5 (D) 3l/4

3. A transverse sinusoidal wave moves along a string in the positive x-direction at a speed of 10 cm/s. The wavelength of the wave is 0.5 m and its amplitude is 10 cm. At a particular time t, the snap-shot of the wave is shown in figure. The velocity of point P when its displacement is 5 cm is **[JEE 2008; 3/163]**
Figure :



(A) $\frac{\sqrt{3}\,\pi}{50}\hat{j}$ m/s (B) $-\frac{\sqrt{3}\,\pi}{50}\hat{j}$ m/s (C) $\frac{\sqrt{3}\,\pi}{50}\hat{i}$ m/s (D) $-\frac{\sqrt{3}\,\pi}{50}\hat{i}$ m/s

4. A 20cm long string, having a mass of 1.0 g, is fixed at both the ends. The tension in the string is 0.5 N. The string is set into vibrations using an external vibrator of frequency 100 Hz. Find the separation (**in cm**) between the successive nodes on the string. **[JEE 2009; 4/160, –1]**

5. When two progressive waves y_1 = 4 sin (2x – 6t) and y_2 = 3 sin $\left(2x - 6t - \frac{\pi}{2}\right)$ are superimposed, the amplitude of the resultant wave is : **[JEE 2010; 3/163, –1]**

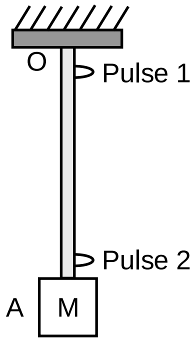
6*.✍ A horizontal stretched string, fixed at two ends, is vibrating in its fifth harmonic according to the equation, y(x, t) = (0.01 m) sin [(62.8 m^{-1})x] cos [(628 s^{-1})t]. Assuming π = 3.14, the correct statement(s) is (are) : **[JEE (Advanced) 2013, 3/60]**
(A) The number of nodes is 5.
(B) The length of the string is 0.25 m.
(C) The maximum displacement of the midpoint of the string its equilibrium position is 0.01 m.
(D) The fundamental frequency is 100 Hz.

7*.✍ One end of a taut string of length 3m along the x-axis is fixed at x = 0. The speed of the waves in the string is 100m/s. The other end of the string is vibrating in the y-direction so that stationary waves are set up in the string. The possible waveform(s) of these stationary waves is (are) **[JEE (Advanced) 2014, P-1, 3/60]**

(A) y(t) = A sin $\frac{\pi x}{6}\cos\frac{50\pi t}{3}$ (B) y(t) = A sin $\frac{\pi x}{3}\cos\frac{100\pi t}{3}$

(C) y(t) = A sin $\frac{5\pi x}{6}\cos\frac{250\pi t}{3}$ (D) y(t) = A sin $\frac{5\pi x}{2}$ cos 250πt

8*. A block M hangs vertically at the bottom end of a uniform rope of constant mass per unit length. The top end of the rope is attached to a fixed rigid support at O. A transverse wave pulse (Pulse 1) of wavelength λ_0 is produced at point O on the rope. The pulse takes time T_{OA} to reach point A. If the wave pulse of wavelength λ_0 is produced at point A (Pulse 2) without disturbing the position of M it takes time T_{AO} to reach point O. Which of the following options is/are correct **[JEE (Advanced) 2017, P-1, 4/61, –2]**



(A) The velocities of the two pulses (Pulse 1 and Pulse 2) are the same at the midpoint of rope
(B) The velocity of any pulse along the rope is independent of its frequency and wavelength
(C) The wavelength of Pulse 1 becomes longer when it reaches point A
(D) The time $T_{AO} = T_{OA}$

PART - II : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. A string is stretched between fixed points separated by 75 cm. It is observed to have resonant frequencies of 420 Hz and 315 Hz. There are no other resonant frequencies between these two. Then, the lowest resonant frequency for this string is: **[AIEEE 2006 ; 3/180]**
(1) 10.5 Hz (2) 105 Hz (3) 1.05 Hz (4) 1050 Hz

