Exercise-1

Marked questions are recommended for Revision.

PART - I : SUBJECTIVE QUESTIONS

Section (A) : Relation between the roots and coefficients ; Quadratic Equation

A-1. For what value of 'a', the equation $(a^2 - a - 2)x^2 + (a^2 - 4)x + (a^2 - 3a + 2) = 0$, will have more than two solutions ? Does there exist a real value of 'x' for which the above equation will be an identity in 'a' ?

A-2. If α and β are the roots of the equation $2x^2 + 3x + 4 = 0$, then find the values of

(i) $\alpha^2 + \beta^2$ (ii) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

A-3. If α and β are the roots of the equation $ax^2 + bx + c = 0$, then find the equation whose roots are given by

(i) $\alpha + \frac{1}{\beta}, \beta + \frac{1}{\alpha}$ (ii) $\alpha^2 + 2, \beta^2 + 2$

A-4. If $\alpha \neq \beta$ but $\alpha^2 = 5\alpha - 3$, $\beta^2 = 5\beta - 3$, then find the equation whose roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$.

A-5. In copying a quadratic equation of the form $x^2 + px + q = 0$, the coefficient of x was wrongly written as – 10 in place of – 11 and the roots were found to be 4 and 6. Find the roots of the correct equation.

A-6. **(i)** Find the value of the expression $2x^3 + 2x^2 - 7x + 72$ when $x = \frac{3 + 5\sqrt{-1}}{2}$.

(ii) Find the value of the expression $2x^3 + 2x^2 - 7x + 72$ when $x = \frac{-1 + \sqrt{15}}{2}$

(iii) Solve the following equation $2^{2x} + 2^{x+2} - 32 = 0$

A-7. Let a, b, c be real numbers with $a \neq 0$ and let α, β be the roots of the equation $ax^2 + bx + c = 0$. Express the roots of $a^3x^2 + abcx + c^3 = 0$ in terms of α, β

A-8. If α, β are roots of $x^2 - px + q = 0$ and $\alpha - 2, \beta + 2$ are roots of $x^2 - px + r = 0$, then prove that $16q + (r + 4 - q)^2 = 4p^2$.

A-9. If one root of the equation $ax^2 + bx + c = 0$ is equal to n^{th} power of the other root, then show that $(ac^n)^{1/(n+1)} + (a^nc)^{1/(n+1)} + b = 0$.

A-10. If the sum of the roots of quadratic equation $(a + 1)x^2 + (2a + 3)x + (3a + 4) = 0$ is –1, then find the product of the roots.

A-11. Find the least prime integral value of '2a' such that the roots α, β of the equation $2x^2 + 6x + a = 0$ satisfy the inequality $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} < 2$.

Section (B) : Relation between roots and coefficients ; Higher Degree Equations

B-1. If α and β be two real roots of the equation $x^3 + px^2 + qx + r = 0$ $(r \neq 0)$ satisfying the relation $\alpha\beta + 1 = 0$, then prove that $r^2 + pr + q + 1 = 0$.

B-2. If α, β, γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, then find the value of $\left(\alpha - \frac{1}{\beta\gamma}\right)\left(\beta - \frac{1}{\gamma\alpha}\right)\left(\gamma - \frac{1}{\alpha\beta}\right)$.

B-3. **(i)** Solve the equation $24x^3 - 14x^2 - 63x + \lambda = 0$, one root being double of another. Hence find the value(s) of λ.

(ii) Solve the equation $18x^3 + 81x^2 + \lambda x + 60 = 0$, one root being half the sum of the other two. Hence find the value of λ.

B-4. If α, β, γ are roots of equation $x^3 - 6x^2 + 10x - 3 = 0$, then find cubic equation with roots $2\alpha + 1$, $2\beta + 1$, $2\gamma + 1$.

B-5. If α, β and γ are roots of $2x^3 + x^2 - 7 = 0$, then find the value of $\sum_{\alpha,\beta,\gamma}\left(\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right)$.

B-6. Find the roots of $4x^3 + 20x^2 - 23x + 6 = 0$ if two of its roots are equal.

Section (C) : Nature of Roots

C-1. If $2 + i\sqrt{3}$ is a root of the equation $x^2 + px + q = 0$ (where $p, q \in R$ and $i^2 = -1$), then find the ordered pair (p, q).

C-2. If the roots of the equation $x^2 - 2cx + ab = 0$ are real and unequal, then prove that the roots of $x^2 - 2(a + b)x + a^2 + b^2 + 2c^2 = 0$ will be imaginary.

C-3. For what values of k the expression $kx^2 + (k + 1)x + 2$ will be a perfect square of a linear polynomial.

C-4. Show that if roots of equation $(a^2 - bc)x^2 + 2(b^2 - ac)x + c^2 - ab = 0$ are equal, then either $b = 0$ or $a^3 + b^3 + c^3 = 3abc$

C-5. If $a, b, c \in R$, then prove that the roots of the equation $\frac{1}{x-a} + \frac{1}{x-b} + \frac{1}{x-c} = 0$ are always real and cannot have roots if $a = b = c$.

C-6. If the roots of the equation $\frac{1}{(x+p)} + \frac{1}{(x+q)} = \frac{1}{r}$ are equal in magnitude but opposite in sign, then show that $p + q = 2r$ and that the product of the roots is equal to $(-1/2)(p^2 + q^2)$.

C-7. **(i)** If $-2 + i\beta$ is a root of $x^3 + 63x + \lambda = 0$ (where $\beta \in R - \{0\}$, $\lambda \in R$ and $i^2 = -1$), then find roots of equation.

(ii) If $\frac{-1}{2} + i\beta$, is a root of $2x^3 + bx^2 + 3x + 1 = 0$ (where $b, \beta \in R - \{0\}$ and $i^2 = -1$), then find the value(s) of b.

C-8. Solve the equation $x^4 + 4x^3 + 5x^2 + 2x - 2 = 0$, one root being $-1 + \sqrt{-1}$.

C-9. Draw graph of $y = 12x^3 - 4x^2 - 3x + 1$. Hence find number of positive zeroes.

Section (D) : Range of quadratic expression and sign of quadratic expression

D-1. Draw the graph of the following expressions :
(i) $y = x^2 + 4x + 3$ (ii) $y = 9x^2 + 6x + 1$ (iii) $y = -2x^2 + x - 1$

D-2. Find the range of following quadratic expressions :
(i) $f(x) = -x^2 + 2x + 3 \quad \forall\, x \in R$
(ii) $f(x) = x^2 - 2x + 3 \quad \forall\, x \in [0, 3]$
(iii) $f(x) = x^2 - 4x + 6 \quad \forall\, x \in (0, 1]$

D-3. If x be real, then find the range of the following rational expressions :

(i) $y = \frac{x^2 + x + 1}{x^2 + 1}$ **(ii)** $y = \frac{x^2 - 2x + 9}{x^2 - 2x - 9}$

D-4. Find the range of values of k, such that f(x) = $\frac{kx^2+2(k+1)x+(9k+4)}{x^2-8x+17}$ is always negative.

D-5. $x^2 + (a - b)x + (1 - a - b) = 0$, a, b $\in$ R. Find the condition on ' a ' for which
(i) Both roots of the equation are real and unequal $\forall$ b $\in$ R .
(ii) Roots are imaginary $\forall$ b $\in$ R

Section (E) : Location of Roots

E-1. If both roots of the equation $x^2 - 6ax + 2 - 2a + 9a^2 = 0$ exceed 3, then show that a > 11/9.

E-2. Find all the values of 'K' for which one root of the equation $x^2 - (K+1)x + K^2 + K - 8 = 0$, exceeds 2 & the other root is smaller than 2.

E-3. Find all the real values of 'a', so that the roots of the equation
$(a^2 - a + 2)x^2 + 2(a - 3)x + 9(a^4 - 16) = 0$ are of opposite sign.

E-4. Find all the values of 'a', so that exactly one root of the equation $x^2 - 2ax + a^2 - 1 = 0$, lies between the numbers 2 and 4, and no root of the equation is either equal to 2 or equal to 4.

E-5. If α & β are the two distinct roots of $x^2 + 2(K - 3)x + 9 = 0$, then find the values of K such that $\alpha, \beta \in (-6, 1)$.

Section (F) : Common Roots & Graphs of Polynomials

F-1. If one of the roots of the equation $ax^2 + bx + c = 0$ be reciprocal of one of the roots of $a_1x^2 + b_1x + c_1 = 0$, then prove that $(a a_1 - c c_1)^2 = (b c_1 - a b_1)(b_1 c - a_1 b)$.

F-2. Find the value of 'a' so that $x^2 - 11x + a = 0$ and $x^2 - 14x + 2a = 0$ have a common root.

F-3. If $ax^2 + bx + c = 0$ and $bx^2 + cx + a = 0$ have a common root and a, b, c are non-zero real numbers, then find the value of $\frac{a^3+b^3+c^3}{abc}$.

F-4. If $x^2 + px + q = 0$ and $x^2 + qx + p = 0$, $(p \neq q)$ have a common root, show that $1 + p + q = 0$; show that their other roots are the roots of the equation $x^2 + x + pq = 0$.

