Exercise-1

✍ **Marked questions are recommended for Revision.**

PART - I : SUBJECTIVE QUESTIONS

Section (A) : Algebra of Complex Numbers and Its Representation and Demoivre's Theorem

A-1. Find the real values of x and y for which the following equation is satisfied :

(i) $\frac{(1+i)\,x-2i}{3+i}+\frac{(2-3i)\,y+i}{3-i} = i$

(ii) $\frac{x}{1+2i} + \frac{y}{3+2i} = \frac{5+6i}{8i-1}$

(iii) $(2 + 3i)\, x^2 - (3 - 2i)\, y = 2x - 3y + 5i$

(iv) $4x^2 + 3xy + (2xy - 3x^2)\, i = 4y^2 - (x^2/2) + (3xy - 2y^2)\, i$

A-2_. Let $z = \frac{1+2(\sin\theta)i}{1-(\sin\theta)i}$

(i) Find the number of values of $\theta\in[0, 4\pi]$ such that z is purely imaginary.
(ii) Find the sum of all values of $\theta\in[0, 4\pi]$ such that z is purely real.

A-3. (i) Find the real values of x and y for which $z_1 = 9y^2 - 4 - 10ix$ and $z_2 = 8y^2 - 20i$ are conjugate complex of each other.

(ii) Find the value of $x^4 - x^3 + x^2 + 3x - 5$ if $x = 2 + 3i$

A-4. Find

(i) the square root of 7 + 24 i (ii) $\sqrt{i}+\sqrt{-i}$

A-5. Solve the following for z :

$z^2 - (3 - 2i)\, z = (5i - 5)$

A-6. Simplify and express the result in the form of a + bi :

(i) $-i\,(9 + 6i)\,(2 - i)^{-1}$ (ii) $\left(\frac{4i^3-i}{2i+1}\right)^2$

(iii) $\frac{1}{(1-\cos\theta)+2\,i\sin\theta}$ (iv) $\left(\sqrt{3}+i\right)e^{-i\frac{\pi}{6}}$

A-7. Convert the following complex numbers in Eulers form

(i) $z = -\pi$ (ii) $z = 5i$ (iii) $z = -\sqrt{3} - i$ (iv) $z = -2\left(\cos\frac{\pi}{5}+i\sin\frac{\pi}{5}\right)$

A-8. Find the modulus, argument and the principal argument of the complex numbers.

(i) $z = 1 + \cos\frac{18\pi}{25} + i\sin\frac{18\pi}{25}$ (ii) $z = -2\,(\cos 30° + i \sin 30°)$

(iii) $(\tan 1 - i)^2$ (iv) $\frac{i-1}{i\left(1-\cos\frac{2\pi}{5}\right)+\sin\frac{2\pi}{5}}$

A-9.✍ Dividing polynomial f(z) by z – i, we get the remainder i and dividing it by z + i, we get the remainder 1 + i. Find the remainder upon the division of f(z) by $z^2 + 1$.

A-10. If $(\sqrt{3} + i)^{100} = 2^{99}(a + ib)$, then find
(i) $a^2 + b^2$ (ii) b

A-11. If n is a positive integer, prove the following
(i) $(1 + \cos\theta + i\sin\theta)^n + (1 + \cos\theta - i\sin\theta)^n = 2^{n+1}\cos^n\frac{\theta}{2}\cos\frac{n\theta}{2}$.

(ii) $(1 + i)^n + (1 - i)^n = 2^{\frac{n}{2}+1} . \cos\frac{n\pi}{4}$

A-12. Show that $e^{i2m\theta}\left(\frac{i\cot\theta + 1}{i\cot\theta - 1}\right)^m = 1$.

A-13. If $x_r = \cos\left(\frac{\pi}{3^r}\right) + i\sin\left(\frac{\pi}{3^r}\right)$, prove that $x_1 x_2 x_3$....... upto infinity = i.

Section (B) : Argument / Modulus / Conjugate Properties and Triangle Inequality

B-1. If $z = x + iy$ is a complex number such that $z = (a + ib)^2$ then
(i) find $\bar{z}$
(ii) show that $x^2 + y^2 = (a^2 + b^2)^2$

B-2. If z_1 and z_2 are conjugate to each other, then find $\arg(-z_1z_2)$.

