Exercise-1

PART - I : SUBJECTIVE QUESTIONS

Section (A) : Definition of LHL/RHL and Indeterminate forms

A-1. Examine the graph of y = f(x) as shown and evaluate the following limits :
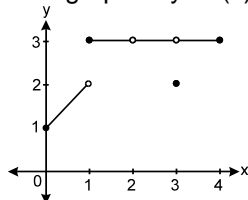
(i) $\lim\limits_{x \to 1} f(x)$ (ii) $\lim\limits_{x \to 2} f(x)$ (iii) $\lim\limits_{x \to 3} f(x)$

(iv) $\lim\limits_{x \to 1.99} f(x)$ (v) $\lim\limits_{x \to 3^-} f(x)$

A-2. Evaluate the following limits :

(i) $\lim\limits_{x \to 2} (x + \sin x)$ (ii) $\lim\limits_{x \to 3} (\tan x - 2^x)$ (iii) $\lim\limits_{x \to \frac{3}{4}} x \cos x$

(iv) $\lim\limits_{x \to 5} x^x$ (v) $\lim\limits_{x \to 1} \dfrac{e^x}{\sin x}$

A-3. Evaluate the following limits,
where [.] represents greatest integer function and { . } represents fractional part function

(i) $\lim\limits_{x \to \frac{\pi}{2}} [\sin x]$ (ii) $\lim\limits_{x \to 2} \left\{ \dfrac{x}{2} \right\}$ (iii)✎ $\lim\limits_{x \to \pi}$ sgn [tan x]

(iv) $\lim\limits_{x \to 1} \sin^{-1} (\ell n \, x)$

A-4. (i) If f(x) = $\begin{cases} x+1 & , & x < 1 \\ 2x-3 & , & x \geq 1 \end{cases}$, evaluate $\lim\limits_{x \to 1} f(x)$.

(ii) Let f(x) = $\begin{cases} x+\lambda & , & x < 1 \\ 2x-3 & , & x \geq 1 \end{cases}$, if $\lim\limits_{x \to 1} f(x)$ exist, then find value of λ.

A-5.✎ If f(x) = $\begin{cases} x^2+2 & , & x \geq 2 \\ 1-x & , & x < 2 \end{cases}$ and g(x) = $\begin{cases} 2x & , & x > 1 \\ 3-x & , & x \leq 1 \end{cases}$, evaluate $\lim\limits_{x \to 1} f(g(x))$.

A-6. Which of the followings are indeterminate forms. Also state the type.

(i) $\lim\limits_{x \to 0^+} \dfrac{[x]}{x}$, where [.] denotes the greatest integer function

(ii) $\lim\limits_{x \to -\infty} \sqrt{x^2+1} - x$ (iii) $\lim\limits_{x \to \left(\frac{\pi}{2}\right)^-} (\tan x)^{\tan 2x}$

(iv)✎ $\lim\limits_{x \to 1^+} \left(\{x\} \right)^{\frac{1}{\ell n x}}$ where { . } denotes the fractional part function

SECTION (B) : Evaluation of limits of form 0/0, ∞/∞, ∞ – ∞, 0 × ∞, Use of L-Hospital Rule & Expansion

B-1. Evaluate each of the following limits, if exists

(i) $\lim\limits_{x \to -1} \dfrac{x^3 - 3x + 1}{x - 1}$

(ii) $\lim\limits_{x \to 1} \dfrac{4x^3 - x^2 + 2x - 5}{x^6 + 5x^3 - 2x - 4}$

(iii) $\lim\limits_{x \to a} \dfrac{\sqrt{a + 2x} - \sqrt{3x}}{\sqrt{3a + x} - 2\sqrt{x}}$, $a \neq 0$

B-2. Evaluate the following limits, if exists

(i) $\lim\limits_{x \to 0} \dfrac{1 - \cos 4x}{1 - \cos 5x}$

(ii) $\lim\limits_{x \to \frac{\pi}{6}} \dfrac{\sqrt{3}\sin x - \cos x}{x - \dfrac{\pi}{6}}$

(iii) $\lim\limits_{x \to 0} \dfrac{\tan 3x - 2x}{3x - \sin^2 x}$

(iv) $\lim\limits_{x \to 0} \dfrac{(a + x)^2 \sin(a + x) - a^2 \sin a}{x}$

(v) $\lim\limits_{x \to 0} \dfrac{e^{bx} - e^{ax}}{x}$, where $0 < a < b$

(vi) $\lim\limits_{x \to 0} \dfrac{x\left(e^{2+x} - e^2\right)}{1 - \cos x}$

(vii) $\lim\limits_{x \to 0} \dfrac{\ell n(1 + 3x)}{3^x - 1}$

(viii) $\lim\limits_{x \to 0} \dfrac{\ell n\ (2 + x) + \ell n\ \ 0.5}{x}$

(ix) Find $n \in N$, if $\lim\limits_{x \to 2} \dfrac{x^n - 2^n}{x - 2} = 80$.

(x) $\lim\limits_{x \to 0} \dfrac{\sqrt{\dfrac{1 - \cos 2x}{2}}}{x}$

(xi) $\lim\limits_{x \to 1} \dfrac{(\ell n\ (1 + x) - \ell n 2)(3.4^{x-1} - 3x)}{[(7 + x)^{\frac{1}{3}} - (1 + 3x)^{\frac{1}{2}}].\sin(x - 1)}$

B-3. Evaluate the following limits.

(i) $\lim\limits_{x \to \infty} \left(\dfrac{1}{x^2} + \dfrac{2}{x^2} + ... + \dfrac{x}{x^2} \right)$

(ii) $\lim\limits_{n \to \infty} \dfrac{\sqrt{n^3 - 2n^2 + 1} + \sqrt[3]{n^4 + 1}}{\sqrt[4]{n^6 + 6n^5 + 2} - \sqrt[5]{n^7 + 3n^3 + 1}}$, $n \in N$

(iii) $\lim\limits_{x \to \infty} \left(\sqrt{x^2 - 8x} + x \right)$

(iv) $\lim\limits_{x \to -\infty} \dfrac{x^5 \tan\left(\dfrac{1}{\pi x^2} \right) + 3\ |x|^2 + \ 7}{|x|^3 + 7|x| + \ 8}$

B-4. Evaluate the following limits.

(i) $\lim\limits_{x \to \infty} \left[(x + 1)^{\frac{2}{3}} - (x - 1)^{\frac{2}{3}} \right]$

(ii) $\lim\limits_{x \to a} \dfrac{(x + 2)^{\frac{5}{2}} - (a + 2)^{\frac{5}{2}}}{x - a}$

(iii) $\lim\limits_{x \to \infty} \cos \left(\sqrt{x + 1} \right) - \cos \left(\sqrt{x} \right)$

(iv) $\lim\limits_{x \to \infty} \left[((x + 1)\ (x + 2)\ (x + 3)\ (x + 4))^{\frac{1}{4}} - \ x \right]$

B-5. Evaluate the following limits using expansions :

(i) $\lim\limits_{x \to 2} \dfrac{(x + 2)^{\frac{1}{2}} - (15x + 2)^{\frac{1}{5}}}{(7x + 2)^{\frac{1}{4}} - x}$

(ii) $\lim\limits_{x \to 0} \dfrac{e^x - 1 - \sin x - \dfrac{\tan^2 x}{2}}{x^3}$

B-6 If $\lim\limits_{x \to 0} \dfrac{a + b\sin x - \cos x + ce^x}{x^3}$ exists, find the values of a, b, c. Also find the limit

B-7. Find the values of a and b so that:

(i) $\displaystyle\lim_{x \to 0} \frac{1 + \; a \, x \; \sin x \; - b \; \cos x}{x^4}$ may have a finite limit.

(ii)✍ $\displaystyle\lim_{x \to \infty} \left(\sqrt{x^4 + ax^3 + 3x^2 + bx + 2} - \sqrt{x^4 + 2x^3 - cx^2 + 3x - d} \right)$ $= 4$

(iii) $\displaystyle\lim_{x \to 0} \frac{axe^x - \; b \; \ell n \; (1 + x) \; + \; cxe^{-x}}{x^2 \sin x} \; = \; 2$

B-8. Find the following limit using expansion $\displaystyle\lim_{x \to 0} \left(\frac{\ell n \; (1+x)^{(1+x)}}{x^2} - \frac{1}{x} \right)$:

B-9.✍ Prove that $\displaystyle\lim_{x \to 4} \frac{(\cos \alpha)^x \; - \; (\sin \alpha)^x \; - \; \cos 2\alpha}{x - 4} = \cos^4 \alpha \; \square n \, (\cos \alpha) - \sin^4 \alpha \; \square n \, (\sin \alpha), \; \alpha \in \left(0 , \dfrac{\pi}{2} \right)$

B-10_. Find the value of $\displaystyle\lim_{h \to 0} \frac{\tan(a + 2h) - 2\tan(a + h) + \tan a}{h^2}$.

SECTION (C) : Limit of form 0^0, ∞^0 , 1^∞ , $\displaystyle\lim_{x \to \infty} \frac{x}{e^x}$, $\displaystyle\lim_{x \to \infty} \frac{\ell n \; x}{x}$, Sandwitch theorem and Miscellaneous problems on limits.

C-1 Evaluate the following limits :

(i)✍ $\displaystyle\lim_{x \to 0^+} \left(x \right)^{x^2}$ (ii) $\displaystyle\lim_{x \to \frac{\pi}{2}^-} (\tan x)^{\cos x}$

(iii) $\displaystyle\lim_{x \to 1^-} ([x])^{1-x}$, where [.] denotes greatest integer function

(iv) $\displaystyle\lim_{x \to \frac{\pi}{2}^+} e^{\tan x}$

C-2. Evaluate the following limits :

(i) $\displaystyle\lim_{x \to \frac{\pi}{4}} (\tan x)^{\tan 2x}$ (ii)✍ $\displaystyle\lim_{x \to \infty} \left(\frac{1 + 2x}{1 + 3x} \right)^x$

(iii) $\displaystyle\lim_{x \to 1} \left(1 + \ell n x \right)^{\sec \frac{\pi x}{2}}$ (iv) $\displaystyle\lim_{x \to 0} \left(\tan\left(\frac{\pi}{4} + x \right) \right)^{\frac{1}{x}}$

C-3.✍ If $\displaystyle\lim_{x \to 1} \left(1 + ax + bx^2 \right)^{\frac{c}{x-1}} = e^3$, then find conditions on a, b and c.

C-4. Evaluate following limits :

(i)✍ $\displaystyle\lim_{x \to \infty} \frac{x \; \; \ell n \; \left(1 + \dfrac{\ell n \; x}{x} \right)}{\ell n \; \; x}$ (ii) $\displaystyle\lim_{x \to \infty} \frac{e^x \sin\left(\dfrac{x^n}{e^x} \right)}{x^n}$

C-5.✍ Evaluate $\displaystyle\lim_{n \to \infty} \frac{[1 \; . \; 2x] + [2 \; . \; 3x] + ... + [n \; . \; (n+1) \; x]}{n^3}$, where [.] denotes greatest integer function.

C-6. If $f(x) = \displaystyle\lim_{n \to \infty} \frac{x^{2n} - 1}{x^{2n} + 1}$, $n \in N$ find range of $f(x)$.

Section (D) : Continuity at a point

D-1. Determine the values of a, b & c for which the function f (x) =
$$\begin{cases} \dfrac{\sin (a + 1)\, x\, +\, \sin x}{x} & \text{for } x < 0 \\ c & \text{for } x = 0 \\ \dfrac{(x + bx^2)^{1/2} - x^{1/2}}{b\, x^{3/2}} & \text{for } x > 0 \end{cases}$$

is continuous at x = 0.

D-2. Find the values of ' a ' & ' b ' so that the function, f (x) =
$$\begin{cases} \dfrac{1 - \sin^3 x}{3\,\cos^2 x} & , \ x < \pi/2 \\ a & , \ x = \pi/2 \\ \dfrac{b\ (1 - \sin x)}{(\pi - 2x)^2} & , \ x > \pi/2 \end{cases}$$ is continuous at

x = π/2.

D-3. If f(x) = {x} & g(x) = [x] (where {. } & [.] denotes the fractional part and the integral part functions respectively), then discuss the continuity of :
(i)   h(x) = f(x). g(x) at x = 1 and 2   (ii)   h(x) = f(x) + g(x) at x = 1
(iii)   h(x) = f(x) − g(x) at x = 1   (iv)   h(x) = g(x) + $\sqrt{f(x)}$ at x = 1 and 2

D-4. Suppose that f(x) = x³ − 3x² − 4x + 12 and h(x) = $\begin{cases} \dfrac{f(x)}{x - 3} & , \ x \neq 3 \\ K & , \ x = 3 \end{cases}$, then

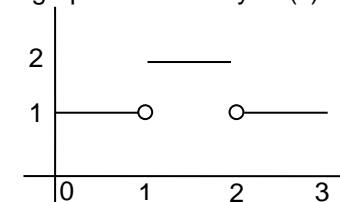(a)   find all zeros of f
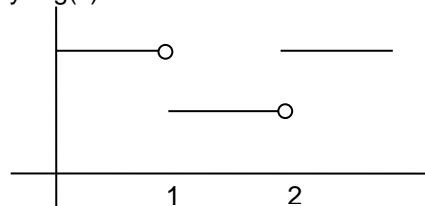(b)   find the value of K that makes h continuous at x = 3
(c)   using the value of K found in (b), determine whether h is an even function.

D-5. If f(x) = $\dfrac{\sin 3x\ +\ A\ \sin\ 2x\ +\ B\ \sin\ x}{x^5}$ (x ≠ 0) is continuous at x = 0. Find A & B. Also find f (0).

D-6_. If graph of function y = f(x) is

and graph of function
y = g(x)is

then discuss the continuity of f(x) g(x) at x = 3 and x = 2.

Section (E) : Continuity in an interval, Continuity of composite functions, IMVT

E-1. Find interval for which the function given by the following expressions are continuous :

(i)   f(x) = $\dfrac{3x + 7}{x^2 - 5x + 6}$ (ii)   f(x) = $\dfrac{1}{|x| - 1} - \dfrac{x^2}{2}$

(iii)   f(x) = $\dfrac{\sqrt{x^2 + 1}}{1 + \sin^2 x}$ (iv)   f(x) = $\tan\left(\dfrac{\pi x}{2}\right)$

E-2. If f(x) = x + {– x} + [x], where [.] is the integral part & { . } is the fractional part function. Discuss the continuity of f in [–2, 2]. Also find nature of each discontinuity.

E-3. If f(x) = $\dfrac{x^2 + 1}{x^2 - 1}$ and g(x) = tan x, then discuss the continuity of fog (x).

E-4. Let f(x) = $\begin{bmatrix} 1 + x & , & 0 \le x \le 2 \\ 3 - x & , & 2 < x \le 3 \end{bmatrix}$. Determine the composite function g(x) = f (f(x)) & hence find the point of discontinuity of g , if any.

E-5. Find the point of discontinuity of y = f(u), where f(u) = $\dfrac{3}{2u^2 + 5u - 3}$ and u = $\dfrac{1}{x + 2}$.

E-6. Show that the function f(x) = $\dfrac{x^3}{4}$ – sin π x + 3 takes the value $\dfrac{7}{3}$ within the interval

[–2, 2].

E-7_. If g(x) = (|x – 1| + |4x – 11|) [x² – 2x – 2], then find the number of point of discontinuity of g(x) in $\left(\dfrac{1}{2}, \dfrac{5}{2} \right)$

{where [.] denotes GIF}

Section (F) : Derivability at a point

F-1. Test the continuity & differentiability of the function defined as under at x = 1 & x = 2.

$$f(x) = \begin{cases} x & ; & x < 1 \\ 2 - x & ; & 1 \le x \le 2 \\ -2 + 3x - x^2 & ; & x > 2 \end{cases}$$

F-2. A function f is defined as follows: f(x) = $\begin{cases} 1 & \text{for} & -\infty < x < 0 \\ 1 + \sin x & \text{for} & 0 \le x < \dfrac{\pi}{2} \\ 2 + \left(x - \dfrac{\pi}{2} \right)^2 & \text{for} & \dfrac{\pi}{2} \le x < \infty \end{cases}$

Discuss the continuity & differentiability at x = 0 & x = π/2.

