Exercise-1

✍ **Marked questions are recommended for Revision.**

PART - I : SUBJECTIVE QUESTIONS

Section (A) : General Term & Coefficient of x^k in $(ax + b)^n$

A-1. Expand the following :

(i) $\left(\frac{2}{x} - \frac{x}{2}\right)^5$, $(x \neq 0)$ (ii) $\left(y^2 + \frac{2}{y}\right)^4$, $(y \neq 0)$

A-2. In the binomial expansion of $\left(\sqrt[3]{2} + \frac{1}{\sqrt[3]{3}}\right)^n$, the ratio of the 7th term from the begining to the 7th term from the end is 1 : 6 ; find n.

A-3. Find the degree of the polynomial $\left(x + (x^3 - 1)^{1/2}\right)^5 + \left(x - (x^3 - 1)^{1/2}\right)^5$.

A-4. Find the coefficient of

(i) x^6y^3 in $(x + y)^9$ (ii) $a^5 b^7$ in $(a - 2b)^{12}$

A-5. Find the co-efficient of x^7 in $\left(ax^2 + \frac{1}{b\ x}\right)^{11}$ and of x^{-7} in $\left(ax - \frac{1}{b\ x^2}\right)^{11}$ and find the relation between 'a' & 'b' so that these co-efficients are equal. (where $a, b \neq 0$).

A-6. Find the term independent of 'x' in the expansion of the expression,

$(1 + x + 2x^3)\left(\frac{3}{2}x^2 - \frac{1}{3\ x}\right)^9$.

A-7. (i) Find the coefficient of x^5 in $(1 + 2x)^6(1 - x)^7$.

(ii) Find the coefficient of x^4 in $(1 + 2x)^4 (2 - x)^5$

A-8. In the expansion of $\left(x^3 - \frac{1}{x^2}\right)^n$, $n \in N$, if the sum of the coefficients of x^5 and x^{10} is 0, then n is :

Section (B) : Middle term, Remainder & Numerically/Algebraically Greatest terms

B-1. Find the middle term(s) in the expansion of

(i) $\left(\frac{x}{y} - \frac{y}{x}\right)^7$ (ii) $(1 - 2x + x^2)^n$

B-2. Prove that the co-efficient of the middle term in the expansion of $(1 + x)^{2n}$ is equal to the sum of the co-efficients of middle terms in the expansion of $(1 + x)^{2n-1}$.

B-3. (i) Find the remainder when 7^{98} is divided by 5

(ii) Using binomial theorem prove that $6^n - 5n$ always leaves the remainder 1 when divided by 25.

(iii) Find the last digit, last two digits and last three digits of the number $(27)^{27}$.

B-4.✍ Which is larger : $(99^{50} + 100^{50})$ or $(101)^{50}$.

B-5. (i) Find numerically greatest term(s) in the expansion of $(3 - 5x)^{15}$ when $x = \frac{1}{5}$

(ii) Which term is the numerically greatest term in the expansion of $(2x + 5y)^{34}$, when x = 3 & y = 2 ?

B-6. Find the term in the expansion of $(2x - 5)^6$ which have

(i) Greatest binomial coefficient
(ii) Greatest numerical coefficient
(iii) Algebraically greatest coefficient
(iv) Algebraically least coefficient

Section (C) : Summation of series, Variable upper index & Product of binomial coefficients

C-1. If $C_0, C_1, C_2, \ldots C_n$ are the binomial coefficients in the expansion of $(1 + x)^n$ then prove that :

(i) $\frac{(3.2-1)}{2}C_1 + \frac{3^2.2^2-1}{2^2}C_2 + \frac{3^3.2^3-1}{2^3}C_3 + \ldots\ldots + \frac{3^n.2^n-1}{2^n}C_n = \frac{2^{3n}-3^n}{2^n}$

(ii) $\frac{C_1}{C_0} + 2\frac{C_2}{C_1} + 3\frac{C_3}{C_2} + \ldots\ldots + n\frac{C_n}{C_{n-1}} = \frac{n(n+1)}{2}$

(iii) $(C_0 + C_1)(C_1 + C_2)(C_2 + C_3)(C_3 + C_4)\ldots\ldots(C_{n-1} + C_n) = \frac{C_0C_1C_2\ldots\ldots C_{n-1}(n+1)^n}{n!}$.

(iv) $C_o - 2C_1 + 3C_2 - 4C_3 + \ldots + (-1)^n(n+1)C_n = 0$

(v) $4C_0 + \frac{4^2}{2}.C_1 + \frac{4^3}{3}C_2 + \ldots\ldots + \frac{4^{n+1}}{n+1}C_n = \frac{5^{n+1}-1}{n+1}$

(vi) $\frac{2^2.C_0}{1.2} + \frac{2^3.C_1}{2.3} + \frac{2^4.C_2}{3.4} + \ldots + \frac{2^{n+2}.C_n}{(n+1)(n+2)} = \frac{3^{n+2}-2n-5}{(n+1)(n+2)}$

C-2. Prove that

$$2.C_o + \frac{2^2.C_1}{2} + \frac{2^3.C_2}{3} + \frac{2^4.C_3}{4} + \ldots\ldots \frac{2^{n+1}.C_n}{n+1} = \frac{3^{n+1}-1}{n+1}$$

C-3. Prove that ${}^nC_r + {}^{n-1}C_r + {}^{n-2}C_r + \ldots\ldots + {}^rC_r = {}^{n+1}C_{r+1}$

C-4. If $(1 + x)^n = C_0 + C_1x + C_2x^2 + \ldots\ldots + C_nx^n$, prove that

(i) $C_0C_3 + C_1C_4 + \ldots\ldots + C_{n-3}C_n = \frac{(2n)!}{(n+3)!\,(n-3)!}$

(ii) $C_0C_r + C_1C_{r+1} + \ldots\ldots + C_{n-r}C_n = \frac{(2n)!}{(n+r)!\,(n-r)!}$

(iii) $C_0^2 - C_1^2 + C_2^2 - C_3^2 + \ldots\ldots + (-1)^nC_n^2 = 0$ or $(-1)^{n/2}C_{n/2}$ according as n is odd or even.

Section (D) : Negative & fractional index, Multinomial theorem

D-1. Find the co-efficient of x^6 in the expansion of $(1 - 2x)^{-5/2}$.

D-2. (i) Find the coefficient of x^{12} in $\frac{4+2x-x^2}{(1+x)^3}$

(ii) Find the coefficient of x^{100} in $\frac{3-5x}{(1-x)^2}$

D-3. Assuming ' x ' to be so small that x^2 and higher powers of ' x ' can be neglected, show that, $\frac{\left(1+\frac{3}{4}x\right)^{-4}(16-3x)^{1/2}}{(8+x)^{2/3}}$ is approximately equal to, $1 - \frac{305}{96}x$.

D-4. (i) Find the coefficient of $a^5 b^4 c^7$ in the expansion of $(bc + ca + ab)^8$.
(ii) Sum of coefficients of odd powers of x in expansion of $(9x^2 + x - 8)^6$

D-5. Find the coefficient of x^7 in $(1 - 2x + x^3)^5$.

