Exercise-1

✍ **Marked questions are recommended for Revision.**

PART - I : SUBJECTIVE QUESTIONS

Section (A): Matrix, Algebra of Matrix, Transpose, symmetric and skew symmetric matrix

A-1. Construct a 3 × 2 matrix whose elements are given by $a_{ij} = 2i - j$.

A-2. If $\begin{bmatrix} x-y & 1 & z \\ 2x-y & 0 & w \end{bmatrix} = \begin{bmatrix} -1 & 1 & 4 \\ 0 & 0 & 5 \end{bmatrix}$, find x, y, z, w.

A-3. Let $A + B + C = \begin{bmatrix} 4 & -1 \\ 0 & 1 \end{bmatrix}$, $4A + 2B + C = \begin{bmatrix} 0 & -1 \\ -3 & 2 \end{bmatrix}$ and $9A + 3B + C = \begin{bmatrix} 0 & -2 \\ 2 & 1 \end{bmatrix}$ then find A

A-4. If $A = \begin{bmatrix} 1 & 2 \\ 3 & -4 \\ 5 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 5 & 6 \\ 7 & -8 & 2 \end{bmatrix}$, will AB be equal to BA. Also find AB & BA.

A-5. If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, then show that $A^3 = \begin{bmatrix} 7 & -12 \\ 3 & -5 \end{bmatrix}$

A-6. If $A = \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$ show that $(I + A) = (I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

A-7. Given $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$. If $x \in R$ Then for what values of y, $F(x + y) = F(x)\,F(y)$.

A-8. Let $A = [a_{ij}]_{n \times n}$ where $a_{ij} = i^2 - j^2$. Show that A is skew-symmetric matrix.

A-9. If $C = \begin{bmatrix} 1 & 4 & 6 \\ 7 & 2 & 5 \\ 9 & 8 & 3 \end{bmatrix}\begin{bmatrix} 0 & 2 & 3 \\ -2 & 0 & 4 \\ -3 & -4 & 0 \end{bmatrix}\begin{bmatrix} 1 & 7 & 9 \\ 4 & 2 & 8 \\ 6 & 5 & 3 \end{bmatrix}$, then trace of $C + C^3 + C^5 + \ldots\ldots + C^{99}$ is

Section (B) : Determinant of Matrix

B-1. If the minor of three-one element (i.e. M_{31}) in the determinant $\begin{vmatrix} 0 & 1 & \sec\alpha \\ \tan\alpha & -\sec\alpha & \tan\alpha \\ 1 & 0 & 1 \end{vmatrix}$ is 1 then find the value of α. $(0 \le \alpha \le \pi)$.

B-2. Using the properties of determinants, evalulate:

(i) $\begin{vmatrix} 23 & 6 & 11 \\ 36 & 5 & 26 \\ 63 & 13 & 37 \end{vmatrix}$ (ii) $\begin{vmatrix} 0 & c & b \\ -c & 0 & a \\ -b & -a & 0 \end{vmatrix}$

(iii) $\begin{vmatrix} 103 & 115 & 114 \\ 111 & 108 & 106 \\ 104 & 113 & 116 \end{vmatrix} + \begin{vmatrix} 113 & 116 & 104 \\ 108 & 106 & 111 \\ 115 & 114 & 103 \end{vmatrix}$. (iv) $\begin{vmatrix} \sqrt{13}+\sqrt{3} & 2\sqrt{5} & \sqrt{5} \\ \sqrt{15}+\sqrt{26} & 5 & \sqrt{10} \\ 3+\sqrt{65} & \sqrt{15} & 5 \end{vmatrix}$

B-3. Prove that :

(i) $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a - b)(b - c)(c - a)(a + b + c)$

(ii) $\begin{vmatrix} a & b+c & a^2 \\ b & c+a & b^2 \\ c & a+b & c^2 \end{vmatrix} = -(a + b + c)(a - b)(b - c)(c - a)$

(iii) $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc$

(iv) If $\begin{vmatrix} 1 & a^2 & a^4 \\ 1 & b^2 & b^4 \\ 1 & c^2 & c^4 \end{vmatrix} = (a + b)(b + c)(c + a)\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$.

B-4. If a, b, c are positive and are the p^{th}, q^{th}, r^{th} terms respectively of a G.P., show without expanding that,
$\begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix} = 0$.

B-5. Find the non – zero roots of the equation,

(i) $\Delta = \begin{vmatrix} a & b & ax+b \\ b & c & bx+c \\ ax+b & bx+c & c \end{vmatrix} = 0$. (ii) $\begin{vmatrix} 15-2x & 11 & 10 \\ 11-3x & 17 & 16 \\ 7-x & 14 & 13 \end{vmatrix} = 0$

B-6. If $S_r = \alpha^r + \beta^r + \gamma^r$ then show that $\begin{vmatrix} S_0 & S_1 & S_2 \\ S_1 & S_2 & S_3 \\ S_2 & S_3 & S_4 \end{vmatrix} = (\alpha - \beta)^2 (\beta - \gamma)^2 (\gamma - \alpha)^2$.

B-7. Show that $\begin{vmatrix} a_1 l_1 + b_1 m_1 & a_1 l_2 + b_1 m_2 & a_1 l_3 + b_1 m_3 \\ a_2 l_1 + b_2 m_1 & a_2 l_2 + b_2 m_2 & a_2 l_3 + b_2 m_3 \\ a_3 l_1 + b_3 m_1 & a_3 l_2 + b_3 m_2 & a_3 l_3 + b_3 m_3 \end{vmatrix} = 0$.

B-8. If $\begin{vmatrix} e^x & \sin x \\ \cos x & \ln(1+x) \end{vmatrix} = A + Bx + Cx^2 + ...$, then find the value of A and B.

Section (C) : Cofactor matrix, adj matrix and inverse of matrix

C-1. If A = $\begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$, B = $\begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix}$, C = $\begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}$ and AB – CD = 0 find D.

C-2. (i) Prove that (adj adj A) = $|A|^{n-2}$ A
(ii) Find the value of |adj adj adj A| in terms of |A|

C-3. If $A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ & $B = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$, find $(AB)^{-1}$

C-4. If A is a symmetric and B skew symmetric matrix and (A + B) is non-singular and C = $(A + B)^{-1}$ (A – B), then prove that
(i) $C^T (A + B) C = A + B$ (ii) $C^T (A - B) C = A - B$

C-5. If $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & a & 1 \end{bmatrix}$, $A^{-1} = \begin{bmatrix} 1/2 & -1/2 & 1/2 \\ -4 & 3 & c \\ 5/2 & -3/2 & 1/2 \end{bmatrix}$, then find values of a & c.

Section (D) : Charactristic equation and system of equations

D-1. For the matrix A = $\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$ find a & b so that $A^2 + aA + bI = 0$. Hence find A^{-1} .

D-2. Find the total number of possible square matrix A of order 3 with all real entries, whose adjoint matrix B has characterstics polynomial equation as $\lambda^3 - \lambda^2 + \lambda + 1 = 0$.

D-3. If $A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix}$, show that $A^3 = (5A - I)(A - I)$

D-4. Apply Cramer's rule to solve the following simultaneous equations.
(i) $2x + y + 6z = 46$
$5x - 6y + 4z = 15$
$7x + 4y - 3z = 19$
(ii) $x + 2y + 3z = 2$
$x - y + z = 3$
$5x - 11y + z = 17$

D-5. Solve using Cramer's rule: $\frac{4}{x+5} + \frac{3}{y+7} = -1$ & $\frac{6}{x+5} - \frac{6}{y+7} = -5$.

D-6. Find those values of c for which the equations:
$2x + 3y = 3$
$(c+2)x + (c+4)y = c+6$
$(c+2)^2 x + (c+4)^2 y = (c+6)^2$ are consistent.
Also solve above equations for these values of c.

D-7. Solve the following systems of linear equations by matrix method.

(i) $2x - y + 3z = 8$
$-x + 2y + z = 4$
$3x + y - 4z = 0$

(ii) $x + y + z = 9$
$2x + 5y + 7z = 52$
$2x + y - z = 0$

D-8. Investigate for what values of λ, μ the simultaneous equations
$x + y + z = 6$; $x + 2y + 3z = 10$ & $x + 2y + \lambda z = \mu$ have;
(a) A unique solution
(b) An infinite number of solutions.
(c) No solution.

D-9. Determine the product $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and use it to solve the system of equations $x - y + z = 4$, $x - 2y - 2z = 9$, $2x + y + 3z = 1$.

D-10. Compute A^{-1}, if $A = \begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix}$. Hence solve the matrix equations

$$\begin{bmatrix} 3 & 0 & 3 \\ 2 & 1 & 0 \\ 4 & 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix} + \begin{bmatrix} 2y \\ z \\ 3y \end{bmatrix}.$$

D-11. Which of the following statement(s) is/are true:

S1 : The system of equations $\begin{matrix} 4x - 5y - 2z = 2 \\ 5x - 4y + 2z = 3 \\ 2x + 2y + 8z = 1 \end{matrix}$ is Inconsistent.

S2 : A matrix 'A' has 6 elements. The number of possible orders of A is 6.

S3 : For any 2 × 2 matrix A, if A (adjA) = $\begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$, then |A| = 10.

S4 : If A is skew symmetric, then B′AB is also skew symmetric.