F-3. Prove that f (x) = |x| cos x is not differentiable at x = 0

F-4. Show that the function f (x) = $\begin{cases} x^m \sin \left(\dfrac{1}{x} \right) & ; & x > 0 \\ 0 & ; & x = 0 \end{cases}$ is,

(i)   differentiable at x = 0, if m > 1.
(ii)   continuous but not differentiable at x = 0, if 0 < m ≤ 1.
(iii)   neither continuous nor differentiable, if m ≤ 0.

F-5. Examine the differentiability of f (x) = $\sqrt{1 - e^{-x^2}}$ at x = 0.

F-6. If f(x) = $\begin{cases} ax^2 - b & \text{if } |x| < 1 \\ -\dfrac{1}{|x|} & \text{if } |x| \ge 1 \end{cases}$ is derivable at x = 1. Find the values of a & b.

Section (G) : Derivability in an interval

G-1. Draw a graph of the function, $y = [x] + |1 - x|$, $-1 \le x \le 3$. Determine the points, if any, where this function is not differentiable, where $[\cdot]$ denotes the greatest integer function.

G-2. Discuss the continuity & derivability of $f(x) = \begin{cases} \left| x - \dfrac{1}{2} \right| & ; \ 0 \le x < 1 \\ x \cdot [x] & ; \ 1 \le x \le 2 \end{cases}$

where $[x]$ indicates the greatest integer x.

G-3. Discuss continuity and differentiability of $y = f(x)$ in $[-2, 5]$ where [.] denotes GIF & {.} denotes FPF

$$f(x) = \begin{cases} [x] & , \ x \in [-2, \ 0] \\ \{x\} & , \ x \in (0, \ 2) \\ \dfrac{x^2}{4} & , \ x \in [2, \ 3) \\ \dfrac{1}{\log_4(x-3)} & , \ x \in [3, \ 5] \end{cases}$$

G-4. Check differentiability of $f(x) = \text{sgn}(x^{2/3}) + \left[\cos\left(\dfrac{x^2}{1+x^2} \right) \right] + |x - 1|^{5/3}$ in $[-2, 2]$ where [.] denotes GIF.

G-5_. Discuss the continuity and differentiability of $h(x) = f(x)\, g(x)$ in $(0, 3)$ if

$$f(x) = \frac{e^x - e}{[x] + 1} \ \{\text{where [.] denot GIF}\} \quad \text{and} \quad g(x) = \begin{cases} \dfrac{|x-1| + |x-2|}{2} & , \ x \in (0, 1) \\ |x-1| + |x-2| & , \ x \in [1, 2) \\ \dfrac{3(|x-1| + |x-2|)}{2} & , \ x \in [2, 3) \end{cases}$$

Section (H) : Functional equations and Miscellaneous

H-1. If $f : R \to R$ satisfies $f(x + y) = f(x) + f(y)$, for all $x, y \in R$ and $f(1) = 2$, then $\displaystyle\sum_{r=1}^{7} f(r)$ is :

H-2. If $f'(2) = 4$ then, evaluate $\displaystyle\lim_{x \to 0} \frac{f(1 + \cos x) - f(2)}{\tan^2 x}$ ..

H-3. Let a function $f : R \to R$ be given by $f(x + y) = f(x)\, f(y)$ for all $x, y \in R$ and $f(x) \ne 0$ for any $x \in R$. If the function $f(x)$ is differentiable at $x = 0$, show that $f'(x) = f'(0)\, f(x)$ for all $x \in R$. Also, determine $f(x)$.

H-4. Let $f(x)$ be a polynomial function satisfying the relation $f(x) \cdot f\left(\dfrac{1}{x}\right) = f(x) + f\left(\dfrac{1}{x}\right) \ \forall \ x \in R - \{0\}$ and $f(3) = -26$. Determine $f'(1)$.

H-5. Let function $f(x)$ satisfying the relation $f(x + y) + f(x - y) = 2f(x) \cdot f(y)$, then prove that it is even function

H-6. Let $f(x)$ be a bounded function. $L_1 = \displaystyle\lim_{x \to \infty} (f'(x) - \lambda f(x))$ and $L_2 = \displaystyle\lim_{x \to \infty} f(x)$ where $\lambda > 0$. If L_1, L_2 both exist and $L_1 = L$, then prove that $L_2 = -\dfrac{L}{\lambda}$.

H-7. Let R be the set of real numbers and $f: R \to R$ be such that for all x & y in R $|f(x) - f(y)| \le |x - y|^3$. Prove that $f(x)$ is constant.

PART - II : ONLY ONE OPTION CORRECT TYPE

Section (A) : Definition of LHL/RHL and Indeterminate forms

A-1. $\lim\limits_{x \to 0} \sin^{-1}((\sec x))$ is equal to

(A) $\dfrac{\pi}{2}$ 　　　　(B) 1 　　　　(C) zero 　　　　(D) none of these

A-2. Consider the following statements :

S₁ : $\lim\limits_{x \to 0^-} \dfrac{[x]}{x}$ is an indeterminate form (where [.] denotes greatest integer function).

S₂ : $\lim\limits_{x \to \infty} \dfrac{\sin (3^x)}{3^x} = 0$

S₃ : $\lim\limits_{x \to \infty} \sqrt{\dfrac{x - \sin x}{x + \cos^2 x}}$ does not exist.

S₄ : $\lim\limits_{n \to \infty} \dfrac{(n+2)! + (n+1)!}{(n+3)!}$ $(n \in N) = 0$

S₄ : $\lim\limits_{n \to \infty} \dfrac{(n+2)! + (n+1)!}{(n+3)!}$ $(n \in N) = 0$

State, in order, whether S_1, S_2, S_3, S_4 are true or false
(A) FTFT 　　　　(B) FTTT 　　　　(C) FTFF 　　　　(D) TTFT

A-3. $\lim\limits_{x \to 1} (1 - x + [x - 1] + [1 - x])$ is equal to (where [.] denotes greatest integer function)

(A) 0 　　　　(B) 1 　　　　(C) -1 　　　　(D) does not exist

A-4_. $\lim\limits_{x \to 0^-} \dfrac{\cos^{-1}(\cos x)}{\sin^{-1}(\sin x)}$ is equal to :

(A) 0 　　　　(B) 1 　　　　(C) –1 　　　　(D) Does not exist

SECTION (B) : Evaluation of limits of form 0/0, ∞/∞, ∞ − ∞, 0 × ∞, Use of L-Hospital Rule & Expansion

B-1. $\lim\limits_{x \to 3} \dfrac{(x^3 + 27) \; \ln \; (x - 2)}{(x^2 - 9)}$ is equal to

(A) -8 　　　　(B) 8 　　　　(C) 9 　　　　(D) -9

B-2. $\lim\limits_{x \to 0} \dfrac{(4^x - 1)^3}{\sin \left(\dfrac{x}{p} \right) \; \ln \left(1 + \dfrac{x^2}{3} \right)}$ is equal to

(A) 9 p (□n 4) 　　(B) 3 p (□n 4)³ 　　(C) 12 p (□n 4)³ 　　(D) 27 p (□n 4)²

B-3. $\lim\limits_{x \to 2} \dfrac{\sin \left(e^{x-2} - 1 \right)}{\ln \; (x - 1)}$ is equal to

(A) 0 　　　　(B) -1 　　　　(C) 2 　　　　(D) 1

B-4. The value of $\displaystyle\lim_{x \to 0} \frac{\sin(\ell n(1+x))}{\ell n(1+\sin x)}$ is equal to

(A) 0   (B) $\dfrac{1}{2}$ (C) $\dfrac{1}{4}$ (D) 1

B-5. $\displaystyle\lim_{x \to 1} \frac{\sqrt{1-\cos 2(x-1)}}{x-1}$

(A) exists and it equals $\sqrt{2}$

(B) exists and it equals $-\sqrt{2}$

(C) does not exist because $x-1 \to 0$

(D) does not exist because left hand limit is not equal to right hand limit.

B-6. The value of $\displaystyle\lim_{x \to 0} \frac{\sqrt[3]{1+x^2} - \sqrt[4]{1-2x}}{x+x^2}$ is equal to

(A) $\dfrac{1}{2}$ (B) 1   (C) –1   (D) $-\dfrac{1}{2}$

B-7. The value of $\displaystyle\lim_{x \to 0} \frac{\sqrt{2} - \sqrt{1+\cos x}}{\sin^2 x}$ is equal to

(A) $\dfrac{\sqrt{2}}{2}$ (B) $\dfrac{\sqrt{2}}{4}$ (C) $\dfrac{-\sqrt{2}}{8}$ (D) $\dfrac{\sqrt{2}}{8}$

B-8. $\displaystyle\lim_{x \to 0^+} \frac{\cos^{-1}(1-x)}{\sqrt{x}}$ is equal to

(A) $\dfrac{1}{\sqrt{2}}$ (B) $\sqrt{2}$ (C) 1   (D) 0

B-9. $\displaystyle\lim_{x \to 1} \frac{\left(\displaystyle\sum_{k=1}^{100} x^k\right) - 100}{x-1}$ is equal to

(A) 0   (B) 5050   (C) 4550   (D) -5050

B-10. $\displaystyle\lim_{x \to \infty} \frac{x^3 \sin \dfrac{1}{x} + x + 1}{x^2 + x + 1}$ is equal to

(A) 0   (B) $\dfrac{1}{2}$ (C) 1   (D) none of these

B-11. $\displaystyle\lim_{x \to -\infty} \frac{x^2 \sin\left(\dfrac{1}{x}\right)}{\sqrt{9x^2 + x + 1}}$ is equal to

(A) $\dfrac{1}{3}$ (B) $-\dfrac{1}{3}$ (C) 0   (D) does not exist

B-12. $\displaystyle\lim_{n \to \infty} \frac{5^{n+1} + 3^n - 2^{2n}}{5^n + 2^n + 3^{2n+3}}$, $n \in N$ is equal to

(A) 5   (B) 3   (C) 1   (D) zero

B-13. $\lim\limits_{n \to \infty} n \cos\left(\dfrac{\pi}{4n}\right) \sin\left(\dfrac{\pi}{4n}\right)$, $n \in N$ is equal to:

(A) $\dfrac{\pi}{3}$ 　　　　　(B) $\dfrac{\pi}{4}$ 　　　　　(C) $\dfrac{\pi}{6}$ 　　　　　(D) none of these

B-14. $\lim\limits_{x \to \frac{\pi}{2}} \left[\dfrac{x - \dfrac{\pi}{2}}{\cos x}\right]$ is equal to (where [.] represents greatest integer function)

(A) -1 　　　　　(B) 0 　　　　　(C) -2 　　　　　(D) does not exist

B-15. $\lim\limits_{n \to \infty} \dfrac{-3n + (-1)^n}{4n - (-1)^n}$ is equal to ($n \in N$)

(A) $-\dfrac{3}{4}$ 　　　　　　　　(B) $-\dfrac{3}{4}$ if n is even ; $\dfrac{3}{4}$ if n is odd

(C) not exist if n is even ; $-\dfrac{3}{4}$ if n is odd 　　　(D) 1 if n is even ; does not exist if n is odd

B-16_. $\lim\limits_{x \to 1}\left(\dfrac{2}{1 - x^2} + \dfrac{1}{x - 1}\right)$ is equal to :

(A) $\dfrac{1}{2}$ 　　　　　(B) $-\dfrac{1}{2}$ 　　　　　(C) -1 　　　　　(D) Does not exist

B-17. $\lim\limits_{x \to \infty}\left(x - x^2 \ln\left(1 + \dfrac{1}{x}\right)\right)$ is equal to :

(A) $\dfrac{1}{2}$ 　　　　　(B) $\dfrac{3}{2}$ 　　　　　(C) $\dfrac{1}{3}$ 　　　　　(D) 1

B-18. $\lim\limits_{x \to 0} \dfrac{e^{-\frac{x^2}{2}} - \cos x}{x^3 \sin x}$ is equal to

(A) $\dfrac{1}{4}$ 　　　　　(B) $\dfrac{1}{6}$ 　　　　　(C) $\dfrac{1}{12}$ 　　　　　(D) $\dfrac{1}{8}$

B-19. $\lim\limits_{x \to 0} \dfrac{\sin(6x^2)}{\ln \cos(2x^2 - x)}$ is equal to

(A) 12 　　　　　(B) -12 　　　　　(C) 6 　　　　　(D) -6

B-20_. $\lim\limits_{h \to 0} \dfrac{\sin(a + 3h) - 3\sin(a + 2h) + 3\sin(a + h) - \sin a}{h^3}$ is equal to :

(A) cosa 　　　　　(B) −cosa 　　　　　(C) sina 　　　　　(D) sina cosa

SECTION (C) : Limit of form 0^0, ∞^0, 1^∞, $\displaystyle\lim_{x\to\infty} \frac{x}{e^x}$, $\displaystyle\lim_{x\to\infty} \frac{\ell n\ x}{x}$, Sandwitch theorem and Miscellaneous problems on limits.

C-1. $\displaystyle\lim_{x\to\infty} \left(\frac{x+2}{x-2}\right)^{x+1}$ is equal to

(A) e^4 (B) e^{-4} (C) e^2 (D) none of these

C-2. $\displaystyle\lim_{x\to 0^+} \left(1+\tan^2\sqrt{x}\right)^{\frac{5}{x}}$ is equal to

(A) e^5 (B) e^2 (C) e (D) none of these

C-3. The value of $\displaystyle\lim_{x\to \frac{\pi}{4}} (1+[x])^{\frac{1}{\ell n\ (\tan x)}}$ is equal to (where [.] denotes the greatest integer function)

(A) 0   (B) 1   (C) e (D) e^{-1}

C-4. $\displaystyle\lim_{x\to\infty} \left(\frac{x^2-2x+1}{x^2-4x+2}\right)^x$ is equal to

(A) 1   (B) 2   (C) e^2 (D) e

C-5. The limiting value of $(\cos x)^{\frac{1}{\sin x}}$ at $x = 0$ is:

(A) 1   (B) e (C) 0   (D) none of these

C-6. $\displaystyle\lim_{x\to a} \left(2-\frac{a}{x}\right)^{\tan\left(\frac{\pi x}{2a}\right)}$ is equal to

(A) $e^{-\frac{a}{\pi}}$ (B) $e^{-\frac{2a}{\pi}}$ (C) $e^{-\frac{2}{\pi}}$ (D) 1

C-7. $\displaystyle\lim_{n\to\infty} \left(\cos\frac{x}{\sqrt{n}}\right)^n$ is

(A) e^{-x^2} (B) $e^{\frac{-x^2}{2}}$ (C) e^{x^2} (D) $e^{\frac{x^2}{2}}$

C-8. If [x] denotes greatest integer less than or equal to x, then $\displaystyle\lim_{n\to\infty} \frac{1}{n^4} \left([1^3\ x] + [2^3\ x] +...+ [n^3\ x] \right)$ is equal to

(A) $\dfrac{x}{2}$ (B) $\dfrac{x}{3}$ (C) $\dfrac{x}{6}$ (D) $\dfrac{x}{4}$

Section (D) : Continuity at a point

D-1. A function f(x) is defined as below f(x) = $\dfrac{\cos(\sin x)-\cos x}{x^2}$, $x \neq 0$ and f(0) = a

f(x) is continuous at x = 0 if 'a' equals

(A) 0   (B) 4   (C) 5   (D) 6

D-2. Let f(x) = $\left|\left(x+\dfrac{1}{2}\right)[x]\right|$, when $-2 \leq x \leq 2$. where [.] represents greatest integer function. Then

(A) f(x) is continuous at x = 2   (B) f(x) is continuous at x = 1

(C) f(x) is continuous at x = -1 (D) f(x) is discontinuous at x = 0

D-3. The function f(x) is defined by $f(x) = \begin{cases} \log_{(4x-3)}\left(x^2 - 2x + 5\right) & , \quad \text{if} \quad \frac{3}{4} < x < 1 \quad \text{or} \quad x > 1 \\ 4 & , \qquad\qquad \text{if} \quad x = 1 \end{cases}$

(A) is continuous at x = 1
(B) is discontinuous at x = 1 since f(1⁺) does not exist though f(1⁻) exists
(C) is discontinuous at x = 1 since f(1⁻) does not exist though f(1⁺) exists
(D) is discontinuous since neither f(1⁻) nor f(1⁺) exists.