PART - II : ONLY ONE OPTION CORRECT TYPE

Section (A) : General Term & Coefficient of x^k in $(ax + b)^n$

A-1. The $(m + 1)^{th}$ term of $\left(\frac{x}{y}+\frac{y}{x}\right)^{2m+1}$ is:
(A) independent of x (B) a constant
(C) depends on the ratio x/y and m (D) none of these

A-2. The total number of distinct terms in the expansion of, $(x + a)^{100} + (x - a)^{100}$ after simplification is :
(A) 50 (B) 202 (C) 51 (D) none of these

A-3. The value of, $\dfrac{18^3+7^3+3\,.\,18\,.\,7\,.\,25}{3^6+6.\,243.\,2+15\,.\,81\,.\,4+20.\,27.\,8+15.\,9\,.\,16+6.3.\,32+64}$ is :
(A) 1 (B) 2 (C) 3 (D) none

A-4. In the expansion of $\left(3-\sqrt{\frac{17}{4}+3\sqrt{2}}\right)^{15}$ the 11th term is a :
(A) positive integer (B) positive irrational number
(C) negative integer (D) negative irrational number.

A-5. If the second term of the expansion $\left[a^{1/13}+\frac{a}{\sqrt{a^{-1}}}\right]^n$ is $14a^{5/2}$, then the value of $\frac{{}^nC_3}{{}^nC_2}$ is:
(A) 4 (B) 3 (C) 12 (D) 6

A-6. In the expansion of $(7^{1/3} + 11^{1/9})^{6561}$, the number of terms free from radicals is:
(A) 730 (B) 729 (C) 725 (D) 750

A-7. The value of m, for which the coefficients of the $(2m + 1)^{th}$ and $(4m + 5)^{th}$ terms in the expansion of $(1 + x)^{10}$ are equal, is
(A) 3 (B) 1 (C) 5 (D) 8

A-8. The co-efficient of x in the expansion of $(1 - 2x^3 + 3x^5)\left(1+\frac{1}{x}\right)^8$ is :
(A) 56 (B) 65 (C) 154 (D) 62

A-9. Given that the term of the expansion $(x^{1/3} - x^{-1/2})^{15}$ which does not contain x is 5 m, where $m \in N$, then m=
(A) 1100 (B) 1010 (C) 1001 (D) 1002

A-10. The term independent of x in the expansion of $\left(x-\frac{1}{x}\right)^4\left(x+\frac{1}{x}\right)^3$ is:
(A) – 3 (B) 0 (C) 1 (D) 3

Section (B) : Middle term, Remainder & Numerically/Algebraically Greatest terms

B-1. If $k \in R^+$ and the middle term of $\left(\frac{k}{2}+2\right)^8$ is 1120, then value of k is:

(A) 3 (B) 2 (C) 1 (D) 4

B-2. The remainder when 2^{2003} is divided by 17 is :

(A) 1 (B) 2 (C) 8 (D) 7

B-3. The last two digits of the number 3^{400} are:

(A) 81 (B) 43 (C) 29 (D) 01

B-4. The last three digits in 10 ! are :

(A) 800 (B) 700 (C) 500 (D) 600

B-5. The value of $\sum_{r=1}^{10} r \cdot \frac{{}^nC_r}{{}^nC_{r-1}}$ is equal to

(A) 5 (2n – 9) (B) 10 n (C) 9 (n – 4) (D) n –2

B-6. $\sum_{r=0}^{n-1} \frac{{}^nC_r}{{}^nC_r + {}^nC_{r+1}} =$

(A) $\frac{n}{2}$ (B) $\frac{n+1}{2}$ (C) $(n+1)\frac{n}{2}$ (D) $\frac{n\ (n-1)}{2\ (n+1)}$

B-7. Find numerically greatest term in the expansion of $(2 + 3x)^9$, when x = 3/2.

(A) ${}^9C_6 . 2^9 . (3/2)^{12}$ (B) ${}^9C_3 . 2^9 . (3/2)^6$ (C) ${}^9C_5 . 2^9 . (3/2)^{10}$ (D) ${}^9C_4 . 2^9 . (3/2)^8$

B-8. The greatest integer less than or equal to $(\sqrt{2} + 1)^6$ is

(A) 196 (B) 197 (C) 198 (D) 199

Section (C) : Summation of series, Variable upper index & Product of binomial coefficients

C-1. $\frac{{}^{11}C_0}{1} + \frac{{}^{11}C_1}{2} + \frac{{}^{11}C_2}{3} + ... + \frac{{}^{11}C_{10}}{11} =$

(A) $\frac{2^{11}-1}{11}$ (B) $\frac{2^{11}-1}{6}$ (C) $\frac{3^{11}-1}{11}$ (D) $\frac{3^{11}-1}{6}$

C-2. The value of $\frac{C_0}{1.3} - \frac{C_1}{2.3} + \frac{C_2}{3.3} - \frac{C_3}{4.3} + ... + (-1)^n \frac{C_n}{(n+1)\ .\ 3}$ is :

(A) $\frac{3}{n+1}$ (B) $\frac{n+1}{3}$ (C) $\frac{1}{3(n+1)}$ (D) none of these

C-3. The value of the expression ${}^{47}C_4 + \sum_{j=1}^{5} {}^{52-j}C_3$ is equal to :

(A) ${}^{47}C_5$ (B) ${}^{52}C_5$ (C) ${}^{52}C_4$ (D) ${}^{49}C_4$

C-4. The value of $\binom{50}{0}\binom{50}{1}+\binom{50}{1}\binom{50}{2}+.........+\binom{50}{49}\binom{50}{50}$ is, where ${}^nC_r=\binom{n}{r}$

(A) $\binom{100}{50}$ (B) $\binom{100}{51}$ (C) $\binom{50}{25}$ (D) $\binom{50}{25}^2$

Section (D) : Negative & fractional index, Multinomial theorem

D-1. If $|x| < 1$, then the co-efficient of x^n in the expansion of $(1 + x + x^2 + x^3 + ...)^2$ is

(A) n (B) n – 1 (C) n + 2 (D) n + 1

D-2. The co–efficient of x^4 in the expansion of $(1 – x + 2x^2)^{12}$ is:

(A) ${}^{12}C_3$ (B) ${}^{13}C_3$ (C) ${}^{14}C_4$ (D) ${}^{12}C_3 + 3\ {}^{13}C_3 + {}^{14}C_4$

D-3. If $(1 + x)^{10} = a_0 + a_1x + a_2x^2 + ...+ a_{10}x^{10}$, then value of $(a_0 – a_2 + a_4 – a_6 + a_8 – a_{10})^2 + (a_1 – a_3 + a_5 – a_7 + a_9)^2$ is

(A) 2^{10} (B) 2 (C) 2^{20} (D) None of these

PART - III : MATCH THE COLUMN

1. **Column – I** / **Column – II**

(A) If $(r + 1)^{th}$ term is the first negative term in the expansion of $(1 + x)^{7/2}$, then the value of r (where $0 < x < 1$) is — (p) divisible by 2

(B) If the sum of the co-efficients in the expansion of $(1 + 2x)^n$ is 6561, and T_r is the greatest term in the expansion for x = 1/2 then r is — (q) divisible by 5

(C) ${}^nC_r$ is divisible by n, $(1 < r < n)$ if n is — (r) divisible by 10

(D) The coefficient of x^4 in the expression $(1 + 2x + 3x^2 + 4x^3 + ...\text{up to } \infty)^{1/2}$ is c, $(c \in N)$, then c + 1 (where $|x| < 1$) is — (s) a prime number

Exercise-2

Marked questions are recommended for Revision.