2. A wave travelling along the x- axis is described by the equation y(x, t) = 0.005 cos (αx –βt). If the wavelength and the time period of the wave are 0.08 m and 2.0 s, respectively, then α and β in appropriate units are **[AIEEE 2008 ; 3/105, –1]**
(1) $\alpha = \frac{0.08}{\pi}, \beta = \frac{2.0}{\pi}$ (2) $\alpha = \frac{0.04}{\pi}, \beta = \frac{1.0}{\pi}$ (3) $\alpha = 12.50\pi, \beta = \frac{\pi}{2.0}$ (4) α = 25.00 π, $\beta = \pi$

3. The equation of a wave on a string of linear mass density 0.04 kg m^{-1} is given by y = 0.02 (m) sin $\left[2\pi\left(\frac{t}{0.04(s)} - \frac{x}{0.50(m)}\right)\right]$. The tension in the string is : **[AIEEE 2010; 144/4 –1]**
(1) 4.0 N (2) 12.5 N (3) 0.5 N (4) 6.25 N

4. The transverse displacement y(x, t) of a wave on a string is given by $y(x,t) = e^{-\left(ax^2 + bt^2 + 2\sqrt{ab}\,xt\right)}$. This represents a : **[AIEEE - 2011; 1 MAY; 4/120, –1]**
(1) wave moving in +x–direction with speed $\sqrt{\frac{a}{b}}$ (2) wave moving in –x-direction with speed $\sqrt{\frac{b}{a}}$
(3) standing wave of frequency $\sqrt{b}$ (4) standing wave of frequency $\frac{1}{\sqrt{b}}$

5. A travelling wave represented by y = A sin ((ωt – kx) is superimposed on another wave represented by y = A sin (ωt + kx). The resultant is : **[AIEEE 2011, 11 MAY; 4/120, –1]**
(1) A wave travelling along + x direction
(2) A wave travelling along – x direction
(3) A standing wave having nodes at x = $\frac{n\lambda}{2}$, n = 0, 1, 2 ....
(4) A standing wave having nodes at x = $\left(n + \frac{1}{2}\right)\frac{\lambda}{2}$; n = 0, 1, 2 .....

6. A sonometer wire of length 1.5 m is made of steel. The tension in it produces an elastic strain of 1%. What is the fundamental frequency of steel if density and elasticity of steel are 7.7 × 10^3 kg/m^3 and 2.2 × 10^{11} N/m^2 respectively ? **[JEE (Main) 2013; 4/120, –1]**
(1) 188.5 Hz (2) 178.2 Hz (3) 200.5 Hz (4) 770 Hz

7. A uniform string of length 20 m is suspended from a rigid support. A short wave pulse is introduced at its lowest end. It starts moving up the string. The time taken to reach the support is : (take g = 10 ms^{-2}) **[JEE (Main) 2016; 4/120, –1]**
(1) 2 s (2) $2\sqrt{2}$ s (3) $\sqrt{2}$ s (4) $2\pi\sqrt{2}$ s

Answers

EXERCISE - 1

PART - I

Section (A) :

A-1. (a) 10 mm (b) 5π cm^{-1} (c) $\frac{2}{5}$ cm

(d) 30 Hz (e) $\frac{1}{30}$ s (f) 12 cm/s.

A-2. 1.41 y_m

A-3. (a) 10π rad/s (b) $\pi/2$ rad/m

(c) $y = (12 \times 10^{-2}\ \text{m}) \sin\left(\frac{\pi}{2}x - 10\pi t\right)$

(d) $\frac{6}{5}\pi$ m/s (e) $12\pi^2$ m/s^2

A-4. $0.5 \sin\left(\frac{\pi}{3}t - \frac{\pi}{18}x + \frac{7\pi}{9}\right)$

A-5. (a) D, E, F; (b) A, B, H ; (c) C, G ; (d) A, E

Section (B) :

B-1. $300\sqrt{3}$ m/s **B-2.** $\frac{1}{50}$ sec

B-3. $\frac{3}{10\sqrt{5}}$ m.

B-4. 100 Hz, 4 cm, 4 m/s **B-5.** 4

B-6. (a) 200 m/s; (b) 240 m/s

Section (C) :

C-1. (a) $y = (7.50\ \text{cm}) \sin\left(\frac{4\pi}{3}x - 314t + \phi\right)$

(b) $\frac{2025\pi^2}{32} \simeq 625$ W

C-2. 50 mW

Section (D) :

D-1. $y' = 0.8\ a \sin. \frac{2\pi}{\lambda}\left(vt + x + \frac{\lambda}{2}\right)$

D-2. (a) Zero (b) 0.300 m.