D-4_. If $f(x) = x \sin\left(\frac{\pi}{2}(x + 2[x])\right)$, then f(x)is {where [.] denotes GIF}

(A) Discontinuous at x = 2   (B) Discontinuous at x = 1
(C) Continuous at x = 1   (D) Continuous at x = 3

Section (E) : Continuity in an interval, Continuity of composite functions, IMVT

E-1. $f(x) = \begin{cases} \dfrac{\sqrt{(1 + px)} - \sqrt{(1 - px)}}{x} & , \quad -1 \le x < 0 \\ \dfrac{2x + 1}{x - 2} & , \quad 0 \le x \le 1 \end{cases}$ is continuous in the interval $[-1, 1]$, then 'p' is

equal to:
(A) − 1   (B) − 1/2   (C) 1/2   (D) 1

E-2. Let f(x) = Sgn(x) and g(x) = x (x² − 5x + 6). The function f(g(x)) is discontinuous at
(A) infinitely many points   (B) exactly one point
(C) exactly three points   (D) no point

E-3. If $y = \dfrac{1}{t^2 + t - 2}$ where $t = \dfrac{1}{x - 1}$, then the number of points of discontinuities of y = f(x), x ∈ R is
(A) 1   (B) 2   (C) 3   (D) infinite

E-4. The equation 2 tan x + 5x − 2 = 0 has
(A) no solution in [0, π/4]   (B) at least one real solution in [0, π/4]
(C) two real solution in [0, π/4]   (D) None of these

Section (F) : Derivability at a point

F-1. If $f(x) = x\left(\sqrt{x} - \sqrt{x+1}\right)$, then indicate the correct alternative(s):

(A)   f(x) is continuous but not differentiable at x = 0
(B)   f(x) is differentiable at x = 0
(C)   f(x) is not differentiable at x = 0
(D)   none

F-2. If $f(x) = \begin{cases} \dfrac{x(3e^{1/x} + 4)}{2 - e^{1/x}} & , \quad x \ne 0 \\ 0 & , \quad x = 0 \end{cases}$, then f(x) is

(A) continuous as well differentiable at x = 0
(B) continuous but not differentiable at x = 0
(C) neither differentiable at x = 0 nor continuous at x = 0
(D) none of these

F-3. If f(x) = $\dfrac{x}{\sqrt{x+1}-\sqrt{x}}$ be a real valued function, then

(A) f(x) is continuous, but f′(0) does not exist (B) f(x) is differentiable at x = 0
(C) f(x) is not continuous at x = 0 (D) f(x) is not differentiable at x = 0

F-4. The function f(x) = $\sin^{-1}(\cos x)$ is:
(A) discontinuous at x = 0 (B) continuous at x = 0
(C) differentiable at x = 0 (D) none of these

F-5. If f(x) = $\begin{cases} x + \{x\} + x\sin\{x\} & \text{for } x \neq 0 \\ 0 & \text{for } x = 0 \end{cases}$, where { . } denotes the fractional part function, then:

(A) f is continuous & differentiable at x = 0 (B) f is continuous but not differentiable at x = 0
(C) f is continuous & differentiable at x = 2 (D) none of these.

F-6. Given f(x) = $\begin{cases} \log_a\left(a\,|[x] + [-x]|\right)^x \left(\dfrac{a^{\left(\frac{2}{[x]+[-x]}-5\right)}}{3 + a^{\frac{1}{|x|}}}\right) & \text{for } |x| \neq 0 \; ; \; a > 1 \\ \\ 0 & \text{for } x = 0 \end{cases}$

where [.] represents the integral part function, then:
(A) f is continuous but not differentiable at x = 0
(B) f is continuous & differentiable at x = 0
(C) the differentiability of 'f' at x = 0 depends on the value of a
(D) f is continuous & differentiable at x = 0 and for a = e only.

F-7. If f (x) = $\begin{cases} \dfrac{x^2 - 1}{x^2 + 1} & , \; 0 < x \leq 2 \\ \dfrac{1}{4}\left(x^3 - x^2\right) & , \; 2 < x \leq 3 \\ \dfrac{9}{4}\left(|x - 4| + |2 - x|\right) & , \; 3 < x < 4 \end{cases}$, then:

(A) f (x) is differentiable at x = 2 & x = 3 (B) f (x) is non-differentiable at x = 2 & x = 3
(C) f (x) is differentiable at x = 3 but not at x = 2 (D) f (x) is differentiable at x = 2 but not at x = 3.

Section (G) : Derivability in an interval

G-1. The set of all points where the function f(x) = $\dfrac{x}{1 + |x|}$ is differentiable is:

(A) $(-\infty, \infty)$ (B) $[\,0, \infty\,)$ (C) $(-\infty, 0) \cup (0, \infty)$ (D) $(0, \infty)$

G-2. If f (x) is differentiable everywhere, then :
(A) $|f|$ is differentiable everywhere (B) $|f|^2$ is differentiable everywhere
(C) $f\,|f|$ is not differentiable at some point (D) $f + |f|$ is differentiable everywhere

G-3. Let f (x) be defined in $[-2, 2]$ by

f (x) = $\begin{cases} \max\left(\sqrt{4-x^2}, \sqrt{1+x^2}\right) & , \; -2 \leq x \leq 0 \\ \min\left(\sqrt{4-x^2}, \sqrt{1+x^2}\right) & , \; 0 < x \leq 2 \end{cases}$, then f (x) :

(A) is continuous at all points
(B) is not continuous at more than one point .
(C) is not differentiable only at one point
(D) is not differentiable at more than one point

G-4. The number of points at which the function $f(x) = \max.\{a - x, a + x, b\}$, $-\infty < x < \infty$, $0 < a < b$ cannot be differentiable is:

    (A) 1   (B) 2   (C) 3   (D) none of these

G-5. Let $f(x) = x - x^2$ and $g(x) = \begin{cases} \max \ f(t), \ 0 \le t \le x, \ 0 \le x \le 1 \\ \sin \ \pi x, \ x > 1 \end{cases}$, then in the interval $[0, \infty)$

    (A) g(x) is everywhere continuous except at two points
    (B) g(x) is everywhere differentiable except at two points
    (C) g(x) is everywhere differentiable except at x = 1
    (D) none of these

G-6. Consider the following statements :

    S$_1$: Number of points where $f(x) = |\, x \, \text{sgn} \, (1 - x^2) \,|$ is non-differentiable is 3.

    S$_2$: Defined $f(x) = \begin{bmatrix} a\sin\dfrac{\pi}{2}(x+1) \ , \ & x \le 0 \\ \dfrac{\tan x - \sin x}{x^3} \ , \ & x > 0 \end{bmatrix}$, In order that f(x) be continuous at x = 0, 'a' should be

        equal to $\ \dfrac{1}{2}$

    S$_3$: The set of all points, where the function $\sqrt[3]{x^2 \, |\, x\,|}$ is differentiable is $(-\infty, 0) \cup (0, \infty)$

    S$_4$: Number of points where $f(x) = \dfrac{1}{\sin^{-1}(\sin x)}$ is non-differentiable in the interval $(0, 3\pi)$ is 3.

    State, in order, whether S_1, S_2, S_3, S_4 are true or false

    (A) TTTF   (B) TTTT   (C) FTTF   (D) TFTT

G-7. Consider the following statements :

    S$_1$: Let $f(x) = \dfrac{\sin \ (\pi \ [x - \pi])}{1 + [x]^2}$, where $[\ .\ ]$ stands for the greatest integer function. Then f(x) is

        discontinuous at $x = n + \pi, n \in I$

    S$_2$: The function $f(x) = p[x + 1] + q\,[x - 1]$, (where [.] denotes the greatest integer function) is continuous at x = 1 if p + q = 0

    S$_3$: Let $f(x) = |[x]\, x|$ for $-1 \le x \le 2$, where [.] is greatest integer function, then f is not differentiable at x = 2.

    S$_4$: If f(x) takes only rational values for all real x and is continuous, then f'(10) = 10.

    (A) FTTT   (B) TTTF   (C) FTTF   (D) FFTF

G-8. For what triplets of real numbers (a, b, c) with $a \ne 0$ the function

    $f(x) = \begin{cases} x \ , \ & x \le 1 \\ ax^2 + bx + c \ , \ & \text{otherwise} \end{cases}$ is differentiable for all real x?

    (A) $\{(a, 1-2a, a)\,|\, a \in R, a \ne 0 \}$ (B) $\{(a, 1-2a, c)\,|\, a, c \in R, a \ne 0 \}$
    (C) $\{(a, b, c)\,|\, a, b, c \in R, a + b + c = 1 \}$ (D) $\{(a, 1-2a, 0)\,|\, a \in R, a \ne 0\}$

Section (H) : Functional Equations and Miscellaneous

H-1. Given that $f'(2) = 6$ and $f'(1) = 4$, then $\displaystyle\lim_{h\to 0} \dfrac{f(2h + 2 + h^2) - f(2)}{f(h - h^2 + 1) - f(1)} \ =$

    (A) does not exist   (B) is equal to –3/2   (C) is equal to 3/2   (D) is equal to 3

H-2. If $f(x + y) = f(x) \cdot f(y)$, $\forall$ x & y $\in$ N and f(1) = 2, then the value of $\displaystyle\sum_{n=1}^{10} f(n)$ is

(A) 2036   (B) 2046   (C) 2056   (D) 2066

H-3. If f(1) = 1 and f(n + 1) = 2f(n) + 1 if n $\geq$ 1, then f(n) is equal to

(A) $2^n + 1$ (B) 2^n (C) $2^n - 1$ (D) $2^{n-1} - 1$

H-4. If y = f(x) satisfies the condition $f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2}$ (x $\neq$ 0), then f(x) is equal to

(A) $- x^2 + 2$ (B) $- x^2 - 2$

(C) $x^2 - 2$, x $\in$ R $-$ {0}   (D) $x^2 - 2$, |x| $\in$ [2, ∞)

H-5. A function f : R $\rightarrow$ R satisfies the condition $x^2 f(x) + f(1 - x) = 2x - x^4$. Then f(x) is:

(A) $- x^2 - 1$ (B) $- x^2 + 1$ (C) $x^2 - 1$ (D) $- x^4 + 1$

H-6. If f: R $\rightarrow$ R be a differentiable function, such that f(x + 2y) = f(x) + f(2y) + 4xy $\forall$ x, y $\in$ R. then

(A) f′(1) = f′(0) + 1   (B) f′(1) = f′(0) – 1   (C) f′(0) = f′(1) + 2   (D) f′(0) = f′(1) – 2

PART - III : MATCH THE COLUMN

1.   Let [**.**] denotes the greatest integer function.

Column – I   **Column – II**

(A)   If P(x) = [2 cos x], x $\in$ [$-\pi$, π], then P(x)   (p)   is discontinuous at exactly 7 points

(B)   If Q(x) = [2 sin x], x $\in$ [$-\pi$, π], then Q(x)   (q)   is discontinuous at exactly 4 points

(C)   If R(x) = [2 tan x/2], x $\in \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$, then R(x)   (r)   is non differentiable at some points

(D)   If S(x) = $\left[3\cosec \dfrac{x}{3}\right]$, x $\in \left[\dfrac{\pi}{2}, 2\pi\right]$, then S(x) (s)   is continuous at infinitely many values

2.   **Column – I**   **Column – II**
(A)   f(x) = |x³| is   (p) continuous in (–1, 1)

(B)   f(x) = $\sqrt{|x|}$ is   (q) differentiable in (–1, 1)

(C)   f(x) = |sin⁻¹ x| is   (r) differentiable in (0, 1)

(D)   f(x) = cos⁻¹ |x| is   (s) not differentiable atleast at one point in (–1, 1)

Exercise-2

PART - I : ONLY ONE OPTION CORRECT TYPE

1. $\lim\limits_{x \to a^-} \left(\dfrac{|x|^3}{a} - \left[\dfrac{x}{a}\right]^3 \right)$ (a < 0), where [x] denotes the greatest integer less than or equal to x, is equal to

(A) $a^2 + 1$ (B) $-a^2 - 1$ (C) a^2 (D) $-a^2$

2. $\lim\limits_{n \to \infty} \cos\dfrac{x}{2}\cos\dfrac{x}{2^2}\cos\dfrac{x}{2^3}\cos\dfrac{x}{2^4}.......\cos\dfrac{x}{2^n}$ is equal to (x ≠ 0)

(A) 1   (B) −1   (C) $\dfrac{\sin x}{x}$ (D) $\dfrac{x}{\sin x}$

3.✎ $\lim\limits_{\theta \to 0} \left(\left[\dfrac{n\sin\theta}{\theta}\right] + \left[\dfrac{n\tan\theta}{\theta}\right] \right)$, where [.] represents greatest integer function and n ∈ N, is equal to

(A) 2n   (B) 2n + 1   (C) 2n − 1   (D) does not exist

4. $\lim\limits_{x \to 0} \left[(1 - e^x) \dfrac{\sin x}{|x|} \right]$, where [·] represents greatest integer function, is equal to

(A) −1   (B) 1   (C) $\log_{\sqrt{2}+1}(3 - \sqrt{2})$ (D) does not exist

5.✎ The value of $\lim\limits_{x \to 0} \dfrac{\cos(\sin x) - \cos x}{x^4}$ is equal to

(A) $\lim\limits_{x \to 0}\dfrac{\tan x}{x^3}$ (B) $\dfrac{1}{6}$ (C) $\lim\limits_{x \to 0}\dfrac{\sin x - x}{x^3}$ (D) $\dfrac{1}{3}$

6.✎ The value of $\lim\limits_{x \to \frac{\pi}{2}} \dfrac{\sin x - (\sin x)^{\sin x}}{1 - \sin x + \ell n \; \sin x}$ is

(A) 1   (B) 2   (C) 3   (D) π/2

7.✎ The value of $\lim\limits_{x \to \frac{\pi}{2}} \tan^2 x \left(\sqrt{2\sin^2 x + 3\sin x + 4} - \sqrt{\sin^2 x + 6\sin x + 2} \right)$ is equal to:

(A) $\dfrac{1}{10}$ (B) $\dfrac{1}{11}$ (C) $\dfrac{1}{12}$ (D) $\dfrac{1}{8}$

8.✎ If α and β be the roots of equation $ax^2 + bx + c = 0$, then $\lim\limits_{x \to \alpha} (1 + ax^2 + bx + c)^{\frac{1}{x-\alpha}}$ is equal to

(A) a (α − β)   (B) □n |a (α − β)| (C) $e^{a(\alpha-\beta)}$ (D) $e^{a|\alpha-\beta|}$

9. $\lim\limits_{x \to \infty} \dfrac{e^x \left(\left(2^{x^n}\right)^{\frac{1}{e^x}} - \left(3^{x^n}\right)^{\frac{1}{e^x}} \right)}{x^n}$, n ∈ N, is equal to

(A) 0   (B) $□n\left(\dfrac{2}{3}\right)$ (C) $□n\left(\dfrac{3}{2}\right)$ (D) none of these

10. $\displaystyle \lim_{y \to 0}\left[\lim_{x \to \infty}\frac{\exp\left(x\,\ln\left(1+\dfrac{ay}{x}\right)\right)-\exp\left(x\,\ln\left(1+\dfrac{by}{x}\right)\right)}{y}\right]$ is equal to

(A) a + b   (B) a − b   (C) b − a   (D) − (a + b)

11. The graph of the function $f(x)=\displaystyle\lim_{t\to 0}\left(\dfrac{2x}{\pi}\cot^{-1}\dfrac{x}{t^2}\right)$ is

(A)   (B)   (C)   (D)
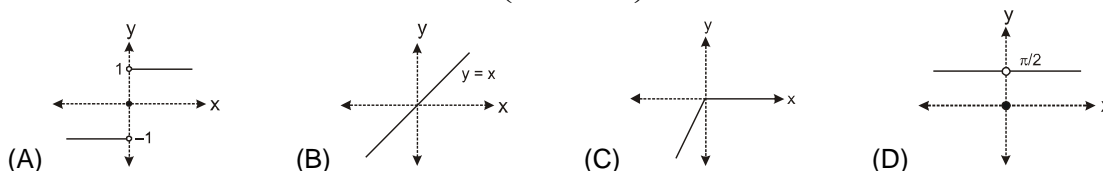
12. Let [x] denote the integral part of $x \in R$ and g(x) = x − [x]. Let f(x) be any continuous function with f(0) = f(1), then the function h(x) = f(g(x)) :
(A) has finitely many discontinuities   (B) is continuous on R
(C) is discontinuous at some x = c   (D) is a constant function.