F-5. Draw the graphs of following :
(i) $y = 2x^3 + 9x^2 - 24x + 15$ (ii) $y = -3x^4 + 4x^3 + 12x^2 - 2$

F-6. Find values of 'k' if equation $x^3 - 3x^2 + 2 = k$ has
(i) 3 real roots (ii) 1 real root

PART - II : ONLY ONE OPTION CORRECT TYPE

Section (A) : Relation between the roots and coefficients quadratic equation

A-1. The roots of the equation $(b - c)x^2 + (c - a)x + (a - b) = 0$ are
(A) $\frac{c-a}{b-c}$, 1 (B) $\frac{a-b}{b-c}$, 1 (C) $\frac{b-c}{a-b}$, 1 (D) $\frac{c-a}{a-b}$, 1

A-2. If α, β are the roots of quadratic equation $x^2 + px + q = 0$ and γ, δ are the roots of $x^2 + px - r = 0$, then $(\alpha - \gamma) . (\alpha - \delta)$ is equal to :
(A) q + r (B) q − r (C) − (q + r) (D) − (p + q + r)

A-3. Two real numbers α & β are such that $\alpha + \beta = 3$, $\alpha - \beta = 4$, then α & β are the roots of the quadratic equation:
(A) $4x^2 - 12x - 7 = 0$ (B) $4x^2 - 12x + 7 = 0$ (C) $4x^2 - 12x + 25 = 0$ (D) none of these

A-4. For the equation $3x^2 + px + 3 = 0$, $p > 0$ if one of the roots is square of the other, then p is equal to:
(A) 1/3 (B) 1 (C) 3 (D) 2/3

A-5. Consider the following statements :
S_1 : If the roots of $x^2 - bx + c = 0$ are two consecutive integers, then value of $b^2 - 4c$ is equal to 1.
S_2 : If α, β are roots of $x^2 - x + 3 = 0$ then value of $\alpha^4 + \beta^4$ is equal 7.
S_3 : If α, β, γ are the roots of $x^3 - 7x^2 + 16x - 12 = 0$ then value of $\alpha^2 + \beta^2 + \gamma^2$ is equal to 17.
State, in order, whether S_1, S_2, S_3 are true or false
(A) TTT (B) FTF (C) TFT (D) FTT

Section (B) : Relation between roots and coefficients ; Higher Degree Equations

B-1. If two roots of the equation $x^3 - px^2 + qx - r = 0$, $(r \neq 0)$ are equal in magnitude but opposite in sign, then:
(A) pr = q (B) qr = p (C) pq = r (D) None of these

B-2. If α, β & γ are the roots of the equation $x^3 - x - 1 = 0$ then, $\frac{1+\alpha}{1-\alpha} + \frac{1+\beta}{1-\beta} + \frac{1+\gamma}{1-\gamma}$ has the value equal to:
(A) zero (B) -1 (C) -7 (D) 1

B-3. Let α, β, γ be the roots of $(x - a)(x - b)(x - c) = d$, $d \neq 0$, then the roots of the equation $(x - \alpha)(x - \beta)(x - \gamma) + d = 0$ are :
(A) a + 1, b + 1, c + 1 (B) a, b, c (C) a – 1, b – 1, c – 1 (D) $\frac{a}{b}, \frac{b}{c}, \frac{c}{a}$

B-4_. If α, β, γ are the roots of the equation $x^3 + ax + b = 0$ then value of $\frac{\alpha^3 + \beta^3 + \gamma^3}{\alpha^2 + \beta^2 + \gamma^2}$ is equal to :
(A) $\frac{3b}{2a}$ (B) $\frac{-3b}{2a}$ (C) 3b (D) 2b

B-5_. If two of the roots of equation $x^4 - 2x^3 + ax^2 + 8x + b = 0$ are equal in magnitude but opposite in sign, then value of 4a + b is equal to :
(A) 16 (B) 8 (C) –16 (D) –8

Section (C) : Nature of Roots

C-1. If one roots of equation $x^2 - \sqrt{3}\,x + \lambda = 0$, $\lambda \in R$ is $\sqrt{3} + 2$ then other root is
(A) $\sqrt{3} - 2$ (B) -2 (C) $2 - \sqrt{3}$ (D) 2

C-2. If roots of equation $2x^2 + bx + c = 0$; $b, c \in R$, are real & distinct then the roots of equation $2cx^2 + (b - 4c)x + 2c - b + 1 = 0$ are
(A) imaginary (B) equal (C) real and distinct (D) can't say

C-3. Let one root of the equation $x^2 + \square x + m = 0$ is square of other root. If $m \in R$ then
(A) $\ell \in \left(-\infty, \frac{1}{4}\right] \cup \{1\}$ (B) $\square \in (-\infty, 0]$ (C) $\ell \in \left(-\infty, \frac{1}{9}\right]$ (D) $\ell \in \left(\frac{1}{4}, 1\right]$

C-4. If a, b, c are integers and $b^2 = 4(ac + 5d^2)$, $d \in N$, then roots of the quadratic equation $ax^2 + bx + c = 0$ are
(A) Irrational (B) Rational & different (C) Complex conjugate (D) Rational & equal

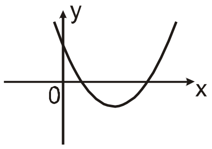
C-5. Let a, b and c be real numbers such that $4a + 2b + c = 0$ and $ab > 0$. Then the equation $ax^2 + bx + c = 0$ has
(A) real roots (B) imaginary roots (C) exactly one root (D) none of these

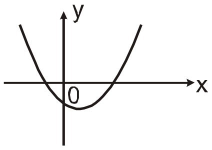
C-6. Consider the equation $x^2 + 2x - n = 0$, where $n \in N$ and $n \in [5, 100]$. Total number of different values of 'n' so that the given equation has integral roots, is
(A) 4 (B) 6 (C) 8 (D) 3

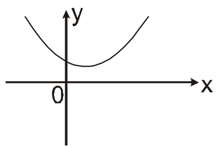
Section (D) : Range of quadratic expression and sign of quadratic expression

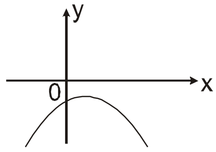
D-1. If α & β ($\alpha < \beta$) are the roots of the equation $x^2 + bx + c = 0$, where $c < 0 < b$, then
(A) $0 < \alpha < \beta$ (B) $\alpha < 0 < \beta^2 < \alpha^2$
(C) $\alpha < \beta < 0$ (D) $\alpha < 0 < \alpha^2 < \beta^2$

D-2. Which of the following graph represents the expression $f(x) = a x^2 + b x + c$ $(a \neq 0)$ when $a > 0$, $b < 0$ & $c < 0$?

(A) 

(B) 

(C) 

(D) 

D-3. The expression $y = ax^2 + bx + c$ has always the same sign as of 'a' if :
(A) $4ac < b^2$ (B) $4ac > b^2$ (C) $4ac = b^2$ (D) $ac < b^2$

D-4. The entire graph of the expression $y = x^2 + kx - x + 9$ is strictly above the x-axis if and only if
(A) $k < 7$ (B) $-5 < k < 7$ (C) $k > -5$ (D) none of these

D-5. If $a, b \in R$, $a \neq 0$ and the quadratic equation $ax^2 - bx + 1 = 0$ has imaginary roots then $a + b + 1$ is:
(A) positive (B) negative (C) zero (D) depends on the sign of b

D-6. If a and b are the non-zero distinct roots of $x^2 + ax + b = 0$, then the least value of $x^2 + ax + b$ is
(A) $\frac{3}{2}$ (B) $\frac{9}{4}$ (C) $-\frac{9}{4}$ (D) 1

D-7. If $y = -2x^2 - 6x + 9$, then
(A) maximum value of y is –11 and it occurs at x = 2
(B) minimum value of y is –11 and it occurs at x = 2
(C) maximum value of y is 13.5 and it occurs at x = –1.5
(D) minimum value of y is 13.5 and it occurs at x = –1.5

D-8. If $f(x) = x^2 + 2bx + 2c^2$ and $g(x) = -x^2 - 2cx + b^2$ are such that min f(x) > max g(x), then the relation between b and c, is
(A) no relation (B) $0 < c < b/2$ (C) $c^2 < 2b$ (D) $c^2 > 2b^2$

Section (E) : Location of Roots

E-1. If $b > a$, then the equation $(x - a)(x - b) - 1 = 0$, has:
(A) both roots in $[a, b]$ (B) both roots in $(-\infty, a)$
(C) both roots in $[b, \infty)$ (D) one root in $(-\infty, a)$ & other in (b, ∞)

E-2. If α, β are the roots of the quadratic equation $x^2 - 2p(x - 4) - 15 = 0$, then the set of values of 'p' for which one root is less than 1 & the other root is greater than 2 is:
(A) $(7/3, \infty)$ (B) $(-\infty, 7/3)$ (C) $x \in R$ (D) none of these

E-3. If α, β be the roots of $4x^2 - 16x + \lambda = 0$, where $\lambda \in R$, such that $1 < \alpha < 2$ and $2 < \beta < 3$, then the number of integral solutions of λ is
(A) 5 (B) 6 (C) 2 (D) 3

E-4. Set of real values of k if the equation $x^2 - (k-1)x + k^2 = 0$ has atleast one root in (1,2) is
(A) (2, 4) (B) [–1, 1/3] (C) {3} (D) ϕ