PART - II : ONLY ONE OPTION CORRECT TYPE

Section (A): Matrix, Algebra of Matrix, Transpose, symmetric and skew symmetric matrix,

A-1. $\begin{bmatrix} x^2 + x & x \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} 0 & -1 \\ -x + 1 & x \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ 5 & 1 \end{bmatrix}$ then x is equal to -

(A) – 1 (B) 2 (C) 1 (D) No value of x

A-2. If A = $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and B = $\begin{bmatrix} -5 & 4 & 0 \\ 0 & 2 & -1 \\ 1 & -3 & 2 \end{bmatrix}$, then

(A) AB = $\begin{bmatrix} -5 & 8 & 0 \\ 0 & 4 & -2 \\ 3 & -9 & 6 \end{bmatrix}$ (B) AB = [– 2 – 1 4] (C) AB = $\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$ (D) AB does not exist

A-3. If I = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, J = $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ and B = $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, then B =

(A) $I\cos\theta + J\sin\theta$ (B) $I\cos\theta - J\sin\theta$ (C) $I\sin\theta + J\cos\theta$ (D) $-I\cos\theta + J\sin\theta$

A-4. In an upper triangular matrix $A = [a_{ij}]_{n \times n}$ the elements $a_{ij} = 0$ for
(A) $i < j$ (B) $i = j$ (C) $i > j$ (D) $i \le j$

A-5. If A = diag (2, –1, 3), B = diag (–1, 3, 2), then A^2B =
(A) diag (5, 4, 11) (B) diag (– 4, 3, 18) (C) diag (3, 1, 8) (D) B

A-6. If A is a skew- symmetric matrix, then trace of A is
(A) 1 (B) – 1 (C) 0 (D) none of these

A-7. Let $A = \begin{bmatrix} p & q \\ q & p \end{bmatrix}$ such that det(A) = r where p, q, r all prime numbers, then trace of A is equal to
(A) 6 (B) 5 (C) 2 (D) 3

A-8. $A = \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix}$ and $(A^8 + A^6 + A^4 + A^2 + I)\, V = \begin{bmatrix} 31 \\ 62 \end{bmatrix}$.
(Where I is the (2 × 2) identity matrix), then the product of all elements of matrix V is
(A) 2 (B) 1 (C) 3 (D) –2

A-9. Let $A = \begin{bmatrix} 3x^2 \\ 1 \\ 6x \end{bmatrix}$, B = [a b c] and $C = \begin{bmatrix} (x+2)^2 & 5x^2 & 2x \\ 5x^2 & 2x & (x+1)^2 \\ 2x & (x+2)^2 & 5x^2 \end{bmatrix}$
Where a, b, c and x ∈ R, Given that tr (AB) = tr(C), then the value of (a + b + c).
(A) 7 (B) 2 (C) 1 (D) 4

Section (B) : Determinant of Matrix

B-1. If A and B are square matrices of order 3 such that |A| = – 1, |B| = 3, then |3AB| is equal to
(A) – 9 (B) – 81 (C) – 27 (D) 81

B-2. Let $A = \begin{bmatrix} \cos^{-1}x & \cos^{-1}y & \cos^{-1}z \\ \cos^{-1}y & \cos^{-1}z & \cos^{-1}x \\ \cos^{-1}z & \cos^{-1}x & \cos^{-1}y \end{bmatrix}$ such that |A| = 0, then maximum value of x + y + z is
(A) 3 (B) 0 (C) 1 (D) 2

B-3. The absolute value of the determinant $\begin{vmatrix} -1 & 2 & 1 \\ 3+2\sqrt{2} & 2+2\sqrt{2} & 1 \\ 3-2\sqrt{2} & 2-2\sqrt{2} & 1 \end{vmatrix}$ is:
(A) $16\sqrt{2}$ (B) $8\sqrt{2}$ (C) 8 (D) none

B-4. If α, β & γ are the roots of the equation $x^3 + px + q = 0$, then the value of the determinant $\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix} =$
(A) p (B) q (C) $p^2 - 2q$ (D) none

B-5. If a, b, c > 0 & x, y, z ∈ R, then the determinant $\begin{vmatrix} (a^x + a^{-x})^2 & (a^x - a^{-x})^2 & 1 \\ (b^y + b^{-y})^2 & (b^y - b^{-y})^2 & 1 \\ (c^z + c^{-z})^2 & (c^z - c^{-z})^2 & 1 \end{vmatrix} =$
(A) $a^x b^y c^z$ (B) $a^{-x} b^{-y} c^{-z}$ (C) $a^{2x} b^{2y} c^{2z}$ (D) zero

B-6. If a, b & c are non-zero real numbers, then D = $\begin{vmatrix} b^2c^2 & bc & b+c \\ c^2a^2 & ca & c+a \\ a^2b^2 & ab & a+b \end{vmatrix}$ =

(A) abc (B) $a^2b^2c^2$ (C) $bc + ca + ab$ (D) zero

B-7. The determinant $\begin{vmatrix} b_1+c_1 & c_1+a_1 & a_1+b_1 \\ b_2+c_2 & c_2+a_2 & a_2+b_2 \\ b_3+c_3 & c_3+a_3 & a_3+b_3 \end{vmatrix}$ =

(A) $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ (B) 2 $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ (C) 3 $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ (D) none of these

B-8. If x, y, z $\in$ R & $\Delta = \begin{vmatrix} x & x+y & x+y+z \\ 2x & 5x+2y & 7x+5y+2z \\ 3x & 7x+3y & 9x+7y+3z \end{vmatrix} = -16$ then value of x is

(A) – 2 (B) – 3 (C) 2 (D) 3

B-9. The determinant $\begin{vmatrix} \cos(\theta+\phi) & -\sin(\theta+\phi) & \cos 2\phi \\ \sin\theta & \cos\theta & \sin\phi \\ -\cos\theta & \sin\theta & \cos\phi \end{vmatrix}$ is:

(A) 0 (B) independent of θ (C) independent of ϕ (D) independent of θ & ϕ both

B-10. Let A be set of all determinants of order 3 with entries 0 or 1, B be the subset of A consisting of all determinants with value 1 and C be the subset of A consisting of all determinants with value –1. Then
STATEMENT -1 : The number of elements in set B is equal to number of elements in set C.
and
STATEMENT-2 : $(B \cap C) \subset A$

(A) STATEMENT-1 is true, STATEMENT-2 is true and STATEMENT-2 is correct explanation for STATEMENT-1
(B) STATEMENT-1 is true, STATEMENT-2 is true and STATEMENT-2 is not correct explanation for STATEMENT-1
(C) STATEMENT-1 is true, STATEMENT-2 is false
(D) STATEMENT-1 is false, STATEMENT-2 is true
(E) Both STATEMENTS are false

Section (C) : Cofactor matrix, adj matrix and inverse of matrix.

C-1. If A = $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$, then adj A =

(A) $\begin{bmatrix} 1 & -2 \\ -2 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$ (C) $\begin{bmatrix} 1 & -2 \\ -2 & -1 \end{bmatrix}$ (D) $\begin{bmatrix} -1 & 2 \\ 2 & -1 \end{bmatrix}$

C-2. Identify statements S_1, S_2, S_3 in order for true(T)/false(F)

S_1 : If A = $\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ then adj A = A'

S_2 : If A = $\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$, then $A^{-1} = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$

S_3 : If B is a non-singular matrix and A is a square matrix, then det $(B^{-1} AB)$ = det (A)

(A) TTF (B) FTT (C) TFT (D) TTT

C-3. If A, B are two n × n non-singular matrices, then

(A) AB is non-singular (B) AB is singular
(C) $(AB)^{-1} = A^{-1} B^{-1}$ (D) $(AB)^{-1}$ does not exist

C-4. Let A = $\begin{bmatrix} 1 & 2 \\ 3 & -5 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ and X be a matrix such that A = BX, then X is equal to

(A) $\frac{1}{2}\begin{bmatrix} 2 & 4 \\ 3 & -5 \end{bmatrix}$ (B) $\frac{1}{2}\begin{bmatrix} -2 & 4 \\ 3 & 5 \end{bmatrix}$ (C) $\begin{bmatrix} 2 & 4 \\ 3 & -5 \end{bmatrix}$ (D) none of these

C-5. Let **A** = $\begin{bmatrix} -1 & 2 & -3 \\ -2 & 0 & 3 \\ 3 & -3 & 1 \end{bmatrix}$ be a matrix, then (det **A**) x (adj $\mathbf{A}^{-1}$) is equal to

(A) $O_{3\times3}$ (B) I_3 (C) $\begin{bmatrix} -1 & 2 & -3 \\ -2 & 0 & 3 \\ 3 & -3 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 3 & -3 & 1 \\ 3 & 0 & -2 \\ -1 & 2 & -3 \end{bmatrix}$

C-6. **STATEMENT-1 :** If A = $\begin{bmatrix} a^2+x^2 & ab-cx & ac+bx \\ ab+xc & b^2+x^2 & bc-ax \\ ac-bx & bc+ax & c^2+x^2 \end{bmatrix}$ and B = $\begin{bmatrix} x & c & -b \\ -c & x & a \\ b & -a & x \end{bmatrix}$, then $|A| = |B|^2$.

STATEMENT-2 : If A^c is cofactor matrix of a square matrix A of order n then $|A^c| = |A|^{n-1}$.

(A) STATEMENT-1 is true, STATEMENT-2 is true and STATEMENT-2 is correct explanation for STATEMENT-1
(B) STATEMENT-1 is true, STATEMENT-2 is true and STATEMENT-2 is not correct explanation for STATEMENT-1
(C) STATEMENT-1 is true, STATEMENT-2 is false
(D) STATEMENT-1 is false, STATEMENT-2 is true
(E) Both STATEMENTS are false

Section (D) : Characteristic equation and system of equations

D-1. If A = $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$ is a root of polynomial $x^3 - 6x^2 + 7x + k = 0$, then the value of k is

(A) 2 (B) 4 (C) –2 (D) 1

D-2 If A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ (where bc ≠ 0) satisfies the equations $x^2 + k = 0$, then

(A) a + d = 0 & k = |A| (B) a – d = 0 & k = |A|
(C) a + d = 0 & k = –|A| (D) a + d ≠ 0 & k = |A|

D-3. If the system of equations x + 2y + 3z = 4, x + py + 2z = 3, x + 4y + μz = 3 has an infinite number of solutions and solution triplet is

(A) p = 2, μ = 3 and (5 – 4λ, λ – 1 , λ) (B) p = 2, μ = 4 and $\left(5 - 4\lambda, \frac{\lambda-1}{2}, 2\lambda\right)$
(C) 3 p = 2 μ and (5 – 4λ, λ – 1, 2λ) (D) p = 4, μ = 2 and $\left(5 - 4\lambda, \frac{\lambda-1}{2}, \lambda\right)$

D-4. Let λ and α be real. Find the set of all values of λ for which the system of linear equations have infinite solution ∀ real values of α.

$\lambda x + (\sin\alpha)y + (\cos\alpha) z = 0$
$x + (\cos\alpha)y + (\sin\alpha) z = 0$
$-x + (\sin\alpha) y + (\cos\alpha) z = 0$

(A) $(-\infty, \sqrt{2}) \cup (\sqrt{2}, \infty)$ (B) – 1 (C) $(-5, -\sqrt{2})$ (D) None of these

D-5. Let A = $\begin{bmatrix} a & o & b \\ 1 & e & 1 \\ c & o & d \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ where a,b, c, d, e $\in \{0, 1\}$

then number of such matrix A for which system of equation AX = O have unique solution.