13. Let $f(x) = \begin{cases} \dfrac{a(1-x\sin x)+b\cos x+5}{x^2} & x<0 \\[2mm] 3 & x=0 \\[2mm] \left(1+\left(\dfrac{cx+dx^3}{x^2}\right)\right)^{1/x} & x>0 \end{cases}$

If f(x) is continuous at x = 0 then find $(a - b - c + e^d)$
(A) 0   (B) 6   (C) –6   (D) 2

14. Let $f(x) = \begin{cases} x^2 & \text{if } x \text{ is irrational} \\ 1 & \text{if } x \text{ is rational} \end{cases}$, then:

(A) f(x) is discontinuous for all x
(B) discontinuous for all x except at x = 0
(C) discontinuous for all x except at x = 1 or − 1
(D) none of these

15. A point (x, y), where function f(x) = [sin [x]] in (0, 2π) is not continuous, is ([**.**] denotes greatest integer ≤ x).
(A) (3, 0)   (B) (2, 0)   (C) (1, 0)   (D) (4, –1)

16. The function f defined by $f(x) = \displaystyle\lim_{t\to\infty}\left\{\dfrac{(1+\sin\pi x)^t-1}{(1+\sin\pi x)^t+1}\right\}$. is

(A) everywhere continuous   (B) discontinuous at all integer values of x
(C) continuous at x = 0   (D) none of these

17. If f(x) = $\begin{cases} \sqrt{x}\left(1+x\sin\dfrac{1}{x}\right), & x > 0 \\ -\sqrt{-x}\left(1+x\sin\dfrac{1}{x}\right), & x < 0 \\ 0, & x = 0 \end{cases}$, then f(x) is

(A) continuous as well as diff. at x = 0   (B) continuous at x = 0, but not diff. at = 0
(C) neither continuous at x = 0 nor diff. at x = 0   (D) none of these

18. The functions defined by f(x) = max {x^2, (x − 1)^2, 2x (1 − x)}, $0 \le x \le 1$
(A) is differentiable for all x
(B) is differentiable for all x except at one point
(C) is differentiable for all x except at two points
(D) is not differentiable at more than two points.

19. [x] denotes the greatest integer less than or equal to x. If f(x) = [x] [sin πx] in (−1,1), then f(x) is:
(A) continuous at x = 0   (B) continuous in (−1, 0)
(C) differentiable in (−1,1)   (D) none

20. Let f(x) = [n + p sin x], x $\in$ (0, π), n $\in$ Z, p is a prime number and [x] is greatest integer less than or equal to x. The number of points at which f(x) is not differentiable is
(A) p   (B) p − 1   (C) 2p + 1   (D) 2p − 1

21. Let f: R $\to$ R be any function and g (x) = $\dfrac{1}{f(x)}$. Then g is

(A) onto if f is onto   (B) one-one if f is one-one
(C) continuous if f is continuous   (D) differentiable if f is differentiable

22. Let f(x) = x^3 − x^2 + x + 1 and g(x) =, $\begin{cases} \max\{f(t) \text{ for } 0 \le t \le x\} & \text{for } 0 \le x \le 1 \\ 3 - x + x^2 & \text{for } 1 < x \le 2 \end{cases}$ then:

(A) g(x) is continuous & derivable at x = 1
(B) g(x) is continuous but not derivable at x = 1
(C) g(x) is neither continuous nor derivable at x = 1
(D) g(x) is derivable but not continuous at x = 1

23. Let f : R $\to$ R be a function such that f$\left(\dfrac{x+y}{3}\right)$ = $\dfrac{f(x)+f(y)}{3}$, f(0) = 0 and f′(0) = 3, then

(A) $\dfrac{f(x)}{x}$ is differentiable in R
(B) f(x) is continuous but not differentiable in R
(C) f(x) is continuous in R
(D) f(x) is bounded in R

24. If a differentiable function f satisfies f$\left(\dfrac{x+y}{3}\right)$ = $\dfrac{4-2(f(x)+f(y))}{3}$ $\forall$ x, y $\in$ R, then f(x) is equal to

(A) $\dfrac{1}{7}$ (B) $\dfrac{2}{7}$ (C) $\dfrac{8}{7}$ (D) $\dfrac{4}{7}$

PART - II : SINGLE AND DOUBLE VALUE INTEGER TYPE

1. Let $f(x) = \dfrac{\sin^{-1}(1-\{x\})}{\sqrt{2\{x\}}} \cdot \dfrac{\cos^{-1}(1-\{x\})}{(1-\{x\})}$, then $\left(\dfrac{\ell\text{im}\limits_{x\to 0^+} f(x)}{\ell\text{im}\limits_{x\to 0^-} f(x)}\right)^2 =$

(where {**.**} denotes the fractional part function)

2. Let $f(x) = \begin{cases} x \sin\left(\dfrac{1}{x}\right) + \sin\left(\dfrac{1}{x^2}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$, then $\ell\text{im}\limits_{x\to\infty} f(x)$ is equal to

3. $\ell\text{im}\limits_{x\to 0}\left(\dfrac{1-\cos x\sqrt{\cos 2x}}{x^2} + \sqrt[3]{\dfrac{1+\cos^3 x + 3\cos^2 x + 3\cos x}{\cos x + 63}}\right)$ is equal to

4. If $\ell\text{im}\limits_{x\to\infty} f(x)$ exists and is finite and nonzero and $\ell\text{im}\limits_{x\to\infty}\left(f(x) + \dfrac{3f(x)-1}{f^2(x)}\right) = 3$, then the value of $\ell\text{im}\limits_{x\to\infty} f(x)$ is equal to

5. If $f(x) = \begin{cases} x-1, & x \geq 1 \\ 2x^2 - 2, & x < 1 \end{cases}$, $g(x) = \begin{cases} x+1, & x > 0 \\ -x^2 + 1, & x \leq 0 \end{cases}$ and $h(x) = |x|$,

then $\ell\text{im}\limits_{x\to 0} f(g(h(x)))$ is equal to

6. If $f(x) = \begin{cases} \sin x, & x \neq n\pi, \quad n = 0, \pm 1, \pm 2,........ \\ 2, & \text{otherwise} \end{cases}$ and

$g(x) = \begin{cases} x^2 + 1, & x \neq 0, 2 \\ 4, & x = 0 \\ 5, & x = 2 \end{cases}$, then $\ell\text{im}\limits_{x\to 0} g(f(x))$ is equal to

7. $\ell\text{im}\limits_{n\to\infty}\left(\dfrac{1}{\sqrt{n^2}} + \dfrac{1}{\sqrt{n^2+1}} + \dfrac{1}{\sqrt{n^2+2}} + ... + \dfrac{1}{\sqrt{n^2+2n}}\right)$ is equal to

8. The value of $\lim\limits_{x\to 0} x^2\left[\dfrac{1}{x^2}\right]$ where [.] denotes G.I.F., is

9. $\ell\text{im}\limits_{x\to 0}\left(\dfrac{\sin^{-1} x - \tan^{-1} x}{x^3} + \dfrac{84x\tan^{-1}\left(\sqrt{2}-1\right)}{\sin \pi x}\right)$ is equal to

10. If $\ell\text{im}\limits_{x\to 0} \dfrac{x^3}{\sqrt{a+x}\,(bx - \sin x)} = 1$, then the value of $(a + b)$ where a > 0, is

11. If $f(x) = \sum\limits_{\lambda=1}^{n}\left(x - \dfrac{5}{\lambda}\right)\left(x - \dfrac{4}{\lambda+1}\right)$, then $\ell\text{im}\limits_{n\to\infty} f(0)$ is equal to

12. Let $f(x) = \begin{cases} (-1)^{[x^2]} & \text{if } x < 0 \\ \displaystyle\lim_{n\to\infty} \dfrac{1}{1+x^n} & \text{if } x \geq 0 \end{cases}$. Then $\displaystyle\lim_{x\to 0^-} 5f(x) + \displaystyle\lim_{x\to 0^+} 7f(x)$ equals (where [.] represents greatest integer function)

13. The value of $\left[\displaystyle\lim_{x\to 0} \dfrac{e - (1+x)^{\frac{1}{x}}}{\tan x} \right]$ where [.] denotes GIF is

14. If $\displaystyle\lim_{x\to 0} \dfrac{e^{-nx} + e^{nx} - 2\cos\dfrac{nx}{2} - kx^2}{(\sin x - \tan x)}$ exists and finite ($n, k \in N$), then the least value of $4k + n \div 2$ is :

15. If $\displaystyle\lim_{n\to\infty} \dfrac{1^2 n + 2^2 (n-1) + 3^2 (n-2) + ... + n^2 . 1}{1^3 + 2^3 + 3^3 + ... + n^3} = \dfrac{a}{b}$ where a and b are coprime numbers then $2a + 3b =$

16. If $\displaystyle\lim_{n\to\infty} \dfrac{n^{98}}{n^x - (n-1)^x} = \dfrac{1}{99}$, then the value of x equals

17. The number of points of discontinuity of $f(x) = \begin{cases} |4x - 5| \ [x] & \text{for } x > 1 \\ [\cos\pi x] & \text{for } x \leq 1 \end{cases}$

(where [x] is the greatest integer not greater than x) in [0, 2] is

18. If $f(x) = \begin{cases} 2x^2 + 12x + 16, & -4 \leq x \leq -2 \\ 2 - |x| & , & -2 < x \leq 1 \\ 4x - x^2 - 2 & , & 1 < x \leq 13 \end{cases}$, then the maximum length of interval for which $f(|x|)$ is continuous is

19. Let $f(x) = \dfrac{1 - \sin x}{(\pi - 2x)^2} \cdot \dfrac{\ln (\sin x)}{\ln (1 + \pi^2 - 4\pi x + 4x^2)}$, $x \neq \dfrac{\pi}{2}$. The value of $f\left(\dfrac{\pi}{2}\right)$ so that the function is continuous at $x = \dfrac{\pi}{2}$ is λ and $|\lambda|\alpha^\beta = 1$ where $\alpha, \beta \in N$ then find product of all possible values of β

20. If the function f(x) defined as $f(x) = \begin{cases} (\sin x + \cos x)^{\text{cosecx}} & , & -\dfrac{\pi}{2} < x < 0 \\ a & , & x = 0 \\ \dfrac{e^{\frac{1}{x}} + e^{\frac{2}{x}} + e^{\frac{3}{x}}}{ae^{-2+\frac{1}{x}} + be^{-1+\frac{3}{x}}} & , & 0 < x < \dfrac{\pi}{2} \end{cases}$

is continuous at x = 0 , then the value of $\log_{e^{1/7}} a + 9b$ is :

21. The number of points of non differentiability of the function $f(x) = |\sin x| + \sin |x|$ in $[-4\pi, 4\pi]$ is

22. If $f(x) = \begin{cases} \dfrac{\sin\ [x^2]\ \pi}{x^2 - 3x + 8} + ax^3 + b & , \quad 0 \le x \le 1 \\ 2\cos\pi\ x + \tan^{-1}x & , \quad 1 < x \le 2 \end{cases}$ is differentiable in $[0, 2]$, then the value of $[a + b + 6]$ is

(Here $[\ .\ ]$ stands for the greatest integer function)

23. If $f(x) = \begin{cases} x^2\ e^{2(x-1)} & \text{for}\ 0 \le x \le 1 \\ a\ \text{sgn}\ (x + 1)\ \cos\ (2x - 2)\ +\ bx^2 & \text{for}\ 1 < x \le 2 \end{cases}$ is differentiable at $x = 1$ then $a^3 + b^3 =$

24. Find number of points of non-differentiability of $f(x) = \lim\limits_{n\to\infty} \dfrac{\{e^x\}^n - 1}{\{e^x\}^n + 1}$ in interval $[0, 1]$ where $\{.\}$ represents fractional part function

25. Let $[x]$ denote the greatest integer less than or equal to x. The number of integral points in $[-1, 1]$ where $f(x) = [x \sin \pi x]$ is differentiable are

26. Let $f''(x)$ be continuous at $x = 0$ and $f''(0) = 4$ then value of $\lim\limits_{x\to 0} \dfrac{2f(x) - 3f(2x) + f(4x)}{x^2}$ is

27. Let $f : R \to R$ is a function satisfying $f(10 - x) = f(x)$ and $f(2 - x) = f(2 + x), \forall x \in R$. If $f(0) = 101$, then the minimum possible number of values of x satisfying $f(x) = 101$ for $x \in [0, 30]$ is

28. Find the natural number 'a' for which $\sum\limits_{k=1}^{n} f(a + k) = 2048(2^n - 1)$, where the function 'f' satisfies the relation $f(x + y) = f(x) \cdot f(y)$ for all natural numbers x & y and further $f(1) = 2$

PART - III : ONE OR MORE THAN ONE OPTIONS CORRECT TYPE

1. Let $f(x) = \dfrac{x^2 - 9x + 20}{x - [x]}$ (where $[x]$ denotes greatest integer less than or equal to x), then

(A) $\lim\limits_{x \to 5^-}\ f(x) = 0$　　　　　　　　　　(B) $\lim\limits_{x \to 5^+} f(x) = 1$

(C) $\lim\limits_{x \to 5} f(x)$ does not exist　　　　　　　(D) none of these

2. If $f(x) = \dfrac{\cos 2 - \cos 2x}{x^2 - |x|}$, then

(A) $\lim\limits_{x \to -1}\ f(x) = 2 \sin 2$　　　　　　　　(B) $\lim\limits_{x \to 1}\ f(x) = 2 \sin 2$

(C) $\lim\limits_{x \to -1}\ f(x) = 2 \cos 2$　　　　　　　　(D) $\lim\limits_{x \to 1} f(x) = 2 \cos 2$

3. If $\square = \lim\limits_{x \to 0} \dfrac{x(1 + a\cos x) - b\sin x}{x^3} = \lim\limits_{x \to 0} \dfrac{1 + a\cos x}{x^2} - \lim\limits_{x \to 0} \dfrac{b\sin x}{x^3}$, where $\square \in R$, then

(A) $(a, b) = (-1, 0)$　　　　　　　　　　(B) a & b are any real numbers

(C) $\square = 0$　　　　　　　　　　　　　　(D) $\square = \dfrac{1}{2}$

4. Let $f(x) = \dfrac{|x + \pi|}{\sin x}$, then

(A) $f(-\pi^+) = -1$　　　　　　　　　　(B) $f(-\pi^-) = 1$

(C) $\lim\limits_{x \to -\pi}\ f(x)$ does not exist　　　　　(D) $\lim\limits_{x \to \pi}\ f(x)$ does not exist

5. Let $f(x) = \begin{cases} 1 + \dfrac{2x}{a}, & 0 \le x < 1 \\ ax, & 1 \le x < 2 \end{cases}$, if $\displaystyle\lim_{x \to 1} f(x)$ exists, then value of a is :

(A) 1   (B) -1 (C) 2   (D) -2

6. Let α, β be the roots of equation $ax^2 + bx + c = 0$, where $1 < \alpha < \beta$ and $\displaystyle\lim_{x \to x_0} \dfrac{\left|ax^2 + bx + c\right|}{ax^2 + bx + c} = 1$, then which of the following statements is correct

(A) $a > 0$ and $x_0 < 1$ (B) $a > 0$ and $x_0 > \beta$

(C) $a < 0$ and $\alpha < x_0 < \beta$ (D) $a < 0$ and $x_0 < 1$

7. Let $\phi(x) = \dfrac{a_0 x^m + a_1 x^{m+1} + ... + a_k x^{m+k}}{b_0 x^n + b_1 x^{n+1} + ... + b_\ell x^{n+\ell}}$, where $a_0 \ne 0$, $b_0 \ne 0$ and $m, n \in N$, then which of the following statements is/are correct.