Section (F) : Common Roots & Graphs of Polynomials

F-1. If the equations k $(6x^2 + 3) + rx + 2x^2 - 1 = 0$ and $6k\,(2x^2 + 1) + px + 4x^2 - 2 = 0$ have both roots common, then the value of $(2r - p)$ is

(A) 0 (B) 1/2 (C) 1 (D) none of these

F-2. If $3x^2 - 17x + 10 = 0$ and $x^2 - 5x + \lambda = 0$ has a common root, then sum of all possible real values of λ is

(A) 0 (B) $-\dfrac{29}{9}$ (C) $\dfrac{26}{9}$ (D) $\dfrac{29}{3}$

F-3. If a, b, p, q are non–zero real numbers, then two equations $2a^2x^2 - 2\,abx + b^2 = 0$ and $p^2x^2 + 2\,pqx + q^2 = 0$ have :

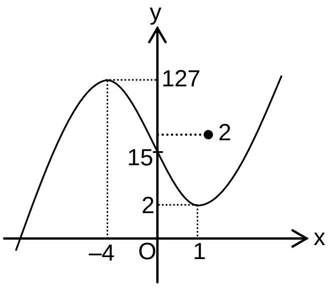
(A) no common root

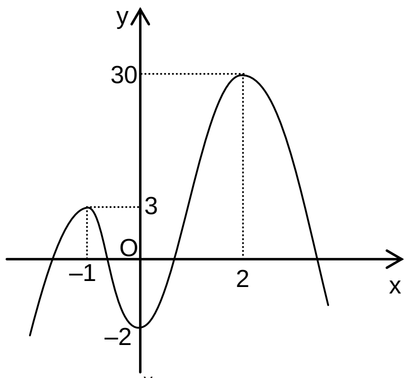
(B) one common root if $2a^2 + b^2 = p^2 + q^2$

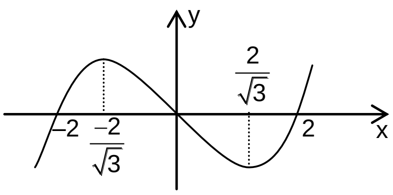
(C) two common roots if 3 pq = 2 ab

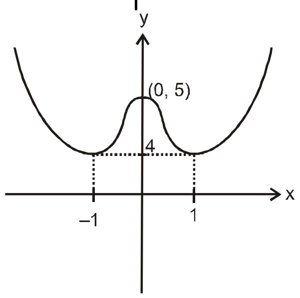
(D) two common roots if 3 qb = 2 ap

F-4. The graphs of $y = \dfrac{x^3 - 4x}{4}$ is

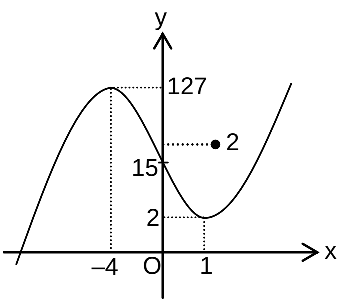
(A) 

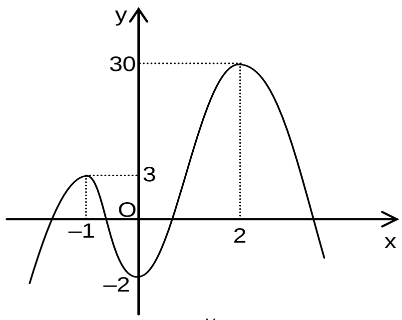
(B) 

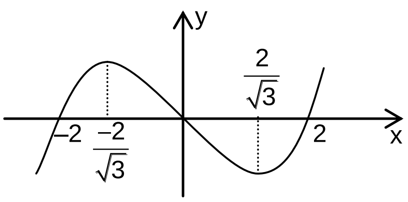
(C) 

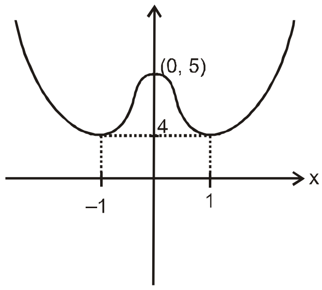
(D) 

F-5. The graphs of $y = x^4 - 2x^2 + 5$ is

(A) 

(B) 

(C) 

(D) 

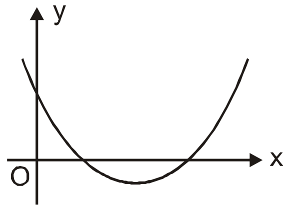
PART - III : MATCH THE COLUMN

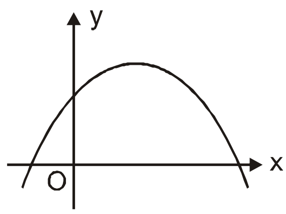
1.

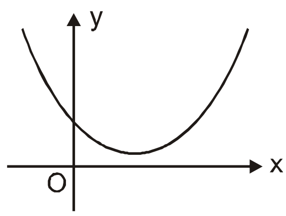
Column – I		**Column – II**	
(A)	If α, $\alpha + 4$ are two roots of $x^2 - 8x + k = 0$, then possible value of k is	(p)	4
(B)	If α, β are roots of $x^2 + 2x - 4 = 0$ and $\frac{1}{\alpha}, \frac{1}{\beta}$ are roots of $x^2 + qx + r = 0$ then value of $\frac{-3}{q+r}$ is	(q)	0
(C)	If α, β are roots of $ax^2 + c = 0$, $ac \neq 0$, then $\alpha^3 + \beta^3$ is equal to	(r)	12
(D)	If roots of $x^2 - kx + 36 = 0$ are Integers then number of values of k =	(s)	10

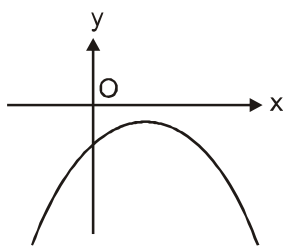
2. If graph of the expression $f(x) = ax^2 + bx + c$ $(a \neq 0)$ are given in column-II, then Match the items in column-I with in column-II (where $D = b^2 - 4ac$)

Column-I | **Column-II**

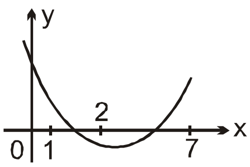
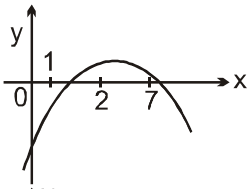
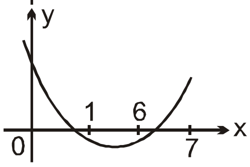
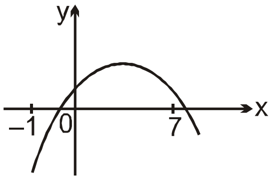
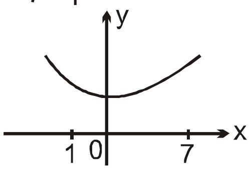
(A) $\frac{abc}{D} > 0$ (p) 

(B) $\frac{abc}{D} < 0$ (q) 

(C) $abc > 0$ (r) 

(D) $abc < 0$ (s) 

3. Let $y = Q(x) = ax^2 + bx + c$ be a quadratic expression. Match the inequalities in **Column-I** with possible graphs in **Column-II.**

Column-I		**Column-II**	
(A)	$Q(x) > 0, \forall x \in (2, 7)$	(p)	
(B)	$Q(x) > 0, \forall x \in (-\infty, 1)$	(q)	
(C)	$Q(x) < 0, \forall x \in (1, 6)$	(r)	
(D)	$Q(x) < 0, \forall x \in (-\infty, -1)$	(s)	
		(t)	

Exercise-2

Marked questions are recommended for Revision.

PART - I : ONLY ONE OPTION CORRECT TYPE

1. Let $a > 0$, $b > 0$ & $c > 0$. Then both the roots of the equation $ax^2 + bx + c = 0$

(A) are real & negative
(B) have negative real parts
(C) are rational numbers
(D) have positive real parts

2. If the roots of the equation $x^2 + 2ax + b = 0$ are real and distinct and they differ by atmost $2m$, then b lies in the interval

(A) $(a^2 - m^2, a^2)$ (B) $[a^2 - m^2, a^2)$ (C) $(a^2, a^2 + m^2)$ (D) none of these

3. The set of possible values of λ for which $x^2 - (\lambda^2 - 5\lambda + 5)x + (2\lambda^2 - 3\lambda - 4) = 0$ has roots, whose sum and product are both less than 1, is

(A) $\left(-1, \frac{5}{2}\right)$ (B) $(1, 4)$ (C) $\left[1, \frac{5}{2}\right]$ (D) $\left(1, \frac{5}{2}\right)$

4. If $p, q, r, s \in R$, then equaton $(x^2 + px + 3q)(-x^2 + rx + q)(-x^2 + sx - 2q) = 0$ has

(A) 6 real roots
(B) atleast two real roots
(C) 2 real and 4 imaginary roots
(D) 4 real and 2 imaginary roots

5. If coefficients of biquadratic equation are all distinct and belong to the set {–9, – 5, 3, 4, 7}, then equation has
(A) atleast two real roots
(B) four real roots, two are conjugate surds and other two are also conjugate surds
(C) four imaginary roots
(D) None of these

6. Find the set of all real values of λ such that the root of the equation $x^2 + 2(a + b + c)x + 3\lambda\,(ab + bc + ca) = 0$ are always real for any choice of a, b, c (where a, b, c represents sides of scalene triangle).
(A) $\left(-\infty, \frac{4}{3}\right)$ (B) $\left(\frac{4}{3}, \infty\right)$ (C) $\left(\frac{1}{3}, \frac{5}{3}\right)$ (D) $\left(\frac{4}{3}, \frac{5}{3}\right)$

7. Let $p, q, r, s \in R$, $x^2 + px + q = 0$, $x^2 + rx + s = 0$ such that $2(q + s) = pr$ then
(A) atleast one of the equation have real roots.
(B) either both equations have imaginary roots or both equations have real roots.
(C) one of equations have real roots and other equation have imaginary roots
(D) atleast one of the equations have imaginary roots.