B-3. If $z\ (\neq -1)$ is a complex number such that $\frac{z-1}{z+1}$ is purely imaginary, then find $|z|$

B-4. If $|z - 2| = 2|z - 1|$, where z is a complex number, prove $|z|^2 = \frac{4}{3}$ Re (z) using
(i) polar form of z, (ii) $z = x + iy$, (iii) modulus, conjugate properties

B-5. For any two complex numbers z_1, z_2 and any two real numbers a, b show that
$|az_1 - bz_2|^2 + |bz_1 + az_2|^2 = (a^2 + b^2)(|z_1|^2 + |z_2|^2)$

B-6. If z_1 and z_2 are two complex numbers such that $|z_1| < 1 < |z_2|$ then prove that $\left|\frac{1 - z_1\bar{z}_2}{z_1 - z_2}\right| < 1$.

B-7. If $k > 0$, $|z| = |w| = k$ and $\alpha = \frac{z - \bar{w}}{k^2 + z\bar{w}}$, then find $\text{Re}(\alpha)$.

B-8. (i) If $w = \frac{z+i}{z-i}$ is purely real then find arg z.

(ii) If $w = \frac{z+4i}{z+2i}$ is purely imaginary then find $|z + 3i|$.

B-9. If $a = e^{i\alpha}$, $b = e^{i\beta}$, $c = e^{i\gamma}$ and $\cos\alpha + \cos\beta + \cos\gamma = 0 = \sin\alpha + \sin\beta + \sin\gamma$, then prove the following
(i) $a + b + c = 0$ (ii) $ab + bc + ca = 0$
(iii) $a^2 + b^2 + c^2 = 0$ (iv) $\Sigma\cos 2\alpha = 0 = \Sigma\sin 2\alpha$

B-10. If $|z - 1 + i| + |z + i| = 1$ then find range of principle argument of z.

Section (C) : Geometry of Complex Number and Rotation Theorem

C-1. If $|z - 2 + i| = 2$, then find the greatest and least value of $|z|$.

C-2. If $|z + 3| \leq 3$ then find minimum and maximum values of
(i) $|z|$ (ii) $|z - 1|$ (iii) $|z + 1|$

C-3. Interpret the following locus in $z \in C$.
(i) $1 < |z - 2i| < 3$ (ii) $\text{Im}(z) \geq 1$
(iii) $\text{Arg}(z - 3 - 4i) = \pi/3$ (iv) $\text{Re}\left(\frac{z + 2i}{iz + 2}\right) \leq 4 \ (z \neq 2i)$

C-4. If O is origin and affixes of P, Q, R are respectively z, iz, z + iz. Locate the points on complex plane. If $\Delta PQR = 200$ then find
(i) $|z|$ (ii) sides of quadrilateral OPRQ

C-5. The three vertices of a triangle are represented by the complex numbers, 0, z_1 and z_2. If the triangle is equilateral, then show that $z_1^2 + z_2^2 = z_1z_2$. Further if z_0 is circumcentre then prove that $z_1^2 + z_2^2 = 3z_0^2$.

C-6. Let z_1 and z_2 be two roots of the equation $z^2 + az + b = 0$, z being complex. Further, assume that the origin, z_1 and z_2 form an equilateral triangle. Then show that $a^2 = 3b$.

C-7. Let $z_1 = 10 + 6i$ and $z_2 = 4 + 6i$. If z is any complex number such that the argument of $(z - z_1) / (z - z_2)$ is $\pi/4$, then find the length of arc of the locus.

C-8. Let I : $\text{Arg}\left(\frac{z - 8i}{z + 6}\right) = \pm\frac{\pi}{2}$

II : $\text{Re}\left(\frac{z - 8i}{z + 6}\right) = 0$

Show that locus of z in I or II lies on $x^2 + y^2 + 6x - 8y = 0$. Hence show that locus of z can also be represented by $\frac{z - 8i}{z + 6} + \frac{\bar{z} + 8i}{\bar{z} + 6} = 0$. Further if locus of z is expressed as $|z + 3 - 4i| = R$, then find R.