(A) 16 (B) 6 (C) 5 (D) none

D-6. If the system of equations ax + y + z = 0,. x + by + z = 0 and x + y + cz = 0, where a, b, c $\neq$ 1, has a non–trivial solution, then the value of $\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c}$ is

(A) 1 (B) 2 (C) 3 (D) 4

PART - III : MATCH THE COLUMN

1. **Column I** **Column II**

(A) $\begin{bmatrix} 1 & x & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 3 & 2 & 5 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ = 0 then x = (p) 2

(B) If A is a square matrix of order 3 × 3 and k is a scalar, then adj (kA) = k^m adj A, then m is (q) – 2

(C) If A = $\begin{bmatrix} 2 & \mu \\ \mu^2 & 3 \end{bmatrix}$ and B = $\begin{bmatrix} \gamma & 7 \\ 49 & \delta \end{bmatrix}$ here (A – B) is upper triangular matrix then number of possible values of μ are (r) 1

(D) If $\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix}$ = k abc (a + b + c)^3 then the value of k is (s) $-\frac{9}{8}$

2. **Column – I** **Column – II**

(A) If A and B are square matrices of order 3 × 3, where |A| = 2 and |B| = 1, then $|(A^{-1})\ .\ adj\ (B^{-1})\ .\ adj\ (2A^{-1})|$ = (p) 7

(B) If A is a square matrix such that $A^2 = A$ and $(I + A)^3 = I + kA$, then k is equal to (q) 8

(C) Matrix $\begin{bmatrix} a & b & (a\alpha - b) \\ b & c & (b\alpha - c) \\ 2 & 1 & 0 \end{bmatrix}$ is non invertible ($b^2 \neq ac$) if -2α is (r) 0

(D) If A = $[a_{ij}]_{3\times3}$ is a scalar matrix with $a_{11} = a_{22} = a_{33} = 2$ and A(adjA) = kI_3 then k is (s) – 1

Exercise-2

✍ **Marked questions are recommended for Revision.**

PART - I : ONLY ONE OPTION CORRECT TYPE

1. Two matrices A and B have in total 6 different elements (none repeated) . How many different matrices A and B are possible such that product AB is defined.

(A) 5(6!) (B) 3(6!) (C) 12(6!) (D) 8 (6!)

2. If AB = O for the matrices

A = $\begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$ and B = $\begin{bmatrix} \cos^2\phi & \cos\phi\sin\phi \\ \cos\phi\sin\phi & \sin^2\phi \end{bmatrix}$ then $\theta - \phi$ is

(A) an odd multiple of $\frac{\pi}{2}$ (B) an odd multiple of π

(C) an even multiple of $\frac{\pi}{2}$ (D) 0

3.✍ If X = $\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, then value of X^n is, (where n is natural number)

(A) $\begin{bmatrix} 3n & -4n \\ n & -n \end{bmatrix}$ (B) $\begin{bmatrix} 2+n & 5-n \\ n & -n \end{bmatrix}$ (C) $\begin{bmatrix} 3^n & (-4)^n \\ 1^n & (-1)^n \end{bmatrix}$ (D) $\begin{bmatrix} 2n+1 & -4n \\ n & -(2n-1) \end{bmatrix}$

4. If A and B are two matrices such that AB = B and BA = A, then $A^2 + B^2$ =

(A) 2AB (B) 2BA (C) A + B (D) AB

5.✍ Find number of all possible ordered sets of two (n × n) matrices **A** and **B** for which **AB** – **BA** = I

(A) infinite (B) n^2 (C) n! (D) zero

6. If B, C are square matrices of order n and if A = B + C, BC = CB, C^2 = O, then which of following is true for any positive integer N.

(A) $A^{N+1} = B^N (B + (N + 1) C)$ (B) $A^N = B^N (B + (N + 1) C)$

(C) $A^{N+1} = B (B + (N + 1) C)$ (D) $A^{N+1} = B^N (B + (N + 2) C)$

7. How many 3 × 3 skew symmetric matrices can be formed using numbers –2, –1, 1, 2, 3, 4, 0 (any number can be used any number of times but 0 can be used at most 3 times)

(A) 8 (B) 27 (C) 64 (D) 54

8. If A is a skew - symmetric matrix and n is an even positive integer, then A^n is

(A) a symmetric matrix (B) a skew-symmetric matrix

(C) a diagonal matrix (D) none of these

9.✍ Number of 3 × 3 non symmetric matrix A such that $A^T = A^2 - I$ and $|A| \neq 0$, equals to

(A) 0 (B) 2 (C) 4 (D) Infinite

10.✍ Matrix **A** is such that $\mathbf{A}^2 = 2\mathbf{A} - I$, where I is the identity matrix. Then for n ≥ 2, $\mathbf{A}^n$ =

(A) $n\mathbf{A} - (n-1)I$ (B) $n\mathbf{A} - I$ (C) $2^{n-1}\mathbf{A} - (n-1)I$ (D) $2^{n-1}\mathbf{A} - I$

11. If P = $\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$, A = $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and Q = PAP^T and x = $P^TQ^{2005}P$, then x is equal to

(A) $\begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$

(B) $\begin{bmatrix} 4+2005\sqrt{3} & 6015 \\ 2005 & 4-2005\sqrt{3} \end{bmatrix}$

(C) $\frac{1}{4}\begin{bmatrix} 2+\sqrt{3} & 1 \\ -1 & 2-\sqrt{3} \end{bmatrix}$

(D) $\frac{1}{4}\begin{bmatrix} 2005 & 2-\sqrt{3} \\ 2+\sqrt{3} & 2005 \end{bmatrix}$

12. Let Δ = $\begin{vmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\theta\sin\phi & \sin\theta\cos\phi & 0 \end{vmatrix}$, then

(A) Δ is independent of θ
(B) Δ is indepedent of φ
(C) Δ is a constant
(D) none of these

13. Δ = $\begin{vmatrix} 1+a^2+a^4 & 1+ab+a^2b^2 & 1+ac+a^2c^2 \\ 1+ab+a^2b^2 & 1+b^2+b^4 & 1+bc+b^2c^2 \\ 1+ac+a^2c^2 & 1+bc+b^2c^2 & 1+c^2+c^4 \end{vmatrix}$ is equal to

(A) $(a-b)^2 (b-c)^2 (c-a)^2$
(B) $2(a-b)(b-c)(c-a)$
(C) $4(a-b)(b-c)(c-a)$
(D) $(a+b+c)^3$

14. If D = $\begin{vmatrix} a^2+1 & ab & ac \\ ba & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix}$ then D =

(A) $1 + a^2 + b^2 + c^2$
(B) $a^2 + b^2 + c^2$
(C) $(a+b+c)^2$
(D) none

15. Value of the Δ = $\begin{vmatrix} a^3-x & a^4-x & a^5-x \\ a^5-x & a^6-x & a^7-x \\ a^7-x & a^8-x & a^9-x \end{vmatrix}$ is

(A) 0
(B) $(a^3-1)(a^6-1)(a^9-1)$
(C) $(a^3+1)(a^6+1)(a^9+1)$
(D) $a^{15}-1$

16. If $\Delta_1 = \begin{vmatrix} 2a & b & e \\ 2d & e & f \\ 4x & 2y & 2z \end{vmatrix}$, $\Delta_2 = \begin{vmatrix} f & 2d & e \\ 2z & 4x & 2y \\ e & 2a & b \end{vmatrix}$, then the value of $\Delta_1 - \Delta_2$ is

(A) $x + \frac{y}{2} + z$
(B) 2
(C) 0
(D) 3

17. From the matrix equation AB = AC, we conclude B = C provided:

(A) A is singular
(B) A is non–singular
(C) A is symmetric
(D) A is a square

18. Let A = $\begin{bmatrix} -2 & 7 & \sqrt{3} \\ 0 & 0 & -2 \\ 0 & 2 & 0 \end{bmatrix}$ and $A^4 = \lambda.I$, then λ is

(A) – 16
(B) 16
(C) 8
(D) –8

19. If A is 3 × 3 square matrix whose characterstic polynomial equations is $\lambda^3 - 3\lambda^2 + 4 = 0$ then trace of adjA is
(A) 0 (B) 3 (C) 4 (D) – 3

20. If a, b, c are non zeros, then the system of equations

$$(\alpha + a)x + \alpha y + \alpha z = 0$$
$$\alpha x + (\alpha + b)y + \alpha z = 0$$
$$\alpha x + \alpha y + (\alpha + c)z = 0$$

has a non-trivial solution if
(A) $\alpha^{-1} = -(a^{-1} + b^{-1} + c^{-1})$ (B) $\alpha^{-1} = a + b + c$
(C) $\alpha + a + b + c = 1$ (D) none of these

PART - II : SINGLE AND DOUBLE VALUE INTEGER TYPE

1. Let X be the solution set of the equation
$A^x = I$, where $A = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$ and I is the unit matrix and $X \subset N$ then the minimum value of $\sum_x (\cos^x\theta + \sin^x\theta)$, $\theta \in R$ is :

2. If A is a diagonal matrix of order 3 × 3 is commutative with every square matrix of order 3 × 3 under multiplication and tr(A) = 12, then the value of |A| is :

3. **A,** is a (3×3) diagonal matrix having integral entries such that det(A) = 120, number of such matrices is 10n. Then n is :

4. If $\begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} \geq 0$, where a, b, c $\in R^+$, then $\dfrac{a+b}{c}$ is

5. If a_1, a_2, a_3, 5, 4, a_6, a_7, a_8, a_9 are in H.P. and $\mathbf{D} = \begin{vmatrix} a_1 & a_2 & a_3 \\ 5 & 4 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix}$, then the value of **21D** is

6. If $\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = k(\alpha a + \beta b + \gamma c)^3$, then $(2\alpha + \beta - \gamma)^k$ is ($\alpha, \beta, \gamma, k \in z^+$)

7. If A is a square matrix of order 3 and A′ denotes transpose of matrix A, A′ A = I and det A = 1, then det (A – I) must be equal to

8. Suppose A is a matrix such that $A^2 = A$ and $(I + A)^6 = I + kA$, then k is

9. If $\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix} = 64$, then (ab + bc + ac) is :

10. Let $f(x) = \begin{vmatrix} 1+\sin^2 x & \cos^2 x & 4\sin 2x \\ \sin^2 x & 1+\cos^2 x & 4\sin 2x \\ \sin^2 x & \cos^2 x & 1+4\sin 2x \end{vmatrix}$ then the maximum value of f(x) is

11. If $U_n = \begin{vmatrix} n & 1 & 5 \\ n^2 & 2N+1 & 2N+1 \\ n^3 & 3N^2 & 3N+1 \end{vmatrix}$ and $\sum_{n=1}^{N} U_n = \lambda \sum_{n=1}^{N} n^2$, then λ is

12. The absolute value of a for which system of equations, $a^3x + (a + 1)^3y + (a + 2)^3z = 0$, $ax + (a + 1)y + (a + 2)z = 0$, $x + y + z = 0$, has a non–zero solution is:

13. Consider the system of linear equations in x, y, z:

$(\sin 3\theta)x - y + z = 0$

$(\cos 2\theta)x + 4y + 3z = 0$

$2x + 7y + 7z = 0$

Number of values of $\theta \in (0, \pi)$ for which this system has non – trivial solution, is

14. The value of ' 2k ' for which the set of equations $3x + ky - 2z = 0$, $x + ky + 3z = 0$, $2x + 3y - 4z = 0$ has a non – trivial solution over the set of rational is:

15. $\mathbf{A_1} = [a_1]$

$A_2 = \begin{bmatrix} a_2 & a_3 \\ a_4 & a_5 \end{bmatrix}$

$A_3 = \begin{bmatrix} a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} \\ a_{12} & a_{13} & a_{14} \end{bmatrix}$......................$A_n = [........]$

Where $a_r = [\log_2 r]$ ([.] denotes greatest integer). Then trace of A_{10}

16. If $\left\{ \frac{1}{2}(A - A' + I) \right\}^{-1} = \frac{2}{\lambda} \begin{bmatrix} \lambda - 13 & -\frac{\lambda}{3} & \frac{\lambda}{3} \\ -17 & 10 & -1 \\ 7 & -11 & 5 \end{bmatrix}$ for $A = \begin{bmatrix} -2 & 3 & 4 \\ 5 & -4 & -3 \\ 7 & 2 & 9 \end{bmatrix}$, then λ is :

17. Given $A = \begin{bmatrix} 2 & 0 & -\alpha \\ 5 & \alpha & 0 \\ 0 & \alpha & 3 \end{bmatrix}$ For $\alpha \in R - \{a, b\}$, A^{-1} exists and $A^{-1} = A^2 - 5bA + cI$, when $\alpha = 1$. The value of $a + 5b + c$ is :

18. Let a, b, c positive numbers. Find the number of solution of system of equations in x, y and z
$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$; $\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$;$-\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ has finitely many solutions

PART - III : ONE OR MORE THAN ONE OPTIONS CORRECT TYPE

1. Which one of the following is wrong ?
(A) The elements on the main diagonal of a symmetric matrix are all zero
(B) The elements on the main diagonal of a skew - symmetric matrix are all zero
(C) For any square matrix A, A A′ is symmetric
(D) For any square matrix A, $(A + A')^2 = A^2 + (A')^2 + 2AA'$

2. Which of the following is true for matrix A = $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

(A) A + 4I is a symmetric matrix
(B) $A^2 - 4A + 5I_2 = 0$
(C) A – B is a diagonal matrix for any value of α if B = $\begin{bmatrix} \alpha & -1 \\ 2 & 5 \end{bmatrix}$
(D) A – 4I is a skew symmetric matrix

3. Suppose a_1, a_2, a_3 are in A.P. and b_1, b_2, b_3 are in H.P. and let

$$\Delta = \begin{vmatrix} a_1 - b_1 & a_1 - b_2 & a_1 - b_3 \\ a_2 - b_1 & a_2 - b_2 & a_2 - b_3 \\ a_3 - b_1 & a_3 - b_2 & a_3 - b_3 \end{vmatrix}, \text{ then}$$

(A) Δ is independent of a_1, a_2, a_3,
(B) $a_1 - \Delta, a_2 - 2\Delta, a_3 - 3\Delta$ are in A.P.
(C) $b_1 + \Delta, b_2 + \Delta^2, b_3 + \Delta$ are in H.P.
(D) Δ is independent of b_1, b_2, b_3

4. Let $\theta = \frac{\pi}{5}$, X = $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, O is null maxtrix and I is an identity matrix of order 2 × 2, and if $I + X + X^2 + ... + X^n = O$, then n can be

(A) 9
(B) 19
(C) 4
(D) 29

5. If $\Delta = \begin{vmatrix} x & 2y - z & -z \\ y & 2x - z & -z \\ y & 2y - z & 2x - 2y - z \end{vmatrix}$, then

(A) x – y is a factor of Δ
(B) $(x - y)^2$ is a factor of Δ
(C) $(x - y)^3$ is a factor of Δ
(D) Δ is independent of z

6. Let a, b > 0 and $\Delta = \begin{vmatrix} -x & a & b \\ b & -x & a \\ a & b & -x \end{vmatrix}$, then

(A) a + b – x is a factor of Δ
(B) $x^2 + (a + b)x + a^2 + b^2 - ab$ is a factor of Δ
(C) $\Delta = 0$ has three real roots if a = b
(D) a + b + x is a factor of Δ

7. The determinent $\Delta = \begin{vmatrix} b & c & b\alpha + c \\ c & d & c\alpha + d \\ b\alpha + c & c\alpha + d & a\alpha^3 - c\alpha \end{vmatrix}$ is equal to zero if

(A) b, c, d are in A.P.
(B) b, c, d are in G.P.
(C) b, c, d are in H.P.
(D) α is a root of $ax^3 - bx^2 - 3cx - d = 0$

8. The determinant $\Delta = \begin{vmatrix} a^2(1+x) & ab & ac \\ ab & b^2(1+x) & bc \\ ac & bc & c^2(1+x) \end{vmatrix}$ is divisible by

(A) x + 3
(B) $(1 + x)^2$
(C) x^2
(D) $x^2 + 1$

9. If A is a non–singular matrix and A^T denotes the transpose of A, then:

(A) $|A| \neq |A^T|$
(B) $|A. A^T| = |A|^2$
(C) $|A^T. A| = |A^T|^2$
(D) $|A|+|A^T| \neq 0$

10. Let f(x) = $\begin{vmatrix} 2\sin x & \sin^2 x & 0 \\ 1 & 2\sin x & \sin^2 x \\ 0 & 1 & 2\sin x \end{vmatrix}$, then

(A) f(x) is independent of x
(B) $f'(\pi/2) = 0$
(C) $\int_{-\pi/2}^{\pi/2} f(x)dx = 0$
(D) tangent to the curve y = f(x) at x = 0 is y = 0

11. Let $\Delta = \begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix}$, then

(A) $1 - x^3$ is a factor of Δ
(B) $(1 - x^3)^2$ is factor of Δ
(C) $\Delta(x) = 0$ has 4 real roots
(D) $\Delta'(1) = 0$

12. Let f(x) = $\begin{vmatrix} 1/x & \log x & x^n \\ 1 & -1/n & (-1)^n \\ 1 & a & a^2 \end{vmatrix}$, then (where $f^n(x)$ denotes n^{th} derivative of f(x))

(A) f^n (1) is indepedent of a
(B) f^n (1) is indepedent of n
(C) f^n (1) depends on a and n
(D) $y = a(x - f^n(1))$ represents a straight line through the origin

13. If D is a determinant of order three and Δ is a determinant formed by the cofactors of determinant D ; then

(A) $\Delta = D^2$
(B) D = 0 implies $\Delta = 0$
(C) if D = 27, then Δ is perfect cube
(D) if D = 27, then Δ is perfect square

14. Let A, B, C, D be real matrices such that $A^T = BCD$; $B^T = CDA$; $C^T = DAB$ and $D^T = ABC$ for the matrix M = ABCD, then find M^{2016} ?

(A) M
(B) M^2
(C) M^3
(D) M^4

15. Let **A** and **B** be two 2 × 2 matrix with real entries. If **AB** = **O** and tr**(A)** = tr**(B)** = 0 then

(A) **A** and **B** are comutative w.r.t. operation of multiplication.
(B) **A** and **B** are not commutative w.r.t. operation of multiplication.
(C) **A** and **B** are both null matrices.
(D) **BA** = 0

16. If $A^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & -2 & 1 \\ 0 & 0 & -1 \end{bmatrix}$, then

(A) | A | = 2
(B) A is non-singular
(C) Adj. A = $\begin{bmatrix} 1/2 & -1/2 & 0 \\ 0 & -1 & 1/2 \\ 0 & 0 & -1/2 \end{bmatrix}$
(D) A is skew symmetric matrix

17. If A and B are square matrices of order 3, then the true statement is/are (where I is unit matrix).

(A) det (– A) = – det A
(B) If AB is singular then atleast one of A or B is singular
(C) det (A + I) = 1 + det A
(D) det (2A) = 2^3 det A

18. Let **M** be a 3 × 3 non-singular matrix with det**(M)** = 4. If $\mathbf{M}^{-1}$ adj(adj **M**) = $\mathbf{k}^2\mathbf{I}$, then the value of **'k'** may be

(A) +2
(B) 4
(C) –2
(D) –4

19. If AX = B where A is 3 × 3 and X and B are 3×1 matrices then which of the following is correct?
(A) If |A| = 0 then AX = B has infinite solutions
(B) If AX = B has infinite solutions then |A| = 0
(C) If (adj(A)) B = 0 and |A| ≠ 0 then AX = B has unique solution
(D) If (adj(A)) B ≠ 0 & |A| = 0 then AX = B has no solution

PART - IV : COMPREHENSION

Comprehension # 1

Let $\mathcal{A}$ be the set of all 3×3 symmetric matrices whose entries are 1,1,1,0,0,0,–1, –1, –1. B is one of the matrix in set $\mathcal{A}$ and

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \qquad U = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad V = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}.$$

1. Number of such matrices B in set $\mathcal{A}$ is λ, then λ lies in the interval
(A) (30, 40) (B) (38, 40) (C) (34, 38) (D) (25, 35)

2. Number of matrices B such that equation BX = U has infinite solutions
(A) is at least 6 (B) is not more than 10 (C) lie between 8 to 16 (D) is zero.

3. The equation BX = V
(A) is inconsistent for atleast 3 matrices B.
(B) is inconsistent for all matrices B.
(C) is inconsistent for at most 12 matrices B.
(D) has infinite number of solutions for at least 3 matrices B.

Comprehension # 2

Some special square matrices are defined as follows :

Nilpotent matrix : A square matrix A is said to be nilpotent (of order 2) if, $A^2 = O$. A square matrix is said to be nilpotent of order p, if p is the least positive integer such that $A^p = O$.

Idempotent matrix : A square matrix A is said to be idempotent if, $A^2 = A$.

e.g. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is an idempotent matrix.

Involutory matrix : A square matrix A is said to be involutory if $A^2 = I$, I being the identity matrix.

e.g. $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is an involutory matrix.

Orthogonal matrix : A square matrix A is said to be an orthogonal matrix if $A'A = I = AA'$.

4. If A and B are two square matrices such that AB = A & BA = B, then A & B are
(A) Idempotent matrices (B) Involutory matrices
(C) Orthogonal matrices (D) Nilpotent matrices

5. If the matrix $\begin{bmatrix} 0 & 2\beta & \gamma \\ \alpha & \beta & -\gamma \\ \alpha & -\beta & \gamma \end{bmatrix}$ is orthogonal, then

(A) $\alpha = \pm \frac{1}{\sqrt{2}}$ (B) $\beta = \pm \frac{1}{\sqrt{6}}$ (C) $\gamma = \pm \frac{1}{\sqrt{3}}$ (D) all of these

6. The matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$ is

(A) idempotent matrix (B) involutory matrix
(C) nilpotent matrix (D) none of these

Exercise-3

✍ **Marked questions are recommended for Revision.**
*** Marked Questions may have more than one correct option.**

PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

Comprehension # 1 (Q. No. 1 to Q. No. 3)

Let $\mathcal{A}$ be the set of all 3 × 3 symmetric matrices all of whose entries are either 0 or 1. Five of these entries are 1 and four of them are 0.