8. The equation, $\pi^x = -2x^2 + 6x - 9$ has:
(A) no solution (B) one solution (C) two solutions (D) infinite solutions

9. If $(\lambda^2 + \lambda - 2)x^2 + (\lambda + 2)x < 1$ for all $x \in R$, then λ belongs to the interval
(A) (–2, 1) (B) $\left[-2, \frac{2}{5}\right)$ (C) $\left(\frac{2}{5}, 1\right)$ (D) none of these

10. Let conditions C_1 and C_2 be defined as follows : $C_1 : b^2 - 4ac \geq 0$, C_2 : a, –b, c are of same sign. The roots of $ax^2 + bx + c = 0$ are real and positive, if
(A) both C_1 and C_2 are satisfied (B) only C_2 is satisfied
(C) only C_1 is satisfied (D) none of these

11. If 'x' is real, then $\frac{x^2 - x + c}{x^2 + x + 2c}$ can take all real values if :
(A) $c \in [0, 6]$ (B) $c \in [-6, 0]$ (C) $c \in (-\infty, -6) \cup (0, \infty)$ (D) $c \in (-6, 0)$

12. If both roots of the quadratic equation $(2 - x)(x + 1) = p$ are distinct & positive, then p must lie in the interval:
(A) $(2, \infty)$ (B) (2, 9/4) (C) $(-\infty, -2)$ (D) $(-\infty, \infty)$

13. If two roots of the equation $(a - 1)(x^2 + x + 1)^2 - (a + 1)(x^4 + x^2 + 1) = 0$ are real and distinct, then 'a' lies in the interval
(A) (–2, 2) (B) $(-\infty, -2) \cup (2, \infty)$ (C) $(2, \infty)$ (D) $(-\infty, -2)$

14. The equations $x^3 + 5x^2 + px + q = 0$ and $x^3 + 7x^2 + px + r = 0$ have two roots in common. If the third root of each equation is represented by x_1 and x_2 respectively, then the ordered pair (x_1, x_2) is:
(A) (–5, –7) (B) (1, –1) (C) (–1, 1) (D) (5, 7)

15. If a, b, c are real and $a^2 + b^2 + c^2 = 1$, then $ab + bc + ca$ lies in the interval:
(A) $\left[\frac{1}{2}, 2\right]$ (B) [0, 2] (C) $\left[-\frac{1}{2}, 1\right]$ (D) $\left[-1, \frac{1}{2}\right]$

PART - II : SINGLE AND DOUBLE VALUE INTEGER TYPE

1. Find number of integer roots of equation x (x + 1) (x + 2) (x + 3) = 120.

2. Find product of all real values of x satisfying $(5+2\sqrt{6})^{x^2-3}+(5-2\sqrt{6})^{x^2-3} = 10$

3. If a, b are the roots of $x^2 + px + 1 = 0$ and c, d are the roots of $x^2 + qx + 1 = 0$. Then find the value of $(a - c)(b - c)(a + d)(b + d)/(q^2 - p^2)$.

4. α, β are roots of the equation $\lambda(x^2 - x) + x + 5 = 0$. If λ_1 and λ_2 are the two values of λ for which the roots α, β are connected by the relation $\frac{\alpha}{\beta}+\frac{\beta}{\alpha} = 4$, then the value of $\left(\frac{\frac{\lambda_1}{\lambda_2}+\frac{\lambda_2}{\lambda_1}}{14}\right)$ is

5. Let one root of equation $(\square - m)x^2 + \square x + 1 = 0$ be double of the other. If $\square$ be real and $8m \le k$ then find the least value of k.

6. Let α, β be the roots of the equation $x^2 + ax + b = 0$ and γ, δ be the roots of $x^2 - ax + b - 2 = 0$. If $\alpha\beta\gamma\delta = 24$ and $\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}+\frac{1}{\delta}=\frac{5}{6}$, then find the value of a.

7. If $a > b > 0$ and $a^3 + b^3 + 27ab = 729$ then the quadratic equation $ax^2 + bx - 9 = 0$ has roots α, β $(\alpha < \beta)$. Find the value of $4\beta - a\alpha$.

8. Let α and β be roots of $x^2 - 6(t^2 - 2t + 2)x - 2 = 0$ with $\alpha > \beta$. If $a_n = \alpha^n - \beta^n$ for $n \ge 1$, then find the minimum value of $\frac{a_{100} - 2a_{98}}{a_{99}}$ (where $t \in R$)

9. If $\alpha, \beta, \gamma, \delta$ are the roots of the equation $x^4 - Kx^3 + Kx^2 + Lx + M = 0$, where K, L & M are real numbers, then the minimum value of $\alpha^2 + \beta^2 + \gamma^2 + \delta^2$ is – n. Find the value of n.

10. Consider $y = \frac{2x}{1+x^2}$, where x is real , then the range of expression $y^2 + y - 2$ is [a, b]. Find the value of (b – 4a).

11. If the roots of the equation $x^3 + Px^2 + Qx - 19 = 0$ are each one more than the roots of the equaton $x^3 - Ax^2 + Bx - C = 0$, where A, B, C, P & Q are constants, then the value of A + B + C is equal to :

12. If one root of the equation $t^2 - (12x)t - (f(x) + 64x) = 0$ is twice of other, then find the maximum value of the function f(x), where $x \in R$.

13. The values of k, for which the equation $x^2 + 2(k - 1)x + k + 5 = 0$ possess atleast one positive root, are $(-\infty, -b]$. Find value of b.

14. Find the least value of 7a for which atleast one of the roots of the equation $x^2 - (a - 3)x + a = 0$ is greater than 2.

15. If the quadratic equations $3x^2 + ax + 1 = 0$ & $2x^2 + bx + 1 = 0$ have a common root, then the value of the expression $5ab - 2a^2 - 3b^2$ is

16. The equations $x^2 - ax + b = 0$, $x^3 - px^2 + qx = 0$, where a, b, p, q $\in R - \{0\}$ have one common root & the second equation has two equal roots. Find value of $\frac{ap}{q+b}$.

17. If $x - y$ and $y - 2x$ are two factors of the expression $x^3 - 3x^2y + \lambda xy^2 + \mu y^3$, then $\frac{16\lambda}{11} + 4\mu$ is

PART - III : ONE OR MORE THAN ONE OPTIONS CORRECT TYPE

1. Possible values of 'p' for which the equation $(p^2 - 3p + 2)x^2 - (p^2 - 5p + 4)x + p - p^2 = 0$ does not possess more than two roots is/are

(A) 0 (B) 1 (C) 2 (D) 4

2. If a, b are non-zero real numbers and α, β the roots of $x^2 + ax + b = 0$, then

(A) α^2, β^2 are the roots of $x^2 - (2b - a^2)x + a^2 = 0$

(B) $\frac{1}{\alpha}, \frac{1}{\beta}$ are the roots of $bx^2 + ax + 1 = 0$

(C) $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$ are the roots of $bx^2 + (2b - a^2)x + b = 0$

(D) $(\alpha - 1), (\beta - 1)$ are the roots of the equation $x^2 + x(a + 2) + 1 + a + b = 0$

3. If α, β are the roots of $ax^2 + bx + c = 0$ $(a \neq 0)$ and $\alpha + \delta$, $\beta + \delta$ are the roots of, $Ax^2 + Bx + C = 0$ $(A \neq 0)$ for some constant δ, then

(A) $\delta = \frac{1}{2}\left(\frac{B}{A} - \frac{b}{a}\right)$ (B) $\delta = \frac{1}{2}\left(\frac{b}{a} - \frac{B}{A}\right)$

(C) $\frac{b^2 - 4ac}{a^2} = \frac{B^2 - 4AC}{A^2}$ (D) $\frac{b^2 + 4ac}{a^2} = \frac{B^2 + 4AC}{A^2}$

4. If one root of the equation $4x^2 + 2x - 1 = 0$ is 'α', then

(A) α can be equal to $\frac{-1+\sqrt{5}}{4}$ (B) α can be equal to $\frac{1+\sqrt{5}}{4}$

(C) other root is $4\alpha^3 - 3\alpha$. (D) other root is $4\alpha^3 + 3\alpha$

5. If α, β are roots of $x^2 + 3x + 1 = 0$, then

(A) $(7 - \alpha)(7 - \beta) = 0$ (B) $(2 - \alpha)(2 - \beta) = 11$

(C) $\frac{\alpha^2}{3\alpha+1} + \frac{\beta^2}{3\beta+1} = -2$ (D) $\left(\frac{\alpha}{1+\beta}\right)^2 + \left(\frac{\beta}{\alpha+1}\right)^2 = 18$

6. If both roots of $x^2 - 32x + c = 0$ are prime numbers then possible values of c are

(A) 60 (B) 87 (C) 247 (D) 231

7. Let $f(x) = x^2 - a(x + 1) - b = 0$, a, b $\in R - \{0\}$, $a + b \neq 0$. If α and β are roots of equation $f(x) = 0$, then the value of $\frac{1}{\alpha^2 - a\alpha} + \frac{1}{\beta^2 - a\beta} - \frac{2}{a+b}$ is equal to

(A) 0 (B) f(a) + a + b (C) f(b) + a + b (D) $f\left(\frac{a}{2}\right) + \frac{a^2}{4} + a + b$

8. If f(x) is a polynomial of degree three with leading coefficient 1 such that f(1) = 1, f(2) = 4, f(3) = 9, then

(A) f(4) = 22 (B) $f\left(\frac{6}{5}\right) = \left(\frac{6}{5}\right)^3$

(C) $f(x) = x^3$ holds for exactly two values of x. (D) f(x) = 0 has a root in interval (0, 1).