PART - I : ONLY ONE OPTION CORRECT TYPE

1. In the expansion of

$$\left(3\sqrt{\frac{a}{b}}+3\sqrt{\frac{b}{\sqrt{a}}}\right)^{21}$$

, the term containing same powers of a & b is

(A) 11^{th} term (B) 13^{th} term (C) 12^{th} term (D) 6^{th} term

2. Consider the following statements :

S_1 : Number of dissimilar terms in the expansion of $(1 + x + x^2 + x^3)^n$ is $3n + 1$

S_2 : $(1 + x)(1 + x + x^2)(1 + x + x^2 + x^3)......(1 + x + x^2 + ... + x^{100})$ when written in the ascending power of x then the highest exponent of x is 5000.

S_3 : $\sum_{k=1}^{n-r} {}^{n-k}C_r = {}^nC_{r+1}$

S_4 : If $(1 + x + x^2)^n = a_0 + a_1x + a_2 x^2 + ... + a_{2n} x^{2n}$, then $a_0 + a_2 + a_4 + ... + a_{2n} = \frac{3^n - 1}{2}$

State, in order, whether S_1, S_2, S_3, S_4 are true or false

(A) TFTF (B) TTTT (C) FFFF (D) FTFT

3. If $\frac{{}^nC_r + 4\ {}^nC_{r+1} + 6\ {}^nC_{r+2} + 4\ {}^nC_{r+3} + {}^nC_{r+4}}{{}^nC_r + 3\,{}^nC_{r+1} + 3\ {}^nC_{r+2} + {}^nC_{r+3}} = \frac{n+k}{r+k}$ then the value of k is :

(A) 1 (B) 2 (C) 4 (D) 5

4. The co-efficient of x^5 in the expansion of $(1 + x)^{21} + (1 + x)^{22} + ... + (1 + x)^{30}$ is :

(A) ${}^{51}C_5$ (B) ${}^9C_5$ (C) ${}^{31}C_6 - {}^{21}C_6$ (D) ${}^{30}C_5 + {}^{20}C_5$

5. The coefficient of x^{52} in the expansion $\sum_{m=0}^{100} {}^{100}C_m (x - 3)^{100-m} . 2^m$ is :

(A) ${}^{100}C_{47}$ (B) ${}^{100}C_{48}$ (C) $-{}^{100}C_{52}$ (D) $-{}^{100}C_{100}$

6. The sum of the coefficients of all the integral powers of x in the expansion of $\left(1+2\sqrt{x}\right)^{40}$ is :

(A) $3^{40} + 1$ (B) $3^{40} - 1$ (C) $\frac{1}{2}(3^{40} - 1)$ (D) $\frac{1}{2}(3^{40} + 1)$

7. $\sum_{r=0}^{n} (-1)^r\ {}^nC_r . \frac{(1+r\ln 10)}{(1+\ln 10^n)^r} =$

(A) 0 (B) 1/2 (C) 1 (D) None of these

8. The coefficient of the term independent of x in the expansion of $\left(\frac{x+1}{x^{\frac{2}{3}} - x^{\frac{1}{3}} + 1} - \frac{x-1}{x - x^{\frac{1}{2}}}\right)^{10}$ is :

(A) 70 (B) 112 (C) 105 (D) 210

9. Coefficient of x^{n-1} in the expansion of, $(x + 3)^n + (x + 3)^{n-1}(x + 2) + (x + 3)^{n-2}(x + 2)^2 + ... + (x + 2)^n$ is :

(A) ${}^{n+1}C_2(3)$ (B) ${}^{n-1}C_2(5)$ (C) ${}^{n+1}C_2(5)$ (D) ${}^nC_2(5)$

10. Let $f(n) = 10^n + 3.4^{n+2} + 5$, $n \in N$. The greatest value of the integer which divides f(n) for all n is :

(A) 27 (B) 9 (C) 3 (D) None of these

11. If $(1 + x)^n = \sum_{r=0}^{n} a_r x^r$ and $b_r = 1 + \frac{a_r}{a_{r-1}}$ and $\prod_{r=1}^{n} b_r = \frac{(101)^{100}}{100!}$, then n equals to :

(A) 99 (B) 100 (C) 101 (D) 102

12. Number of rational terms in the expansion of $\left(1+\sqrt{2}+\sqrt{5}\right)^6$ is :

(A) 7 (B) 10 (C) 6 (D) 8

13. If $S = {}^{404}C_4 - {}^4C_1 . {}^{303}C_4 + {}^4C_2 . {}^{202}C_4 - {}^4C_3 . {}^{101}C_4 = (101)^k$ then k equals to :

(A) 1 (B) 2 (C) 4 (D) 6

14. ${}^{10}C_0{}^2 - {}^{10}C_1{}^2 + {}^{10}C_2{}^2 - .... - \left({}^{10}C_9\right)^2 + \left({}^{10}C_{10}\right)^2 =$

(A) 0 (B) $\left({}^{10}C_5\right)^2$ (C) $-{}^{10}C_5$ (D) $2\,{}^9C_5$

15. The sum $\sum_{r=0}^{n} (r + 1)\, C_r^2$ is equal to *:*

(A) $\frac{(n+2)(2n-1)!}{n!\,(n-1)!}$ (B) $\frac{(n+2)(2n+1)!}{n!\,(n-1)!}$ (C) $\frac{(n+2)(2n+1)!}{n!\,(n+1)!}$ (D) $\frac{(n+2)(2n-1)!}{n!\,(n+1)!}$

16. If $(1 + x + x^2 + x^3)^5 = a_0 + a_1x + a_2x^2 + ... + a_{15}x^{15}$, then a_{10} equals to :

(A) 99 (B) 101 (C) 100 (D) 110

17. If $a_n = \sum_{r=0}^{n} \frac{1}{{}^nC_r}$, the value of $\sum_{r=0}^{n} \frac{n-2r}{{}^nC_r}$ is :

(A) $\frac{n}{2} a_n$ (B) $\frac{1}{4} a_n$ (C) na_n (D) 0

18. The sum of: $3.{}^nC_0 - 8.{}^nC_1 + 13.{}^nC_2 - 18.{}^nC_3 + ...$ upto (n+1) terms is $(n \geq 2)$:

(A) zero (B) 1 (C) 2 (D) none of these

19. If $\sum_{r=0}^{n-1} \left(\frac{{}^nC_r}{{}^nC_r + {}^nC_{r+1}} \right)^3 = \frac{4}{5}$ then n =

(A) 4 (B) 6 (C) 8 (D) None of these

20. The number of terms in the expansion of $\left(x^2 + 1 + \frac{1}{x^2}\right)^n$, $n \in N$, is :

(A) 2n (B) 3n (C) 2n + 1 (D) 3n + 1

PART - II : SINGLE AND DOUBLE VALUE INTEGER TYPE

1. If $\frac{1}{1!10!} + \frac{1}{2!9!} + \frac{1}{3!8!} + ... + \frac{1}{10!1!} = \frac{2}{k!}(2^{k-1} - 1)$ then find the value of k.