C-9. Show that $z\bar{z} + (4 - 3i)z + (4 + 3i)\bar{z} + 5 = 0$ represents circle. Hence find centre and radius.

C-10. If z_1 & z_2 are two complex numbers & if $\arg\frac{z_1 + z_2}{z_1 - z_2} = \frac{\pi}{2}$ but $|z_1 + z_2| \neq |z_1 - z_2|$ then identify the figure formed by the points represented by 0, z_1, z_2 & $z_1 + z_2$.

Section (D) : Cube root and n^th Root of Unity.

D-1. If $\omega \ (\neq 1)$ be a cube root of unity and $(1 + \omega^4)^n = (1 + \omega^2)^n$ then find the least positive integral value of n

D-2. When the polynomial $5x^3 + Mx + N$ is divided by $x^2 + x + 1$, the remainder is 0. Then find M + N.

D-3. Show that $(1 - \omega + \omega^2)(1 - \omega^2 + \omega^4)(1 - \omega^4 + \omega^8)$ .... to 2n factors $= 2^{2n}$

D-4. Let ω is non-real root of $x^3 = 1$

(i) If $P = \omega^n$, ($n \in N$) and

$Q = ({}^{2n}C_0 + {}^{2n}C_3 + ...) + ({}^{2n}C_1 + {}^{2n}C_4 + ...)\omega + ({}^{2n}C_2 + {}^{2n}C_5 + ...)\omega^2$ then find $\frac{P}{Q}$.

(ii) If $P = 1 - \frac{\omega}{2} + \frac{\omega^2}{4} - \frac{\omega^3}{8}......$ upto ∞ terms and $Q = \frac{1-\omega^2}{2}$ then find value of PQ.

D-5. If $x = 1 + i\sqrt{3}$; $y = 1 - i\sqrt{3}$ and z = 2, then prove that $x^p + y^p = z^p$ for every prime p > 3.

D-6. Solve $(z - 1)^4 - 16 = 0$. Find sum of roots. Locate roots, sum of roots and centroid of polygon formed by roots in complex plane.

D-7. Find the value(s) of the following

(i) $\left(\frac{1}{2} + \frac{\sqrt{-3}}{2}\right)^3$ (ii) $\left(\frac{1}{2} + \frac{\sqrt{-3}}{2}\right)^{3/4}$

Hence find continued product if two or more distinct values exists.

D-8. If 1, $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ be the roots of $x^5 - 1 = 0$, then find the value

of $\frac{\omega - \alpha_1}{\omega^2 - \alpha_1} \cdot \frac{\omega - \alpha_2}{\omega^2 - \alpha_2} \cdot \frac{\omega - \alpha_3}{\omega^2 - \alpha_3} \cdot \frac{\omega - \alpha_4}{\omega^2 - \alpha_4}$ (where ω is imaginary cube root of unity.)

D-9. $a = \cos\frac{2\pi}{7} + i\sin\frac{2\pi}{7}$ then find the quadratic equation whose roots are $\alpha = a + a^2 + a^4$ and $\beta = a^3 + a^5 + a^6$

PART - II : ONLY ONE OPTION CORRECT TYPE

Section (A) : Algebra of Complex Numbers and Its Representation and Demoivre's Theorem

A-1. If z is a complex number such that |z| = 4 and arg(z) = $\frac{5\pi}{6}$, then z is equal to

(A) $-2\sqrt{3} + 2i$ (B) $2\sqrt{3} + i$ (C) $2\sqrt{3} - 2i$ (D) $-\sqrt{3} + i$

A-2. The complex numbers sin x + i cos 2x and cos x – i sin 2x are conjugate to each other, for

(A) $x = n\pi$ (B) x = 0 (C) $x = \frac{n\pi}{2}$ (D) no value of x

A.3. The least value of n ($n \in N$), for which $\left(\frac{1+i}{1-i}\right)^n$ is real, is

(A) 1 (B) 2 (C) 3 (D) 4

A-4. In G.P. the first term & common ratio are both $\frac{1}{2}\left(\sqrt{3}+i\right)$, then the modulus of n^{th} term is :

(A) 1 (B) 2^n (C) 4^n (D) 3^n

A-5. If $z = (3 + 7i)(p + iq)$, where $p, q \in I - \{0\}$, is purely imaginary, then minimum value of $|z|^2$ is

