Exercise-1

Marked Questions can be used as Revision Questions.

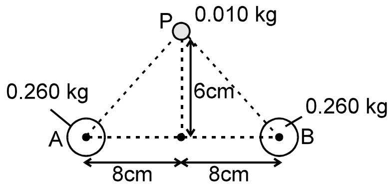
PART - I : SUBJECTIVE QUESTIONS

Section (A) : Universal law of gravitation

A-1. The typical adult human brain has a mass of about 1.4 kg. What force does a full moon exert on such a brain when it is directly above with its centre 378000 km away ? (Mass of the moon = 7.34×10^{22} kg)

A-2. Two uniform solid spheres of same material and same radius 'r' are touching each other. If the density is 'ρ' then find out gravitational force between them.

A-3. Two uniform spheres, each of mass 0.260 kg are fixed at points 'A' and 'B' as shown in the figure. Find the magnitude and direction of the initial acceleration of a sphere with mass 0.010 kg if it is released from rest at point 'P' and acted only by forces of gravitational attraction of sphere at 'A' and 'B'(give your answer in terms of G).



Section (B) : Gravitational field and potential

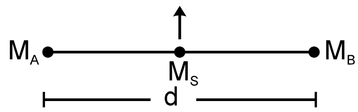
B-1. The gravitational potential in a region is given by V = (20x + 40y) J/kg. Find out the gravitational field (in newton/kg) at a point having co-ordinates (2, 4). Also find out the magnitude of the gravitational force on a particle of 0.250 kg placed at the point (2, 4).

B-2. Radius of the earth is 6.4×10^6 m and the mean density is 5.5×10^3 kg/m^3. Find out the gravitational potential at the earth's surface.

Section (C) : Gravitational Potential Energy and Self Energy

C-1. A body which is initially at rest at a height R above the surface of the earth of radius R, falls freely towards the earth. Find out its velocity on reaching the surface of earth. (Take g = acceleration due to gravity on the surface of the Earth).

C-2. Two planets A and B are fixed at a distance d from each other as shown in the figure. If the mass of A is M_A and that of B is M_B, then find out the minimum velocity of a satellite of mass M_S projected from the mid point of two planets to infinity.



Section (D) : Kepler's law for Satellites, Orbital speed and Escape speed

D-1. A satellite is established in a circular orbit of radius r and another in a circular orbit of radius 1.01 r. How much nearly percentage the time period of second-satellite will be larger than the first satellite.

D-2. Two identical stars of mass M, orbit around their centre of mass. Each orbit is circular and has radius R, so that the two stars are always on opposite sides on a diameter.
(a) Find the gravitational force of one star on the other.
(b) Find the orbital speed of each star and the period of the orbit.
(c) Find their common angular speed.
(d) Find the minimum energy that would be required to separate the two stars to infinity.
(e) If a meteorite passes through this centre of mass perpendicular to the orbital plane of the stars. What value must its speed exceed at that point if it escapes to infinity from the star system.

D-3. Two earth satellites A and B each of equal mass are to be launched into circular orbits about earth's centre. Satellite 'A' is to orbit at an altiude of 6400 km and B at 19200 km. The radius of the earth is 6400 km. Determine-
(a) the ratio of the potential energy
(b) the ratio of kinetic energy
(c) which one has the greater total energy

D-4. The Saturn is about six times farther from the Sun than The Mars. Which planet has :
(a) the greater period of revolution ? (b) the greater orbital speed and

(c) the greater angular speed ?

Section (E) : The Earth and Other Planets Gravity

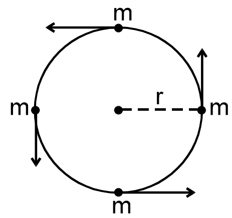
E-1. The acceleration due to gravity at a height $(1/20)^{th}$ the radius of the earth above earth's surface is $9\ m/s^2$. Find out its approximate value at a point at an equal distance below the surface of the earth.

E-2. If a pendulum has a period of exactly 1.00 sec. at the equator, what would be its period at the south pole ? Assume the earth to be spherical and rotational effect of the Earth is to be taken.

PART - II : ONLY ONE OPTION CORRECT TYPE

Section (A) : Universal law of gravitation

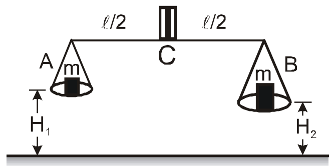
A-1. Four similar particles of mass m are orbiting in a circle of radius r in the same direction and same speed because of their mutual gravitational attractive force as shown in the figure. Speed of a particle is given by



(A) $\left[\frac{Gm}{r}\left(\frac{1+2\sqrt{2}}{4}\right)\right]^{\frac{1}{2}}$ (B) $\sqrt[3]{\frac{Gm}{r}}$

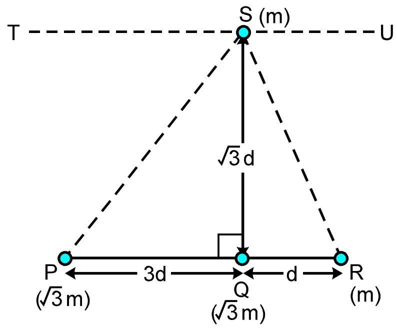
(C) $\sqrt{\frac{Gm}{r}\left(1+2\sqrt{2}\right)}$ (D) zero

A-2. Two blocks of masses m each are hung from a balance as shown in the figure. The scale pan A is at height H_1 whereas scale pan B is at height H_2. Net torque of weights acting on the system about point 'C', will be (length of the rod is ℓ and H_1 & H_2 are << R) ($H_1 > H_2$)



(A) $mg\left(\frac{1-2H_1}{R}\right)\ell$ (B) $\frac{mg}{R}(H_1-H_2)\ell$ (C) $\frac{2mg}{R}(H_1+H_2)\ell$ (D) $2mg\frac{H_2H_1}{H_1+H_2}\ell$

A-3. Three particles P, Q and R are placed as per given figure. Masses of P, Q and R are $\sqrt{3}$ m, $\sqrt{3}$ m and m respectively. The gravitational force on a fourth particle 'S' of mass m is equal to



(A) $\frac{\sqrt{3}GM^2}{2d^2}$ in ST direction only

(B) $\frac{\sqrt{3}Gm^2}{2d^2}$ in SQ direction and $\frac{\sqrt{3}Gm^2}{2d^2}$ in SU direction

(C) $\frac{\sqrt{3}Gm^2}{2d^2}$ in SQ direction only

(D) $\frac{\sqrt{3}Gm^2}{2d^2}$ in SQ direction and $\frac{\sqrt{3}Gm^2}{2d^2}$ in ST direction

A-4. Three identical stars of mass M are located at the vertices of an equilateral triangle with side L. The speed at which they will move if they all revolve under the influence of one another's gravitational force in a circular orbit circumscribing the triangle while still preserving the equilateral triangle :