9. Let $P(x) = x^{32} - x^{25} + x^{18} - x^{11} + x^4 - x^3 + 1$. Which of the following are **CORRECT** ?
(A) Number of real roots of P(x) = 0 are zero.
(B) Number of imaginary roots of P(x) = 0 are 32.
(C) Number of negative roots of P(x) = 0 are zero.
(D) Number of imaginary roots of P(x) + P(–x) = 0 are 32.

10. If α, β are the real and distinct roots of $x^2 + px + q = 0$ and α^4, β^4 are the roots of $x^2 - rx + s = 0$, then the equation $x^2 - 4qx + 2q^2 - r = 0$ has always (given $\alpha \neq -\beta$)
(A) two real roots (B) two negative roots
(C) two positive roots (D) one positive root and one negative root

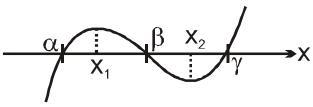
11. $x^2 + x + 1$ is a factor of $ax^3 + bx^2 + cx + d = 0$, then the real root of above equation is (a, b, c, d $\in$ R)
(A) – d/a (B) d/a (C) (b – a)/a (D) (a – b)/a

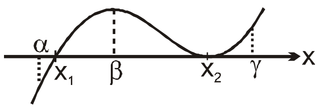
12. If $-5 + i\beta$, $-5 + i\gamma$, (where $\beta^2 \neq \gamma^2$; $\beta, \gamma \in R$ and $i^2 = -1$) are roots of $x^3 + 15x^2 + cx + 860 = 0$, $c \in R$, then
(A) c = 222
(B) all the three roots are imaginary
(C) two roots are imaginary but not complex conjugate of each other.
(D) $-5 + 7i\sqrt{3}$, $-5 - 7i\sqrt{3}$ are imaginary roots.

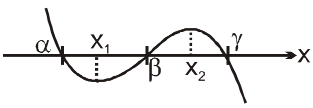
13. Let $f(x) = ax^2 + bx + c > 0$, $\forall x \in R$ or $f(x) < 0$, $\forall x \in R$. Which of the following is/are **CORRECT** ?
(A) If a + b + c > 0 then f(x) > 0, $\forall x \in R$ (B) If a + c < b then f(x) < 0, $\forall x \in R$
(C) If a + 4c > 2b then f(x) < 0, $\forall x \in R$ (D) ac > 0.

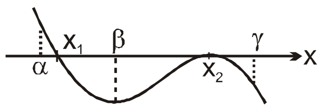
14. Let $x_1 < \alpha < \beta < \gamma < x_4$, $x_1 < x_2 < x_3$. If f(x) is a cubic polynomial with real coefficients such that $(f(\alpha))^2 + (f(\beta))^2 + (f(\gamma))^2 = 0$, $f(x_1) f(x_2) < 0$, $f(x_2) f(x_3) < 0$ and $f(x_1) f(x_3) > 0$ then which of the following are **CORRECT** ?
(A) $\alpha \in (x_1, x_2)$, $\beta \in (x_2, x_3)$ and $\gamma \in (x_3, x_4)$ (B) $\alpha \in (x_1, x_3)$, $\beta, \gamma \in (x_3, x_4)$
(C) $\alpha, \beta \in (x_1, x_2)$ and $\gamma \in (x_4, \infty)$ (D) $\alpha \in (x_1, x_3)$, $\beta \in (x_2, x_3)$ and $\gamma \in (x_2, x_4)$

15. If f(x) is cubic polynomial with real coefficients, $\alpha < \beta < \gamma$ and $x_1 < x_2$ be such that $f(\alpha) = f(\beta) = f(\gamma) = f'(x_1) = f'(x_2) = 0$ then possible graph of y = f(x) is (assuming y-axis vertical)

(A) 

(B) 

(C) 

(D) 

16. Let $f(x) = \frac{3}{x-2} + \frac{4}{x-3} + \frac{5}{x-4}$, then f(x) = 0 has
(A) exactly one real root in (2, 3) (B) exactly one real root in (3, 4)
(C) 3 different roots (D) atleast one negative root

17. If the quadratic equations $ax^2 + bx + c = 0$ ($a, b, c \in R$, $a \neq 0$) and $x^2 + 4x + 5 = 0$ have a common root, then a, b, c must satisfy the relations:

(A) $a > b > c$
(B) $a < b < c$
(C) $a = k$; $b = 4k$; $c = 5k$ ($k \in R$, $k \neq 0$)
(D) $b^2 - 4ac$ is negative.

18. If the quadratic equations $x^2 + abx + c = 0$ and $x^2 + acx + b = 0$ have a common root, then the equation containing their other roots is/are :

(A) $x^2 + a(b + c)x - a^2bc = 0$
(B) $x^2 - a(b + c)x + a^2bc = 0$
(C) $a(b + c)x^2 - (b + c)x + abc = 0$
(D) $a(b + c)x^2 + (b + c)x - abc = 0$

19. Consider the following statements.

S_1 : The equation $2x^2 + 3x + 1 = 0$ has irrational roots.

S_2 : If $a < b < c < d$, then the roots of the equation $(x - a)(x - c) + 2(x - b)(x - d) = 0$ are real and distinct.

S_3 : If $x^2 + 3x + 5 = 0$ and $ax^2 + bx + c = 0$ have a common root and $a, b, c \in N$, then the minimum value of $(a + b + c)$ is 10.

S_4 : The value of the biquadratic expression $x^4 - 8x^3 + 18x^2 - 8x + 2$, when $x = 2 + \sqrt{3}$, is 1

Which of the following are **CORRECT** ?

(A) S_2 and S_4 are true.
(B) S_1 and S_3 are false.
(C) S_1 and S_2 are true.
(D) S_3 and S_4 are false.

20. If the equations $x^2 + ax + 12 = 0$, $x^2 + bx + 15 = 0$ & $x^2 + (a + b)x + 36 = 0$ have a common positive root, then which of the following are true ?

(A) $ab = 56$
(B) common positive root is 3
(C) sum of uncommon roots is 21.
(D) $a + b = 15$.

21. If $x^2 + \lambda x + 1 = 0$, $\lambda \in (-2, 2)$ and $4x^3 + 3x + 2c = 0$ have common root then $c + \lambda$ can be

(A) $\frac{1}{2}$
(B) $-\frac{1}{2}$
(C) 0
(D) $\frac{3}{2}$

PART - IV : COMPREHENSION

Comprehension # 1 (Q. No. 1 & 2)

If $x, y \in R$ then some problems can be solved by direct observing extreme cases

e.g. (i) $(x - 3)^2 + (y - 2)^2 = 0$ is possible only for $x = 3$ and $y = 2$

(ii) if $x \geq 3$, $y \geq 2$ and $xy \leq 6$ then $x = 3$ & $y = 2$

1. The least value of expression $x^2 + 2xy + 2y^2 + 4y + 7$ is :

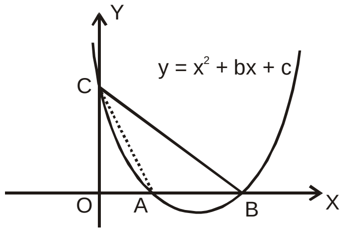
(A) 1
(B) 2
(C) 3
(D) 4

2. Let $P(x) = 4x^2 + 6x + 4$ and $Q(y) = 4y^2 - 12y + 25$. If x, y satisfy equation $P(x).Q(y) = 28$, then the value of $11y - 26x$ is -

(A) 6
(B) 36
(C) 8
(D) 42

Comprehension # 2 (Q. No. 3 & 4)

In the given figure ΔOBC is an isosceles right triangle in which AC is a median, then answer the following questions :



3. Roots of y = 0 are
(A) {2, 1} (B) {4, 2} (C) {1, 1/2} (D) {8, 4}

4. The equation whose roots are $(\alpha + \beta)$ & $(\alpha - \beta)$, where α, β $(\alpha > \beta)$ are roots obtained in previous question, is
(A) $x^2 - 4x + 3 = 0$ (B) $x^2 - 8x + 12 = 0$ (C) $4x^2 - 8x + 3 = 0$ (D) $x^2 - 16x + 48 = 0$

Comprehension # 3 (Q. No. 5 to 7)

Consider the equation $x^4 - \lambda x^2 + 9 = 0$. This can be solved by substituting $x^2 = t$ such equations are called as pseudo quadratic equations.