(A) 0 (B) 58 (C) $\frac{3364}{3}$ (D) 3364

A-6. If $z = x + iy$ and $z^{1/3} = a - ib$ then $\frac{x}{a} - \frac{y}{b} = k\left(a^2 - b^2\right)$ where k =

(A) 1 (B) 2 (C) 3 (D) 4

A-7. If $z = \frac{\pi}{4}(1 + i)^4\left(\frac{1-\sqrt{\pi}\,i}{\sqrt{\pi}+i} + \frac{\sqrt{\pi}-i}{1+\sqrt{\pi}\,i}\right)$, then $\left(\frac{|z|}{amp(z)}\right)$ equals

(A) 1 (B) π (C) 3π (D) 4

A-8. The set of values of $a \in R$ for which $x^2 + i(a - 1)x + 5 = 0$ will have a pair of conjugate imaginary roots is

(A) R (B) {1}
(C) $\{a : a^2 - 2a + 21 > 0\}$ (D) {0}

A-9. Let z is a complex number satisfying the equation, $z^3 - (3 + i)z + m + 2i = 0$, where $m \in R$. Suppose the equation has a real root α, then find the value of $\alpha^4 + m^4$

(A) 32 (B) 16 (C) 8 (D) 64

A-10. The expression $\left(\frac{1+i\tan\alpha}{1-i\tan\alpha}\right)^n - \frac{1+i\tan n\alpha}{1-i\tan n\alpha}$ when simplified reduces to :

(A) zero (B) $2\sin n\alpha$ (C) $2\cos n\alpha$ (D) none

A-11. If $(\cos\theta + i\sin\theta)(\cos 2\theta + i\sin 2\theta) \ldots (\cos n\theta + i\sin n\theta) = 1$, then the value of θ is

(A) $\frac{3m\pi}{n(n+1)}$, $m \in Z$ (B) $\frac{2m\pi}{n(n+1)}$, $m \in Z$ (C) $\frac{4m\pi}{n(n+1)}$, $m \in Z$ (D) $\frac{m\pi}{n(n+1)}$, $m \in Z$

A-12. Let principle argument of complex number be re-defined between $(\pi, 3\pi]$, then sum of principle arguments of roots of equation $z^6 + z^3 + 1 = 0$ is

(A) 0 (B) 3π (C) 6π (D) 12π

Section (B) : Argument / Modulus / Conjugate Properties and Triangle Inequality

B-1. If $|z| = 1$ and $\omega = \frac{z-1}{z+1}$ (where $z \neq -1$), the Re(ω) is

(A) 0 (B) $-\frac{1}{|z+1|^2}$ (C) $\left|\frac{z}{z+1}\right| \cdot \frac{1}{|z+1|^2}$ (D) $\frac{\sqrt{2}}{|z+1|^2}$

B-2. If $a^2 + b^2 = 1$, then $\frac{(1 + b + ia)}{(1 + b - ia)} =$

(A) 1 (B) 2 (C) b + ia (D) a + ib

B-3. If $(2 + i)(2 + 2i)(2 + 3i) \ldots\ldots (2 + ni) = x + iy$, then the value of $5.8.13. \ldots\ldots.(4 + n^2)$

(A) $(x^2 + y^2)$ (B) $\sqrt{(x^2 + y^2)}$ (C) $2(x^2 + y^2)$ (D) $(x + y)$

B-4. If $z = x + iy$ satisfies amp $(z - 1)$ = amp $(z + 3)$ then the value of $(x - 1) : y$ is equal to

(A) 2 : 1 (B) 1 : 3 (C) – 1 : 3 (D) does not exist

B-5. If $z_1 = -3 + 5i$; $z_2 = -5 - 3i$ and z is a complex number lying on the line segment joining z_1 & z_2, then arg(z) can be :

(A) $-\frac{3\pi}{4}$ (B) $-\frac{\pi}{4}$ (C) $\frac{\pi}{6}$ (D) $\frac{5\pi}{6}$

B-6. If $(1 + i)z = (1 - i)\bar{z}$ then z is

(A) $t(1 - i), t \in R$ (B) $t(1 + i), t \in R$ (C) $\frac{t}{1+i}, t \in R^+$ (D) $\frac{t}{1-i}, t \in R^+$