1.✍ The number of matrices in $\mathcal{A}$ is **[IIT-JEE 2009, Paper-1, (4, –1), 80]**
(A) 12 (B) 6 (C) 9 (D) 3

2.✍ The number of matrices A in $\mathcal{A}$ for which the system of linear equations $A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ has a unique solution, is **[IIT-JEE 2009, Paper-1, (4, –1), 80]**
(A) less than 4 (B) at least 4 but less than 7
(C) at least 7 but less than 10 (D) at least 10

3.✍ The number of matrices A in $\mathcal{A}$ for which the system of linear equations $A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ is inconsistent, is
(A) 0 (B) more than 2 (C) 2 (D) 1
[IIT-JEE 2009, Paper-1, (4, –1), 80]

Comprehension # 2 (Q. No. 4 to 6)

Let p be an odd prime number and T_p be the following set of 2 × 2 matrices :

$$T_p = \left\{ A = \begin{bmatrix} a & b \\ c & a \end{bmatrix} : a, b, c \in \{0, 1, 2, \ldots, p-1\} \right\}$$

4. The number of A in T_p such that A is either symmetric or skew-symmetric or both, and det (A) divisible by p is **[IIT-JEE 2010, Paper-1, (3, –1), 84]**

(A) $(p-1)^2$ (B) $2(p-1)$ (C) $(p-1)^2 + 1$ (D) $2p - 1$

5. The number of A in T_p such that the trace of A is not divisible by p but det (A) is divisible by p is
[Note : The trace of matrix is the sum of its diagonal entries.] **[IIT-JEE 2010, Paper-1, (3, –1), 84]**
(A) $(p-1)(p^2-p+1)$ (B) $p^3-(p-1)^2$ (C) $(p-1)^2$ (D) $(p-1)(p^2-2)$

6. The number of A in T_p such that det (A) is not divisible by p is **[IIT-JEE 2010, Paper-1, (3, –1), 84]**
(A) $2p^2$ (B) $p^3 - 5p$ (C) $p^3 - 3p$ (D) $p^3 - p^2$

7.✍ The number of all possible values of θ, where $0 < \theta < \pi$, for which the system of equations

$$(y + z)\cos 3\theta = (xyz)\sin 3\theta$$
$$x \sin 3\theta = \frac{2\cos 3\theta}{y} + \frac{2\sin 3\theta}{z}$$
$$(xyz)\sin 3\theta = (y + 2z)\cos 3\theta + y \sin 3\theta$$

have a solution (x_0, y_0, z_0) with $y_0 z_0 \neq 0$, is **[IIT-JEE 2010, Paper-1, (3, 0), 84]**

8. Let k be a positive real number and let **[IIT-JEE 2010, Paper-2, (3, 0), 79]**

$$A = \begin{bmatrix} 2k-1 & 2\sqrt{k} & 2\sqrt{k} \\ 2\sqrt{k} & 1 & -2k \\ -2\sqrt{k} & 2k & -1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & 2k-1 & \sqrt{k} \\ 1-2k & 0 & 2\sqrt{k} \\ -\sqrt{k} & -2\sqrt{k} & 0 \end{bmatrix}$$

. If det (adj A) + det (adj B) = 10^6, then [k] is equal to

(**Note** : adj M denotes the adjoint of a square matrix M and [k] denotes the largest integer less than or equal to k].

9. Let M and N be two 2n × 2n non-singular skew-symmetric matrices such that MN = NM. If P^T denotes the transpose of P, then $M^2 N^2 (M^T N)^{-1} (MN^{-1})^T$ is equal to

(A) M^2 (B) $-N^2$ (C) $-M^2$ (D) MN

Comprehension # 3 (10 to 12) **[IIT-JEE 2011, Paper-1, (3, –1), 80]**

Let a, b and c be three real numbers satisfying

$$[a\ \ b\ \ c]\begin{bmatrix} 1 & 9 & 7 \\ 8 & 2 & 7 \\ 7 & 3 & 7 \end{bmatrix} = [0\ \ 0\ \ 0] \qquad \text{...........(E)}$$

10. If the point P(a, b, c), with reference to (E), lies on the plane 2x + y + z = 1, then the value of 7a + b + c is

(A) 0 (B) 12 (C) 7 (D) 6

11. Let ω be a solution of $x^3 - 1 = 0$ with Im (ω) > 0. if a = 2 with b and c satisfying (E), then the value of $\dfrac{3}{\omega^a} + \dfrac{1}{\omega^b} + \dfrac{3}{\omega^c}$ is equal to

(A) – 2 (B) 2 (C) 3 (D) – 3

12. Let b = 6, with a and c satisfying (E). If α and β are the roots of the quadratic equation $ax^2 + bx + c = 0$, then $\sum_{n=0}^{\infty}\left(\dfrac{1}{\alpha}+\dfrac{1}{\beta}\right)^n$ is

(A) 6 (B) 7 (C) $\dfrac{6}{7}$ (D) ∞

13. Let $\omega \neq 1$ be a cube root of unity and S be the set of all non-singular matrices of the form, $\begin{bmatrix} 1 & a & b \\ \omega & 1 & c \\ \omega^2 & \omega & 1 \end{bmatrix}$

where each of a, b and c is either ω or ω^2. Then the number of distinct matrices in the set S is **[IIT-JEE 2011, Paper-2, (3, –1), 80]**

(A) 2 (B) 6 (C) 4 (D) 8

14. Let M be a 3 × 3 matrix satisfying

$M\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$, $M\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$, and $M\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 12 \end{bmatrix}$. Then the sum of the diagonal entries of M is

[IIT-JEE 2011, Paper-2, (4, 0), 80]

15. Let P = $[a_{ij}]$ be a 3 × 3 matrix and let Q = $[b_{ij}]$, where $b_{ij} = 2^{i+j} a_{ij}$ for $1 \leq i, j \leq 3$. If the determinant of P is 2, then the determinant of the matrix Q is **[IIT-JEE 2012, Paper-1, (3, –1), 70]**

(A) 2^{10} (B) 2^{11} (C) 2^{12} (D) 2^{13}

16. If P is a 3 × 3 matrix such that $P^T = 2P + I$, where P^T is the transpose of P and I is the 3 × 3 identity matrix, then there exists a column matrix $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ such that

[IIT-JEE 2012, Paper-2, (3, –1), 66]

(A) $PX = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ (B) PX = X (C) PX = 2X (D) PX = – X

17*. If the adjoint of a 3 × 3 matrix P is $\begin{bmatrix} 1 & 4 & 4 \\ 2 & 1 & 7 \\ 1 & 1 & 3 \end{bmatrix}$, then the possible value(s) of the determinant of P is (are)

[IIT-JEE 2012, Paper-2, (4, 0), 66]

(A) –2 (B) –1 (C) 1 (D) 2

18.* For 3×3 matrices M and N, which of the following statement(s) is (are) **NOT** correct ?

[JEE (Advanced) 2013, Paper-1, (4, – 1)/60]

(A) $N^T M N$ is symmetric or skew symmetric, according as M is symmetric or skew symmetric
(B) M N – N M is skew symmetric for all symmetric matrices M and N
(C) M N is symetric for all symmetric matrices M and N
(D) (adj M) (adj N) = adj(MN) for all invertible matrices M and N

19*. Let M be a 2 × 2 symmetric matrix with integer entries. Then M is invertible if

[JEE (Advanced) 2014, Paper-1, (3, 0)/60]

(A) the first column of M is the transpose of the second row of M
(B) the second row of M is the transpose of first column of M
(C) M is a diagonal matrix with nonzero entries in the main diagonal
(D) the product of entries in the main diagonal of M is not the square of an integer

20*. Let M and N be two 3 × 3 matrices such that MN = NM. Further, if $M \neq N^2$ and $M^2 = N^4$, then

[JEE (Advanced) 2014, Paper-1, (3, 0)/60]

(A) determinant of $(M^2 + MN^2)$ is 0
(B) there is a 3 × 3 non-zero matrix U such that $(M^2 + MN^2)U$ is the zero matrix
(C) determinant of $(M^2 + MN^2) \geq 1$
(D) for a 3 × 3 matrix U, if $(M^2 + MN^2)U$ equals the zero matrix then U is the zero matrix

21*. Let X and Y be two arbitrary, 3 × 3, non-zero, skew-symmetric matrices and Z be an arbitrary 3 × 3, non-zero, symmetric matrix. Then which of the following matrices is (are) skew symmetric ?

[JEE (Advanced) 2015, Paper-1 (4, –2)/88]

(A) $Y^3Z^4 - Z^4Y^3$ (B) $X^{44} + Y^{44}$ (C) $X^4Z^3 - Z^3X^4$ (D) $X^{23} + Y^{23}$

22*. Which of the following values of α satisfy the equation $\begin{vmatrix} (1+\alpha)^2 & (1+2\alpha)^2 & (1+3\alpha)^2 \\ (2+\alpha)^2 & (2+2\alpha)^2 & (2+3\alpha)^2 \\ (3+\alpha)^2 & (3+2\alpha)^2 & (3+3\alpha)^2 \end{vmatrix} = -648\alpha$?

[JEE (Advanced) 2015, Paper-1 (4, –2)/88]

(A) – 4 (B) 9 (C) – 9 (D) 4

23*. Let $P = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & \alpha \\ 3 & -5 & 0 \end{bmatrix}$, where $\alpha \in R$. Suppose $Q = [q_{ij}]$ is a matrix such that $PQ = kI$, where $k \in R$, $k \neq 0$ and I is the identity matrix of order 3. If $q_{23} = -\frac{k}{8}$ and $\det(Q) = \frac{k^2}{2}$, then

[JEE (Advanced) 2016, Paper-1 (4, –2)/62]

(A) $\alpha = 0, k = 8$ (B) $4\alpha - k + 8 = 0$ (C) $\det(P \operatorname{adj}(Q)) = 2^9$ (D) $\det(Q \operatorname{adj}(P)) = 2^{13}$

24. The total number of distinct x ∈ R for which $\begin{vmatrix} x & x^2 & 1+x^3 \\ 2x & 4x^2 & 1+8x^3 \\ 3x & 9x^2 & 1+27x^3 \end{vmatrix} = 10$ is

[JEE (Advanced) 2016, Paper-1 (3, 0)/62]

25. Let P = $\begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 16 & 4 & 1 \end{bmatrix}$ and I be the identity matrix of order 3. If Q = $[q_{ij}]$ is a matrix such that $P^{50} - Q = I$, then $\dfrac{q_{31}+q_{32}}{q_{21}}$ equals **[JEE (Advanced) 2016, Paper-2 (3, –1)/62]**

(A) 52 (B) 103 (C) 201 (D) 205

26*. Let a, λ, μ ∈ R. Consider the system of linear equations

$$ax + 2y = \lambda$$
$$3x - 2y = \mu$$

[JEE (Advanced) 2016, Paper-2 (4, –2)/62]

Which of the following statement(s) is(are) correct ?