(A)   If $m > n$ then, $\displaystyle\lim_{x \to 0} \phi(x)$ is equal to 0

(B)   If $m = n$ then, $\displaystyle\lim_{x \to 0} \phi(x)$ is equal to $\dfrac{a_0}{b_0}$

(C)   If $m < n$ and $n - m$ is even, $\dfrac{a_0}{b_0} > 0$, then $\displaystyle\lim_{x \to 0} \phi(x)$ is equal to ∞

(D)   If $m < n$ and $n - m$ is even, $\dfrac{a_0}{b_0} < 0$, then $\displaystyle\lim_{x \to 0} \phi(x)$ is equal to $-\infty$

8. Given a real valued function f such that

$$f(x) = \begin{cases} \dfrac{\tan^2[x]}{(x^2 - [x]^2)} & , \quad x > 0 \\ \\ 1 & , \quad x = 0 \\ \\ \sqrt{\{x\} \; \cot\{x\}}, & x < 0 \end{cases}$$

where [.] represents greatest integer function and {.} represents fractional part function, then

(A) $\displaystyle\lim_{x \to 0} f(x) = 1$ (B) $\displaystyle\lim_{x \to 0^-} f(x) = \sqrt{\cot 1}$

(C) $\cot^{-1}\left(\displaystyle\lim_{x \to 0^-} f(x)\right)^2 = 1$ (D) $\displaystyle\lim_{x \to 0^+} f(x) = 0$

9. If $f(x) = \dfrac{\sqrt{x^2 + 2}}{3x - 6}$, then

(A) $\displaystyle\lim_{x \to -\infty} f(x) = -\dfrac{1}{3}$ (B) $\displaystyle\lim_{x \to \infty} f(x) = \dfrac{1}{3}$ (C) $\displaystyle\lim_{x \to -\infty} f(x) = \dfrac{1}{3}$ (D) $\displaystyle\lim_{x \to \infty} f(x) = -\dfrac{1}{3}$

10. If $\displaystyle\lim_{x \to 0} \dfrac{\sin 2x + a \sin x}{x^3} = p$ (finite), then

(A) $a = -2$ (B) $a = -1$ (C) $p = -2$ (D) $p = -1$

11. $\displaystyle\lim_{x \to \infty} \dfrac{(ax + 1)^n}{x^n + A}$ is equal to

(A) a^n if $n \in N$ (B) ∞ if $n \in Z^-$ & $a = A = 0$

(C) $\dfrac{1}{1 + A}$ if $n = 0$ (D) a^n if $n \in Z^-$, $A = 0$ & $a \ne 0$

12. If $\ell = \lim\limits_{x \to \infty} (\sin \sqrt{x+1} - \sin \sqrt{x})$ and $m = \lim\limits_{x \to -\infty} [\sin \sqrt{x+1} - \sin \sqrt{x}]$, where [.] denotes the greatest integer function, then :

(A) $\ell = 0$ (B) m = 0

(C) m is undefined   (D) ℓ is undefined

13. If $f(x) = |x|^{\sin x}$, then

(A) $\lim\limits_{x \to 0^-} f(x) = 1$ (B) $\lim\limits_{x \to 0^+} f(x) = 1$

(C) $\lim\limits_{x \to 0} f(x) = 1$ (D) limit does not exist at x = 0

14. If $\lim\limits_{x \to 0} (\cos x + a \sin bx)^{\frac{1}{x}} = e^2$, then the possible values of 'a' & 'b' are :

(A) a = 1, b = 2   (B) a = 2, b = 1   (C) a = 3, b = 2/3   (D) a = 2/3, b = 3

15. If $\lim\limits_{x \to 0} (1 + ax + bx^2)^{\frac{2}{x}} = e^3$, then possible values of a and b is/are :

(A) a = 3, b = 0   (B) $a = \frac{3}{2}$, $b = \frac{1}{2}$ (C) $a = \frac{3}{2}$, $b = \frac{3}{2}$ (D) $a = \frac{3}{2}$, b = 0

16. $\lim\limits_{x \to 0^+} \log_{\sin \frac{x}{2}} \sin x$ is equal to

(A) 1   (B) 0   (C) $\lim\limits_{x \to 0} x^{\sin x}$ (D) $\lim\limits_{x \to 0^+} (\tan x)^{\sin x}$

17. $\lim\limits_{x \to \infty} \dfrac{x^n}{e^x} = 0$, $n \in$ integer number, is true for

(A) no value of n   (B) all values of n

(C) negative values of n   (D) positive values of n

18. If $f(x) = \lim\limits_{n \to \infty} \dfrac{\log(x+2) - x^{2n} \sin x}{x^{2n} + 1}$ $(n \in N)$, then

(A) $\lim\limits_{x \to 1^+} f(x) = -\sin 1$ (B) $\lim\limits_{x \to 1^-} f(x) = \log 3$

(C) $\lim\limits_{x \to 1} f(x) = \sin 1$ (D) $f(1) = \dfrac{\log 3 - \sin 1}{2}$

19. Which of the following function(s) defined below has/have single point continuity.

(A) $f(x) = \begin{cases} 1 & \text{if } x \in Q \\ 0 & \text{if } x \notin Q \end{cases}$ (B) $g(x) = \begin{cases} x & \text{if } x \in Q \\ 1-x & \text{if } x \notin Q \end{cases}$

(C) $h(x) = \begin{cases} x & \text{if } x \in Q \\ 0 & \text{if } x \notin Q \end{cases}$ (D) $k(x) = \begin{cases} x & \text{if } x \in Q \\ -x & \text{if } x \notin Q \end{cases}$

20. The function $f(x) = \begin{cases} |x-3| & , x \geq 1 \\ \left(\dfrac{x^2}{4}\right) - \left(\dfrac{3x}{2}\right) + \left(\dfrac{13}{4}\right) & , x < 1 \end{cases}$ is:

(A) continuous at x = 1   (B) differentiable at x = 1

(C) continuous at x = 3   (D) differentiable at x = 3

21. If $f(x) = \dfrac{1}{2}x - 1$, then on the interval $[0, \pi]$

(A) $\tan(f(x))$ and $\dfrac{1}{f(x)}$ are both continuous

(B) $\tan(f(x))$ and $\dfrac{1}{f(x)}$ are both discontinuous

(C) $\tan(f(x))$ and $f^{-1}(x)$ are both continuous

(D) $\tan(f(x))$ is continuous but $\dfrac{1}{f(x)}$ is not.

22. Let $f(x)$ and $g(x)$ be defined by $f(x) = [x]$ and $g(x) = \begin{cases} 0, & x \in I \\ x^2, & x \in R - I \end{cases}$ (where [.] denotes the greatest integer function), then

(A) $\lim\limits_{x \to 1} g(x)$ exists, but g is not continuous at x = 1

(B) $\lim\limits_{x \to 1} f(x)$ does not exist and f is not continuous at x = 1

(C) gof is continuous for all x

(D) fog is continuous for all x

23. Let $f(x) = [x] + \sqrt{x - [x]}$, where [.] denotes the greatest integer function. Then

(A) $f(x)$ is continuous on R^+ (B) $f(x)$ is continuous on R

(C) $f(x)$ is continuous on $R - I$ (D) discontinuous at x = 1

24. The points at which the function, $f(x) = |x - 0.5| + |x - 1| + \tan x$ does not have a derivative in the interval (0, 2) are:

(A) 1   (B) $\pi/2$ (C) $\pi/4$ (D) 1/2

25. $f(x) = (\sin^{-1}x)^2 \cdot \cos(1/x)$ if $x \neq 0$; $f(0) = 0$, $f(x)$ is:

(A) continuous no where in $-1 \leq x \leq 1$ (B) continuous everywhere in $-1 \leq x \leq 1$

(C) differentiable no where in $-1 \leq x \leq 1$ (D) differentiable everywhere in $-1 < x < 1$

26. If $f(x) = a_0 + \sum\limits_{k=1}^{n} a_k |x|^k$, where a_i's are real constants, then $f(x)$ is

(A) continuous at x = 0 for all a_i (B) differentiable at x = 0 for all $a_i \in R$

(C) differentiable at x = 0 for all $a_{2k-1} = 0$ (D) none of these

27. Let $f : R \to R$ be a function such that $f(0) = 1$ and for any x, y $\in R$, $f(xy + 1) = f(x) f(y) - f(y) - x + 2$. Then f is

(A) one-one   (B) onto   (C) many one   (D) into

28. Suppose that f is a differentiable function with the property that $f(x + y) = f(x) + f(y) + xy$ and

$\lim\limits_{h \to 0} \dfrac{1}{h} f(h) = 3$

where [.] represents greatest integer function, then

(A) f is a linear function   (B) $2f(1) = \left[\lim\limits_{x \to 0}(1 + 2x)^{1/x} \right]$

(C) $f(x) = 3x + \dfrac{x^2}{2}$ (D) $f'(1) = 4$

29. Let 'f' be a real valued function defined for all real numbers x such that for some positive constant 'a' the equation $f(x+a) = \frac{1}{2} + \sqrt{f(x) - (f(x))^2}$ holds for all x. Then f(x) is periodic function with period equal to

(A) 2 a   (B) 4 a   (C) 6 a   (D) 8 a

PART - IV : COMPREHENSION

Comprehension # 1

Consider $f(x) = \dfrac{\sin x + ae^x + be^{-x} + c \ \ln \ (1+x)}{x^3}$, where a, b, c are real numbers.

1. If $\lim\limits_{x \to 0^+} f(x)$ is finite, then the value of a + b + c is

(A) 0   (B) 1   (C) 2   (D) – 2

2. If $\lim\limits_{x \to 0^+} f(x) = \ell$ (finite), then the value of ℓ is

(A) – 2   (B) $-\dfrac{1}{2}$ (C) – 1   (D) $-\dfrac{1}{3}$

3. Using the values of a, b, c as found in Q.No. 1 or Q. No.2 above, the value of $\lim\limits_{x \to 0^+} x \, f(x)$ is

(A) 0   (B) $\dfrac{1}{2}$ (C) $-\dfrac{1}{2}$ (D) 2

Comprehension # 2

If both $\lim\limits_{x \to c^-} f(x)$ and $\lim\limits_{x \to c^+} f(x)$ exist finitely and are equal , then the function f is said to have removable discontinuity at x = c

If both the limits i.e. $\lim\limits_{x \to c^-} f(x)$ and $\lim\limits_{x \to c^+} f(x)$ exist finitely and are not equal, then the function f is said to have non-removable discontinuity at x = c and in this case $|\lim\limits_{x \to c^+} f(x) - \lim\limits_{x \to c^-} f(x)|$ is called jump of the discontinuity.

4. Which of the following function has non-removable discontinuity at the origin ?

(A) $f(x) = \dfrac{1}{\ln |x|}$ (B) $f(x) = x \sin \dfrac{\pi}{x}$ (C) $f(x) = \dfrac{1}{1 + 2^{\cot x}}$ (D) $f(x) = \cos\left(\dfrac{|\sin x|}{x}\right)$

5. Which of the following function not defined at x = 0 has removable discontinuity at the origin ?

(A) $f(x) = \dfrac{1}{1 + 2^{\frac{1}{x}}}$ (B) $f(x) = \tan^{-1} \dfrac{1}{x}$ (C) $f(x) = \dfrac{e^{\frac{1}{x}} - 1}{e^{\frac{1}{x}} + 1}$ (D) $f(x) = \dfrac{1}{\ln |x|}$

6. If $f(x) = \begin{bmatrix} \tan^{-1}(\tan x) \ ; & x \le \dfrac{\pi}{4} \\ \pi \ [x] + 1 & ; \ x > \dfrac{\pi}{4} \end{bmatrix}$, then jump of discontinuity is

(where [.] denotes greatest integer function)

(A) $\dfrac{\pi}{4} - 1$ (B) $\dfrac{\pi}{4} + 1$ (C) $1 - \dfrac{\pi}{4}$ (D) $-1 - \dfrac{\pi}{4}$

Comprehension # 3

Let $f(x) = \begin{cases} x\ g(x) & , & x \le 0 \\ x + ax^2 - x^3 & , & x > 0 \end{cases}$, where $g(t) = \lim_{x \to 0} (1 + a \tan x)^{t/x}$, a is positive constant, then

7. If a is even prime number, then $g(2) =$

(A) e^2 (B) e^3 (C) e^4 (D) none of these

8. Set of all values of a for which function $f(x)$ is continuous at $x = 0$

(A) $(-1, 10)$ (B) $(-\infty, \infty)$ (C) $(0, \infty)$ (D) none of these

9. If $f(x)$ is differentiable at $x = 0$, then $a \in$

(A) $(-5, -1)$ (B) $(-10, 3)$ (C) $(0, \infty)$ (D) none of these

Comprehension # 4

Let $f : R \to R$ be a function defined as,

$$f(x) = \begin{cases} 1 - |x| & , & |x| \le 1 \\ 0 & , & |x| > 1 \end{cases} \quad \text{and} \quad g(x) = f(x - 1) + f(x + 1), \ \forall \ x \in R. \text{ Then}$$

10. The value of $g(x)$ is :

(A) $g(x) = \begin{cases} 0 & , & x \le -3 \\ 2 + x & , & -3 \le x \le -1 \\ -x & , & -1 < x \le 0 \\ x & , & 0 < x \le 1 \\ 2 - x & , & 1 < x \le 3 \\ 0 & , & x > 3 \end{cases}$

(B) $g(x) = \begin{cases} 0 & , & x \le -2 \\ 2 + x & , & -2 \le x \le -1 \\ -x & , & -1 < x \le 0 \\ x & , & 0 < x \le 1 \\ 2 - x & , & 1 < x \le 2 \\ 0 & , & x > 2 \end{cases}$

(C) $g(x) = \begin{cases} 0 & , & x \le 0 \\ 2 + x & , & 0 < x < 1 \\ -x & , & 1 \le x \le 2 \\ x & , & 2 < x < 3 \\ 2 - x & , & 3 \le x < 4 \\ 0 & , & 4 \le x \end{cases}$

(D) none of these

11. The function $g(x)$ is continuous for, $x \in$

(A) $R - \{0, 1, 2, 3, 4\}$ (B) $R - \{-2, -1, 0, 1, 2\}$ (C) R (D) none of these

12. The function $g(x)$ is differentiable for, $x \in$

(A) R

(B) $R - \{-2, -1, 0, 1, 2\}$

(C) $R - \{0, 1, 2, 3, 4\}$

(D) none of these

Exercise-3

PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

✍ **Marked questions are recommended for Revision.**

* Marked Questions may have more than one correct option.