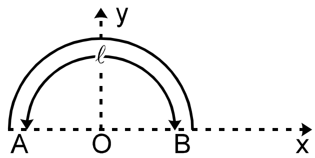
(A) $\sqrt{\frac{2GM}{L}}$ (B) $\sqrt{\frac{GM}{L}}$ (C) $2\sqrt{\frac{GM}{L}}$ (D) not possible at all

Section (B) : Gravitational field and potential

B-1. Let gravitation field in a space be given as E = – (k/r). If the reference point is at distance d_i where potential is V_i then relation for potential is :

(A) $V = k\ \ell n\ \frac{1}{V_i} + 0$ (B) $V = k\ \ell n\ \frac{r}{d_i} + V_i$ (C) $V = \ell n\ \frac{r}{d_i} + kV_i$ (D) $V = \ell n\ \frac{r}{d_i} + \frac{V_i}{k}$

B-2. Gravitational field at the centre of a semicircle formed by a thin wire AB of mass m and length ℓ as shown in the figure is :



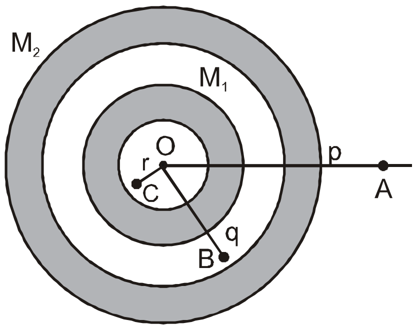
(A) $\frac{Gm}{\ell^2}$ along +x axis (B) $\frac{Gm}{\pi\ell^2}$ along +y axis

(C) $\frac{2\pi Gm}{\ell^2}$ along + x axis (D) $\frac{2\pi Gm}{\ell^2}$ along + y axis

B-3. A very large number of particles of same mass m are kept at horizontal distances of 1m, 2m, 4m, 8m and so on from (0, 0) point. The total gravitational potential at this point (0, 0) is :

(A) – 8G m (B) – 3G m (C) – 4G m (D) – 2G m

B-4. Two concentric shells of uniform density of mass M_1 and M_2 are situated as shown in the figure. The forces experienced by a particle of mass m when placed at positions A, B and C respectively are (given OA = p, OB = q and OC = r).



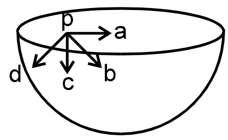
(A) zero, $G\frac{M_1 m}{q^2}$ and $G\frac{(M_1+m_2)m}{p^2}$

(B) $G\frac{(M_1+M_2)m}{p^2}$, $G\frac{(M_1+M_2)m}{q^2}$ and $G\frac{M_1 m}{r^2}$

(C) $G\frac{M_1 m}{q^2}$, $G\frac{M_1 m}{q^2}$ and zero

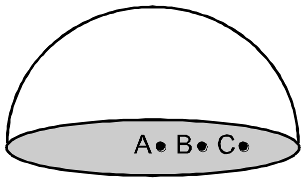
(D) $\frac{G(M_1+M_2)m}{p^2}$, $G\frac{M_1 m}{q^2}$ and zero

B-5. Figure show a hemispherical shell having uniform mass density. The direction of gravitational field intensity at point P will be along:



(A) a (B) b (C) c (D) d

B-6. Mass M is uniformly distributed only on curved surface of a thin hemispherical shell. *A*, *B* and *C* are three points on the circular base of hemisphere, such that *A* is the centre. Let the gravitational potential at points A, B and C be V_A, V_B, V_C respectively. Then



(A) $V_A > V_B > V_C$ (B) $V_C > V_B > V_A$

(C) $V_B > V_A$ and $V_B > V_C$ (D) $V_A = V_B = V_C$

Section (C) : Gravitational Potential Energy and Self Energy

C-1. A body starts from rest at a point, distance R_0 from the centre of the earth of mass M, radius R. The velocity acquired by the body when it reaches the surface of the earth will be

(A) $GM\left(\frac{1}{R}-\frac{1}{R_0}\right)$ (B) $2\ GM\left(\frac{1}{R}-\frac{1}{R_0}\right)$ (C) $\sqrt{2GM\left(\frac{1}{R}-\frac{1}{R_0}\right)}$ (D) $2GM\sqrt{\left(\frac{1}{R}-\frac{1}{R_0}\right)}$

C-2. (a) Three equal masses each of mass 'm' are placed at the three-corners of an equilateral triangle of side 'a'. If a fourth particle of equal mass is placed at the centre of triangle, then net force acting on it, is equal to :

(A) $\frac{Gm^2}{a^2}$ (B) $\frac{4Gm^2}{3a^2}$ (C) $\frac{3Gm^2}{a^2}$ (D) zero

(b) In above problem, if fourth particle is at the mid-point of a side, then net force acting on it, is equal to:

(A) $\frac{Gm^2}{a^2}$ (B) $\frac{4Gm^2}{3a^2}$ (C) $\frac{3Gm^2}{a^2}$ (D) zero

(c) If above given three particles system of equilateral triangle side a is to be changed to side of 2a, then work done on the system is equal to :

(A) $\frac{3Gm^2}{a}$ (B) $\frac{3Gm^2}{2a}$ (C) $\frac{4Gm^2}{3a}$ (D) $\frac{Gm^2}{a}$

(d) In the above given three particle system, if two particles are kept fixed and third particle is released. Then speed of the particle when it reaches to the mid-point of the side connecting other two masses:

(A) $\sqrt{\frac{2Gm}{a}}$ (B) $2\sqrt{\frac{Gm}{a}}$ (C) $\sqrt{\frac{Gm}{a}}$ (D) $\sqrt{\frac{Gm}{2a}}$

Section : (D) Kepler's law for Satellites, Orbital Velocity and Escape Velocity

D-1. Periodic-time of satellite revolving around the earth is - (ρ is density of earth)

(A) Proportional to $\frac{1}{\rho}$ (B) Proportional to $\frac{1}{\sqrt{\rho}}$

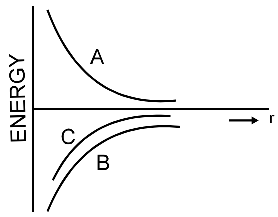
(C) Proportional ρ (D) does not depend on ρ.

D-2. An artificial satellite of the earth releases a package. If air resistance is neglected the point where the package will hit (with respect to the position at the time of release) will be

(A) ahead (B) exactly below

(C) behind (D) it will never reach the earth

D-3. The figure shows the variation of energy with the orbit radius of a body in circular planetary motion. Find the correct statement about the curves A, B and C



(A) A shows the kinetic energy, B the total energy and C the potential energy of the system

(B) C shows the total energy, B the kinetic energy and A the potential energy of the system

(C) C and A are kinetic and potential energies respectively and B is the total energy of the system

(D) A and B are the kinetic and potential energies respectively and C is the total energy of the system.