5. If the equation has four real and distinct roots, then λ lies in the interval
(A) $(-\infty, -6) \cup (6, \infty)$ (B) $(0, \infty)$ (C) $(6, \infty)$ (D) $(-\infty, -6)$

6. If the equation has no real root, then λ lies in the interval
(A) $(-\infty, 0)$ (B) $(-\infty, 6)$ (C) $(6, \infty)$ (D) $(0, \infty)$

7. If the equation has only two real roots, then set of values of λ is
(A) $(-\infty, -6)$ (B) $(-6, 6)$ (C) $\{6\}$ (D) ϕ

Comprehension # 4

To solve equation of type,
$ax^{2m} + bx^{2m-1} + cx^{2m-2} + \dots\dots\dots + kx^m + \dots\dots\dots + cx^2 + bx + a = 0$, $(a \neq 0) \rightarrow (I)$
divide by x^m and rearrange terms to obtain

$$a\left(x^m + \frac{1}{x^m}\right) + b\left(x^{m-1} + \frac{1}{x^{m-1}}\right) + c\left(x^{m-2} + \frac{1}{x^{m-2}}\right) + \dots\dots\dots + k = 0$$

Substitutions like

$t = x + \frac{1}{x}$ or $t = x - \frac{1}{x}$ helps transforming equation into a reduced degree equation.

8. Roots of equation $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$ are
(A) $2 \pm \sqrt{3}, 3 \pm \sqrt{2}$ (B) $2 \pm \sqrt{3}, 3 \pm 2\sqrt{2}$ (C) $3 \pm \sqrt{2}, 3 \pm 2\sqrt{2}$ (D) $8 \pm \sqrt{3}, 3 \pm \sqrt{2}$

9. Roots of equation $x^5 - 5x^4 + 9x^3 - 9x^2 + 5x - 1 = 0$ are
(A) $1, \frac{3 \pm \sqrt{5}}{2}, \frac{1 \pm i\sqrt{3}}{2}$ (B) $1, \frac{5 \pm \sqrt{3}}{2}, \frac{3 \pm i}{2}$ (C) $1, \frac{3 \pm \sqrt{5}}{2}, \frac{3 \pm i}{2}$ (D) $1, \frac{5 \pm \sqrt{3}}{2}, \frac{1 \pm i\sqrt{3}}{2}$

10. Roots of equation $x^6 - 4x^4 + 4x^2 - 1 = 0$ are
(A) $\pm 1, \frac{1 \pm i\sqrt{5}}{2}, \frac{-1 \pm \sqrt{5}}{2}$ (B) $\pm 1, \frac{1 \pm \sqrt{5}}{2}, \frac{-1 \pm i\sqrt{5}}{2}$
(C) $\pm 1, \frac{1 \pm \sqrt{5}}{2}, \frac{-1 \pm \sqrt{5}}{2}$ (D) $\pm 1, \frac{-1 \pm \sqrt{5}}{2}, \frac{-1 \pm i\sqrt{5}}{2}$.

Exercise-3

✍ Marked questions are recommended for Revision.
*** Marked Questions may have more than one correct option.**

PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

1. The smallest value of k, for which both the roots of the equation $x^2 - 8kx + 16(k^2 - k + 1) = 0$ are real, distinct and have values atleast 4, is **[IIT-JEE 2009, Paper-2, (4, –1)/ 80]**

2. Let p and q be real numbers such that $p \neq 0$, $p^3 \neq q$ and $p^3 \neq -q$. If α and β are nonzero complex numbers satisfying $\alpha + \beta = -p$ and $\alpha^3 + \beta^3 = q$, then a quadratic equation having $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ as its roots is **[IIT-JEE 2010, Paper-1, (3, –1)/ 84]**
(A) $(p^3 + q)x^2 - (p^3 + 2q)x + (p^3 + q) = 0$
(B) $(p^3 + q)x^2 - (p^3 - 2q)x + (p^3 + q) = 0$
(C) $(p^3 - q)x^2 - (5p^3 - 2q)x + (p^3 - q) = 0$
(D) $(p^3 - q)x^2 - (5p^3 + 2q)x + (p^3 - q) = 0$

3. Let α and β be the roots of $x^2 - 6x - 2 = 0$, with $\alpha > \beta$. If $a_n = \alpha^n - \beta^n$ for $n \geq 1$, then the value of $\frac{a_{10} - 2a_8}{2a_9}$ is **[IIT-JEE 2011, Paper-1, (3, –1), 80]**
(A) 1 (B) 2 (C) 3 (D) 4

4. A value of b for which the equations **[IIT-JEE 2011, Paper-2, (3, –1), 80]**
$x^2 + bx - 1 = 0$
$x^2 + x + b = 0$
have one root in common is
(A) $-\sqrt{2}$ (B) $-i\sqrt{3}$ (C) $i\sqrt{5}$ (D) $\sqrt{2}$

5. The quadratic equation p(x) = 0 with real coefficients has purely imaginary roots. Then the equation p(p(x)) = 0 has **[JEE (Advanced) 2014, Paper-2, (3, –1)/60]**
(A) only purely imaginary roots
(B) all real roots
(C) two real and two purely imaginary roots
(D) neither real nor purely imaginary roots

6*. Let S be the set of all non-zero real numbers α such that the quadratic equation $\alpha x^2 - x + \alpha = 0$ has two distinct real roots x_1 and x_2 satisfying the inequality $|x_1 - x_2| < 1$. Which of the following intervals is(are) a subset(s) of S ? **[JEE (Advanced) 2015, P-2 (4, –2)/ 80]**
(A) $\left(-\frac{1}{2}, -\frac{1}{\sqrt{5}}\right)$ (B) $\left(-\frac{1}{\sqrt{5}}, 0\right)$ (C) $\left(0, \frac{1}{\sqrt{5}}\right)$ (D) $\left(\frac{1}{\sqrt{5}}, \frac{1}{2}\right)$

7. Let $-\frac{\pi}{6} < \theta < -\frac{\pi}{12}$. Suppose α_1 and β_1 are the roots of the equation $x^2 - 2x \sec\theta + 1 = 0$ and α_2 and β_2 are the roots of the equation $x^2 + 2x \tan\theta - 1 = 0$. If $\alpha_1 > \beta_1$ and $\alpha_2 > \beta_2$, then $\alpha_1 + \beta_2$ equals
[JEE (Advanced) 2016, Paper-1, (3, –1)/62]
(A) $2(\sec\theta - \tan\theta)$ (B) $2\sec\theta$ (C) $-2\tan\theta$ (D) 0

Comprehension (Q-8 & 9)
Let p, q be integers and let α, β be the roots of the equation, $x^2 - x - 1 = 0$ where $\alpha \neq \beta$. For n = 0,1,2,...., let $a_n = p\alpha^n + q\beta^n$.

FACT : If a and b are rational numbers and $a + b\sqrt{5} = 0$, then a = 0 = b.

8. $a_{12} =$ **[JEE(Advanced) 2017, Paper-2,(3, 0)/61]**
(A) $a_{11} + 2a_{10}$ (B) $2a_{11} + a_{10}$ (C) $a_{11} - a_{10}$ (D) $a_{11} + a_{10}$

9. If $a_4 = 28$, then p + 2q = **[JEE(Advanced) 2017, Paper-2,(3, 0)/61]**
(A) 14 (B) 7 (C) 21 (D) 12

PART - II : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. Sachin and Rahul attempted to solve a quadratic equaiton. Sachin made a mistake in writing down the constant term and ended up in roots (4, 3). Rahul made a mistake in writing down coefficient of x to get roots (3, 2). The correct roots of equation are : **[AIEEE- 2011, II, (4, –1), 120]**

(1) 6, 1 (2) 4, 3 (3) –6 , –1 (4) –4 , –3

2. Let for $a \neq a_1 \neq 0$, $f(x) = ax^2 + bx + c$, $g(x) = a_1x^2 + b_1x + c_1$ and $p(x) = f(x) – g(x)$. If $p(x) = 0$ only for $x = –1$ and $p(–2) = 2$, then the value of $p(2)$ is : **[AIEEE- 2011, II, (4, –1), 120]**

(1) 3 (2) 9 (3) 6 (4) 18

3. The equation $e^{\sin x} – e^{–\sin x} – 4 = 0$ has : **[AIEEE- 2012 (4, –1), 120]**

(1) infinite number of real roots (2) no real roots

(3) exactly one real root (4) exactly four real roots

4. If the equations $x^2 + 2x + 3 = 0$ and $ax^2 + bx + c = 0$, $a,b,c \in R$, have a common root, then a : b : c is **[AIEEE - 2013, (4, –1), 120]**

(1) 1 : 2 : 3 (2) 3 : 2 : 1 (3) 1 : 3 : 2 (4) 3 : 1 : 2

5. If $a \in R$ and the equation $– 3(x – [x])^2 + 2 (x – [x]) + a^2 = 0$ (where [x] denotes the greatest integer $\leq x$) has no intgeral solution, then all possible values of a lie in the interval : **[JEE(Main)2014,(4, – 1), 120]**

(1) (–2, –1) (2) $(–\infty, –2) \cup (2, \infty)$ (3) $(–1, 0) \cup (0, 1)$ (4) (1, 2)