D-3. (a) 2π (b) π (c) 4.0 mm, zero

Section (E) :

E-1. (a) 2.5 Hz; (b) 5 Hz; (c) 7.5 Hz.

E-2. (a) $\frac{250}{\sqrt{3}}$ m/s; (b) 60.0 cm; (c) $\frac{1250}{3\sqrt{3}}$ Hz

E-3. $\frac{400}{3}$ Hz

E-4. (a) 0.25 cm ; (b) 1.2×10^2 cm/s;
(c) 3.0 cm ; (d) 0

E-5. (a) 100 Hz (b) 700 Hz

E-6. **(i)** $y_1 = 1.5 \cos\{(\pi/20)x - 72\pi t\}$,
$y_2 = 1.5 \cos\{(\pi/20)x + 72\pi t\}$

(ii) 10 , 30 , 50 cm and 0 , 20 , 40 , 60 cm

(iii) 0

E-7. 240 Hz.

PART - II

Section (A) :

A-1. (A) **A-2.** (C) **A-3.** (B)
A-4. (B) **A-5.** (B) **A-6.** (B)
A-7. (B) **A-8.** (B) **A-9.** (C)

Section (B) :

B-1. (C) **B-2.** (A) **B-3.** (C)
B-4. (D)

Section (C) :

C-1. (A) **C-2.** (D) **C-3.** (C)
C-4. (D)

Section (D) :

D-1. (C) **D-2.** (A) **D-3.** (C)
D-4. (C) **D-5.** (A) **D-6.** (D)
D-7. (C)

Section (E) :

E-1. (B) **E-2.** (C) **E-3.** (D)
E-4. (B) **E-5.** (C) **E-6.** (A)
E-7. (A)

PART - III

1. (A) $\rightarrow$ r ; (B) $\rightarrow$ p, t ; (C) $\rightarrow$ s ; (D) $\rightarrow$ q

2. (A) $\rightarrow$ p,q,s; (B) $\rightarrow$ s; (C) $\rightarrow$ q,r,s; (D) $\rightarrow$ s,t

EXERCISE - 2

PART - I

1.	(B)	**2.**	(A)	**3.**	(D)
4.	(B)	**5.**	(A)		
6.	(a) (B);	(b) (C);	(c) (D)		
7.	(C)	**8.**	(A)	**9.**	(D)
10.	(D)	**11.**	(D)	**12.**	(C)
13.	(C)	**14.**	(B)	**15.**	(C)
16.	(B)	**17.**	(B)	**18.**	(A)
19.	(B)	**20.**	(D)	**21.**	(D)

PART - II

1.	10	**2.**	30	**3.**	2
4.	2	**5.**	8	**6.**	15
7.	5	**8.**	3	**9.**	2, 3
10.	4	**11.**	4	**12.**	50
13.	80	**14.**	36	**15.**	96
16.	11	**17.**	6	**18.**	5
19.	18	**20.**	8		

PART - III

1.	(AD)	**2.**	(ABCD)	**3.**	(BCD)
4.	(AC)	**5.**	(AD)	**6.**	(AC)
7.	(AC)	**8.**	(CD)	**9.**	(ABCD)
10.	(ACD)	**11.**	(AD)	**12.**	(ABCD)
13.	(ABCD)	**14.**	(ABC)	**15.**	(CD)

PART - IV

1.	(B)	**2.**	(C)	**3.**	(A)
4.	(B)	**5.**	(C)	**6.**	(B)
7.	(D)	**8.**	(B)	**9.**	(B)

EXERCISE - 3

PART - I

1. Equation of wave in string

$y = 0.1 \sin\left(30\ \ t\ \ \pm\frac{3}{2}x+\phi\right)$ [where ϕ is initial phase]

2.	(A)	**3.**	(A)	**4.**	5
6.	(BC)	**7.**	(ACD)		
8.	(BD)				

PART - II

1.	(2)	**2.**	(4)	**3.**	(4)
4.	(2)	**5.**	(4)	**6.**	(2)
7.	(2)				