D-4. A planet of mass m revolves around the sun of mass M in an elliptical orbit. The minimum and maximum distance of the planet from the sun are r_1 & r_2 respectively. If the minimum velocity of the planet is $\sqrt{\frac{2GMr_1}{(r_1+r_2)r_2}}$ then it's maximum velocity will be :

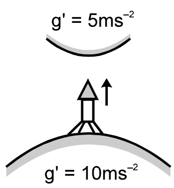
(A) $\sqrt{\frac{2GMr_2}{(r_1+r_2)r_1}}$ (B) $g\sqrt{\frac{2GMr_1}{(r_1+r_2)r_2}}$ (C) $\sqrt{\frac{2Gmr_2}{(r_1+r_2)r_1}}$ (D) $\sqrt{\frac{2GM}{r_1+r_2}}$

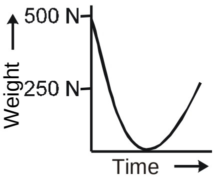
D-5. The escape velocity for a body projected vertically upwards from the surface of earth is 11 km/s. If the body is projected at an angle of 45º with the vertical, the escape velocity will be :

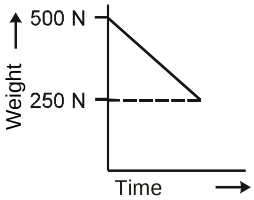
(A) $11\sqrt{2}$ km/s (B) 22 km/s (C) 11 km/s (D) $11/\sqrt{2}$ m/s

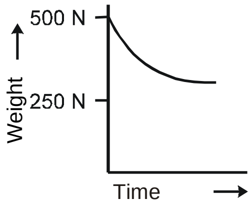
Section (E) : Earth and Other Planets Gravity

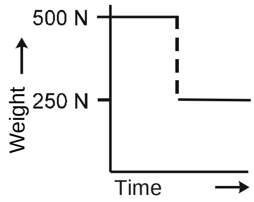
E-1. If acceleration due to gravity on the surface of earth is 10 ms^{-2} and let acceleration due to gravitational acceleration at surface of another planet of our solar system be 5 ms^{-2}. An astronaut weighing 50 kg on earth goes to this planet in a spaceship with a constant velocity. The weight of the astronaut with time of flight is roughly given by



(A) 

(B) 

(C) 

(D) 

PART - III : MATCH THE COLUMN

1.✍ A particle is taken to a distance r (> R) from centre of the earth. R is radius of the earth. It is given velocity V which is perpendicular to. With the given values of V in column I you have to match the values of total energy of particle in column II and the resultant path of particle in column III. Here 'G' is the universal gravitational constant and 'M' is the mass of the earth.

Column I (Velocity)	**Column II (Total energy)**	**Column III (Path)**
(A) $V = \sqrt{GM/r}$	(p) Negative	(t) Elliptical
(B) $V = \sqrt{2GM/r}$	(q) Positive	(u) Parabolic
(C) $V > \sqrt{2GM/r}$	(r) Zero	(v) Hyperbolic
(D) $\sqrt{GM/r} < V < \sqrt{2GM/r}$	(s) Infinite	(w) Circular

2. Let V and E denote the gravitational potential and gravitational field respectively at a point due to certain uniform mass distribution described in four different situations of column-I. Assume the gravitational potential at infinity to be zero.The value of E and V are given in column-II. Match the statement in column-I with results in column-II.

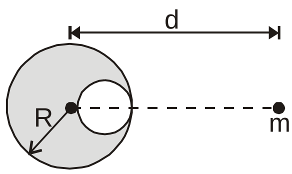
Column-I	**Column-II**
(A) At centre of thin spherical shell	(p) $E = 0$
(B) At centre of solid sphere	(q) $E \neq 0$
(C) A solid sphere has a non-concentric spherical cavity. At the centre of the spherical cavity	(r) $V \neq 0$
(D) At centre of line joining two point masses of equal magnitude	(s) $V = 0$

Exercise-2

✍ **Marked Questions can be used as Revision Questions.**

PART - I : ONLY ONE OPTION CORRECT TYPE

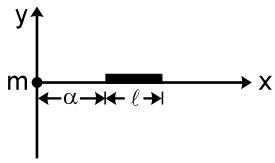
1.✍ A spherical hollow cavity is made in a lead sphere of radius R, such that its surface touches the outside surface of the lead sphere and passes through its centre. The mass of the sphere before hollowing was M. With what gravitational force will the hollowed-out lead sphere attract a small sphere of mass ‘m’, which lies at a distance d from the



centre of the lead sphere on the straight line connecting the centres of the spheres and that of the hollow, if d = 2R :

(A) $\frac{7GMm}{18R^2}$ (B) $\frac{7GMm}{36R^2}$ (C) $\frac{7GMm}{9R^2}$ (D) $\frac{7GMm}{72R^2}$

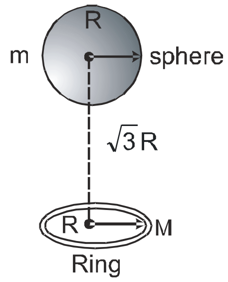
2. A straight rod of length □ extends from x = α to x = □ + α as shown in the figure. If the mass per unit length is $(a + bx^2)$. The gravitational force it exerts on a point mass m placed at x = 0 is given by



(A) $Gm\left(a\left(\frac{1}{\alpha}-\frac{1}{\alpha+\ell}\right)+b\ell\right)$ (B) $\frac{Gm(a+bx^2)}{\ell^2}$

(C) $Gm\left(\alpha\left(\frac{1}{a}-\frac{1}{a+\ell}\right)+b\ell\right)$ (D) $Gm\left(a\left(\frac{1}{\alpha+\ell}-\frac{1}{\alpha}\right)+b\ell\right)$

3. A uniform ring of mass M is lying at a distance $\sqrt{3}$ R from the centre of a uniform sphere of mass m just below the sphere as shown in the figure where R is the radius of the ring as well as that of the sphere. Then gravitational force exerted by the ring on the sphere is :



(A) $\frac{GMm}{8R^2}$ (B) $\frac{GMm}{3R^2}$

(C) $\sqrt{3}\frac{GMm}{R^2}$ (D) $\sqrt{3}\frac{GMm}{8R^2}$

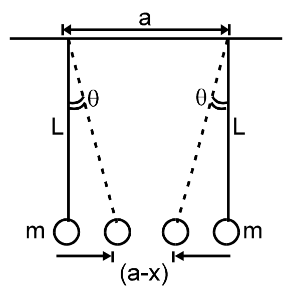
4. The gravitational potential of two homogeneous spherical shells A and B (separated by large distance) of same surface mass density at their respective centres are in the ratio 3 : 4. If the two shells coalesce into single one such that surface mass density remains same, then the ratio of potential at an internal point of the new shell to shell A is equal to :

(A) 3 : 2 (B) 4 : 3 (C) 5 : 3 (D) 3 : 5

5. A projectile is fired from the surface of earth of radius R with a speed $k\nu_e$ in radially outward direction (where ν_e is the escape velocity and k < 1). Neglecting air resistance, the maximum hight from centre of earth is

(A) $\frac{R}{k^2+1}$ (B) k^2R (C) $\frac{R}{1-k^2}$ (D) kR

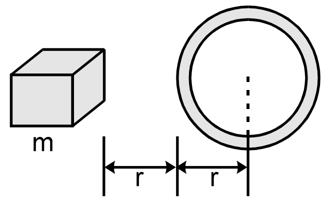
6. Two small balls of mass m each are suspended side by side by two equal threads of length L as shown in the figure. If the distance between the upper ends of the threads be a, the angle θ that the threads will make with the vertical due to attraction between the balls is



(A) $\tan^{-1}\frac{(a-x)g}{mG}$ (B) $\tan^{-1}\frac{mG}{(a-x)^2g}$

(C) $\tan^{-1}\frac{(a-x)^2g}{mG}$ (D) $\tan^{-1}\frac{(a^2-x^2)g}{mG}$

7. A block of mass m is lying at a distance r from a spherical shell of mass m and radius r as shown in the figure. Then



(A) only gravitational field inside the shell is zero

(B) gravitational field and gravitational potential both are zero inside the shell

(C) gravitational potential as well as gravitational field inside the shell are not zero

(D) can't be ascertained.