6. Let α and β be the roots of equation $px^2 + qx + r = 0$, $p \neq 0$. If p, q ,r are in the A.P. and $\frac{1}{\alpha} + \frac{1}{\beta} = 4$, then the value of $|\alpha – \beta|$ is : **[JEE(Main)2014,(4, – 1), 120]**

(1) $\frac{\sqrt{34}}{9}$ (2) $\frac{2\sqrt{13}}{9}$ (3) $\frac{\sqrt{61}}{9}$ (4) $\frac{2\sqrt{17}}{9}$

7. Let α and β be the roots of equation $x^2 – 6x – 2 = 0$. If $a_n = \alpha^n – \beta^n$, for $n \geq 1$, then the value of $\frac{a_{10} – 2a_8}{2a_9}$ is equal to : **[JEE(Main)2015,(4, – 1), 120]**

(1) 6 (2) – 6 (3) 3 (4) –3

8. The number of all possible positive integral values of α for which the roots of the quadratic equation, $6x^2 – 11x + \alpha = 0$ are rational numbers is : **[JEE(Main) 2019, Online (09-01-19),P-2 (4, – 1), 120]**

(1) 3 (2) 4 (3) 5 (4) 2

9. If λ be the ratio of the roots of the quadratic equation in x, $3m^2x^2 + m(m – 4)x + 2 = 0$, then the least value of m for which $\lambda + \frac{1}{\lambda} = 1$, is : **[JEE(Main) 2019, Online (12-01-19),P-1 (4, – 1), 120]**

(1) $–2 + \sqrt{2}$ (2) $4 – 3\sqrt{2}$ (3) $2 – \sqrt{3}$ (4) $4 – 2\sqrt{3}$

Answers

EXERCISE - 1

PART - I

Section (A) :

A-1. a = 2; No real value of x **A-2.** (i) $-\frac{7}{4}$ (ii) $-\frac{7}{8}$

A-3. (i) $ac\,x^2 + b(a + c)\,x + (a + c)^2 = 0$ (ii) $a^2x^2 + (2ac - 4a^2 - b^2)\,x + 2b^2 + (c - 2a)^2 = 0$

A-4. $3x^2 - 19x + 3 = 0$ **A-5.** 8, 3 **A-6.** (i) 4 (ii) 72 (iii) 2

A-7. $\alpha^2\beta$ and $\alpha\beta^2$ **A-10.** 2 **A-11.** 11

Section (B) :

B-2. $-\frac{(r+1)^3}{r^2}$ **B-3.** **(i)** roots are $\frac{3}{4}, \frac{3}{2}, \frac{-5}{3}$, $\lambda = 45$ or $\frac{-1}{2}, -1, \frac{25}{12}$, $\lambda = -25$.

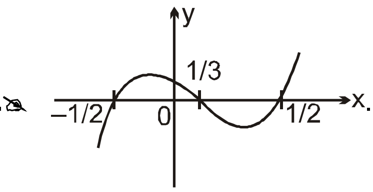
(ii) roots are $\frac{-4}{3}, -\frac{3}{2}, \frac{-5}{3}$, $\lambda = 121$ **B-4.** $x^3 - 15x^2 + 67x - 77 = 0.$

B-5. -3 **B-6.** $\mathbf{\frac{1}{2}, \frac{1}{2}, -6}$

Section (C) :

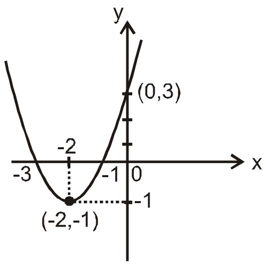
C-1. (– 4, 7) **C-3.** $3 \pm 2\sqrt{2}$ **C-7.** **(i)** $4,\ -2 \pm i5\sqrt{3}$ **(ii)** 3 or 4

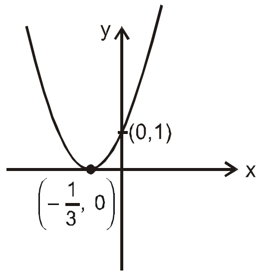
C-8. $-1 \pm \sqrt{2}, -1 \pm \sqrt{-1}$

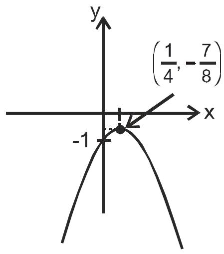
C-9. 

. Two positive roots

Section (D) :

D-1. (i) 

(ii) 

(iii) 

D-2. (i) $(-\infty, 4]$ (ii) [2, 6] (iii) [3, 6) **D-3.** **(i)** $\left[\frac{1}{2}, \frac{3}{2}\right]$ **(ii)** $\left(-\infty, \frac{-4}{5}\right] \cup (1, \infty)$

D-4. $\left(-\infty, -\frac{1}{2}\right)$ **D-5.** **(i)** $a > 1$ **(ii)** $a \in \phi$.

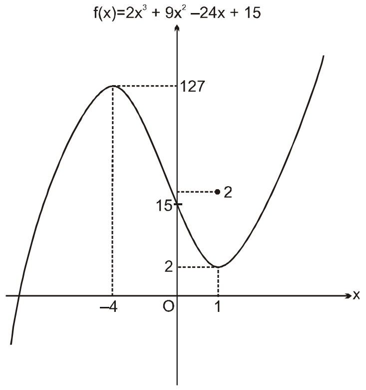
Section (E) :

E-2. $K \in (-2, 3)$ **E-3.** $a \in (-2, 2)$ **E-4.** $a \in (1, 5) - \{3\}$ **E-5.** $6 < K < 6.75$

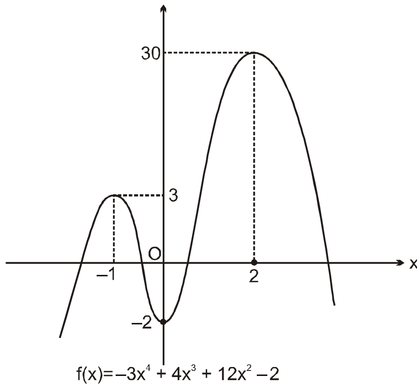
Section (F) :

F-2. a = 0, 24 **F-3.** 3

F-5. (i)



(ii)



F-6. (i) $k\in[-2,2]$ (ii) $k\in(-\infty,-2) \cup (2, \infty)$

PART - II

Section (A) :

A-1. (B) **A-2.** (C) **A-3.** (A) **A-4.** (C) **A-5.** (A)

Section (B) :

B-1. (C) **B-2.** (C) **B-3.** (B) **B-4.** (A) **B-5.** (C)

Section (C) :

C-1. (B) **C-2.** (C) **C-3.** (A) **C-4.** (A) **C-5.** (A) **C-6.** (C)

Section (D) :

D-1. (B) **D-2.** (B) **D-3.** (B) **D-4.** (B) **D-5.** (A) **D-6.** (C)

D-7. (C) **D-8.** (D)

Section (E) :

E-1. (D) **E-2.** (B) **E-3.** (D) **E-4** (D)

Section (F) :

F-1. (A) **F-2.** (C) **F-3.** (A) **F-4.** (C) **F-5.** (D)

PART - III

1. (A) $\to$ (r), (B) $\to$ (p), (C) $\to$ (q), (D) $\to$ (s) **2.** (A $\to$ r); (B $\to$ p,q,s); (C $\to$ s); (D $\to$ p,q,r)

3. (A) q, s, t (B) p, t (C) r (D) q, s.

EXERCISE - 2

PART – I

1.	(B)	**2.**	(B)	**3.**	(D)	**4.**	(B)	**5.**	(A)	6.	(A)	**7.**	(A)
8.	(A)	**9.**	(B)	**10.**	(A)	**11.**	(D)	**12.**	(B)	**13.**	(B)	**14.**	(A)
15.	(C)												

PART - II

1.	2	**2.**	8	**3.**	1	**4.**	73	**5.**	9	**6.**	10	**7.**	13
8.	6	**9.**	1	**10.**	9	**11.**	18	**12.**	32	**13.**	1	**14.**	63
15.	1	**16.**	2	**17.**	1								

PART - III

1.	(ACD)	**2.**	(BCD)	**3.**	(BC)	**4.**	(AC)	**5.**	(BCD)	**6.**	(BC)	**7.**	(ABD)
8.	(ABCD)	**9.**	(ABCD)	**10.**	(AD)	**11.**	(AD)	**12.**	(AD)	**13.**	(ABD)	**14.**	(AD)
15.	(AC)	**16.**	(AB)	**17.**	(CD)	**18.**	(BD)	**19**.	(AB)	**20.**	(ABC)	**21.**	(AB)

PART - IV

1.	(C)	**2.**	(B)	**3.**	(A)	**4.**	(A)	**5.**	(C)	**6.**	(B)	**7.**	(D)
8.	(B)	**9.**	(A)	**10.**	(C)								

EXERCISE - 3

PART – I

1.	2	**2.**	(B)	**3.**	(C)	**4.**	(B)	**5.**	(D)	**6.**	(AD)	**7.**	(C)
8.	(D)	**9.**	(D)										

PART - II

1.	(1)	**2.**	(4)	**3.**	(2)	**4.**	(1)	**5.**	(3)	**6.**	(2)	**7.**	(3)
8.	(1)	**9.**	(2)										

Advance Level Problems (ALP):-

1. Find the number of values of x satisfying the relation

$$\alpha_1^3\left(\frac{\prod_{i=2}^{n}(x-\alpha_i)}{\prod_{i=2}^{n}(\alpha_1-\alpha_i)}\right)+\sum_{j=2}^{n-1}\left(\left(\frac{\prod_{i=1}^{j-1}(x-\alpha_i)\prod_{i=j+1}^{n}(x-\alpha_i)}{\prod_{i=1}^{j-1}(\alpha_j-\alpha_i)\prod_{i=j+1}^{n}(\alpha_j-\alpha_i)}\right)\alpha_j^3\right)+\left(\frac{\prod_{i=1}^{n-1}(x-\alpha_i)}{\prod_{i=1}^{n-1}(\alpha_n-\alpha_i)}\right)\alpha_n^3-x^3=0 \text{ (where } n\ge 5).$$

2. Prove that roots of $a^2x^2 + (b^2 + a^2 - c^2)x + b^2 = 0$ are not real, if $a + b > c$ and $|a - b| < c$. (where a, b, c are positive real numbers)

3. Solve the inequality, $\frac{1}{x-1}-\frac{4}{x-2}+\frac{4}{x-3}-\frac{1}{x-4}<\frac{1}{30}$.

4. If three real and distinct numbers a, b, c are in G.P. (i.e., $b^2 = ac$) and $a + b + c = xb$, then prove that $x < -1$ or $x > 3$.