2. If the 6th term in the expansion of $\left[\frac{1}{x^{8/3}} + x^2 \log_{10} x\right]^8$ is 5600, then x =

3. The number of values of ' x ' for which the fourth term in the expansion,

$$\left(5^{\frac{2}{5}\log_5 \sqrt{4^x + 44}} + \frac{1}{5^{\log_5 \sqrt[3]{2^{x-1} + 7}}}\right)^8$$ is 336, is :

4. If second, third and fourth terms in the expansion of $(x + a)^n$ are 240, 720 and 1080 respectively, then n is equal to

5. Let the co-efficients of x^n in $(1 + x)^{2n}$ & $(1 + x)^{2n-1}$ be P & Q respectively, then $\left(\frac{P+Q}{Q}\right)^5 =$

6. In the expansion of $\left(3^{\frac{-x}{4}} + 3^{\frac{5x}{4}}\right)^n$, the sum of the binomial coefficients is 256 and four times the term with greatest binomial coefficient exceeds the square of the third term by 21n, then find 4x.

7. If $\sum_{k=1}^{19} \frac{(-2)^k}{k!(19-k)!} = \frac{-\lambda}{19!}$ then find λ.

8. The value of p, for which coefficient of x^{50} in the expression $(1 + x)^{1000} + 2x(1 + x)^{999} + 3x^2(1 + x)^{998} + \ldots + 1001\,x^{1000}$ is equal to ${}^{1002}C_p$, is :

9. If $\{x\}$ denotes the fractional part of 'x', then $82\left\{\frac{3^{1001}}{82}\right\} =$

10. The index 'n' of the binomial $\left(\frac{x}{5} + \frac{2}{5}\right)^n$ if the only 9^{th} term of the expansion has numerically the greatest coefficient ($n \in N$), is :

11. The number of values of 'r' satisfying the equation, ${}^{39}C_{3r-1} - {}^{39}C_{r^2} = {}^{39}C_{r^2-1} - {}^{39}C_{3r}$ is :

12. Find the value of
${}^6C_0 . {}^{12}C_6 - {}^6C_1\,{}^{11}C_6 + {}^6C_2\,{}^{10}C_6 - {}^6C_3 . {}^9C_6 + {}^6C_4 . {}^8C_6 - {}^6C_5 . {}^7C_6 + {}^6C_6 . {}^6C_6$

13. If n is a positive integer & $C_k = {}^nC_k$, find the value of $\left(\sum_{k=1}^{n} \frac{k^3}{n(n+1)^2.(n+2)}\left(\frac{C_k}{C_{k-1}}\right)^2\right)^{-1}$ is :

14. The value of the expression $\left(\sum_{r=0}^{10} {}^{10}C_r\right)\left(\sum_{K=0}^{10} (-1)^K \frac{{}^{10}C_K}{2^K}\right)$ is :

15. The value of λ if $\sum_{m=97}^{100} {}^{100}C_m . {}^mC_{97} = 2^{\lambda} . {}^{100}C_{97}$, is :

16. If $(1 + x + x^2 + \ldots + x^p)^6 = a_0 + a_1x + a_2x^2 + \ldots + a_{6p}\,x^{6p}$, then the value of :
$\frac{1}{p(p+1)^6}\,[a_1 + 2a_2 + 3a_3 + \ldots + 6p\,a_{6p}]$ is :

17. If $({}^{2n}C_1)^2 + 2.({}^{2n}C_2)^2 + 3.({}^{2n}C_3)^2 + \ldots + 2n.({}^{2n}C_{2n})^2 = 18 . {}^{4n-1}C_{2n-1}$, then n is :

18. If $\sum_{r=0}^{n} \frac{2r+3}{r+1}.{}^nC_r = \frac{(n+k).2^{n+1}-1}{n+1}$ then 'k' is

19. If $\sum_{r=0}^{n} \frac{(-1)^r.C_r}{(r+1)(r+2)(r+3)} = \frac{1}{a(n+b)}$, then a + b is

20. $\sum_{k=1}^{3n} {}^{6n}C_{2k-1}(-3)^k$ is equal to :

21. If x is very large as compare to y, then the value of k in $\sqrt{\frac{x}{x+y}}\sqrt{\frac{x}{x-y}} = 1 + \frac{y^2}{kx^2}$

PART - III : ONE OR MORE THAN ONE OPTIONS CORRECT TYPE

1. In the expansion of $\left(\sqrt[3]{4} + \frac{1}{\sqrt[4]{6}}\right)^{20}$

(A) the number of irrational terms is 19 (B) middle term is irrational
(C) the number of rational terms is 2 (D) 9th term is rational

2. The coefficient of x^4 in $\left(\frac{1+x}{1-x}\right)^2$, | x | < 1, is

(A) 4 (B) –4 (C) $10 + {}^4C_2$ (D) 16

3. $7^9 + 9^7$ is divisible by :
(A) 16 (B) 24 (C) 64 (D) 72

4. The sum of the series $\sum_{r=1}^{n} (-1)^{r-1} \cdot {}^nC_r(a-r)$ is equal to :

(A) 5 if a = 5 (B) –5 if a = 5 (C) –5 if a = –5 (D) 5 if a = –5

5. Let $a_n = \frac{1000^n}{n!}$ for $n \in N$, then a_n is greatest, when
(A) n = 997 (B) n = 998 (C) n = 999 (D) n = 1000

6. ${}^nC_0 - 2.3\, {}^nC_1 + 3.3^2\, {}^nC_2 - 4.3^3\, {}^nC_3 + ... + (-1)^n (n+1)\, {}^nC_n\, 3^n$ is equal to

(A) $2^n\left(\frac{3n}{2}+1\right)$ if n is even (B) $2^n\left(n+\frac{3}{2}\right)$ if n is even

(C) $-\,2^n\left(\frac{3n}{2}+1\right)$ if n is odd (D) $2^n\left(n+\frac{3}{2}\right)$ if n is odd

7. Element in set of values of r for which, ${}^{18}C_{r-2} + 2.\,{}^{18}C_{r-1} + {}^{18}C_r \geq {}^{20}C_{13}$ is :
(A) 9 (B) 5 (C) 7 (D) 10

8. The expansion of $(3x + 2)^{-1/2}$ is valid in ascending powers of x, if x lies in the interval.
(A) (0, 2/3) (B) (–3/2, 3/2) (C) (–2/3, 2/3) (D) (–∞, –3/2) (3/2, ∞)

9. If $(1 + 2x + 3x^2)^{10} = a_0 + a_1x + a_2x^2 + ... + a_{20}x^{20}$, then :
(A) $a_1 = 20$ (B) $a_2 = 210$ (C) $a_4 = 8085$ (D) $a_{20} = 2^2 . 3^7 . 7$

10. In the expansion of $(x + y + z)^{25}$

(A) every term is of the form ${}^{25}C_r \cdot {}^{r}C_k \cdot x^{25-r} \cdot y^{r-k} \cdot z^k$ (B) the coefficient of $x^8\, y^9\, z^9$ is 0

(C) the number of terms is 325 (D) none of these

11. If $(1 + x + 2x^2)^{20} = a_0 + a_1x + a_2x^2 + \dots\dots\dots + a_{40}x^{40}$, then $a_0 + a_2 + a_4 \dots\dots + a_{38}$ is equal to :

(A) $2^{19}(2^{30} + 1)$ (B) $2^{19}(2^{20} - 1)$ (C) $2^{39} - 2^{19}$ (D) $2^{39} + 2^{19}$

12. $n^n\left(\frac{n+1}{2}\right)^{2n}$ is $(n \in N)$

(A) Less than $\left(\frac{n+1}{2}\right)^{3}$ (B) Greater than or equal to $\left(\frac{n+1}{2}\right)^{3}$

(C) Less than $(n!)^3$ (D) Greater than or equal to $(n!)^3$.