8. In a spherical region, the density varies inversely with the distance from the centre. Gravitational field at a distance r from the centre is :

(A) proportional to r (B) proportional to $\frac{1}{r}$ (C) proportional to r^2 (D) same everywhere

9. In above problem, the gravitational potential is -

(A) linearly dependent on r (B) proportional to $\frac{1}{r}$

(C) proportional to r^2 (D) same every where.

10. A point P lies on the axis of a fixed ring of mass M and radius R, at a distance 2R from its centre O. A small particle starts from P and reaches O under gravitational attraction only. Its speed at O will be

(A) zero (B) $\sqrt{\frac{2GM}{R}}$ (C) $\sqrt{\frac{2GM}{R}(\sqrt{5}-1)}$ (D) $\sqrt{\frac{2GM}{R}(1-\frac{1}{\sqrt{5}})}$

11. A body of mass m is lifted up from the surface of earth to a height three times the radius of the earth. The change in potential energy of the body is (g = gravity field at the surface of the earth)

(A) mgR (B) $\frac{3}{4}$mgR (C) $\frac{1}{3}$mgR (D) $\frac{2}{3}$ mgR

12. Assuming that the moon is a sphere of the same mean density as that of the earth and one quarter of its radius, the length of a seconds pendulum on the moon (its length on the earth's surface is 99.2 cm) is

(A) 24.8 cm (B) 49.6 cm (C) 99.2 (D) $\frac{99.2}{\sqrt{2}}$ cm

13. A satellite can be in a geostationary orbit around a planet at a distance r from the centre of the planet. If the angular velocity of the planet about its axis doubles, a satellite can now be in a geostationary orbit around the planet if its distance from the centre of the planet is

(A) $\frac{r}{2}$ (B) $\frac{r}{2\sqrt{2}}$ (C) $\frac{r}{(4)^{1/3}}$ (D) $\frac{r}{(2)^{1/3}}$

14. A geostationary satellite orbits around the earth in a circular orbit of radius 36000 km. Then, the time period of a spy satellite orbiting a few hundred kilometers above the earth's surface (R_{Earth} = 6400 km) will approximately be (R_{earth} = 6400 km) :
(A) 1/2 hr (B) 1 hr (C) 2 hr (D) 4 hr

15. A satellite of mass m revolves around earth of radius R at a height x from its surface. If g is the acceleration due to gravity on the surface of the earth, the orbital speed of the satellite is :

(A) gx (B) $\frac{gR}{R-x}$ (C) $\frac{gR^2}{R+x}$ (D) $\left(\frac{gR^2}{R+x}\right)^{1/2}$

16. If Newton's inverse square law of gravitation had some dependence on radial distance other than r^{-2}, which on of kepler's three laws of planetary motion would remain unchanged? **[Olympiad (Stage-1) 2017]**
(A) First law on nature of orbits
(B) Second law on constant areal velocity
(C) Third law on dependence of orbital time period on orbit's semi major axis
(D) None of the above

PART - II : SINGLE AND DOUBLE VALUE INTEGER TYPE

1. A projectile is fired vertically up from the bottom of a crater (big hole) on the moon. The depth of the crater is R/100, where R is the radius of the moon. If the initial velocity of the projectile is the same as the escape velocity from the moon surface. The maximum aproximate height attained by the projectile above the lunar (moon) surface is xR. Find value of x.

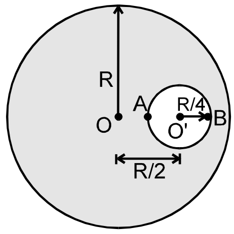
2. The time period of a satellite of earth is 5 hours. If the separation between the earth and the satellite is increased to 4 times the previous value, the new time period becomes (in hrs).

3. If g is the acceleration due to gravity on the earth's surface, the gain in the potential energy of an object of mass m raised from the surface of the earth to a height equal to the radius R of the earth $\frac{NmgR}{2}$ is. Find the value of N :

4. The gravitational field in a region is given by $\vec{E} = (3\hat{i} - 4\hat{j})$ N/kg. Find out the work done (in joule) in displacing a particle of mass 1 kg by 1 m along the line 4y = 3x + 9.

5. In a solid sphere of radius 'R' and density 'ρ' there is a spherical cavity of radius R/4 as shown in figure. A particle of mass 'm' is released from rest from point 'B' (inside the cavity). Find out Velocity (in mm/sec.) of the particle at the instant when it strikes the cavity

($R = 3$m, $\rho = \frac{10}{\pi} \times 10^3$ kg/m^3, $G = \frac{20}{3} \times 10^{-11}$ Nm^{2}kg^{-2})



6. A ring of radius R = 8m is made of a highly dense-material. Mass of the ring is $m_R = 2.7 \times 10^9$ kg distributed uniformly over its circumference. A particle of mass (dense) $m_p = 3 \times 10^8$ kg is placed on the axis of the ring at a distance x_0 = 6m from the centre. Neglect all other forces except gravitational interaction. Determine speed (in cm/sec.) of the particle at the instant when it passes through centre of ring. :

7. Our sun, with mass 2×10^{30} kg revolves on the edge of our milky way galaxy, which can be assumed to be spherical, having radius 10^{20} m. Also assume that many stars, identical to our sun are uniformly distributed in the spherical milky way galaxy. If the time period of the sun is 10^{15} second and number of stars in the galaxy are nearly $3 \times 10^{(a)}$, find value of 'a' (take $\pi^2 = 10$, G $\frac{20}{3} \times 10^{-11}$ in MKS)

8. Assume earth to be a sphere of uniform mass density. The energy needed to completely disassemble the planet earth against the gravitational pull amongst its constituent particles is $x \times 10^{31}$ J. Find the value of x. given the product of mass of earth and radius of earth to be 2.5×10^{31} kg-m and g = 10 m/s

9. The two stars in a certain binary star system move in circular orbits. The first star, α moves in an orbit of radius 1.00×10^9 km. The other star, β moves in an orbit of radius 5.00×10^8 km. What is the ratio of masses of star β to the star α ?