B-7. Let z and ω be two non-zero complex numbers such that $|z| = |\omega|$ and arg $z = \pi -$ arg ω, then z equals

(A) ω (B) $-\omega$ (C) $\bar{\omega}$ (D) $-\bar{\omega}$

B-8. If z_1 and z_2 are two non-zero complex numbers such that $\left|\frac{z_1}{z_2}\right| = 2$ and $\arg(z_1z_2) = \frac{3\pi}{2}$, then $\frac{\bar{z}_1}{z_2}$ is equal to

(A) 2 (B) –2 (C) –2i (D) 2i

B-9. Number of complex numbers z such that $|z| = 1$ and $|z/\bar{z} + \bar{z}/z| = 1$ is $(\arg(z) \in [0, 2\pi])$

(A) 4 (B) 6 (C) 8 (D) more than 8

B-10. If $|z_1| = |z_2|$ and $\arg(z_1/z_2) = \pi$, then $z_1 + z_2$ is equal to

(A) 1 (B) 3 (C) 0 (D) 2

B-11. The number of solutions of the system of equations $Re(z^2) = 0$, $|z| = 2$ is

(A) 4 (B) 3 (C) 2 (D) 1

B-12. If $|z^2 - 1| = |z^2| + 1$, then z lies on :

(A) the real axis (B) the imaginary axis (C) a circle (D) an ellipse

B-13. If $|z - 2i| + |z - 2| \geq ||z| - |z - 2 - 2i||$, then locus of z is

(A) circle (B) line segment (C) point (D) complete x-y plane

Section (C) : Geometry of Complex Number and Rotation Theorem

C-1. The complex number $z = x + iy$ which satisfy the equation $\left|\frac{z-5i}{z+5i}\right| = 1$ lie on :

(A) the x-axis (B) the straight line $y = 5$
(C) a circle passing through the origin (D) the y-axis

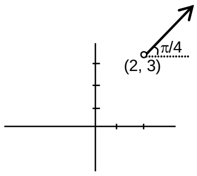
C-2. The inequality $|z - 4| < |z - 2|$ represents :

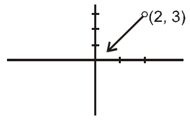
(A) $Re(z) > 0$ (B) $Re(z) < 0$ (C) $Re(z) > 2$ (D) $Re(z) > 3$

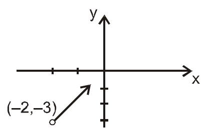
C-3. Let A, B, C represent the complex numbers z_1, z_2, z_3 respectively on the complex plane. If the circumcentre of the triangle ABC lies at the origin, then the orthocentre is represented by the complex number :

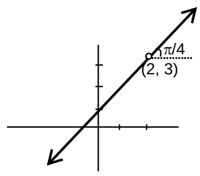
(A) $z_1 + z_2 - z_3$ (B) $z_2 + z_3 - z_1$ (C) $z_3 + z_1 - z_2$ (D) $z_1 + z_2 + z_3$

C-4. If Arg (z – 2 – 3i) = $\frac{\pi}{4}$, then the locus of z is

(A)


(B)


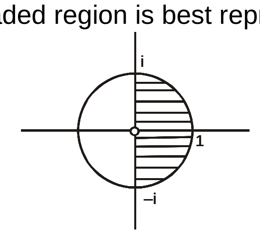
(C)


(D)


C-5. The system of equations $\left.\begin{array}{ll}|z+1-i| & =2 \\ \text{Re } z & \geq 1\end{array}\right\}$, where z is a complex number has :

(A) no solution
(B) exactly one solution
(C) two distinct solutions
(D) infinite solution

C-6. The locus of z which lies in shaded region is best represented by



(A) $|z| \leq 1, \ \frac{-\pi}{2} \leq \arg z \leq \frac{\pi}{2}$
(B) $|z| = 1, \ \frac{-\pi}{2} \leq \arg z \leq 0$
(C) $|z| \geq 0, \ 0 \leq \arg z \leq \frac{\pi}{2}$
(D) $|z| \leq 1, \ \frac{-\pi}{2} \leq \arg z \leq \pi$