13. If recursion polynomials $P_k(x)$ are defined as $P_1(x) = (x - 2)^2$, $P_2(x) = ((x - 2)^2 - 2)^2$
$P_3(x) = ((x - 2)^2 - 2)^2 - 2)^2$ .... (In general $P_k(x) = (P_{k-1}(x) - 2)^2$, then the constant term in $P_k(x)$ is

(A) 4 (B) 2 (C) 16 (D) a perfect square

PART - IV : COMPREHENSION

Comprehension # 1 (Q. No. 1 to 3)

Consider, sum of the series $\sum\sum_{0 \le i < j \le n} f(i)\, f(j)$

In the given summation, i and j are not independent.

In the sum of series $\sum_{i=1}^{n}\sum_{j=1}^{n} f(i)\ \ f(j) = \sum_{i=1}^{n}\left(f(i)\left(\sum_{j=1}^{n} f(j)\right)\right)$ i and j are independent. In this summation, three types of terms occur, those when i < j, i > j and i = j.

Also, sum of terms when i < j is equal to the sum of the terms when i > j if f(i) and f(j) are symmetrical. So, in that case

$$\sum_{i=1}^{n}\sum_{j=1}^{n} f(i)f(j) = \sum\sum_{0\le i<j\le n} f(i)f(j) + \sum\sum_{0\le i<j\le n} f(i)f(j) + \sum\sum_{i=j} f(i)f(j)$$

$$= 2\sum\sum_{0\le i<j\le n} f(i)f(j) + \sum\sum_{i=j} f(i)f(j)$$

$$\Rightarrow \sum\sum_{0\le i<j\le n} f(i)f(j) = \frac{\sum_{i=0}^{n}\sum_{j=0}^{n} f(i)f(j) - \sum\sum_{i=j} f(i)f(j)}{2}$$

When f(i) and f(j) are not symmetrical, we find the sum by listing all the terms.

1. $\sum\sum_{0\le i<j\le n} {}^{n}C_i\ \ {}^{n}C_j$ is equal to

(A) $\frac{2^{2n} - {}^{2n}C_n}{2}$ (B) $\frac{2^{2n} + {}^{2n}C_n}{2}$ (C) $\frac{2^{2n} - {}^{n}C_n}{2}$ (D) $\frac{2^{2n} + {}^{n}C_n}{2}$

2. Let ${}^0C_0 = 1$, then $\sum_{m=0}^{n}\ \sum_{p=0}^{m} {}^{n}C_m \ .\ {}^{m}C_p$ is equal to

(A) $2^n - 1$ (B) 3^n (C) $3^n - 1$ (D) 2^n

3. $\sum\sum_{0\le i\le j\le n}\left({}^nC_i + {}^nC_j\right)$

(A) $(n + 2)2^n$ (B) $(n + 1)2^n$ (C) $(n – 1)2^n$ (D) $(n + 1)2^n – 1$

Comprehension # 2 (Q. No. 4 to 6)

Let P be a product given by $P = (x + a_1)(x + a_2) \ldots\ldots\ldots (x + a_n)$

and Let $S_1 = a_1 + a_2 + \ldots\ldots + a_n = \sum_{i=1}^{n} a_i$, $S_2 = \sum\sum_{i<j} a_i.a_j$, $S_3 = \sum\sum_{i<j<k}\sum a_i.a_j.a_k$ and so on,

then it can be shown that

$P = x^n + S_1 x^{n-1} + S_2 x^{n-2} + \ldots\ldots\ldots + S_n$.

4. The coefficient of x^8 in the expression $(2 + x)^2 (3 + x)^3 (4 + x)^4$ must be

(A) 26 (B) 27 (C) 28 (D) 29

5. The coefficient of x^{203} in the expression $(x – 1)(x^2 – 2)(x^3 – 3) \ldots\ldots\ldots\ldots (x^{20} – 20)$ must be

(A) 11 (B) 12 (C) 13 (D) 15

6. The coefficient of x^{98} in the expression of $(x – 1)(x – 2) \ldots\ldots\ldots (x – 100)$ must be

(A) $1^2 + 2^2 + 3^2 + \ldots\ldots\ldots + 100^2$

(B) $(1 + 2 + 3 + \ldots\ldots\ldots + 100)^2 – (1^2 + 2^2 + 3^2 + \ldots\ldots\ldots + 100^2)$

(C) $\frac{1}{2}[(1 + 2 + 3 + \ldots\ldots\ldots + 100)^2 – (1^2 + 2^2 + 3^2 + \ldots\ldots\ldots + 100^2)]$

(D) None of these

Comprehension # 3 (Q.No. 7 to 9)

Let $(7 + 4\sqrt{3})^n = I + f = {}^nC_0.7^n + {}^nC_1.7^{n-1}.(4\sqrt{3})^1 + \ldots\ldots\ldots$ ....(i)

where I & f are its integral and fractional parts respectively.

It means $0 < f < 1$

Now, $0 < 7 – 4\sqrt{3} < 1 \Rightarrow 0 < (7 – 4\sqrt{3})^n < 1$

Let $(7 – 4\sqrt{3})^n = f' = {}^nC_0.7^n – {}^nC_1.7^{n-1}.(4\sqrt{3})^1 + \ldots\ldots\ldots$ ....(ii)

$\Rightarrow 0 < f' < 1$

Adding (i) and (ii) (so that irrational terms cancelled out)

$I + f + f' = (7 + 4\sqrt{3})^n + (7 – 4\sqrt{3})^n$

$= 2[{}^nC_0\, 7^n + {}^nC_2\, 7^{n-2}(4\sqrt{3})^2 + \ldots\ldots\ldots\ldots]$

$I + f + f'$ = even integer $\Rightarrow$ ($f + f'$ must be an integer)

$0 < f + f' < 2 \Rightarrow f + f' = 1$

with help of above analysis answer the following questions

7. If $\left(3\sqrt{3} + 5\right)^n = p + f$, where p is an integer and f is a proper fraction, then find the value of $\left(3\sqrt{3} - 5\right)^n$, $n \in N$, is

(A) 1 – f, if n is even (B) f, if n is even (C) 1 – f, if n is odd (D) f, if n is odd

8. If $\left(9 + \sqrt{80}\right)^n = I + f$, where I, n are integers and $0 < f < 1$, then :

(A) I is an odd integer (B) I is an even integer (C) $(I + f)(1 – f) = 1$ (D) $1 – f = \left(9 - \sqrt{80}\right)^n$

9. The integer just above $(\sqrt{3} + 1)^{2n}$ is, for all $n \in N$.

(A) divisible by 2^n (B) divisible by 2^{n+1} (C) divisible by 8 (D) divisible by 16

Exercise-3

✍ **Marked questions are recommended for Revision.**

PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

*** Marked Questions may have more than one correct option.**

1. Coefficient of t^{24} in $(1 + t^2)^{12} (1 + t^{12}) (1 + t^{24})$ is: **[IIT-JEE-2003, Scr, (3, – 1), 84]**

(A) ${}^{12}C_6 + 3$ (B) ${}^{12}C_6 + 1$ (C) ${}^{12}C_6$ (D) ${}^{12}C_6 + 2$

2.✍ Prove that $2^k \binom{n}{0}\binom{n}{k} - 2^{k-1}\binom{n}{1}\binom{n-1}{k-1} + 2^{k-2}\binom{n}{2}\binom{n-2}{k-2} - \ldots\ldots + (-1)^k\binom{n}{k}\binom{n-k}{0} = \binom{n}{k}$.