1*. Let L = $\displaystyle\lim_{x \to 0} \dfrac{a - \sqrt{a^2 - x^2} - \dfrac{x^2}{4}}{x^4}$, a > 0. If L is finite, then

[IIT-JEE-2009, Paper-1, (4, –1), 80]

(A) a = 2   (B) a = 1   (C) L = $\dfrac{1}{64}$ (D) L = $\dfrac{1}{32}$

2.✍ If $\displaystyle\lim_{x \to 0} \left[1 + x \ln(1 + b^2)\right]^{\frac{1}{x}} = 2b \sin^2 \theta$, b > 0 and $\theta \in (-\pi, \pi]$, then the value of θ is

[IIT-JEE 2011, Paper-2, (3, –1), 80]

(A) $\pm\ \dfrac{\pi}{4}$ (B) $\pm\ \dfrac{\pi}{3}$ (C) $\pm\ \dfrac{\pi}{6}$ (D) $\pm\ \dfrac{\pi}{2}$

3*. Let f : **R** → **R** be a function such that f(x + y) = f(x) + f(y), $\forall$ x, y $\in$ **R**. If f(x) is differentiable at x = 0, then
(A) f(x) is differentiable only in a finite interval containing zero   **[IIT-JEE 2011, Paper-1, (4, 0), 80]**
(B) f(x) is continuous $\forall$ x $\in$ **R**
(C) f′(x) is constant $\forall$x $\in$ **R**
(D) f(x) is differentiable except at finitely many points

4*.✍ If f(x) = $\begin{cases} -x - \dfrac{\pi}{2} & , \quad x \le -\dfrac{\pi}{2} \\ -\cos x & , \quad -\dfrac{\pi}{2} < x \le 0 \\ x - 1 & , \quad 0 < x \le 1 \\ \ell n\ x & , \quad x > 1 \end{cases}$, then   **[IIT-JEE 2011, Paper-2, (4, 0), 80]**

(A) f(x) is continuous at x = $-\dfrac{\pi}{2}$ (B) f(x) is not differentiable at x = 0

(C) f(x) is differentiable at x = 1   (D) f(x) is differentiable at x = $-\dfrac{3}{2}$

5.✍ Let f : (0, 1) → **R** be defined by f(x) = $\dfrac{b - x}{1 - bx}$, where b is a constant such that 0 < b < 1. Then

[IIT-JEE 2011, Paper-2, (4, 0), 80]

(A) f is not invertible on (0, 1)   (B) f ≠ f^{-1} on (0, 1) and f′(b) = $\dfrac{1}{f'(0)}$

(C) f = f^{-1} on (0, 1) and f′(b) = $\dfrac{1}{f'(0)}$ (D) f^{-1} is differentiable on (0, 1)

6.✍ If $\displaystyle\lim_{x \to \infty} \left(\dfrac{x^2 + x + 1}{x + 1} - ax - b \right) = 4$, then   **[IIT-JEE 2012, Paper-1, (3, –1), 70]**

(A) a = 1, b = 4   (B) a = 1, b = –4
(C) a = 2, b = –3   (D) a = 2, b = 3

7. Let $\alpha(a)$ and $\beta(a)$ be the roots of the equation $\left(\sqrt[3]{1+a}-1\right)x^2+\left(\sqrt{1+a}-1\right)x+\left(\sqrt[6]{1+a}-1\right)=0$ where $a > -1$. Then $\lim\limits_{a \to 0^+} \alpha(a)$ and $\lim\limits_{a \to 0^+} \beta(a)$ are **[IIT-JEE 2012, Paper-2, (3, –1), 66]**

(A) $-\dfrac{5}{2}$ and 1 (B) $-\dfrac{1}{2}$ and –1 (C) $-\dfrac{7}{2}$ and 2 (D) $-\dfrac{9}{2}$ and 3

8. Let $f(x) = \begin{cases} x^2\left|\cos\dfrac{\pi}{x}\right|, & x \neq 0 \\ 0, & x = 0 \end{cases}$, $x \in IR$, then f is

(A) differentiable both at x = 0 and at x = 2 **[IIT-JEE 2012, Paper-1, (3, –1), 70]**
(B) differentiable at x = 0 but not differentiable at x = 2
(C) not differentiable at x = 0 but differentiable at x = 2
(D) differentiable neither at x = 0 nor at x = 2

9*. For every integer n, let a_n and b_n be real numbers. Let function f : IR → IR be given by

$f(x) = \begin{cases} a_n + \sin\pi\ x, & \text{for } x \in [2n,\ 2n+1] \\ b_n + \cos\pi x, & \text{for } x \in (2n-1,\ 2n) \end{cases}$, for all integers n.

If f is continuous, then which of the following hold(s) for all n ? **[IIT-JEE 2012, Paper-2, (4, 0), 66]**

(A) $a_{n-1} - b_{n-1} = 0$ (B) $a_n - b_n = 1$ (C) $a_n - b_{n+1} = 1$ (D) $a_{n-1} - b_n = -1$

10*. For every pair of continuous functions f, g:[0, 1] → R such that
max {f(x) : x ∈ [0,1]} = max {g(x) : x ∈ [0, 1]}, **[JEE (Advanced) 2014, Paper-1, (3, 0)/60]**
the correct statement(s) is (are) :
(A) $(f(c))^2 + 3f(c) = (g(c))^2 + 3g(c)$ for some c ∈ [0, 1]
(B) $(f(c))^2 + f(c) = (g(c))^2 + 3g(c)$ for some c ∈ [0, 1]
(C) $(f(c))^2 + 3f(c) = (g(c))^2 + g(c)$ for some c ∈ [0, 1]
(D) $(f(c))^2 = (g(c))^2$ for some c ∈ [0, 1]

11. The largest value of the non-negative integer a for which $\lim\limits_{x \to 1}\left\{\dfrac{-ax+\sin(x-1)+a}{x+\sin(x-1)-1}\right\}^{\frac{1-x}{1-\sqrt{x}}} = \dfrac{1}{4}$ is

[JEE (Advanced) 2014, Paper-1, (3, 0)/60]

12. Let f : R → R and g : R → R be respectively given by f(x) = |x| + 1 and g(x) = x^2 + 1. Define h : R → R
by $h(x) = \begin{cases} \max\ \{f(x),g(x)\} & \text{if } x \leq 0, \\ \min\ \{f(x),g(x)\} & \text{if } x > 0. \end{cases}$ The number of points at which h(x) is not differentiable is

[JEE (Advanced) 2014, Paper-1, (3, 0)/60]

13. Let f_1 : R → R, f_2 : [0, ∞) → R, f_3 : R → R and f_4 : R → [0, ∞) be defined by

$f_1(x) = \begin{cases} |x| & \text{if } x < 0, \\ e^x & \text{if } x \geq 0; \end{cases}$

$f_2(x) = x^2$;

$f_3(x) = \begin{cases} \sin x & \text{if } x < 0, \\ x & \text{if } x \geq 0 \end{cases}$

and $f_4(x) = \begin{cases} f_2(f_1(x)) & \text{if } x < 0, \\ f_2(f_1(x)) - 1 & \text{if } x \geq 0 \end{cases}$

List I

P. f_4 is

Q. f_3 is

R. $f_2 o f_1$ is

S. f_2 is

List II

1. onto but not one-one

2. neither continuous nor one-one

3. differentiable but not one-one

4. continuous and one-one

[JEE (Advanced) 2014, Paper-2, (3, –1)/60]

	P	Q	R	S
(A)	3	1	4	2
(B)	1	3	4	2
(C)	3	1	2	4
(D)	1	3	2	4

14*. Let g: R → R be a differentiable function with g(0) = 0, g'(0) = 0 and g'(1) ≠ 0. Let

$$f(x) = \begin{cases} \dfrac{x}{|x|}g(x), & x \neq 0 \\ 0, & x = 0 \end{cases}$$ and $h(x) = e^{|x|}$ for all x ∈ R. Let (foh)(x) denote f(h(x)) and (hof)(x) denote h(f(x)).

Then which of the following is(are) true?

(A) f is differentiable at x = 0

(B) h is differentiable at x = 0

(C) foh is differentiable at x = 0

(D) hof is differentiable at x = 0

[JEE (Advanced) 2015, P-1 (4, –2)/ 88]

15*. Let $f(x) = \sin\left(\dfrac{\pi}{6}\sin\left(\dfrac{\pi}{2}\sin x\right)\right)$ for all x ∈ R and $g(x) = \dfrac{\pi}{2}\sin x$ for all x ∈ R. Let (fog) (x) dentoe f(g(x))

and (gof) (x) denote g(f(x)). Then which of the following is(are)true?

(A) Range of f is $\left[-\dfrac{1}{2}, \dfrac{1}{2}\right]$

(B) Range of fog is $\left[-\dfrac{1}{2}, \dfrac{1}{2}\right]$

(C) $\lim\limits_{x \to 0}\dfrac{f(x)}{g(x)} = \dfrac{\pi}{6}$

(D) There is an x ∈ R such that (gof)(x) = 1

[JEE (Advanced) 2015, P-1 (4, –2)/ 88]

16. Let m and n be two positive integers greater than 1. If $\lim\limits_{\alpha \to 0}\left(\dfrac{e^{\cos(\alpha^n)} - e}{\alpha^m}\right) = -\left(\dfrac{e}{2}\right)$, then the value of $\dfrac{m}{n}$

is

[JEE (Advanced) 2015, P-2 (4, 0) / 80]

17*. Let f : R → R , g : R → R and h : R → R be differentiable functions such that f(x) = x³ + 3x + 2, g(f(x)) = x and h(g(g(x))) = x for all x ∈ R. Then **[JEE (Advanced) 2016, Paper-1, (4, –2)/62]**

(A) g'(2) = $\dfrac{1}{15}$

(B) h'(1) = 666

(C) h(0) = 16

(D) h(g(3)) = 36

18. Let α, β ∈ R be such that $\lim\limits_{x \to 0}\dfrac{x^2 \sin(\beta x)}{\alpha x - \sin x} = 1$. Then 6(α + β) equals

[JEE (Advanced) 2016, Paper-1, (3, 0)/62]

19*. Let $f : \left[-\dfrac{1}{2}, 2\right] \to R$ and $g : \left[-\dfrac{1}{2}, 2\right] \to R$ be functions defined by $f(x) = [x^2 - 3]$ and

$g(x) = |x| \, f(x) + |4x - 7| \, f(x)$, where [y] denotes the greatest integer less than or equal to y for $y \in R$. Then

(A) f is discontinuous exactly at three points in $\left[-\dfrac{1}{2}, 2\right]$ **[JEE (Advanced) 2016, Paper-2, (4, –2)/62]**

(B) f is discontinuous exactly at four point in $\left[-\dfrac{1}{2}, 2\right]$

(C) g is NOT differentiable exactly at four points in $\left(-\dfrac{1}{2}, 2\right)$

(D) g is NOT differentiable exactly at five points in $\left(-\dfrac{1}{2}, 2\right)$.

20*. Let $a, b \in R$ and $f : R \to R$ be defined by $f(x) = a \cos(|x^3 - x|) + b|x| \sin(|x^3 + x|)$. Then f is
(A) differentiable at x = 0 if a = 0 and b = 1   **[JEE (Advanced) 2016, Paper-2, (4, –2)/62]**
(B) differentiable at x = 1 if a = 1 and b = 0
(C) NOT differentiable at x = 0 if a = 1 and b = 0
(D) NOT differentiable at x = 1 if a = 1 and b = 1

21*. Let [x] be the greatest integer less than or equals to x. Then, at which of the following point(s) the function $f(x) = x \cos(\pi(x + [x]))$ is discontinuous ?   **[JEE(Advanced) 2017, Paper-1,(4, –2)/61]**
(A) x = – 1   (B) x = 1   (C) x = 0   (D) x = 2

22*. Let $f(x) = \dfrac{1 - x(1 + |1 - x|)}{|1 - x|} \cos\left(\dfrac{1}{1 - x}\right)$ for $x \neq 1$. Then   **[JEE(Advanced) 2017, Paper-2,(4, –2)/61]**

(A) $\lim_{x \to 1^+} f(x) = 0$ (B) $\lim_{x \to 1^-} f(x)$ does not exist

(C) $\lim_{x \to 1^-} f(x) = 0$ (D) $\lim_{x \to 1^+} f(x)$ does not exist

23. For any positive integer n, define $f_n : (0, \infty) \to R$ as

$f_n(x) = \displaystyle\sum_{j=1}^{n} \tan^{-1}\left(\dfrac{1}{1 + (x + j)(x + j - 1)}\right)$ for all $x \in (0, \infty)$.   **[JEE(Advanced) 2018, Paper-2,(4, –2)/60]**

(Here, the inverse trigonometric function $\tan^{-1} x$ assumes values in $\left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$)

Then, which of the following statement(s) is (are) TRUE ?

(A) $\displaystyle\sum_{j=1}^{5} \tan^2(f_j(0)) = 55$

(B) $\displaystyle\sum_{j=1}^{10} (1 + f_j'(0)) \sec^2(f_j(0)) = 10$

(C) For any fixed positive integer n, $\displaystyle\lim_{x \to \infty} \tan(f_n(x)) = \dfrac{1}{n}$

(D) For any fixed positive integer n, $\displaystyle\lim_{x \to \infty} \sec^2(f_n(x)) = 1$

24. Let $f_1 : R \rightarrow R$, $f_2 : \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right) \rightarrow R$, $f_3 : \left(-1, e^{\frac{\pi}{2}} - 2\right) \rightarrow R$ and $f_4 : R \rightarrow R$ be functions defined by

(i) $f_1(x) = \sin\left(\sqrt{1 - e^{-x^2}}\right)$ **[JEE(Advanced) 2018, Paper-2,(3, –1)/60]**

(ii) $f_2(x) = \begin{cases} \dfrac{|\sin x|}{\tan^{-1} x} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$, where the inverse trigonometric function $\tan^{-1} x$ assumes values in

$\left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$

(iii) $f_3(x) = [\sin(\log_e(x+2))]$, where for $t \in R$, $[t]$ denotes the greatest integer less than or equal to t,

(iv) $f_4(x) = \begin{cases} x^2 \sin\left(\dfrac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

LIST-I
(P) The function f_1 is
(Q) The function f_2 is
(R) The function f_3 is
(S) The function f_4 is
The correct option is:

LIST-II
(1) **NOT** continuous at $x = 0$
(2) continuous at $x = 0$ and **NOT** differentiable at $x = 0$
(3) differentiable at $x = 0$ and its derivative is **NOT** continuous at $x = 0$
(4) differentiable at $x = 0$ and its derivative is continuous at $x = 0$

(A) P → 2; Q → 3; R → 1; S → 4
(B) P → 4; Q → 1; R → 2; S → 3
(C) P → 4; Q → 2; R → 1; S → 3
(D) P → 2; Q → 1; R → 4; S → 3

PART - II : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. Let $f(x) = x|x|$ and $g(x) = \sin x$ **[AIEEE 2009, (8, –2), 144]**

Statement-1 gof is differentiable at $x = 0$ and its derivative is continuous at that point.

Statement-2 gof is twice differentiable at $x = 0$.

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(3) Statement-1 is True, Statement-2 is False

(4) Statement-1 is False, Statement-2 is True

2. Let $f : \mathbf{R} \rightarrow \mathbf{R}$ be a positive increasing function with $\displaystyle\lim_{x \to \infty} \dfrac{f(3x)}{f(x)} = 1$. Then $\displaystyle\lim_{x \to \infty} \dfrac{f(2x)}{f(x)}$.