(A) if a = – 3, then the system has infinitely many solutions for all values of λ and μ

(B) If a ≠ –3, then the system has a a unique solution for all values of λ and μ

(C) If λ + μ = 0, the the system has infinitely many solutions for a = –3

(D) If λ + μ ≠ 0, then the system has no solution for a = –3

27*. Which of the following is(are) NOT the square of a 3 × 3 matrix with real entries?

[JEE(Advanced) 2017, Paper-1,(4, –2)/61]

(A) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ (C) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

28. For a real number α, if the system **[JEE(Advanced) 2017, Paper-1,(3, 0)/61]**

$$\begin{bmatrix} 1 & \alpha & \alpha^2 \\ \alpha & 1 & \alpha \\ \alpha^2 & \alpha & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

of linear equations, has infinitely many solutions, then $1 + \alpha + \alpha^2$ =

29. How many 3 × 3 matrices M with entries from {0, 1, 2} are there, for which the sum of the diagonal entries of $M^T M$ is 5 ? **[JEE(Advanced) 2017, Paper-2,(3, –1)/61]**

(A) 198 (B) 162 (C) 126 (D) 135

30*. Let S be the set of all column matrices $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ such that $b_1, b_2, b_2 \in R$ and the system of equation (in real variables)

$$-x + 2y + 5z = b_1$$
$$2x - 4y + 3z = b_2$$
$$x - 2y + 2z = b_3$$

has at least one solution. Then, which of the following system(s) (in real variables) has (have) at least one solution for each $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \in S$? **[JEE(Advanced) 2018, Paper-2,(4, –2)/60]**

(A) $x + 2y + 3z = b_1$, $4y + 5z = b_2$ and $x + 2y + 6z = b_3$

(B) $x + y + 3z = b_1$, $5x + 2y + 6z = b_2$ and $-2x - y - 3z = b_3$

(C) $-x + 2y - 5z = b_1$, $2x - 4y + 10z = b_2$ and $x - 2y + 5z = b_3$

(D) $x + 2y + 5z = b_1$, $2x + 3z = b_2$ and $x + 4y - 5z = b_3$

31. Let P be a matrix of order 3 × 3 such that all the entries in P are from the set {–1, 0, 1}. Then, the maximum possible value of the determinant of P is _____ . **[JEE(Advanced) 2018, Paper-2,(3, 0)/60]**

PART - II : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. Let A be a 2 × 2 matrix.
Statement-1 : adj(adj (A)) = A.
Statement-2 : |adj A| = |A| **[AIEEE 2009 (4, –1), 144]**
(1) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
(2) Statement-1 is true, Statement-2 is false.
(3) Statement-1 is false, Statement-2 is true.
(4) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

2. Let a, b, c be such that $b(a + c) \neq 0$. If $\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ (-1)^{n+2}a & (-1)^{n+1}b & (-1)^{n}c \end{vmatrix} = 0$. Then the value of 'n' is - **[AIEEE 2009 (4, –1), 144]**
(1) zero (2) any even integer (3) any odd integer (4) any integer

3. The number of 3 × 3 non-singular matrices, with four entries as 1 and all other entries as 0, is
(1) 5 (2) 6 (3) at least 7 (4) less than 4
[AIEEE 2010 (8, –2), 144]

4. Let A be a 2 × 2 matrix with non-zero entries and let $A^2 = I$, where I is 2 × 2 identity matrix.
Tr(A) = sum of diagonal elements of A and |A| = determinant of matrix A. **[AIEEE 2010 (4, –1), 144]**
Statement -1 : Tr(A) = 0
Statement -2 : |A| = 1
(1) Statement -1 is true, Statement-2 is true ; Statement -2 is not a correct explanation for Statement-1.
(2) Statement-1 is true, Statement-2 is false.
(3) Statement -1 is false, Statement -2 is true.
(4) Statement -1 is true, Statement -2 is true; Statement-2 is a correct explanation for Statement-1.

5. Consider the system of linear equations : **[AIEEE 2010 (4, –1), 144]**
$x_1 + 2x_2 + x_3 = 3$
$2x_1 + 3x_2 + x_3 = 3$
$3x_1 + 5x_2 + 2x_3 = 1$
The system has
(1) exactly 3 solutions (2) a unique solution (3) no solution (4) infinite number of solutions

6. Let A and B be two symmetric matrices of order 3. **[AIEEE 2011, I, (4, –1), 120]**
Statement-1 : A(BA) and (AB)A are symmetric matrices.
Statement-2 : AB is symmetric matrix if matrix multiplication of A with B is commutative.
(1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is true, Statement-2 is true; Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1.
(3) Statement-1 is true, Statement-2 is false.
(4) Statement-1 is false, Statement-2 is true.

7. The number of values of k for which the linear equations **[AIEEE 2011, I, (4, –1), 120]**
$4x + ky + 2z = 0$
$kx + 4y + z = 0$
$2x + 2y + z = 0$
posses a non-zero solution is :
(1) 3 (2) 2 (3) 1 (4) zero

8. If $\omega \neq 1$ is the complex cube root of unity and matrix $H = \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix}$, then H^{70} is equal to -

[AIEEE 2011, I, (4, –1), 120]

(1) 0 (2) – H (3) H^2 (4) H

9. If the trivial solution is the only solution of the system of equations **[AIEEE 2011, II, (4, –1), 120]**

x – ky + z = 0

kx + 3y – kz = 0

3x + y – z = 0

then the set of all values of k is :

(1) R – {2, –3} (2) R – { 2 } (3) R – { –3 } (4) {2, –3}

10. **Statement - 1 :** Determinant of a skew-symmetric matrix of order 3 is zero.

[AIEEE 2011, II, (4, –1), 120]

Statement - 2 : For any matrix A, det $(A)^T$ = det(A) and det (–A) = – det(A).

Where det (B) denotes the determinant of matrix B. Then :

(1) Both statements are true (2) Both statements are false

(3) Statement-1 is false and statement-2 is true. (4) Statement-1 is true and statement-2 is false

11. Let $A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}$. If u_1 and u_2 are column matrices such that $Au_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ and $Au_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, then $u_1 + u_2$ is equal to : **[AIEEE-2012, (4, –1)/120]**

(1) $\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$ (2) $\begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$ (3) $\begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix}$ (4) $\begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$

12. Let P and Q be 3 × 3 matrices P ≠ Q. If $P^3 = Q^3$ and $P^2Q = Q^2P$, then determinant of $(P^2 + Q^2)$ is equal to :

[AIEEE-2012, (4, –1)/120]

(1) – 2 (2) 1 (3) 0 (4) – 1

13. The number of values of k, for which the system of equations :

[AIEEE - 2013, (4, – 1) 120]

(k + 1)x + 8y = 4k

kx + (k + 3)y = 3k – 1

has no solution, is

(1) infinite (2) 1 (3) 2 (4) 3

14. If $P = \begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$ is the adjoint of a 3 × 3 matrix A and |A| = 4, then α is equal to :

[AIEEE - 2013, (4, – 1) 120]

(1) 4 (2) 11 (3) 5 (4) 0

15. If $\alpha, \beta \neq 0$ and $f(n) = \alpha^n + \beta^n$ and $\begin{vmatrix} 3 & 1+f(1) & 1+f(2) \\ 1+f(1) & 1+f(2) & 1+f(3) \\ 1+f(2) & 1+f(3) & 1+f(4) \end{vmatrix} = K(1-\alpha)^2(1-\beta)^2(\alpha-\beta)^2$, then K is equal to **[JEE(Main) 2014, (4, – 1), 120]**

(1) 1 (2) –1 (3) $\alpha\beta$ (4) $\frac{1}{\alpha\beta}$

16. If A is an 3 × 3 non-singular matrix such that $AA' = A'A$ and $B = A^{-1}A'$, then BB' equals :

[JEE(Main) 2014, (4, – 1), 120]

(1) B^{-1} (2) $(B^{-1})'$ (3) I + B (4) I

17. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$ is a matrix satisfying the equation $AA^T = 9I$, where I is 3×3 identity matrix, then the ordered pair (a, b) is equal to : **[JEE(Main) 2015, (4, – 1), 120]**

(1) (2, – 1) (2) (–2, 1) (3) (2, 1) (4) (–2, – 1)

18. The set of all value of λ for which the system of linear equations :

$2x_1 - 2x_2 + x_3 = \lambda x_1$

$2x_1 - 3x_2 + 2x_3 = \lambda x_2$

$-x_1 + 2x_2 = \lambda x_3$

has a non-trivial solution, **[JEE(Main) 2015, (4, –1), 120]**

(1) is an empty set (2) is a singleton

(3) contains two elements (4) contains more than two elements

19. The system of linear equations **[JEE(Main) 2016, (4, –1), 120]**

$x + \lambda y - z = 0$

$\lambda x - y - z = 0$

$x + y - \lambda z = 0$

has a non-trivial solution for :

(1) Exactly one value of λ. (2) Exactly two values of λ.

(3) Exactly three values of λ. (4) Infinitely many values of λ.