10. If the radius of earth is R and height of a satellite above earth's surface is R then find the minimum co-latitude (in degree) which can directly receive a signal from satellite. (Satellite is in equitorial plane)

PART - III : ONE OR MORE THAN ONE OPTIONS CORRECT TYPE

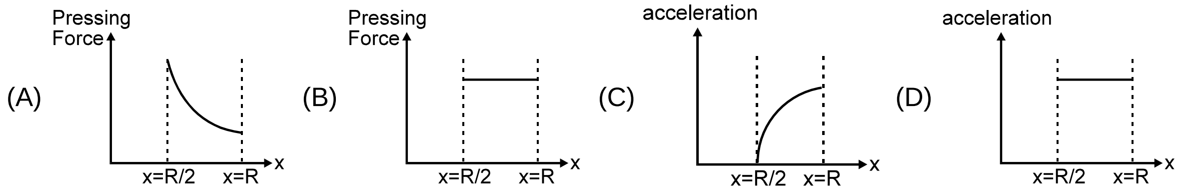
1. For a satellite to appear stationary w.r.t. an observer on earth
(A) It must be rotating about the earth's axis.
(B) It must be rotating in the equatorial plane.
(C) Its angular velocity must be from west to east.
(D) Its time period must be 24 hours.

2. Inside an isolated uniform spherical shell :
(A) The gravitation potential is not zero
(B) The gravitational field is not zero
(C) The gravitational potential is same everywhere

(D) The gravitational field is same everywhere.

3. Which of the following statements are correct about a planet rotating around the sun in an elliptic orbit:
(A) its mechanical energy is constant
(B) its angular momentum about the sun is constant
(C) its areal velocity about the sun is constant
(D) its time period is proportional to r^3

4. A tunnel is dug along a chord of the earth at a perpendicular distance R/2 from the earth's centre. The wall of the tunnel may be assumed to be frictionless. A particle is released from one end of the tunnel. The pressing force by the particle on the wall and the acceleration of the particle varies with x (distance of the particle from the centre) according to :



5. A planet revolving around sun in an elliptical orbit has a constant
(A) kinetic energy
(B) angular momentum about the sun
(C) potential energy
(D) Total energy

6. A satellite close to the earth is in orbit above the equator with a period of revolution of 1.5 hours. If it is above a point P on the equator at some time, it will be above P again after time
(A) 1.5 hours
(B) 1.6 hours if it is rotating from west to east
(C) 24/17 hours if it is rotating from east to west
(D) 24/17 hours if it is rotating from west to east

7. An object is weighed at the equator by a beam balance and a spring balance, giving readings W_b and W_s respectively. It is again weighed in the same manner at the north pole, giving readings of W_b' and W_s' respectively. Assume that intensity of earth gravitational field is the same every where on the earth's surface and that the balances are quite sensitive.
(A) $W_b = W_b'$
(B) $W_b = W_s$
(C) $W_b' = W_s'$
(D) $W_s' > W_s$

8. If a body is projected with speed lesser than escape velocity :
(A) the body can reach a certain height and may fall down following a straight line path
(B) the body can reach a certain height and may fall down following a parabolic path
(C) the body may orbit the earth in a circular orbit
(D) the body may orbit the earth in an elliptic orbit

9. A double star is a system of two stars of masses m and 2m, rotating about their centre of mass only under their mutual gravitational attraction. If r is the separation between these two stars then their time period of rotation about their centre of mass will be proportional to
(A) $r^{3/2}$
(B) r
(C) $m^{1/2}$
(D) $m^{-1/2}$

10. An orbiting satellite in circular orbit will escape if :
(A) its speed is increased by $(\sqrt{2}-1)100\%$
(B) its speed in the orbit is made $\sqrt{1.5}$ times of its initial value
(C) its KE is doubled
(D) it stops moving in the orbit

11. In case of an orbiting satellite if the radius of orbit is decreased :
(A) its Kinetic Energy decreases
(B) its Potential Energy decreases
(C) its Mechanical Energy decreases
(D) its speed decreases

12. In case of earth :

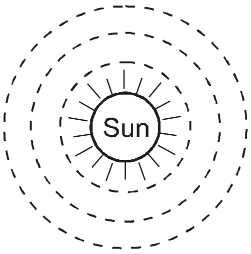
(A) gravitational field is zero, both at centre and infinity
(B) gravitational potential is zero, both at centre and infinity
(C) gravitational potential is same, both at centre and infinity but not zero
(D) gravitational potential is minimum at the centre

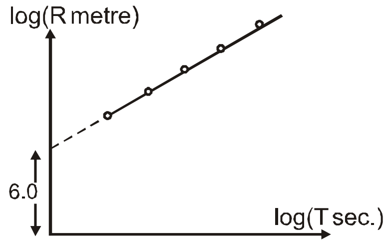
PART - IV : COMPREHENSION

Comprehension - 1

Many planets are revolving around the fixed sun, in circular orbits of different radius (R) and different time period (T). To estimate the mass of the sun, the orbital radius (R) and time period (T) of planets were noted. Then $\log_{10} T$ v/s $\log_{10} R$ curve was plotted.

The curve was found to be approximately straight line (as shown in figure) having y intercept = 6.0 (Neglect the gravitational interaction among the planets [Take $G = \frac{20}{3} \times 10^{-11}$ in MKS, $\pi^2 = 10$]





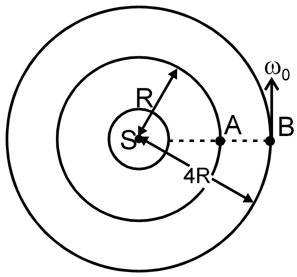
1. The slope of the line should be :

(A) 1 (B) $\frac{3}{2}$ (C) $\frac{2}{3}$ (D) $\frac{19}{4}$

2. Estimate the mass of the sun :

(A) 6×10^{29} kg (B) 5×10^{20} kg (C) 8×10^{25} kg (D) 3×10^{35} kg

3. Two planets A and B, having orbital radius R and 4R are initially at the closest position and rotating in the same direction. If angular velocity of planet B is ω_0, then after how much time will both the planets be again in the closest position ? (Neglect the interaction between planets).



(A) $\frac{2\pi}{7\omega_0}$ (B) $\frac{2\pi}{9\omega_0}$

(C) $\frac{2\pi}{\omega_0}$ (D) $\frac{2\pi}{5\omega_0}$

Comprehension - 2

An artificial satellite is moving in a circular orbit around the earth with a speed equal to half the magnitude of escape velocity from the surface of earth. R is the radius of earth and g is acceleration due to gravity at the surface of earth. (R = 6400 km)

4. Then the distance of satellite from the surface of earth is

(A) 3200 km (B) 6400 km (C) 12800 km (D) 4800 km

5. The time period of revolution of satellite in the given orbit is

(A) $2\pi\sqrt{\frac{2R}{g}}$ (B) $2\pi\sqrt{\frac{4R}{g}}$ (C) $2\pi\sqrt{\frac{8R}{g}}$ (D) $2\pi\sqrt{\frac{6R}{g}}$

6. If the satellite is stopped suddenly in its orbit and allowed to fall freely onto the earth, the speed with which it hits the surface of the earth.