[IIT-JEE-2003, Main, (2, 0), 60]

3. If ${}^{(n-1)}C_r = (k^2 - 3)\ {}^nC_{r+1,}$ then an interval in which k lies is **[IIT-JEE-2004, Scr, (3, – 1), 84]**

(A) $(2, \infty)$ (B) $(-\infty, -2)$ (C) $\left[-\sqrt{3}, \sqrt{3}\right]$ (D) $\left(\sqrt{3}, 2\right]$

4.✍ The value of **[IIT-JEE-2005, Scr, (3, – 1), 84]**

$\binom{30}{0}\binom{30}{10} - \binom{30}{1}\binom{30}{11} + \binom{30}{2}\binom{30}{12} - \ldots\ldots\ldots\ldots + \binom{30}{20}\binom{30}{30}$ is :

(A) $\binom{60}{20}$ (B) $\binom{30}{10}$ (C) $\binom{30}{15}$ (D) None of these

5.✍ For r = 0, 1, ..., 10, let A_r, B_r and C_r denote, respectively, the coefficient of x^r in the expansions of $(1 + x)^{10}$, $(1 + x)^{20}$ and $(1 + x)^{30}$. Then $\sum_{r=1}^{10} A_r(B_{10}B_r - C_{10}A_r)$ is equal to

[IIT-JEE 2010, Paper-2, (5, –2)/79]

(A) $B_{10} - C_{10}$ (B) $A_{10} (B^2_{10} - C_{10} A_{10})$ (C) 0 (D) $C_{10} - B_{10}$

6. The coefficients of three consecutive terms of $(1 + x)^{n+5}$ are in the ratio 5 : 10 : 14. Then n =

[JEE (Advanced) 2013, Paper-1, (4, – 1)/60]

7. Coefficient of x^{11} in the expansion of $(1 + x^2)^4 (1 + x^3)^7 (1 + x^4)^{12}$ is

[JEE (Advanced) 2014, Paper-2, (3, –1)/60]

(A) 1051 (B) 1106 (C) 1113 (D) 1120

8.✍ The coefficient of x^9 in the expansion of $(1 + x) (1 + x^2) (1 + x^3) ......(1 + x^{100})$ is

[JEE (Advanced) 2015, P-2 (4, 0) / 80]

9. Let m be the smallest positive integer such that the coefficient of x^2 in the expansion of $(1 + x)^2 + (1 + x)^3 + ... + (1 + x)^{49} + (1 + mx)^{50}$ is $(3n + 1)\ {}^{51}C_3$ for some positive integer n. Then the value of n is **[JEE (Advanced) 2016, Paper-1, (3, 0)/62]**

10. Let $X = ({}^{10}C_1)^2 + 2({}^{10}C_2)^2 + 3({}^{10}C_3)^2 + ... + 10({}^{10}C_{10})^2$ where ${}^{10}C_r$, $r \in \{1, 2, ..., 10\}$ denote binomial coefficients. Then the value of $\frac{1}{1430}$ X is _______ . **[JEE (Advanced) 2018, Paper-1, (3, 0)/60]**

PART - II : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. Let $S_1 = \sum_{j=1}^{10} j\,(j-1)\ {}^{10}C_j$, $S_2 = \sum_{j=1}^{10} j\ {}^{10}C_j$ and $S_3 = \sum_{j=1}^{10} j^2\ {}^{10}C_j$. **[AIEEE 2009, (4, –1), 144]**

Statement -1 : $S_3 = 55 \times 2^9$.

Statement -2 : $S_1 = 90 \times 2^8$ and $S_2 = 10 \times 2^8$.

(1) Statement-1 is true, Statement-2 is true ; Statement -2 is not a correct explanation for Statement -1.
(2) Statement-1 is true, Statement-2 is false.
(3) Statement -1 is false, Statement -2 is true.
(4) Statement -1 is true, Statement -2 is true; Statement-2 is a correct explanation for Statement-1.

2. The coefficient of x^7 in the expansion of $(1 - x - x^2 + x^3)^6$ is : **[AIEEE 2011, (4, –1), 120]**
(1) 144 (2) – 132 (3) – 144 (4) 132

3. If n is a positive integer, then $\left(\sqrt{3}+1\right)^{2n} - \left(\sqrt{3}-1\right)^{2n}$ is : **[AIEEE 2012, (4, –1), 120]**
(1) an irrational number (2) an odd positive integer
(3) an even positive integer (4) a rational number other than positive integers

4. The term independent of x in expansion of $\left(\frac{x+1}{x^{2/3}-x^{1/3}+1}-\frac{x-1}{x-x^{1/2}}\right)^{10}$ is :**[AIEEE - 2013, (4, –1),120]**
(1) 4 (2) 120 (3) 210 (4) 310

5. If the coefficients of x^3 and x^4 in the expansion of $(1 + ax + bx^2)(1 - 2x)^{18}$ in powers of x are both zero, then (a, b) is equal to **[JEE(Main) 2014, (4, – 1), 120]**
(1) $\left(14,\frac{272}{3}\right)$ (2) $\left(16,\frac{272}{3}\right)$ (3) $\left(16,\frac{251}{3}\right)$ (4) $\left(14,\frac{251}{3}\right)$

6. The sum of coefficients of integral powers of x in the binomial expansion of $(1 - 2\sqrt{x})^{50}$ is **[JEE(Main) 2015, (4, – 1), 120]**
(1) $\frac{1}{2}(3^{50}+1)$ (2) $\frac{1}{2}(3^{50})$ (3) $\frac{1}{2}(3^{50}-1)$ (4) $\frac{1}{2}(2^{50}+1)$

7. If the number of terms in the expansion of $\left(1-\frac{2}{x}+\frac{4}{x^2}\right)^n$, $x \neq 0$, is 28, then the sum of the coefficients of all the terms in this expansion, is **[JEE(Main) 2016, (4, – 1), 120]**
(1) 2187 (2) 243 (3) 729 (4) 64

8. The value of $({}^{21}C_1 - {}^{10}C_1) + ({}^{21}C_2 - {}^{10}C_2) + ({}^{21}C_3 - {}^{10}C_3) + ({}^{21}C_4 - {}^{10}C_4) + \ldots\ldots + ({}^{21}C_{10} - {}^{10}C_{10})$ is **[JEE(Main) 2017, (4, – 1), 120]**
(1) $2^{21} - 2^{11}$ (2) $2^{21} - 2^{10}$ (3) $2^{20} - 2^9$ (4) $2^{20} - 2^{10}$

9. The sum of the co-efficients of all odd degree terms in the expansion of $\left(x+\sqrt{x^3-1}\right)^5+\left(x-\sqrt{x^3-1}\right)^5$, $(x > 1)$ is : **[JEE(Main) 2018, (4, – 1), 120]**
(1) 1 (2) 2 (3) –1 (4) 0

10. If the fractional part of the number $\frac{2^{403}}{15}$ is $\frac{k}{15}$, then k is equal to :

[JEE(Main) 2019, Online (09-01-19),P-1 (4, – 1), 120]