5. If $V_n = \alpha^n + \beta^n$, where α, β are roots of equation $x^2 + x - 1 = 0$. Then prove that $V_n + V_{n-3} = 2V_{n-2}$ and hence evaluate V_7 (n is a whole number)

6. Find all 'm' for which $f(x) \equiv x^2 - (m - 3)x + m > 0$ for all values of 'x' in [1, 2].

7. Find the values of a, for which the quadratic expression $ax^2 + (a - 2)x - 2$ is negative for exactly two integral values of x.

8. (i) Solve for real values of 'x' :
$x^2 - 2a|x - a| - 3a^2 = 0, \quad a \le 0$

(ii) Find the number of real roots of $\left(x+\frac{1}{x}\right)^3+\left(x+\frac{1}{x}\right)=0$

9. If α, β are roots of the equation $x^2 - 34x + 1 = 0$, evaluate $\sqrt[4]{\alpha}-\sqrt[4]{\beta}$, where $\sqrt[4]{\ .\ }$ denotes the principal value.

10. Find the values of 'a' for which the equation
$\left(x^2+x+2\right)^2-(a-3)\left(x^2+x+2\right)\left(x^2+x+1\right)+(a-4)\left(x^2+x+1\right)^2=0$ has atleast one real root.

11. Show that the quadratic equation $x^2 + 7x - 14(q^2 + 1) = 0$ where q is an integer, has no integral roots.

12. Find the integral values of 'a' for which the equation $x^4-\left(a^2-5a+6\right)x^2-\left(a^2-3a+2\right)=0$ has only real roots.

13. If α, β; β, γ and γ, α are the roots of $a_ix^2+b_ix+c_i=0$; i = 1, 2, 3 then show that

$$(\alpha+\beta+\gamma)+(\alpha\beta+\beta\gamma+\alpha\gamma)+\alpha\beta\gamma=\pm\left\{\prod_{i=1}^{3}\left(\frac{a_i-b_i+c_i}{a_i}\right)\right\}^{1/2}-1$$

14. Suppose that $a_1 > a_2 > a_3 > a_4 > a_5 > a_6$ and

$p = a_1 + a_2 + a_3 + a_4 + a_5 + a_6$

$q = a_1a_3 + a_3a_5 + a_5a_1 + a_2a_4 + a_4a_6 + a_6a_2$

$r = a_1a_3a_5 + a_2a_4a_6$,

then show that roots of the equation $2x^3 - px^2 + qx - r = 0$ are real.

15. If $\beta + \cos^2\alpha$, $\beta + \sin^2\alpha$ are the roots of $x^2 + 2bx + c = 0$ and $\gamma + \cos^4\alpha$, $\gamma + \sin^4\alpha$ are the roots of $X^2 + 2BX + C = 0$, then prove that $b^2 - B^2 = c - C$.

16. Find the set of values of 'a' if $(x^2 + x)^2 + a(x^2 + x) + 4 = 0$ has
(i) all four real & distinct roots.
(ii) four roots in which only two roots are real and distinct.
(iii) all four imaginary roots.
(iv) four real roots in which only two are equal.

17. $f(x) = x^2 + bx + c$, where $b,c \in R$, if f(x) is a factor of both $x^4 + 6x^2 + 25$ and $3x^4 + 4x^2 + 28x + 5$ then find f(x).

18. Let $ax^4 + bx^3 + x^2 + (3-a)x + 3 = 0$ and $x^2 + (2-a)x + 3 = 0$ have common roots. If $a \in (-1,5)$ then find $|a+12b|$

19. How many quadratic equations are there which are unchanged by squaring their roots ?

20. Let $P(x) = x^5 + x^2 + 1$ have zeros $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5$ and $Q(x) = x^2 - 2$, then find

(i) $\prod_{i=1}^{5} Q(\alpha_i)$ (ii) $\sum_{i=1}^{5} Q(\alpha_i)$ (iii) $\sum_{1 \le i < j \le 5} \sum Q(\alpha_i)\,Q(\alpha_j)$ (iv) $\sum_{i=1}^{5} Q^2(\alpha_i)$

21. If a, b, c are non–zero, unequal rational numbers then prove that the roots of the equation $(abc^2)x^2 + 3a^2cx + b^2cx - 6a^2 - ab + 2b^2 = 0$ are rational.

22. If a, b, c represents sides of a Δ then prove that equation $x^2 - (a^2 + b^2 + c^2)x + a^2b^2 + b^2c^2 + c^2a^2 = 0$ has imaginary roots.

23. If x_1 is a root of $ax^2 + bx + c = 0$, x_2 is a root of $-ax^2 + bx + c = 0$ where $0 < x_1 < x_2$, show that the equation $ax^2 + 2bx + 2c = 0$ has a root x_3 satisfying $0 < x_1 < x_3 < x_2$.

24. Find the number of positive real roots of $x^4 - 4x - 1 = 0$.

25. If $(1 + k)\tan^2x - 4\tan x - 1 + k = 0$ has real roots $\tan x_1$ and $\tan x_2$, where $\tan x_1 \neq \tan x_2$, then find k.

26. Let Δ^2 be the discriminant and α, β be the roots of the equation $ax^2 + bx + c = 0$. Then find equation whose roots are $2a\alpha + \Delta$ and $2a\beta - \Delta$.

27. Prove that $\frac{\pi^e}{x-e} + \frac{e^\pi}{x-\pi} + \frac{\pi^\pi + e^e}{x-\pi-e} = 0$ has one real root in (e, π) and other in $(\pi, \pi + e)$.

28. If α, β^2 are integers, β^2 is non-zero multiple of 3 and $\alpha + i\beta$, -2α are roots of $x^3 + ax^2 + bx - 316 = 0$, a, b, $\beta \in R$, then find a, b.

29_. Let polynomial $f(x) = ax^4 + bx^3 + cx^2 + dx + e$ have integral coefficient (where a > 0) If there exist four distinct integer $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ ($\alpha_1 < \alpha_2 < \alpha_3 < \alpha_4$) such that $f(\alpha_1) = f(\alpha_2) = f(\alpha_3) = f(\alpha_4) = 5$ and equation $f(x) = 9$ has integeral roots then find

(i) $f\left(\frac{\alpha_1+\alpha_2+\alpha_3+\alpha_4}{4}\right)$ (ii) $f'\left(\frac{\alpha_1+\alpha_2+\alpha_3+\alpha_4}{4}\right)$

(iii) Range of f(x) in $[\alpha_2, \alpha_3]$

(iv) Difference of largest and smallest root of equation f(x) = 9

30. If x and y both are non-negative integral values for which $(xy - 7)^2 = x^2 + y^2$, then find the sum of all possible values of x.

Answer Key (ALP):-

1. Infinite **3.** $(-\infty, -2) \cup (-1, 1) \cup (2, 3) \cup (4, 6) \cup (7, \infty)$ **5.** –29

6. $(-\infty, 10)$ **7.** [1, 2)

8. **(i)** $x = a(1 - \sqrt{2})$, $x = a(\sqrt{6} - 1)$ **(ii)** 0

9. ± 2 **10.** $5 < a \le \frac{19}{3}$ **12.** $a \in \{1, 2\}$

16. (i) $a \in (-\infty, -4)$ (ii) $a \in \left(\frac{65}{4}, \infty\right)$ (iii) $a \in \left(-4, \frac{65}{4}\right)$ (iv) $a \in \phi$

17. $x^2 - 2x + 5$ **18.** 3 **19.** 4 **20.** (i) – 23 (ii) – 10 (iii) 40 (iv) 20

24. 1 **25.** $\left(-\sqrt{5}, -1\right) \cup \left(-1, \sqrt{5}\right)$ **26.** $x^2 + 2bx + b^2 = 0$ or $x^2 + 2bx - 3b^2 + 16ac = 0$

28. a = 0, b = 63 **29.** (i) 9 (ii) 0 (iii) [5, 9] (iv) $2\sqrt{5}$

30. 14