[AIEEE– 2010, (8, –2), 144]

(1) $\dfrac{2}{3}$ (2) $\dfrac{3}{2}$ (3) 3 (4) 1

3. $\displaystyle\lim_{x \to 2} \left(\dfrac{\sqrt{1 - \cos\ \{2(x - 2)\}}}{x - 2}\right)$ **[AIEEE– 2011, I, (4, –1), 120]**

(1) does not exist (2) equals $\sqrt{2}$ (3) equals $-\sqrt{2}$ (4) equals $\dfrac{1}{\sqrt{2}}$

4. Let f: $R \rightarrow [0, \infty)$ be such that $\lim_{x \to 5} f(x)$ exists and $\lim_{x \to 5} \frac{(f(x))^2 - 9}{\sqrt{|x - 5|}} = 0$ **[AIEEE– 2011, II,(4, –1), 120]**

Then $\lim_{x \to 5} f(x)$ equals :

(1) 0   (2) 1   (3) 2   (4) 3

5. The value of p and q for which the function f(x) = $\begin{cases} \dfrac{\sin(p+1)x + \sin x}{x} & , \quad x < 0 \\ q & , \quad x = 0 \\ \dfrac{\sqrt{x + x^2} - \sqrt{x}}{x^{3/2}} & , \quad x > 0 \end{cases}$ is continuous for all x in

R, are : **[AIEEE 2011, I,(4, –1), 120]**

(1) $p = \dfrac{1}{2}$, $q = -\dfrac{3}{2}$ (2) $p = \dfrac{5}{2}$, $q = \dfrac{1}{2}$ (3) $p = -\dfrac{3}{2}$, $q = \dfrac{1}{2}$ (4) $p = \dfrac{1}{2}$, $q = \dfrac{3}{2}$

6. Define F(x) as the product of two real functions $f_1(x) = x$, $x \in R$, and $f_2(x) = \begin{cases} \sin\dfrac{1}{x} & , \quad \text{If} \quad x \neq 0 \\ 0 & , \quad \text{If} \quad x = 0 \end{cases}$

as follows :

$F(x) = \begin{cases} f_1(x) \cdot f_2(x) & , \quad \text{If} \quad x \neq 0 \\ 0 & , \quad \text{If} \quad x = 0 \end{cases}$ **[AIEEE 2011, II,(4, –1), 120]**

Statement - 1 : F(x) is continuous on R.
Statement - 2 : $f_1(x)$ and $f_2(x)$ are continuous on R.
(1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is true, Statement-2 is true; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is true, Statement-2 is false
(4) Statement-1 is false, Statement-2 is true

7. If function f(x) is differentiable at x = a, then $\lim_{x \to a} \dfrac{x^2 f(a) - a^2 f(x)}{x - a}$ is : **[AIEEE 2011, II,(4, –1), 120]**

(1) $-a^2 f'(a)$ (2) $af(a) - a^2 f'(a)$ (3) $2af(a) - a^2 f'(a)$ (4) $2af(a) + a^2 f'(a)$

8. If f : R $\rightarrow$ R is a function defined by f(x) = [x] cos $\left(\dfrac{2x - 1}{2} \right) \pi$, where[x] denotes the greatest integer

function, then f is : **[AIEEE- 2012, (4, –1), 120]**
(1) continuous for every real x.
(2) discontinuous only at x = 0.
(3) discontinuous only at non-zero integral values of x.
(4) continuous only at x = 0.

9. Consider the function, f(x) = |x – 2| + |x – 5|, x $\in$ R . **[AIEEE- 2012, (4, –1), 120]**

Statement-1 : f'(4) = 0

Statement-2 : f is continuous in [2, 5], differentiable in (2, 5) and f(2) = f(5).

(1) Statement-1 is false, Statement-2 is true.

(2) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for Statement-1.

(3) Statement-1 is true, statement-2 is true; statement-2 is **not** a correct explanation for Statement-1.

(4) Statement-1 is true, statement-2 is false.

10. $\lim\limits_{x\to 0}\dfrac{(1-\cos 2x)(3+\cos x)}{x\tan 4x}$ is equal to   **[AIEEE - 2013, (4, –1),360]**

(1) $-\dfrac{1}{4}$ (2) $\dfrac{1}{2}$ (3) 1   (4) 2

11. $\lim\limits_{x\to 0}\dfrac{\sin(\pi\cos^2 x)}{x^2}$ is equal to :   **[JEE(Main) 2014, (4, – 1), 120]**

(1) $-\pi$ (2) π (3) $\pi/2$ (4) 1

12. $\lim\limits_{x\to 0}\dfrac{(1-\cos 2x)(3+\cos x)}{x\tan 4x}$ is equal to   **[JEE(Main) 2015, (4, – 1), 120]**

(1) 4   (2) 3   (3) 2   (4) $\dfrac{1}{2}$

13. If the function $g(x) = \begin{cases} k\sqrt{x+1} & , \quad 0 \le x \le 3 \\ mx+2 & , \quad 3 < x \le 5 \end{cases}$ is differentiable, then the value of k+ m is;

                    [JEE(Main) 2015, (4, – 1), 120]

(1) 2   (2) $\dfrac{16}{5}$ (3) $\dfrac{10}{3}$ (4) 4

14. Let p = $\lim\limits_{x\to 0+}\left(1+\tan^2\sqrt{x}\right)^{\frac{1}{2x}}$ then log p is equal to: **[JEE(Main) 2016, (4, – 1), 120]**

(1) 1   (2) $\dfrac{1}{2}$ (3) $\dfrac{1}{4}$ (4) 2

15. For $x \in R$, f(x) = |log2 – sinx| and g(x) = f(f(x)), then   **[JEE(Main) 2016, (4, – 1), 120]**
(1) g′(0) = cos(log2)
(2) g′(0) = –cos(log2)
(3) g is differentiable at x = 0 and g′(0) = –sin(log2)
(4) g is not differentiable at x = 0

16. $\lim\limits_{x\to\frac{\pi}{2}}\dfrac{\cot x - \cos x}{(\pi - 2x)^3}$ equals   **[JEE(Main) 2017, (4, – 1), 120]**

(1) $\dfrac{1}{24}$ (2) $\dfrac{1}{16}$ (3) $\dfrac{1}{8}$ (4) $\dfrac{1}{4}$

17. For each $t \in R$ let [t] be the greatest integer less than or equal to t. Then $\lim\limits_{x\to 0^+} x\left(\left[\dfrac{1}{x}\right]+\left[\dfrac{2}{x}\right]+......+\left[\dfrac{15}{x}\right]\right)$

                    [JEE(Main) 2018, (4, – 1), 120]
(1) is equal to 120   (2) does not exist (in R)   (3) is equal to 0   (4) is equal to 15

18. Let S = {t ∈ R : f(x) = |x − π|. (e^{|x|} − 1) sin|x| is not differentiable at t.} Then the set S is equal to :
                    [JEE(Main) 2018, (4, – 1), 120]
(1) {π}   (2) {0, π}   (3) ϕ (an empty set)   (4) {0}

19. ✎ $\lim_{y \to 0} \dfrac{\sqrt{1+\sqrt{1+y^4}} - \sqrt{2}}{y^4}$ **[JEE(Main) 2019, Online (09-01-19),P-1 (4, – 1), 120]**

(1) exists and equals $\dfrac{1}{2\sqrt{2}}$

(2) exists and equals $\dfrac{1}{2\sqrt{2}(\sqrt{2}+1)}$

(3) exists and equals $\dfrac{1}{4\sqrt{2}}$

(4) does not exist

20. For each t∈R, let [t] be the greatest integer less than or equal to t. Then ,

$\lim_{x \to 1+} \dfrac{(1-|x|+\sin|1-x|)\sin\left(\dfrac{\pi}{2}[1-x]\right)}{|1-x|[1-x]}$ **[JEE(Main) 2019, Online (10-01-19),P-1 (4, – 1), 120]**

(1) does not exist   (2) equals 1   (3) equals –1   (4) equals 0

21. Let f(x) = $\begin{cases} -1, & -2 \le x < 0 \\ x^2 - 1, & 0 \le x \le 2 \end{cases}$ and g(x) = |f(x)| + f(|x|). Then, in the interval (–2, 2), g is :

(1) not differentiable at two point

(2) not continuous

(3) not differentiable at one point

(4) differentiable at all points

[JEE(Main) 2019, Online (11-01-19),P-1 (4, – 1), 120]

Answers

EXERCISE - 1

PART – I

Section (A) :

A-1. (i) Limit does not exist (ii) 3 (iii) 3 (iv) 3 (v) 3

A-2. (i) $2 + \sin 2$ (ii) $\tan 3 - 2^3$ (iii) $\dfrac{3}{4} \cos \dfrac{3}{4}$ (iv) 5^5 (v) $\dfrac{e}{\sin 1}$

A-3. (i) 0 (ii) Limit does not exist (iii) Limit does not exist (iv) 0

A-4. (i) Limit does not exist (ii) $\lambda = -2$ **A-5.** 6

A-6. (i) No (ii) No (iii) Yes, ∞^0 form (iv) No

SECTION (B) :

B-1. (i) $-\dfrac{3}{2}$ (ii) $\dfrac{12}{19}$ (iii) $\dfrac{2}{3\sqrt{3}}$

B-2. (i) $\dfrac{16}{25}$ (ii) 2 (iii) $\dfrac{1}{3}$ (iv) $2a \sin a + a^2 \cos a$

(v) $(b - a)$ (vi) $2e^2$ (vii) $\dfrac{3}{\ell n 3}$ (viii) $\dfrac{1}{2}$ (ix) 5

(x) limit does not exist (xi) $-\dfrac{9}{4} \, \ell n \, \dfrac{4}{e}$

B-3. (i) $\dfrac{1}{2}$ (ii) 1 (iii) ∞ (iv) $-\dfrac{1}{\pi}$ **B-4.** (i) 0 (ii) $\dfrac{5}{2} \left(a + 2\right)^{\frac{3}{2}}$ (iii) 0 (iv) 5/2

B-5. (i) $-\dfrac{2}{25}$ (ii) $\dfrac{1}{3}$ **B-6** $a = 2, b = 1, c = -1$ and limit $= -\dfrac{1}{3}$

B-7. (i) $a = -\dfrac{1}{2}$, b = 1 (ii) $a = 2, b \in R, c = 5, d \in R$ (iii) $a = 3, b = 12, c = 9$

B-8. $\dfrac{1}{2}$ **B10.** $2(\sec^2 a) \tan a$

SECTION (C) :

C-1 (i) 1 (ii) 1 (iii) 0 (iv) 0

C-2. (i) e^{-1} (ii) 0 (iii) $e^{-\frac{2}{\pi}}$ (iv) e^2 **C-3** $a + b = 0$ and $bc = 3$

C-4. (i) $\lim\limits_{x \to \infty} \dfrac{x \, \ell n \, \left(1 + \dfrac{\ell n \, x}{x}\right)}{\ell n \, x}$ 1 (ii) 1 **C-5.** $\dfrac{x}{3}$ **C-6.** $\{-1, 0, 1\}$

Section (D) :

D-1. $a = -\dfrac{3}{2}$, $b \neq 0$, $c = \dfrac{1}{2}$ **D-2.** $a = \dfrac{1}{2}$, $b = 4$

D-3. (i) continuous at x = 1   (ii) continuous   (iii)   discontinuous   (iv)   continuous at x = 1, 2

D-4. (a)   −2, 2, 3   (b)   K = 5   (c)   even

D-5. A = − 4, B = 5, f(0) = 1   **D6.**   Continuous at x = 1 but discontinuous at x = 2

Section (E) :

E-1. (i) x ∈ R − {2, 3}   (ii) x ∈ R − {− 1, 1}   (iii)   x ∈ R   (iv) x ∈ R − {(2n + 1), n ∈ I}

E-2. discontinuous at all integral values in [− 2, 2]   **E-3.**   discontinuous at $n\pi \pm \dfrac{\pi}{4}$, $(2n + 1)\dfrac{\pi}{2}$, n ∈ I

E-4. g(x) = 2 + x ; 0 ≤ x ≤ 1,
= 2 − x ; 1 < x ≤ 2,
= 4 − x ; 2 < x ≤ 3,
g is discontinuous at x = 1 & x = 2   **E-5.** $-\dfrac{7}{3}$, − 2, 0   **E-7_.**   2

Section (F) :

F-1. continuous at both points but differentiable only at x = 2

F-2. continuous but not differentiable at x = 0; differentiable & continuous at x = π/2

F-5. not differentiable at x = 0   **F-6.**   a = 1/2, b = 3/2

Section (G) :

G-1. f is not derivable at all integral values in −1 < x ≤ 3

G-2. f is continuous but not derivable at x = 1/2, f is neither differentiable nor continuous at x = 1 & x = 2

G-3. discontinuous and non-differentiable at −1, 0, 1, 3, 4

G-4. Differentiable in [−2, 2]

G-5_. Continuos everwhere in (0, 3) but non differentiable at x= 2

Section (H) :
H-1. 56   **H-2.**   − 2   **H-3.** $f(x) = e^{xf'(0)}$ ∀ x ∈ R   **H-4.**   − 3

PART – II

Section (A) :

A-1. (D)   **A-2.** (A)   **A-3.** (C)   **A-4.** (C)

SECTION (B) :

B-1. (C) **B-2.** (B) **B-3.** (D) **B-4.** (D) **B-5.** (D) **B-6.** (A) **B-7.** (D)

B-8. (B) **B-9.** (B) **B-10.** (C) **B-11.** (B) **B-12.** (D) **B-13.** (B) **B-14.** (C)

B-15. (A) **B16.** (A) **B-17.** (A) **B-18.** (C) **B-19.** (B) **B-20_.** (B)

SECTION (C) :

C-1. (A) **C-2.** (A) **C-3.** (B) **C-4.** (C) **C-5.** (A) **C-6.** (C) **C-7.** (B)

C-8. (D)

Section (D) :

D-1. (A) **D-2.** (D) **D3.** (D) **D-4_.** (B)

Section (E) :

E-1. (B) **E-2.** (C) **E-3.** (C) **E-4.** (B)

Section (F) :

F-1. (B) **F-2.** (B) **F-3.** (B) **F-4.** (B) **F-5.** (D) **F-6.** (B) **F-7.** (B)

Section (G) :

G-1. (A) **G-2.** (B) **G-3.** (D) **G-4.** (B) **G-5.** (C) **G-6.** (A) **G-7.** (C)

G-8. (A)

Section (H) :

H-1. (D) **H-2.** (B) **H-3.** (C) **H-4.** (D) **H-5.** (B) **H-6.** (D)

PART – III

1. (A) → (p, r, s), (B) → (p, r, s), (C) → (q, r, s), (D) → (r, s)
2. (A) → (p, q, r), (B) → (p, r, s), (C) → (p, r, s), (D) → (p, r, s)

EXERCISE - 2

PART – I

1. (B) **2.** (C) **3.** (C) **4.** (A) **5.** (B) **6.** (B) **7.** (C)

8. (C) **9.** (B) **10.** (B) **11.** (C) **12.** (B) **13_.** (B) **14.** (C)

15. (D) **16.** (B) **17.** (B) **18.** (C) **19.** (B) **20.** (D) **21.** (B)

22. (C) **23.** (C) **24.** (D)

PART – II

1.	2	**2.**	1	**3.**	2	**4.**	1	**5.**	0	**6.**	1	**7.**	2
8.	1	**9.**	11	**10.**	37	**11.**	20	**12.**	12	**13.**	1	**14.**	21
15.	11	**16.**	99	**17.**	4	**18.**	26	**19.**	36	**20.**	16	**21.**	7
22.	4	**23.**	7	**24.**	0	**25.**	3	**26.**	12	**27.**	11	**28.**	10

PART – III

1.	(ABC)	**2.**	(AB)	**3.**	(AD)	**4.**	(ABCD)	**5.**	(BC)	**6.**	(ABC)
7.	(ABCD)	**8.**	(BCD)	**9.**	(AB)	**10.**	(AD)	**11.**	(ABCD)	**12.**	(AC)
13.	(ABC)	**14.**	(ABCD)	**15.**	(BCD)	**16.**	(AD)	**17.**	(BCD)	**18.**	(ABD)
19.	(BCD)	**20.**	(ABC)	**21.**	(CD)	**22.**	(ABC)	**23.**	(ABC)	**24.**	(ABD)
25.	(BD)	**26.**	(AC)	**27.**	(AB)	**28.**	(BCD)	**H-9.**	(ABCD)		

PART – IV

1.	(A)	**2.**	(D)	**3.**	(A)	**4.**	(C)	**5.**	(D)	**6.**	(C)
7.	(C)	**8.**	(C)	**9.**	(C)	**10.**	(B)	**11.**	(C)	**12.**	(B)

EXERCISE - 3

PART – I

1*.	(AC)	**2.**	(D)	**3.**	(B, C, D) or (B,C)		**4*.**	(ABCD)	**5.**	(A)	**6.**	(B)
7.	(B)	**8.**	(B)	**9*.**	(BD)	**10*.**	(AD)	**11.**	0	**12.**	3	**13.**	(D)
14*.	(AD)	**15*.**	(ABC)	**16.**	2	**17.**	(B,C)	**18.**	7	**19.**	(B,C)	**20.**	(A,B)
21.	(ABD)	**22*.**	(CD)	**23.**	(D)	**24.**	(D)						

PART - II

1.	(3)	**2.**	(4)	**3.**	(1)	**4.**	(4)	**5.**	(3)	**6.**	(3)	**7.**	(3)
8.	(1)	**9.**	(3)	**10.**	(4)	**11.**	(2)	**12.**	(3)	**13.**	(1)	**14.**	(2)
15.	(1)	**16.**	(2)	**17.**	(1)	**18.**	(3)	**19.**	(3)	**20.**	(4)	**21.**	(3)

Advance Level Problems (ALP)

1. Evaluate : $\displaystyle \lim_{x \to 0} \frac{1 - \cos(a_1 x) \; . \; \cos \; (a_2 x). \; \cos \; (a_3 x).......\cos(a_n x)}{x^2}$, where $a_1, a_2, a_3, ... , a_n \in R$.