C-7. The equation $|z - 1|^2 + |z + 1|^2 = 2$ represents

(A) a circle of radius '1'
(B) a straight line
(C) the ordered pair (0, 0)
(D) None of these

C-8. If $\arg\left(\frac{z-2}{z-4}\right) = \frac{\pi}{3}$ then locus of z is :

(A) equileteral triangle
(B) arc of circle
(C) arc of ellipse
(D) two rays making angle $\frac{\pi}{3}$ between them

C-9. The region of Argand diagram defined by $|z - 1| + |z + 1| \leq 4$ is :

(A) interior of an ellipse
(B) exterior of a circle
(C) interior and boundary of an ellipse
(D) exterior of ellipse

C-10. The vector z = – 4 + 5i is turned counter clockwise through an angle of 180º & stretched 1.5 times. The complex number corresponding to the newly obtained vector is :

(A) $6 - \frac{15}{2} i$
(B) $-6 + \frac{15}{2} i$
(C) $6 + \frac{15}{2} i$
(D) $-6 - \frac{15}{2} i$

C-11. The points z_1, z_2, z_3, z_4 in the complex plane are the vertices of a parallelogram taken in order if and only if :
(A) $z_1 + z_4 = z_2 + z_3$ (B) $z_1 + z_3 = z_2 + z_4$ (C) $z_1 + z_2 = z_3 + z_4$ (D) $z_1 z_3 = z_2 z_4$

C-12. Complex numbers z_1 , z_2 , z_3 are the vertices A, B, C respectively of an isosceles right angled triangle with right angle at C and $(z_1 - z_2)^2 = k(z_1 - z_3)(z_3 - z_2)$, then find k
(A) 1 (B) 2 (C) 3 (D) –2

C-13. If z_1, z_2, z_3 are vertices of an equilateral triangle inscribed in the circle $|z| = 2$ and if $z_1 = 1 + i\sqrt{3}$, then
(A) $z_2 = -2$, $z_3 = 1 + i\sqrt{3}$ (B) $z_2 = 2$, $z_3 = 1 - i\sqrt{3}$
(C) $z_2 = -2$, $z_3 = 1 - i\sqrt{3}$ (D) $z_2 = 1 - i\sqrt{3}$, $z_3 = -1 - i\sqrt{3}$

Section (D) : Cube root of unity and n^th Root of Unity.

D-1. Let z_1 and z_2 be two non real complex cube roots of unity and $|z - z_1|^2 + |z - z_2|^2 = \lambda$ be the equation of a circle with z_1, z_2 as ends of a diameter then the value of λ is
(A) 4 (B) 3 (C) 2 (D) $\sqrt{2}$

D-2. If $x = a + b + c$, $y = a\alpha + b\beta + c$ and $z = a\beta + b\alpha + c$, where α and β are imaginary cube roots of unity, then xyz =
(A) $2(a^3 + b^3 + c^3)$ (B) $2(a^3 - b^3 - c^3)$ (C) $a^3 + b^3 + c^3 - 3abc$ (D) $a^3 - b^3 - c^3$

D-3. If 1, ω, ω^2 are the cube roots of unity, then $\Delta = \begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^n & \omega^{2n} & 1 \\ \omega^{2n} & 1 & \omega^n \end{vmatrix}$ is equal to-
(A) 0 (B) 1 (C) ω (D) ω^2

D-4. If $x^2 + x + 1 = 0$, then the numerical value of
$\left(x+\frac{1}{x}\right)^2 + \left(x^2+\frac{1}{x^2}\right)^2 + \left(x^3+\frac{1}{x^3}\right)^2 + \left(x^4+\frac{1}{x^4}\right)^2 + ... + \left(x^{27}+\frac{1}{x^{27}}\right)^2$ is equal to
(A) 54 (B) 36 (C) 27 (D) 18

D-5. If $a = 1 + \frac{x^3}{3!} + \frac{x^6}{6!} + ...$, $b = x + \frac{x^4}{4!} + \frac{x^7}{7!} + ...$, $c = \frac{x^2}{2!} + \frac{x^5}{5!} + \frac{x^8}{8!} + ...$
then find $a^3 + b^3 + c^3 - 3abc$.
(A) 1 (B) 2 (C) 3 (D) 4