(1) 14 (2) 8 (3) 6 (4) 4

11. If $\sum_{i=1}^{20}\left(\frac{{}^{20}C_{i-1}}{{}^{20}C_i + {}^{20}C_{i-1}}\right)^3 = \frac{k}{21}$, then k equals : **[JEE(Main) 2019, Online (10-01-19),P-1 (4, – 1), 120]**

(1) 50 (2) 400 (3) 200 (4) 100

12. If $\sum_{r=0}^{25}\left\{{}^{50}C_r . {}^{50-r}C_{25-r}\right\} = K\left({}^{50}C_{25}\right)$, then K is equal to :

[JEE(Main) 2019, Online (10-01-19),P-2 (4, – 1), 120]

(1) 2^{25} (2) $2^{25} - 1$ (3) $(25)^2$ (4) 2^{24}

13. Let $S_n = 1 + q + q^2 + \ldots + q^n$ and $T_n = 1 + \left(\frac{q+1}{2}\right) + \left(\frac{q+1}{2}\right)^2 + \ldots + \left(\frac{q+1}{2}\right)^n$.

where q is a real number and $q \neq 1$. If ${}^{101}C_1 + {}^{101}C_2 . S_1 + \ldots + {}^{101}C_{101} . S_{100} = \alpha T_{100}$ then α is equal to

[JEE(Main) 2019, Online (11-01-19),P-2 (4, – 1), 120]

(1) 200 (2) 2^{99} (3) 2^{100} (4) 202

Answers

EXERCISE - 1

PART - I

Section (A) :

A-1. (i) $\left(\frac{2}{x}\right)^5 - 5\left(\frac{2}{x}\right)^3 + 10\left(\frac{2}{x}\right) - 10\left(\frac{x}{2}\right) + 5\left(\frac{x}{2}\right)^3 - \left(\frac{x}{2}\right)^5$ (ii) $y^8 + 8y^5 + 24y^2 + \frac{32}{y} + \frac{16}{y^4}$

A-2. n = 9 **A-3.** 7 **A-4.** (i) ${}^9C_3$ (ii) $-2^7 \cdot {}^{12}C_7$

A-5. ${}^{11}C_5 \frac{a^6}{b^5}, {}^{11}C_6 \frac{a^5}{b^6}$, a b = 1 **A-6.** $\frac{17}{54}$ **A-7.** (i) 171 (ii) –438

A-8 15

Section (B) :

B-1. (i) $-\frac{35x}{y}, \frac{35y}{x}$ (ii) $(-1)^n \frac{(2n)!}{n!\ \ n!} x^n$ **B-3.** (i) 4 (iii) 3, 03, 803

B-4. 101^{50} **B-5.** (i) $T_4 = -455 \times 3^{12}$ and $T_5 = 455 \times 3^{12}$ (ii) 22

B-6. (i) T_4 (ii) T_5, T_6 (iii) T_5 (iv) T_6

Section (D) :

D-1. $\frac{15015}{16}$ **D-2.** (i) 142 (ii) – 197 **D-4.** (i) 280 (ii) 2^5 **D-5.** 20

PART - II

Section (A) :

A-1. (C) **A-2.** (C) **A-3.** (A) **A-4.** (B) **A-5.** (A) **A-6.** (A)

A-7. (B) **A-8.** (C) **A-9.** (C) **A-10.** (B)

Section (B) :

B-1. (B) **B-2.** (C) **B-3.** (D) **B-4.** (A) **B-5.** (A) **B-6.** (A)

B-7. (A) **B-8.** (B)

Section (C) :

C-1. (B) **C-2.** (C) **C-3.** (C) **C-4.** (B)

Section (D) :

D-1. (D) **D-2.** (D) **D-3.** (A)

PART - III

1. (A) → (q, s), (B) → (q,s), (C) → (s), (D) → (p, s)

EXERCISE - 2

PART - I

1.	(B)	**2.**	(A)	**3.**	(C)	**4.**	(C)	**5.**	(B)	**6.**	(D)	**7.**	(A)
8.	(D)	**9.**	(C)	**10.**	(B)	**11.**	(B)	**12.**	(B)	**13.**	(C)	**14.**	(C)
15.	(A)	**16.**	(B)	**17.**	(D)	**18.**	(A)	**19.**	(A)	**20.**	(C)		

PART - II

1.	k = 11	**2.**	10	**3.**	2	**4.**	5	**5.**	3^5	**6.**	2	**7.**	**2**
8.	50	**9.**	3	**10.**	n = 12	**11.**	2	**12.**	1	**13.**	12	**14.**	1
15.	3	**16.**	3	**17.**	**9**	**18.**	2	**19.**	5	**20.**	0	**21.**	2

PART - III

1.	(ABCD)	**2.**	(CD)	**3.**	(AC)	**4.**	(AC)	**5.**	(CD)	**6.**	(AC)
7.	(ACD)	**8.**	(AC)	**9.**	(ABC)	**10.**	(AB)	**11.**	(BC)	**12.**	(BD)
13.	(AD)										

PART - IV

1.	(A)	**2.**	(B)	**3.**	(A)	**4.**	(D)	**5.**	(C)	**6.**	(C)	**7.**	(AD)
8.	(ACD)	**9.**	(ABC)										

EXERCISE - 3

PART - I

1.	(D)	**3.**	(D)	**4.**	(B)	**5.**	(D)	**6.**	**6**	**7.**	(C)	**8.**	8
9.	5	**10.**	646										

PART - II

1.	(2)	**2.**	(3)	**3.**	(1)	**4.**	(3)	**5.**	(2)	**6.**	**1**		
7.	(3) or Bonus			**8.**	(4)	**9.**	(2)	**10.**	(2)	**11.**	(4)	**2.**	(1)
13.	(3)												

Advance level problems (ALP)

1. Find the coefficient of x^{49} in

$\left(x+\frac{C_1}{C_0}\right)\left(x+2^2\frac{C_2}{C_1}\right)\left(x+3^2\frac{C_3}{C_2}\right)........\left(x+50^2\frac{C_{50}}{C_{49}}\right)$ where $C_r = {}^{50}C_r$

2. The expression, $\left(\sqrt{2x^2+1}+\sqrt{2x^2-1}\right)^6+\left(\frac{2}{\sqrt{2x^2+1}+\sqrt{2x^2-1}}\right)^6$ is a polynomial of degree

3. Find the co-efficient of x^5 in the expansion of $(1 + x^2)^5 (1 + x)^4$.

4. Prove that the co-efficient of x^{15} in $(1 + x + x^3 + x^4)^n$ is $\sum_{r=0}^{5} {}^nC_{15-3r}\,{}^nC_r$.

5. If n is even natural and coefficient of x^r in the expansion of $\frac{(1+x)^n}{1-x}$ is 2^n, $(|x| < 1)$, then prove that $r \ge n$

6. Find the coefficient of x^n in polynomial $(x + {}^{2n+1}C_0)(x + {}^{2n+1}C_1)........(x + {}^{2n+1}C_n)$.

7. Find the value of $\sum_{r=1}^{n}\left(\sum_{p=0}^{r-1} {}^nC_r\,{}^rC_p\,2^p\right)$.

Comprehension (Q-8 to Q.10)

For k, n $\in$ N, we define

$B(k, n) = 1.2.3......... k + 2.3.4........(k+1) + ...+ n(n + 1).......(n + k - 1)$, $S_0(n) = n$ and $S_k(n) = 1^k + 2^k + ... + n^k$.