20. If $A = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}$ and A adj A = A A^T, then 5a + b is equal to **[JEE(Main) 2016, (4, –1), 120]**

(1) 5 (2) 4 (3) 13 (4) – 1

21. It S is the set of distinct values of 'b' for which the following system of linear equations

$x + y + z = 1$ **[JEE(Main) 2017, (4, –1), 120]**

$x + ay + z = 1$

$ax + by + z = 0$

has no solution, then S is :

(1) an empty set (2) an infinite set

(3) a finite set containing two or more elements (4) a singleton

22. If $A = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$, then adj $(3A^2 + 12A)$ is equal to **[JEE(Main) 2017, (4, –1), 120]**

(1) $\begin{bmatrix} 72 & -84 \\ -63 & 51 \end{bmatrix}$ (2) $\begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}$ (3) $\begin{bmatrix} 51 & 84 \\ 63 & 72 \end{bmatrix}$ (4) $\begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$

23. If the system of linear equations **[JEE(Main) 2018, (4, –1), 120]**

$x + ky + 3z = 0$

$3x + ky - 2z = 0$

$2x + 4y - 3z = 0$

has a non-zero solution (x, y, z), then $\frac{xz}{y^2}$ is equal to :

(1) – 30 (2) 30 (3) –10 (4) 10

24. If $\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (A + Bx)(x-A)^2$ then the ordered pair (A, B) is equal to :

[JEE(Main) 2018, (4, –1), 120]

(1) (–4, 5) (2) (4, 5) (3) (–4, –5) (4) (–4, 3)

25. The system of linear equations **[JEE(Main) 2019, Online (09-01-19),P-1 (4, – 1), 120]**

x + y + z = 2

2x + 3y + 2z = 5

2x + 3y + $(a^2 + 1)$z = a + 1

(1) is inconsistent when a = 4

(2) has infinitely many solutions for a = 4

(3) is inconsistent when |a| = $\sqrt{3}$

(4) has a unique solution for |a| = $\sqrt{3}$

26. If A = $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, then the matrix A^{-50} when $\theta = \frac{\pi}{12}$, is equal to :

[JEE(Main) 2019, Online (09-01-19),P-1 (4, – 1), 120]

(1) $\begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$

(2) $\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

(3) $\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

(4) $\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$

27. Let d $\in$ R, and A = $\begin{bmatrix} -2 & 4+d & (\sin\theta - 2) \\ 1 & (\sin\theta)+2 & d \\ 5 & (2\sin\theta)-d & (-\sin\theta)+2+2d \end{bmatrix}$, $\theta \in [0, 2\pi]$. If the minimum value of det(A) is 8, then a value of d is : **[JEE(Main) 2019, Online (10-01-19),P-1 (4, – 1), 120]**

(1) –5

(2) 2($\sqrt{2}$ + 2)

(3) 2($\sqrt{2}$ +1)

(4) –7

28. Let A = $\begin{pmatrix} 0 & 2q & r \\ p & q & -r \\ p & -q & r \end{pmatrix}$. If $AA^T = I_3$, then |p| is **[JEE(Main) 2019, Online (11-01-19),P-1 (4, – 1), 120]**

(1) $\frac{1}{\sqrt{5}}$

(2) $\frac{1}{\sqrt{3}}$

(3) $\frac{1}{\sqrt{6}}$

(4) $\frac{1}{\sqrt{2}}$

Answers

EXERCISE # 1

PART-I

Section (A) :

A-1. $\begin{bmatrix} 1 & 0 \\ 3 & 2 \\ 5 & 4 \end{bmatrix}$ **A-2.** (x, y, z, w) = (1, 2, 4, 5) **A-3.** $\begin{bmatrix} 2 & -1/2 \\ 4 & -1 \end{bmatrix}$

A-4. AB = $\begin{bmatrix} 18 & -11 & 10 \\ -16 & 47 & 10 \\ 62 & -23 & 42 \end{bmatrix}$, BA = $\begin{bmatrix} 49 & 24 \\ -7 & 58 \end{bmatrix}$ **A-7.** $y \in R$ **A-9.** Zero

Section (B) :

B-1. 0, $\frac{3\pi}{4}$, π **B-2.** (i) 0 (ii) 0 (iii) 0 (iv) $5(3\sqrt{2} - 5\sqrt{3})$

B-5. (i) x = – 2 b/a (ii) 4 **B-8.** A = 0, B = 0

Section (C) :

C-1. $\begin{bmatrix} -191 & -110 \\ 77 & 44 \end{bmatrix}$ **C-2.** (ii) $|A|^{(n-1)^3}$ **C-3.** $\begin{bmatrix} 9 & -3 & 5 \\ -2 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ **C-5.** a = 1, c = – 1

Section (D) :

D-1. a = – 4, b = 1, $A^{-1} = \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$ **D-2.** 0

D-4. (i) x = 3, y = 4, z = 6 (ii) $x = -\frac{5k}{3} + \frac{8}{3}$, $y = -\frac{2k}{3} - \frac{1}{3}$, z = k, where $k \in R$

D-5. x = – 7, y = – 4 **D-6.** for c = 0, x = – 3, y = 3 ; for c = – 10, $x = -\frac{1}{2}$, $y = \frac{4}{3}$

D-7. **(i)** x = 2, y = 2, z = 2 **(ii)** x = 1, y = 3, z = 5 **D-8. (a)** $\lambda \neq 3$ **(b)** $\lambda = 3, \mu = 10$ **(c)** $\lambda = 3, \mu \neq 10$

D-9. x = 3, y = – 2, z = – 1 **D-10.** x = 1, y = 2, z = 3, $A^{-1} = \frac{1}{17}\begin{bmatrix} 1 & 5 & 1 \\ 8 & 6 & -9 \\ 10 & -1 & -7 \end{bmatrix}$

D-11. S1, S3, S4

PART-II

Section (A) :

A-1. (A) **A-2.** (D) **A-3.** (A) **A-4.** (C) **A-5.** (B) **A-6.** (C) **A-7.** (A)
A-8. (A) **A-9.** (A)

Section (B) :

B-1. (B) **B-2.** (A) **B-3.** (A) **B-4.** (D) **B-5.** (D) **B-6.** (D) **B-7.** (B)
B-8. (C) **B-9.** (B) **B-10.** (B)

Section (C) :

C-1. (A) **C-2.** (C) **C-3.** (A) **C-4.** (A) **C-5.** (C) **C-6.** (A)

Section (D) :

D-1. (A) **D-2** (A) **D-3.** (D) **D-4.** (B) **D-5.** (B) **D-6.** (A)

PART-III

1.	(A) → (s),	(B) → (p),	(C) → (p),	(D) → (p)
2.	(A) → (q),	(B) → (p),	(C) → (s),	(D) → (q)

EXERCISE # 2

PART-I

1.	(D)	**2.**	(A)	**3.**	(D)	**4.**	(C)	**5.**	(D)	**6.**	(A)	**7.**	(C)
8.	(A)	**9.**	(A)	**10.**	(A)	**11.**	(A)	**12.**	(B)	**13.**	(A)	**14.**	(A)
15.	(A)	**6.**	(C)	**17.**	(B)	**18.**	(B)	**19.**	(A)	**20.**	(A)		

PART - II

1.	2	**2.**	64	**3.**	36	**4.**	2	**5.**	50	**6.**	4	**7.**	0
8.	63	**9.**	4	**10.**	6	**11.**	2	**12.**	1	**13.**	**2**	**14.**	33
15.	80	**16.**	39	**17.**	17	**18.**	8						

PART - III

1.	(AD)	**2.**	(BC)	**3.**	(ABCD)	**4.**	(ABD)	**5.**	(AB)	**6.**	(ABC)	**7.**	(BD)
8.	(AC)	**9.**	(BCD)	**10.**	(BCD)	**11.**	(ABD)	**12.**	(ABD)	**13.**	(ABCD)		
14.	(BD)	**15.**	(AD)	**16.**	(BC)	**17.**	(ABD)	**18.**	(AC)	**19.**	(BCD)		

PART - IV

1*.	(AC)	**2*.**	(AC)	**3*.**	(AC)	**4.**	(A)	**5.**	(D)	**6.**	(C)

EXERCISE # 3

PART-I

1.	(A)	**2.**	(B)	**3.**	(B)	**4.**	(D)	**5.**	(C)	**6.**	(D)	**7.**	3
8.	4	**9.**	(C)	**10.**	(D)	**11.**	(A)	**12 .**	(B)	**13.**	(A)	**14.**	**9**
15.	(D)	**16.**	(D)	**17*.**	(AD)	**18.***	(CD)	**19*.**	(CD)	**20*.**	(AB)	**21*.**	(CD)
22*.	(BC)	**23*.**	(BC)	**24.**	2	**25.**	(B)	**26*.**	(BCD)	**27*.**	(AC)	**28.**	1
29.	(A)	**30*.**	(AD)	**31.**	4								

PART - II

1.	(1)	**2.**	(3)	**3.**	(3)	**4.**	(2)	**5.**	(3)	**6.**	(2)	**7.**	(2)
8.	(4)	**9.**	(1)	**10.**	(4)	**11.**	(4)	**12.**	(3)	**13.**	(2)	**14.**	(2)
15.	(1)	**16.**	(4)	**17.**	(4)	**18.**	(3)	**19.**	(3)	**20.**	(1)	**21.**	(4)
22.	(2)	**23.**	(4)	**24.**	(1)	**25.**	(3)	**26.**	**(2)**	**27.**	(1)	**28.**	(4)

Advance Level Problems (ALP)

1. If $a^2 + b^2 + c^2 = 1$, then prove that $\begin{vmatrix} a^2+(b^2+c^2)\cos\phi & ab(1-\cos\phi) & ac(1-\cos\phi) \\ ba(1-\cos\phi) & b^2+(c^2+a^2)\cos\phi & bc(1-\cos\phi) \\ ca(1-\cos\phi) & cb(1-\cos\phi) & c^2+(a^2+b^2)\cos\phi \end{vmatrix}$

is independent of a, b, c

2. If a, b, c, x, y, z $\in$ R, then prove that,

$$\begin{vmatrix} (a-x)^2 & (b-x)^2 & (c-x)^2 \\ (a-y)^2 & (b-y)^2 & (c-y)^2 \\ (a-z)^2 & (b-z)^2 & (c-z)^2 \end{vmatrix} = . \begin{vmatrix} (1+ax)^2 & (1+bx)^2 & (1+cx)^2 \\ (1+ay)^2 & (1+by)^2 & (1+cy)^2 \\ (1+az)^2 & (1+bz)^2 & (1+cz)^2 \end{vmatrix}$$

3. If a_1, a_2, a_3 are distinct real roots of the equation $px^3 + px^2 + qx + r = 0$ such that $\begin{vmatrix} 1+a_1 & 1 & 1 \\ 1 & 1+a_2 & 1 \\ 1 & 1 & 1+a_3 \end{vmatrix} = 0$,

then Prove that $\frac{r}{p} < 0$

4. Prove that $\Delta = \begin{vmatrix} \beta\gamma & \beta\gamma'+\beta'\gamma & \beta'\gamma' \\ \gamma\alpha & \gamma\alpha'+\gamma'\alpha & \gamma'\alpha' \\ \alpha\beta & \alpha\beta'+\alpha'\beta & \alpha'\beta' \end{vmatrix} = (\alpha\beta' - \alpha'\beta)\,(\beta\gamma' - \beta'\gamma)\,(\gamma\alpha' - \gamma'\alpha)$

5. If

$$\begin{aligned} ax_1^2 + by_1^2 + cz_1^2 &= d & \quad ax_2x_3 + by_2y_3 + cz_2z_3 &= f \\ ax_2^2 + by_2^2 + cz_2^2 &= d \quad \text{and} & \quad ax_3x_1 + by_3y_1 + cz_3z_1 &= f \\ ax_3^2 + by_3^2 + cz_3^2 &= d & \quad ax_1x_2 + by_1y_2 + cz_1z_2 &= f \end{aligned}$$

then prove that $\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}^2 = (d-f)^2 \, \frac{d+2f}{abc}$, where a, b, c $\neq$ 0.