(A) $\sqrt{gR}$ (B) $\sqrt{1.5gR}$ (C) $\sqrt{\frac{gR}{2}}$ (D) $\sqrt{\frac{gR}{\sqrt{2}}}$

Comprehension - 3

A pair of stars rotates about their center of mass. One of the stars has a mass M and the other has mass m such that M = 2m. The distance between the centres of the stars is d (d being large compared to the size of either star).

7. The period of rotation of the stars about their common centre of mass (in terms of d, m, G.) is

(A) $\sqrt{\frac{4\pi^2}{Gm}d^3}$ (B) $\sqrt{\frac{8\pi^2}{Gm}d^3}$ (C) $\sqrt{\frac{2\pi^2}{3Gm}d^3}$ (D) $\sqrt{\frac{4\pi^2}{3Gm}d^3}$

8. The ratio of the angular momentum of the two stars about their common centre of mass (L_m/ L_M) is

(A) 1 (B) 2 (C) 4 (D) 9

9. The ratio of kinetic energies of the two stars (K_m/K_M) is

(A) 1 (B) 2 (C) 4 (D) 9

Exercise-3

✍ **Marked Questions can be used as Revision Questions.**
*** Marked Questions may have more than one correct option.**

PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

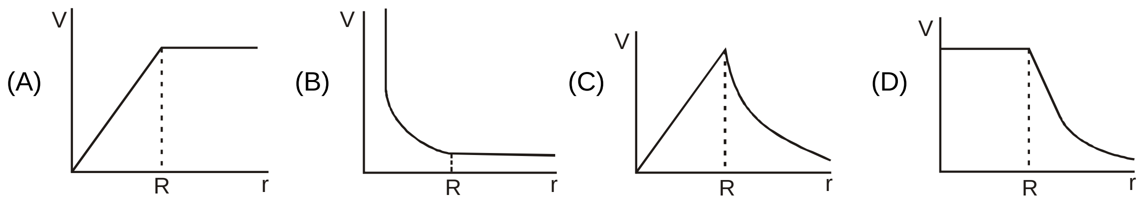
1. **Column** I describes some situations in which a small object moves. **Column** II describes some characteristics of these motions. Match the situations in **Column** I with the characteristics in **Column** II **[IIT-JEE 2007, 6/162]**

Column I	**Column** II
(A) The object moves on the x–axis under a conservative force in such a way that its "speed" and "position" satisfy $v = c_1\sqrt{c_2 - x^2}$, where c_1 and c_2 are positive constants.	(p) The object executes a simple harmonic motion.
(B) The object moves on the x–axis in such a way that its velocity and its displacement from the origin satisfy $v = -kx$, where k is a positive constant.	(q) The object does not change its direction,
(C) The object is attached to one end of a massless spring of a given spring constant. The other end of the spring is attached to the ceiling of an elevator. Initially everything is at rest. The elevator starts going upwards with a constant acceleration a. The motion of the object is observed from the elevator during the period it maintains this acceleration.	(r) The kinetic energy of the object keeps on decreasing.
(D) The object is projected from the earth's surface vertically upwards with a speed $2\sqrt{GM_e/R_e}$, where M_e is the mass of the earth and R_e is the radius of the earth. Neglect forces from objects other than the earth.	(s) The object can change its direction only once.

2. A spherically symmetric gravitational system of particles has a mass density **[JEE 2008, +3, –1/82]**

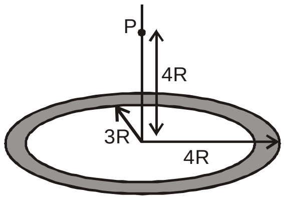
$$\rho = \begin{cases} \rho_0 & \text{for} \quad r \le R \\ 0 & \text{for} \quad r > R \end{cases}$$

where ρ_0 is a constant. A test mass can undergo circular motion under the influence of the gravitational field of particles. Its speed V as a function of distance r ($0 < r < \infty$) from the centre of the system is represented by



3. **STATEMENT-1 :** An astronaut in an orbiting space station above the Earth experiences weightlessness.
and
STATEMENT-2 : An object moving around the Earth under the influence of Earth's gravitational force is in a state of 'free-fall. **[JEE 2008,+3, –1/82]**
(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is a correct explanation for STATEMENT-1
(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is NOT a correct explanation for STATEMENT-1
(C) STATEMENT-1 is True, STATEMENT-2 is False
(D) STATEMENT-1 is False, STATEMENT-2 is True.

4.✍ A thin uniform annular disc (see figure) of mass M has outer radius 4R and inner radius 3R. The work required to take a unit mass from point P on its axis to infinity is : **[JEE 2010, 3/163, –1]**



(A) $\frac{2GM}{7R}\left(4\sqrt{2}-5\right)$ (B) $-\frac{2GM}{7R}\left(4\sqrt{2}-5\right)$ (C) $\frac{GM}{4R}$ (D) $\frac{2GM}{5R}\left(\sqrt{2}-1\right)$

5. A binary star consists of two stars A (mass 2.2 M_S) and B (mass 11 M_S) where M_s is the mass of the sun. They are separated by distance d and are rotating about their centre of mass, which is stationary. The ratio of the total angular momentum of the binary star to the angular momentum of star B about the centre of mass is : **[JEE 2010, 3/163]**

6. Gravitational acceleration on the surface of a planet is $\frac{\sqrt{6}}{11}g$, where g is the gravitational acceleration on the surface of the earth. The average mass density of the planet is $\frac{2}{3}$ times that of the earth. If the escape speed on the surface of the earth is taken to be 11 kms^{-1}, the escape speed on the surface of the planet in kms^{-1} will be : **[JEE 2010, 3/163]**

7. A satellite is moving with a constant speed 'V' in a circular orbit about the earth. An object of mass *'m'* is ejected from the satellite such that it just escapes from the gravitational pull of the earth. At the time of its ejection, the kinetic energy of the object is **[JEE 2011, 3/160, –1]**

(A) $\frac{1}{2}mV^2$ (B) mV^2 (C) $\frac{3}{2}mV^2$ (D) $2mV^2$

8*. Two spherical planets P and Q have the same unfirom density ρ, masses M_P and M_Q, with surface areas A and 4A, respectively. A spherical planet R also has unfirom density ρ and its mass is $(M_P + M_Q)$. The escape velocities from the planets P, Q and R, are V_P, V_Q and V respectivley. Then **[IIT-JEE-2012, Paper-2; 4/66]**

(A) $V_Q > V_R > V_P$ (B) $V_R > V_Q > V_P$ (C) $V_R/V_P = 3$ (D) $V_P / V_Q = 1/2$

9*. Two bodies, each of mass M, are kept fixed with a separation 2L. A particle of mass m is projected from the midpoint of the line joining their centres, perpendicular to the line. The gravitational constant is G. The correct statement(s) is (are) : **[JEE (Advanced) 2013, 3/60, –1]**

(A) The minimum initial velocity of the mass m to escape the gravitational field of the two bodies is $4\sqrt{\frac{GM}{L}}$.