To obtain value B(k, n), we rewrite B(k, n) as follows

$B(k,n) = k!\left[{}^kC_k + {}^{k+1}C_k + {}^{k+2}C_k + ... + {}^{n+k-1}C_k\right] = k!\left({}^{n+k}C_{k+1}\right)$

$= \frac{n(n+1)........(n+k)}{k+1}$

where $\;{}^nC_k = \frac{n!}{k!\,(n-k)!}$

8. Prove that $S_2(n) + S_1(n) = B(2, n)$

9. Prove that $S_3(n) + 3S_2(n) = B(3, n) - 2B(1, n)$

10. If $(1 + x)^p = 1 + {}^pC_1\,x + {}^pC_2x^2 + ... + {}^pC_p\,x^p$, $p \in N$, then show that ${}^{k+1}C_1\,S_k(n) + {}^{k+1}C_2\,S_{k-1}(n) + ... + {}^{k+1}C_k\,S_1(n) + {}^{k+1}C_{k+1}\,S_0(n) = (n + 1)^{k+1} - 1$

11. Show that $25^n - 20^n - 8^n + 3^n$, $n \in I^+$ is divisible by 85.

12. Prove that ${}^nC_1\,({}^nC_2)^2\,({}^nC_3)^3........({}^nC_n)^n \le \left(\frac{2^n}{n+1}\right)^{{}^{n+1}C_2}$.

13. If p is nearly equal to q and n > 1, show that $\frac{(n+1)\ p+(n-1)q}{(n-1)p+(n+1)q}=\left(\frac{p}{q}\right)^{1/n}$. Hence find the approximate value of $\left(\frac{99}{101}\right)^{1/6}$.

14. If $(18x^2 + 12x + 4)^n = a_0 + a_1x + a_2x^2 + ... + a_{2n}\ x^{2n}$, then prove that $a_r = 2^n\ 3^r\ \left({}^{2n}C_r + {}^nC_1\ {}^{2n-2}C_r + {}^nC_2\ {}^{2n-4}C_r + ...\right)$

15. Prove that $1^2.\ C_0 + 2^2.\ C_1 + 3^2.\ C_2 + 4^2.\ C_3 +.... + (n+1)^2\ C_n = 2^{n-2}(n+1)(n+4)$.

16. If $(1 - x)^{-n} = a_0 + a_1\ x + a_2\ x^2 + a_3\ x^3 +........$, find the value of, $a_0 + a_1 + a_2 +....... + a_n$.

17. Find the remainder when $32^{32^{32}}$ is divided by 7.

18. If n is an integer greater than 1, show that : $a - {}^nC_1(a-1) + {}^nC_2(a-2) -..... + (-1)^n\ (a - n) = 0$.

19. If $(1 + x)^n = p_0 + p_1\ x + p_2\ x^2 + p_3\ x^3 +.......$, then prove that :

(a) $p_0 - p_2 + p_4 -....... = 2^{n/2}\cos\frac{n\pi}{4}$ (b) $p_1 - p_3 + p_5 -....... = 2^{n/2}\sin\frac{n\pi}{4}$

20. Show that if the greatest term in the expansion of $(1 + x)^{2n}$ has also the greatest co-efficient, then ' x ' lies between, $\frac{n}{n+1}$ & $\frac{n+1}{n}$.

21. Prove that if ' p ' is a prime number greater than 2, then $\left[(2+\sqrt{5})^p\right] - 2^{p+1}$ is divisible by p, where [.] denotes greatest integer function.

22. If $\sum_{r=0}^{n}(-1)^r.\ {}^nC_r\left[\frac{1}{2^r}+\frac{3^r}{2^{2r}}+\frac{7^r}{2^{3r}}+........\ \text{to m terms}\right]=k\left(1-\frac{1}{2^{mn}}\right)$, then find the value of k.

23. Given $s_n = 1 + q + q^2 +..... + q^n$ & $S_n = 1 + \frac{q+1}{2} + \left(\frac{q+1}{2}\right)^2 +.... + \left(\frac{q+1}{2}\right)^n$, $q \neq 1$, prove that ${}^{n+1}C_1 + {}^{n+1}C_2.s_1 + {}^{n+1}C_3.s_2 +....+ {}^{n+1}C_{n+1}.s_n = 2^n.\ S_n.$

24. If $(1+x)^{15} = C_0 + C_1.\ x + C_2.\ x^2 +.... + C_{15}.\ x^{15}$, then find the value of : $C_2 + 2C_3 + 3C_4 +.... + 14C_{15}$

25. Prove that, $\frac{1}{2}\ {}^nC_1 - \frac{2}{3}\ {}^nC_2 + \frac{3}{4}\ {}^nC_3 - \frac{4}{5}\ {}^nC_4 +..... + \frac{(-1)^{n+1}\ n}{n+1}.\ {}^nC_n = \frac{1}{n+1}$

26. Prove that $\sum_{r=0}^{n} r^2\ {}^nC_r\ p^r\ q^{n-r} = npq + n^2p^2$, if p + q = 1.

27. Prove that : $(n-1)^2.\ C_1 + (n-3)^2.\ C_3 + (n-5)^2.\ C_5 +..... = n(n + 1)2^{n-3}$

28. Prove that ${}^nC_r + 2\ {}^{n+1}C_r + 3\ {}^{n+2}C_r +....... + (n + 1)\ {}^{2n}C_r = {}^nC_{r+2} + (n + 1)\ {}^{2n+1}C_{r+1} - {}^{2n+1}C_{r+2}$

29. Show that, $\sqrt{3} = 1 + \frac{1}{3} + \frac{1}{3}.\frac{3}{6} + \frac{1}{3}.\frac{3}{6}.\frac{5}{9} + \frac{1}{3}.\frac{3}{6}.\frac{5}{9}.\frac{7}{12} + ...$

30. If $(1 + x)^n = C_0 + C_1x + C_2x^2 + ... + C_nx^n$, show that for $m \geq 2$
$C_0 - C_1 + C_2 - .... + (-1)^{m-1}C_{m-1} = (-1)^{m-1}\ {}^{n-1}C_{m-1}$.

31. If $(1+x)^n = C_0 + C_1x + C_2x^2 + ... + C_n x^n$, then show that the sum of the products of the C_i's taken two at a time, represented by $\sum_{0 \leq i < j \leq n}\sum C_i C_j$ is equal to $2^{2n-1} - \frac{2n!}{2(n!)^2}$.

32. If $a_0, a_1, a_2,.....$ be the coefficients in the expansion of $(1 + x + x^2)^n$ in ascending powers of x, then prove that :

(i) $a_0 a_1 - a_1 a_2 + a_2 a_3 - .... = 0$

(ii) $a_0a_2 - a_1a_3 + a_2a_4 - ..... + a_{2n-2}\, a_{2n} = a_{n+1}$

(iii) $E_1 = E_2 = E_3 = 3^{n-1}$; where $E_1 = a_0 + a_3 + a_6 + ...$; $E_2 = a_1 + a_4 + a_7 + ...$ & $E_3 = a_2 + a_5 + a_8 + ...$

Advance level problems (ALP) Answer

1. 22100 **2.** 6 **3.** 60 **6.** 2^{2n} **7.** $4^n - 3^n$ **13.** $\frac{1198}{1202}$

16. $\frac{(2n)!}{(n!)^2}$ **17.** 4 **22.** $\frac{1}{2^n - 1}$ **24.** 212993