6. If

$$\begin{aligned} (x_1-x_2)^2 + (y_1-y_2)^2 &= a^2 \\ (x_2-x_3)^2 + (y_2-y_3)^2 &= b^2, \\ (x_3-x_1)^2 + (y_3-y_1)^2 &= c^2 \end{aligned}$$

prove that $4\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}^2 = (a+b+c)\,(b+c-a)\,(c+a-b)(a+b-c)$.

7. Let $A = \begin{bmatrix} \cos^{-1}x & \cos^{-1}y & \cos^{-1}z \\ \cos^{-1}y & \cos^{-1}z & \cos^{-1}x \\ \cos^{-1}z & \cos^{-1}x & \cos^{-1}y \end{bmatrix}$ such that |A| = 0, then find the maximum value of x + y + z

8. If $y = \frac{u}{v}$, where u & v are functions of 'x', show that, $v^3 \frac{d^2y}{dx^2} = \begin{vmatrix} u & v & 0 \\ u' & v' & v \\ u'' & v'' & 2v' \end{vmatrix}$.

9. If α, β be the real roots of $ax^2+bx+c=0$ and $s_n=\alpha^n+\beta^n$, then prove that $as_n+bs_{n-1}+cs_{n-2}=0$ for all $n \geq 2$, $n \in N$. Hence or otherwise prove that $\begin{vmatrix} 3 & 1+s_1 & 1+s_2 \\ 1+s_1 & 1+s_2 & 1+s_3 \\ 1+s_2 & 1+s_3 & 1+s_4 \end{vmatrix} \geq 0$ for all real a, b, c.

10. Let a > 0, d > 0. Find the value of determinant $\begin{vmatrix} \frac{1}{a} & \frac{1}{a(a+d)} & \frac{1}{(a+d)(a+2d)} \\ \frac{1}{(a+d)} & \frac{1}{(a+d)(a+2d)} & \frac{1}{(a+2d)(a+3d)} \\ \frac{1}{(a+2d)} & \frac{1}{(a+2d)(a+3d)} & \frac{1}{(a+3d)(a+4d)} \end{vmatrix}$

11. Let $\vec{a}_r = x_r\hat{i} + y_r\hat{j} + z_r\hat{k}$, r = 1, 2, 3 be three mutually perpendicular unit vectors, then find the value of $\begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{vmatrix}$

12. If $\begin{vmatrix} x^k & x^{k+2} & x^{k+3} \\ y^k & y^{k+2} & y^{k+3} \\ z^k & z^{k+2} & z^{k+3} \end{vmatrix} = (x-y)(y-z)(z-x)\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$, then find the value of k.

13. If the determinant $\begin{vmatrix} a+p & l+x & u+f \\ b+q & m+y & v+g \\ c+r & n+z & w+h \end{vmatrix}$ splits into exactly K determinants of order 3, each element of which contains only one term, then find the value of K =

14. If a, b, c are all different and $\begin{vmatrix} a & a^3 & a^4-1 \\ b & b^3 & b^4-1 \\ c & c^3 & c^4-1 \end{vmatrix} = 0$, then find the value of $abc(ab+bc+ca)-(a+b+c)$.

15. If a, b, c are complex numbers and $z = \begin{vmatrix} 0 & -b & -c \\ \bar{b} & 0 & -a \\ \bar{c} & \bar{a} & 0 \end{vmatrix}$ then show that z is purely imaginary.

16. If $f(x)=\log_{10}x$ and $g(x)=e^{i\pi x}$ and $h(x) = \begin{vmatrix} f(x)g(x) & [f(x)]^{g(x)} & 1 \\ f(x^2)g(x^2) & [f(x^2)]^{g(x^2)} & 0 \\ f(x^3)g(x^3) & [f(x^3)]^{g(x^3)} & 1 \end{vmatrix}$, then find the value of h(10).

17. If a, b, c, are real numbers, and $D = \begin{vmatrix} a & 1+2i & 3-5i \\ 1-2i & b & -7-3i \\ 3+5i & -7+3i & c \end{vmatrix}$ then show that D is purely real.

18. If $A = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$ then find $\lim_{n\to\infty} \frac{1}{n}A^n$.

19. Let **P** = $\begin{bmatrix} \cos\frac{\pi}{9} & \sin\frac{\pi}{9} \\ -\sin\frac{\pi}{9} & \cos\frac{\pi}{9} \end{bmatrix}$ and α, β, γ be non-zero real numbers such that $\alpha p^6 + \beta p^3 + \gamma I$ is the zero matrix. Then find value of $(\alpha^2 + \beta^2 + \gamma^2)^{(\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)}$

20. Consider an odd order square symmetric matrix $A = [a_{ij}]_{n \times n}$. It's element in any row are 1, 2, ..., n in some order, then prove that $a_{11}, a_{22}, \ldots\ldots, a_{nn}$ are numbers 1, 2, 3, ..., n in some order.

21. Let **A** = $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$; **B** = $\begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$ and **C** = 3**A** + 7**B**

Prove that

(i) $(\mathbf{A} + \mathbf{B})^{2013} = \mathbf{A}^{2013} + \mathbf{B}^{2013}$

(ii) Prove that $\mathbf{A}^n = 3^{n-1}\mathbf{A}$; $\mathbf{B}^n = 3^{n-1}\mathbf{B}$; $\mathbf{C}^n = 3^{2n-1}\mathbf{A} + \mathbf{7.21^{n-1}}\,\mathbf{B}$.

22. Let **'A'** is (4×4) matrix such that sum of elements in each row is 1. Find out sum of all the elements in $\mathbf{A}^{10}$.

23. Let A = $\begin{bmatrix} x+\lambda & x & x \\ x & x+\lambda & x \\ x & x & x+\lambda \end{bmatrix}$, then prove that A^{-1} exists if $3x + \lambda \neq 0$, $\lambda \neq 0$

24. Prove that if A and B are n × n matrices, then $\det(I_n - AB) = \det(I_n - BA)$.

25. Let A be an n × n matrix such that $A^n = \alpha A$ where α is a real number different from 1 and – 1. Prove that the matrix $A + I_n$ is invertible.

26. Let p and q be real numbers such that $x^2 + px + q \neq 0$ for every real number x. Prove that if n is an odd positive integer, then $X^2 + pX + qI_n \neq 0_n$ for all real matrices X of order n × n.

27. Let **A, B, C** be three 3 × 3 matrices with real entries. If **BA** + **BC** + **AC** = I and det**(A + B)** = 0 then find the value of det**(A + B + C – BAC).**

28. If $|z_1| = |z_2| = 1$, then prove that $\begin{bmatrix} z_1 & -z_2 \\ \bar{z}_2 & \bar{z}_1 \end{bmatrix}^{-1} \begin{bmatrix} \bar{z}_1 & z_2 \\ -\bar{z}_2 & z_1 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix}$

29. If A and B are two square matrices such that $B = -A^{-1}BA$, then show that $(A+B)^2 = A^2 + B^2$

30. If $A = \begin{bmatrix} a & 1 & 0 \\ 1 & b & d \\ 1 & b & c \end{bmatrix}$, $B = \begin{bmatrix} a & 1 & 1 \\ 0 & d & c \\ f & g & h \end{bmatrix}$, $U = \begin{bmatrix} f \\ g \\ h \end{bmatrix}$, $V = \begin{bmatrix} a^2 \\ 0 \\ 0 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

and AX = U has infinitely many solution. Prove that BX = V has no unique solution, also prove that if $afd \neq 0$, then BX = V has no solution.

31. If the system of equations x = cy + bz, y = az + cx and z = bx + ay has a non-zero solution and at least one of a, b, c is a proper fraction, prove that $a^3 + b^3 + c^3 < 3$ and $abc > -1$.

32. If $D = \text{diag}\{d_1, d_2, \ldots\ldots, d_n\}$, then prove that $f(D) = \text{diag}\{f(d_1), f(d_2), \ldots\ldots, f(d_n)\}$, where f(x) is a polynomial with scalar coefficient.

33. Given the matrix **A** = $\begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix}$ and **X** be the solution set of the equation A^x = **A**, where $x \in$ **N** – {1}. Evaluate $\Pi \dfrac{x^3+1}{x^3-1}$; where the continued product extends $\forall\ x \in$ **X.**

Comprehension (Q. NO. 34 to 36)

Any non-zero vector, X, is said to be characteristic vector of a matrix A, if there exist a number λ such that AX = λX. And then λ is said to be a charactristic root of the matrix A corresponding to the characteristic vector X and vice versa.
Also AX = λX $\Rightarrow$ $(A - \lambda I)X = 0$
Since X $\neq$ 0 $\Rightarrow$ $|A - \lambda I| = 0$
Thus every characteristic root λ of a matrix A is a root of its characteristic equation.

34. Prove that the two matrices A and P^{-1} AP have the same characteristic roots and hence show that square matrices AB & BA have same characteristic roots if at least one of them is invertible.

35. If q is a characteristic root of a non singular matrix A, then prove that $\dfrac{|A|}{q}$ is a characteristic root of a adj A.

36. Show that if $\alpha_1, \alpha_2, \ldots\ldots, \alpha_n$ are n characteristic roots of a square matrix A of order n, then the roots of the matrix A^2 be $\alpha_1^2, \alpha_2^2, \ldots\ldots\ldots \alpha_n^2$.

37. IF threre are three square matrices A, B, C of same order satisying the equation $A^3 = A^{-1}$ and let $B = -A^{3^n}$ and $C = A^{3^{(n+4)}}$ then prove that det(B + C) = 0, $n \in N$

38. If A is a non-singular matrix satisfying AB – BA = A, then prove that det.(B + I) = det.(B – I)

39. If rank is a number associated with a matrix which is the highest order of non-singular sub matrix then prove that

(i) Rank of the matrix A = $\begin{bmatrix} 1 & 3 & -2 \\ 4 & -1 & 0 \\ 2 & -7 & 4 \end{bmatrix}$ is 2

(ii) If the matrix A = $\begin{bmatrix} y+a & b & c \\ a & y+b & c \\ a & b & y+c \end{bmatrix}$ has rank 3, then $y \neq -(a+b+c)$ and $y \neq 0$

(iii) If A & B are two square matrices of order 3 such that rank of matrix AB is two, then atleast one of A & B is singular.

Answer Key (ALP)

7. 3 **10.** $\dfrac{4d^4}{a(a+d)^2(a+2d)^3(a+3d)^2(a+4d)}$ **11.** ±1 **12.** -1 **13.** 8

14. 0 **16.** 0 **18.** $\begin{bmatrix} 0 & a \\ 0 & 0 \end{bmatrix}$ **19.** 1 **22.** 4 **27.** 0

33. $\dfrac{3}{2}$