(B) The minimum initial velocity of the mass m to escape the gravitational field of the two bodies is $2\sqrt{\frac{GM}{L}}$.

(C) The minimum initial velocity of the mass m to escape the gravitational field of the two bodies is $\sqrt{\frac{2GM}{L}}$.

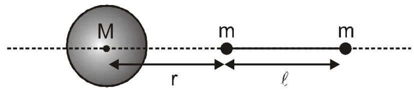
(D) The energy of the mass m remains constant.

10. A planet of radius R = $\frac{1}{10}$ × (radius of Earth) has the same mass density as Earth. Scientists dig a well of depth $\frac{R}{5}$ on it and lower a wire of the same length and of linear mass density 10^{-3} kgm^{-1} into it. If the wire is not touching anywhere, the force applied at the top of the wire by a person holding it in place is (take the radius of Earth = 6 × 10^6 m and the acceleration due to gravity on Earth is 10 ms^{-2})
[JEE (Advanced) 2014, 3/60, –1]

(A) 96 N (B) 108N (C) 120N (D) 150N

11. A Bullet is fired vertically upwards with velocity v from the surface of a spherical planet. When it reaches its maximum height, its acceleration due to the planet's gravity is $1/4^{th}$ of its value at the surface of the planet. If the escape velocity from the planet is $v_{esc} = v\sqrt{N}$, then the value of N is (ignore energy loss due to atmosphere) **[JEE (Advanced) 2015 ; P-1, 4/88]**

12. A large spherical mass M is fixed at one position and two identical point masses m are kept on a line passing through the centre of M (see figure). The point masses are connected by a rigid massless rod of length ℓ and this assembly is free to move along the line connecting them. All three masses interact only through their mutual gravitational interaction. When the point mass nearer to M is at a distance $r = 3\ell$ from M, the tension in the rod is zero for $m = k\left(\frac{M}{288}\right)$. The value of k is **[JEE (Advanced) 2015 ; P-2,4/88]**



13*. A spherical body of radius R consists of a fluid of constant density and is in equilibrium under its own gravity. If P(r) is the pressure at r(r < R), then the correct option(s) is (are)
[JEE (Advanced) 2015 ; P-2,4/88, –2]

(A) $P(r = 0) = 0$ (B) $\frac{P(r = 3R/4)}{P(r = 2R/3)} = \frac{63}{80}$ (C) $\frac{P(r = 3R/5)}{P(r = 2R/5)} = \frac{16}{21}$ (D) $\frac{P(r = R/2)}{P(r = R/3)} = \frac{20}{27}$

14. A rocket is launched normal to the surface of the Earth, away from the Sun, along the line joining the Sun and the Earth. The Sun is 3×10^5 times heavier than the Earth and is at a distance 2.5×10^4 times larger than the radius of the Earth. The escape velocity from Earth's gravitational field is $v_e = 11.2 \text{km s}^{-1}$. The minimum initial velocity (v_s) required for the rocket to be able to leave the Sun-Earth system is closest to : (Ignore the rotation and revolution of the Earth and the presence of any other planet)
[JEE (Advanced) 2017 ; P-2, 3/61, –1]

(A) $v_s = 72 \text{ km s}^{-1}$ (B) $v_s = 22 \text{ km s}^{-1}$ (C) $v_s = 42 \text{ km s}^{-1}$ (D) $v_s = 62 \text{ km s}^{-1}$

15. A planet of mass M, has two natural satellites with masses m_1 and m_2. The radii of their circular orbits are R_1 and R_2 respectively. Ignore the gravitational force between the satellites. Define v_1, L_1, K_1 and T_1 to be, respectively, the orbital speed, angular momentum, kinetic energy and time period of revolution of satellite 1; and v_2, L_2, K_2 and T_2 to be the corresponding quantities of satellite 2. Given $m_1/m_2 = 2$ and $R_1/R_2 = 1/4$, match the ratios in List-I to the numbers in List-II. **[JEE (Advanced) 2018; P-2, 3/60, –1]**

List-I		**List-II**	
P.	$\frac{v_1}{v_2}$	1.	$\frac{1}{8}$
Q.	$\frac{L_1}{L_2}$	2.	1
R.	$\frac{K_1}{K_2}$	3.	2
S.	$\frac{T_1}{T_2}$	4.	8

(A) P → 4 ; Q → 2 ; R → 1 ; S → 3 (B) P → 3 ; Q → 2 ; R → 4 ; S → 1
(C) P → 2 ; Q → 3 ; R → 1 ; S → 4 (D) P → 2 ; Q → 3 ; R → 4 ; S → 1

PART - II : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. If g_E and g_m are the accelerations due to gravity on the surfaces of the earth and the moon respectively and if Millikan's oil drop expriment could be performed on the two surfaces, one will find the ratio $\frac{\text{Electronic charge on the moon}}{\text{electronic charge on the earth}}$ to be **[AIEEE-2007, 3/120]**

(1) 1 (2) 0 (3) g_E/g_M (4) g_M/g_E

2. A planet in a distant solar system is 10 times more massive than the earth and its radius is 10 times smaller. Given that the escape velocity from the earth is 11 km s^{-1}, the escape velocity from the surface of the planet would be : **[AIEEE-2008, 3/105]**

(1) 11 km s^{-1} (2) 110 km s^{-1} (3) 0.11 km s^{-1} (4) 1.1 km s^{-1}

3. The height at which the acceleration due to gravity becomes $\frac{g}{9}$ (where g = the acceleration due to gravity on the surface of the earth) in terms of R, the radius of the earth, is **[AIEEE-2009, 4/144]**

(1) $\frac{R}{\sqrt{2}}$ (2) $\frac{R}{2}$ (3) $\sqrt{2}R$ (4) 2R

4. Two bodies of masses m and 4 m are placed at a distance r. The gravitational potential at a point on the line joining them where the gravitational field is zero is : **[AIEEE - 2011, 4/120, –1]**

(1) zero (2) $-\frac{4Gm}{r}$ (3) $-\frac{6Gm}{r}$ (4) $-\frac{9Gm}{r}$

5. Two particles of equal mass 'm' go around a circle of radius R under the action of their mutual gravitational attraction. The speed of each particle with respect to their centre of mass is : **[AIEEE 2011, 11 May; 4/120, –1]**

(1) $\sqrt{\frac{Gm}{4R}}$ (2) $\sqrt{\frac{Gm}{3R}}$ (3) $\sqrt{\frac{Gm}{2R}}$ (4) $\sqrt{\frac{Gm}{R}}$

6. The mass of a spaceship is 1000 kg. It is to be launched from the earth's surface out into free space. The value of 'g' and 'R' (radius of earth) are 10 m/s^2 and 6400 km respectively. The required energy for this work will be : **[AIEEE 2012 ; 4/120, –1]**

(1) 6.4×10^{11} Joules (2) 6.4×10^{8} Joules (3) 6.4×10^{9} Joules (4) 6.4×10^{10} Joules