2. $f_1(x) = \dfrac{x}{2} + 10$

$f_n(x) = f_1(f_{n-1}(x)) \qquad n \ge 2$

then evaluate $\displaystyle \lim_{n \to \infty} f_n(x)$

3. Let $f : R \to R$ be a real function. The function f is derivable and there exists $n \in N$ and $p \in R$ such that $\displaystyle \lim_{x \to \infty} x^n f(x) = p$, then evaluate $\displaystyle \lim_{x \to \infty} \; (x^{n+1}.f'(x))$.

4. Let $<x_n>$ denotes a sequence, $x_1 = 1$, $x_{n+1} = \sqrt{x_n^2 + 1}$, then evaluate $\displaystyle \lim_{n \to \infty} \left(\frac{x_{n+1}}{x_n} \right)^n$

5. Evaluate $\displaystyle \lim_{n \to \infty} \left(\frac{1}{n^2 + 1} + \frac{2}{n^2 + 2} + \frac{3}{n^2 + 3} + ... + \frac{n}{n^2 + n} \right)$

6. Evaluate : $\displaystyle \lim_{x \to \infty} \; x^3 \left\{ \sqrt{x^2 + \sqrt{1 + x^4}} \; - \; x\sqrt{2} \right\}$

7. Evaluate $\displaystyle \lim_{x \to \infty} \frac{\log_e (\log_e x)}{e^{\sqrt{x}}}$

8. Evaluate $\displaystyle \lim_{x \to \frac{\pi}{2}} \frac{\log_e \left(\sin \; (4m+1)x \right)}{\log_e \left(\sin(4n+1)x \right)}$, where m,n Z

9. $f(x) = \displaystyle \sum_{r=1}^{n} \tan \frac{x}{2^r}.\sec \frac{x}{2^{r-1}} \quad r, n \in N$

$$g(x) = \begin{bmatrix} \displaystyle \lim_{n \to \infty} \dfrac{\log_e \left(f(x) + \tan \dfrac{x}{2^n} \right) - \left(f(x) + \tan \dfrac{x}{2^n} \right)^n \left[\sin \left(\tan \dfrac{x}{2} \right) \right]}{1 + \left(f(x) + \tan \dfrac{x}{2^n} \right)^n} & x \neq \dfrac{\pi}{4} \\ \\ k & x = \dfrac{\pi}{4} \end{bmatrix}$$

where [ ] denotes greatest integer function and domain of $g(x)$ is $\left(0, \dfrac{\pi}{2} \right)$

find 'k' for which $g(x)$ is continuous at $x = \pi/4$

10. Evaluate $\displaystyle \lim_{x \to (e^{-1})^+} \frac{e^{\frac{\ell n(1 + \ell nx)}{x}}}{x - e^{-1}}$

11. Let $P_n = \dfrac{2^3 - 1}{2^3 + 1} . \dfrac{3^3 - 1}{3^3 + 1} .... \dfrac{n^3 - 1}{n^3 + 1}$ Prove that $\displaystyle \lim_{n \to \infty} P_n = \dfrac{2}{3}$.

12. Verify the following limits

(i) $\displaystyle\lim_{x \to 0} \left(\dfrac{(1 + x)^{\frac{1}{x}}}{e} \right)^{\frac{1}{x}} = e^{-\frac{1}{2}}$
(ii) $\displaystyle\lim_{x \to 0} \left[\sin^2 \left(\dfrac{\pi}{2 - ax} \right) \right]^{\sec^2\left(\frac{\pi}{2 - bx}\right)} = e^{-\frac{a^2}{b^2}}$

13. $f(x) = \displaystyle\lim_{n \to \infty} \dfrac{x\sin^n x}{\sin^n x + 1}$. Find domain and range of $f(x)$, where $n \in \mathbb{N}$.

14. Evaluate $\displaystyle\lim_{x \to 0} \left(\dfrac{a_1^x + a_2^x}{b_1^x + b_2^x} \right)^{\frac{1}{x}}$ where a_1, a_2, b_1 and b_2 are positive numbers

15. Evaluate $\displaystyle\lim_{x \to 1} \left(\dfrac{p}{1 - x^p} - \dfrac{q}{1 - x^q} \right)$ where $p, q \in \mathbb{N}$

16. If $f(n, \theta) = \displaystyle\prod_{r=1}^{n} \left(1 - \tan^2 \dfrac{\theta}{2^r} \right)$ and $\displaystyle\lim_{n \to \infty} f(n,\theta) = g(\theta)$, then find the value $\displaystyle\lim_{\theta \to 0}$ of $g(\theta)$

17. Find the value of $\displaystyle\lim_{x \to \pi} \dfrac{1}{(x - \pi)} \left(\sqrt{\dfrac{4\cos^2 x}{2 + \cos x}} - 2 \right)$

18. $\displaystyle\lim_{x \to \infty} x^a \left(\sqrt[3]{x+1} + \sqrt[3]{x-1} - 2\sqrt[3]{x} \right) = \lambda$, $\lambda \neq 0$ then find the value of $a + \lambda$

19. Discuss the continuity of the function $f(x) = \displaystyle\lim_{n \to \infty} \dfrac{(1 + \sin x)^n + \log x}{2 + (1 + \sin x)^n}$

20. If $g(x) = \begin{cases} \dfrac{1 - a^x + xa^x \cdot \ell na}{x^2 a^x} & , \quad x < 0 \\ k & , \quad x = 0 \\ \dfrac{(2a)^x - x\,\ell n\,2a - 1}{x^2} & , \quad x > 0 \end{cases}$

(where $a > 0$), then find 'a' and $g(0)$ so that $g(x)$ is continuous at $x = 0$.

21. $f(x) = \begin{cases} \dfrac{\cos^{-1}(2x\sqrt{1-x^2})}{x - \dfrac{1}{\sqrt{2}}} & x \neq \dfrac{1}{\sqrt{2}} \\[6mm] k & x = \dfrac{1}{\sqrt{2}} \end{cases}$

Then find 'k' if possible for which function is continuous at $x = \dfrac{1}{\sqrt{2}}$

22. Find the value of $f(0)$ so that the function

$f(x) = \dfrac{\cos^{-1}(1 - \{x\}^2) \sin^{-1}(1 - \{x\})}{\{x\} - \{x\}^3}$, $x \neq 0$

({x} denotes fractional part of x) becomes continuous at $x = 0$

23. Let f be a continuous function on $\mathbb{R}$ such that $f\left(\dfrac{1}{4x} \right) = (\sin e^x)\,e^{-x^2} + \dfrac{x^2}{x^2 + 1}$, then find the value of $f(0)$.

24. Examine the continuity at x = 0 of the sum function of the infinite series:

$$\frac{x}{x+1} + \frac{x}{(x+1)(2x+1)} + \frac{x}{(2x+1)(3x+1)} + \ldots\ldots\ldots\infty .$$

25. If f(x) is continuous in [a, b] such that f(a) = b and f(b) = a, then prove that there exists at least one c $\in$ (a, b) such that f(c) = c.

26. If f(x . y) = f(x). f(y) for all x, y and f(x) is continuous at x = 1. Prove that f(x) is continuous for all x except possibly at x = 0. Given f(1) $\neq$ 0.

27. $g(x) = \underset{n \to \infty}{\ell im} \dfrac{x^n f(x) + h(x) + 1}{2x^n + 3x + 3}$,   x $\neq$ 1

$g(1) = \underset{x \to 1}{\ell im} \dfrac{\sin^2(\pi\, 2^x)}{\log_e \sec(\pi\, 2^x)}$ be a continous function at x = 1, then find the value of 4g(1) + 2 f(1) − h(1),

assume that f(x) and h(x) are continuous at x = 1

28. If f(x) = x² − 2|x|, then test the derivability of g(x) in the interval [–2, 3], where

$$g(x) = \begin{cases} \min.\{f(t);\ -2 \le t \le x\}\ , & -2 \le x < 0 \\ \max.\ \{f(t);\ 0 \le t \le x\}\ , & 0 \le x \le 3 \end{cases}$$

29. Discuss the continuity and differentiability of f(x) = [x] + {x}² and also draw its graph. Where [.] and {.} denote the greatest integer function and fractional part function respectively.

30. Discuss the continuity and differentiability of the function f(x) = $\begin{cases} \dfrac{x}{1+|x|} & ;\ |x| \ge 1 \\[3mm] \dfrac{x}{1-|x|} & ;\ |x| < 1 \end{cases}$

31. Discuss the continuity and differentiability of the function f(x) = $\left\{ \underset{n \to \infty}{\lim}\left(\underset{m \to \infty}{\lim} \dfrac{\cos^{2n}(m!\,\pi x) - 1}{\cos^{2n}(m!\,\pi x) + 1}\right)\right\}$,

(where m, n $\in$ N) at rational and irrational points.

32. Given f(x) = $\cos^{-1}\left(\text{sgn}\left(\dfrac{2[x]}{3x - [x]}\right)\right)$, where sgn ( ) denotes the signum function and [**.**] denotes the greatest integer function. Discuss the continuity and differentiability of f (x) at x = ± 1.

33. Discuss the continuity on 0 $\le$ x $\le$ 1 & differentiability at x = 0 for the function.

f(x) = x $\sin \dfrac{1}{x}$ $\sin \dfrac{1}{x\ \sin\frac{1}{x}}$ where x $\neq$ 0, x $\neq$ $\dfrac{1}{r\pi}$ & f(0) = f (1/rπ) = 0, r = 1, 2, 3,.......

34. Let f be a function such that f(xy) = f(x) . f(y) $\forall$ x > 0 , y > 0 . If f(1 + x) = 1 + x (1+g(x)) ,

where $\underset{x \to 0}{\lim}$ g(x) = 0 . Find $\int \dfrac{f(x)}{f'(x)} dx$

35. Let f : R⁺ $\to$ R satisfies the equation

f(xy) = e^{xy − x − y} (e^y f(x) + e^x f(y)) $\forall$ x , y $\in$ R⁺

If f′ (1) = e , then find f(x) .

36. Let f(x) be a real valued function not identically zero such that

f(x + y³) = f(x) + (f(y))³ ∀ x, y ∈ R and f′(0) ≥ 0, then find f(10)

37. Determine a function f satisfying the functional relation $f(x) + f\left(\dfrac{1}{1-x}\right) = \dfrac{2(1-2x)}{x(1-x)}$.

38. If f (x) + f (y) + f (xy) = 2 + f (x) . f (y) , for all real values of x and y and f (x) is a polynomial function with f (4) = 17 and f(1) ≠ 1, then find the value of f (5) .

39. If $| f(p + q) - f(q)| \le \dfrac{p}{q}$ for all p and q ∈ Q & q ≠ 0, show that $\sum_{i=1}^{k}| f(2^k) - f(2^i)| \le \dfrac{k(k-1)}{2}$

40. The function f : R → R satisfies x + f(x) = f(f(x)) for every x∈R. Find all solutions of the equation f(f(x)) = 0.

41. If 2f (x) = f(xy) + f(x/y) ∀ x, y ∈ R⁺, f(1) = 0 and f′(1) = 1, find f(x).

42. If $f(x \times f(y)) = \dfrac{f(x)}{y}$ ∀ x, y ∈ R , y ≠ 0, then prove that $f(x) . f\left(\dfrac{1}{x}\right) = 1$

43. Find the period of f(x) satisfying the condition :
(i)   f(x + p) = 1 + {1 − 3 f(x) + 3 f²(x) − f³ (x)}¹ᐟ³, p > 0
(ii)   f(x − 1) + f(x + 3) = f(x + 1) + f(x + 5)

44. Let f(x) is defined only for x ∈ (0, 5) and defined as f² (x) = 1 ∀ x ∈ (0, 5). Function f(x) is continuous for all x ∈ (0, 5) − {1, 2, 3, 4} (at x = 1, 2, 3, 4 f(x) may or may not be continuous). Find the number of possible function f(x) if it is discontinuous at
(i)   One integral points in (0, 5)
(ii)   two integral points in (0, 5)
(iii)   three integral points in (0, 5)
(iv)   four integral points in (0, 5)

45. Let f(x) is increasing and double differentiable function everywhere such that f(x) = x has 3 distinct root α, β and γ(α < β < γ). $h(x) = \lim_{n\to\infty}(\underbrace{f(f(....(f(x))))}_{n\ times})$
(i)   If f''(x) > 0 ∀ x ∈ (−∞, β) and f''(x) < 0 ∀ x ∈ (β, ∞] and f''(β) = 0, then find h(x)
(ii)   If f(x) ≥ x ∀ x ∈ (−∞, α] ∪ [γ, ∞) and f(x) ≤ x ∀ x ∈ [β, γ] then find h(x)

Answer Key (ALP)

1. $\dfrac{1}{2}\sum_{i=1}^{n} a_i^{\,2}$ **2.** 20   **3.** $-np$ **4.** $\sqrt{e}$ **5.** $\dfrac{1}{2}$ **6.** $\dfrac{1}{4\sqrt{2}}$

7. 0   **8.** $\dfrac{(4m+1)^2}{(4n+1)^2}$ **9.** $k=0$ **10.** 0

13. Domain $= R - \left\{2k\pi - \dfrac{\pi}{2},\ k \in Z\right\}$, Range $= \{0\} \cup \left\{k\pi + \dfrac{\pi}{4},\ k \in Z\right\}$

14. $\sqrt{\dfrac{a_1 a_2}{b_1 b_2}}$ **15.** $\dfrac{p-q}{2}$ **16.** 1   **17.** 0   **18.** $\dfrac{13}{9}$

19. $f(x)$ is discontinuous at integral multiples of π **20.** $\dfrac{1}{\sqrt{2}}$, $\dfrac{1}{8}$ $(\ell n\ 2)^2$

22. no value of $f(0)$ **23.** 1   **24.** Discontinuous   **27.** 5

28. discontinuous at $x = 0$ and not differentiable at $x = 0, 2$

29. $f(x)$ is continuous and non-differentiable for integral points

30. At $x = 0$ differentiable and at $x = \pm 1$ discontinuous

31. discontinuous and non-differentiable

32. f is continuous & derivable at $x = -1$ but f is neither continuous nor derivable at $x = 1$

33. continuous in $0 \le x \le 1$ & not differentiable at $x = 0$

34. $\dfrac{x^2}{2} + c$ **35.** $f(x) = e^x \ell n|x|$ **36.** $f(10) = 10$ **37.** $\dfrac{x+1}{x-1}$ **38.** 26

41. $f(x) = \log(x)$ **43.** (i) $2p$ (ii) 8   44. (i) 24   (ii) 108 (iii) 216 (iv) 162

45. (i) $h(x) = \begin{cases} \alpha & , & x \in (-\infty, \beta) \\ \beta & , & x = \beta \\ \gamma & , & x \in (\beta, \infty) \end{cases}$ (ii) $h(x) = \begin{cases} \alpha & , & x \in (-\infty, \alpha] \\ \beta & , & (\alpha, \gamma) \\ \gamma & , & [\gamma, \infty) \end{cases}$