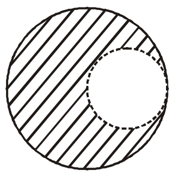
7. What is the minimum energy required to launch a satellite of mass m from the surface of a planet of mass M and radius R in a circular orbit at an altitude of 2R? **[JEE (Main) 2013, 4/120, –1]**

(1) $\frac{5GmM}{6R}$ (2) $\frac{2GmM}{3R}$ (3) $\frac{GmM}{2R}$ (4) $\frac{GmM}{3R}$

8. Four particles, each of mass M and equidistant from each other, move along a circle of radius R under the action of their mutual gravitational attration. The speed of each particle is : **[JEE (Main) 2014, 4/120,–1]**

(1) $\sqrt{\frac{GM}{R}}$ (2) $\sqrt{2\sqrt{2}\frac{GM}{R}}$ (3) $\sqrt{\frac{GM}{R}\left(1+2\sqrt{2}\right)}$ (4) $\frac{1}{2}\sqrt{\frac{GM}{R}\left(1+2\sqrt{2}\right)}$

9. From a solid sphere of mass M and radius R, a spherical portion of radius R/2 is removed, as shown in the figure. Taking gravitational potential V = 0 at r = ∞, the potential at the centre of the cavity thus formed is : (G = gravitational constant) **[JEE (Main) 2015; 4/120, –1]**

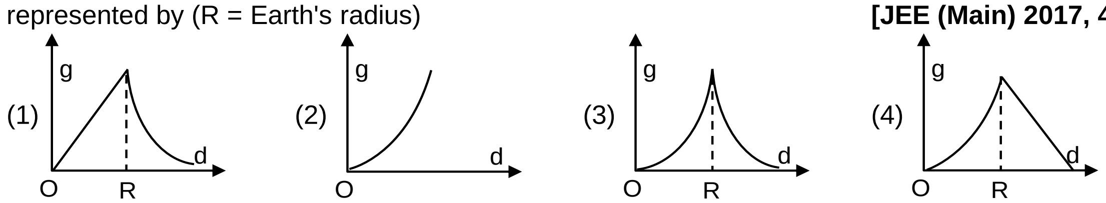


(1) $\frac{-GM}{2R}$ (2) $\frac{-GM}{R}$ (3) $\frac{-2GM}{3R}$ (4) $\frac{-2GM}{R}$

10. A satellite is revolving in a circular orbit at a height 'h' from the earth's surface (radius or earth R ; h << R). The minimum increase in its orbital velocity required, so that the satellite could escape from the earth's gravitational field, is close to : (Neglect the effect of atmosphere.) **[JEE (Main) 2016; 4/120, –1]**

(1) $\sqrt{gR}$ (2) $\sqrt{gR/2}$ (3) $\sqrt{gR}\left(\sqrt{2}-1\right)$ (4) $\sqrt{2gR}$

11. The variation of acceleration due to gravity g with distance d from centre of the earth is best represented by (R = Earth's radius) **[JEE (Main) 2017, 4/120, –1]**



Answers

EXERCISE-1

PART - I

Section (A)

A-1. 4.8×10^{-5} N **A-2.** $\frac{4}{9}\pi^2\rho^2 G r^4$

A-3. 31.2 G m/sec^2 = 2.1×10^{-9} m/s^2, towards mid point

Section (B)

B-1. $-20\hat{i} - 40\hat{j}$, $|\vec{F}| = 5\sqrt{5}$ N, $\vec{F} = -5\hat{i} - 10\hat{j}$

B-2. 6.3×10^7 J/Kg

Section (C)

C-1. $\sqrt{gR}$ **C-2.** $2\sqrt{\frac{G(M_A + M_B)}{d}}$

Section (D)

D-1. 1.5%

D-2. (a) $F = \frac{GM^2}{4R^2}$ (b) $\sqrt{\frac{GM}{4R}}$; $T = 4\pi\sqrt{\frac{R^3}{GM}}$

(c) $\sqrt{\frac{GM}{4R^3}}$ (d) $\frac{GM^2}{4R}$ (e) $\sqrt{\frac{4GM}{R}}$

D-3. (a) $\frac{U_A}{U_B} = \frac{25600}{12800} = 2$

(b) $\frac{K_A}{K_B} = \frac{m_A}{m_B}\frac{r_B}{r_A} = 2$

(c) B is having more energy.

D-4. (a) The Saturn (b) The Mars (c) The Mars

Section (E)

E-1. $\frac{19}{2}$ m/s^2 = 9.5 m/s^2

E-2. $T = 1 - \frac{1}{2}\frac{4\pi^2}{(86400)^2} \times 6400 \times \frac{10^3}{9.8} = 0.998$ s

PART - II

Section (A)

A-1 (A) **A-2.** (B) **A-3.** (C)
A-4. (B)

Section (B)

B-1. (B) **B-2.** (D) **B-3.** (D)
B-4. (D) **B-5.** (C) **B-6.** (D)

Section (C)

C-1. (C) **C-2.** **(a)** (D), **(b)** (B), **(c)** (B), **(d)** (B)

Section : (D)

D-1. (B) **D-2.** (D) **D-3.** (D)
D-4. (A) **D-5.** (C)

Section (E)

E-1. (A)

PART - III

1.

I	II	III
A	p	w
B	r	u
C	q	v
D	p	t

2. (A) – p, r (B) – p, r (C) – q, r (D) – p, r

EXERCISE-2

PART - I

1.	(B)	**2.**	(A)	**3.**	(D)
4.	(C)	**5.**	(C)	**6.**	(B)
7.	(C)	**8.**	(D)	**9.**	(A)
10.	(D)	**11.**	(B)	**12.**	(A)
13.	(C)	**14.**	(C)	**15.**	(D)
16.	(B)				

PART - II

1.	99	**2.**	40	**3.**	1
4.	0	**5.**	2	**6.**	9
7.	11	**8.**	15	**9.**	2
10.	30				

PART - III

1.	(ABCD)	**2.**	(ACD)	**3.**	(ABC)
4.	(BC)	**5.**	(BD)	**6.**	(BC)
7.	(ACD)	**8.**	(ABCD)	**9.**	(AD)
10.	(AC)	**11.**	(BC)	**12.**	(AD)

PART - IV

1.	(C)	**2.**	(A)	**3.**	(A)
4.	(B)	**5.**	(C)	**6.**	(A)
7.	(D)	**8.**	(B)	**9.**	(B)

EXERCISE-3

PART - I

1. (A) $\rightarrow$ (p) ; (B) $\rightarrow$ (q, r) ; (C) $\rightarrow$ (p) ; (D) $\rightarrow$ (q, r)

2.	(C)	**3.**	(A)	**4.**	(A)
5.	6	**6.**	3	**7.**	(B)
8.	(BD)	**9.**	(BD)	**10.**	(B)
11.	2	**12.**	7	**13.**	(BC)
14.	(C)	**15.**	(B)		

PART - II

1.	(1)	**2.**	(2)	**3.**	(4)
4.	(4)	**5.**	(1)	**6.**	(4)
7.	(1)	**8.**	(4)	**9.**	(2)
10.	(3)	**11.**	(1)		