D-6. If equation $(z - 1)^n = z^n = 1 (n \in N)$ has solutions, then n can be :
(A) 2 (B) 3 (C) 6 (D) 9

D-7. If α is non real and $\alpha = \sqrt[5]{1}$ then the value of $2^{|1 + \alpha + \alpha^2 + \alpha^{-2} - \alpha^{-1}|}$ is equal to
(A) 4 (B) 2 (C) 1 (D) 8

D-8. If $\alpha = e^{i8\pi/11}$ then Real $(\alpha + \alpha^2 + \alpha^3 + \alpha^4 + \alpha^5)$ equals to :
(A) (B) 1 (C) $-\frac{1}{2}$ (D) –1

PART - III : MATCH THE COLUMN

1. **Match the column**

Column – I (Complex number Z)		**Column – II (Principal argument of Z)**	
(A)	$Z = \dfrac{(1+i)^5(1+\sqrt{3}i)^2}{-2i(-\sqrt{3}+i)}$.	(p)	π
(B)	$Z = \sin\dfrac{6\pi}{5} + i\left(1+\cos\dfrac{6\pi}{5}\right)$ is	(q)	$-\dfrac{7\pi}{18}$
(C)	$Z = 1 + \cos\left(\dfrac{11\pi}{9}\right) + i\sin\left(\dfrac{11\pi}{9}\right)$	(r)	$\dfrac{9\pi}{10}$
(D)	$Z = \sin x \sin(x - 60) \sin(x + 60)$ where $x \in \left(0, \dfrac{\pi}{3}\right)$ and $x \in R$	(s)	$-\dfrac{5\pi}{12}$
		(t)	0

2.✍

Column I		**Column II (one of the values of z)**	
(A)	$z^4 - 1 = 0$	p.	$z = \cos\dfrac{\pi}{8} + i\sin\dfrac{\pi}{8}$
(B)	$z^4 + 1 = 0$	q.	$z = \cos\dfrac{\pi}{8} - i\sin\dfrac{\pi}{8}$
(C)	$iz^4 + 1 = 0$	r.	$z = \cos\dfrac{\pi}{4} + i\sin\dfrac{\pi}{4}$
(D)	$iz^4 - 1 = 0$	s.	$z = \cos 0 + i\sin 0$

3. Which of the condition/ conditions in column II are satisfied by the quadrilateral formed by z_1, z_2, z_3, z_4 in order given in column I ?

Column - I		**Column-II**	
(A)	Parallelogram	(p)	$z_1 - z_4 = z_2 - z_3$
(B)	Rectangle	(q)	$\lvert z_1 - z_3\rvert = \lvert z_2 - z_4\rvert$
(C)	Rhombus	(r)	$\dfrac{z_1 - z_2}{z_3 - z_4}$ is real
(D)	Square	(s)	$\dfrac{z_1 - z_3}{z_2 - z_4}$ is purely imaginary
		(t)	$\dfrac{z_1 - z_2}{z_3 - z_2}$ is purely imaginary

4.✍ Let z_1 lies on $| z | = 1$ and z_2 lies on $| z | = 2$.

Column – I		**Column – II**	
(A)	Maximum value of $\lvert z_1 + z_2\rvert$	(p)	3
(B)	Minimum value of $\lvert z_1 - z_2\rvert$	(q)	1
(C)	Minimum value of $\lvert 2z_1 + 3z_2\rvert$	(r)	4
(D)	Maximum value of $\lvert z_1 - 2z_2\rvert$	(s)	5

Exercise-2

✍ Marked questions are recommended for Revision.

PART - I : ONLY ONE OPTION CORRECT TYPE

1. $\sin^{-1}\left\{\frac{1}{i}(z-1)\right\}$, where z is nonreal, can be the angle of a triangle if

(A) Re(z) = 1, Im(z) = 2 (B) Re(z) = 1, 0 < Im (z) ≤ 1
(C) Re(z) + Im(z) = 0 (D) Re(z) = 2, 0 < Im (z) ≤ 1

2. If $|z|^2 - 2iz + 2c(1 + i) = 0$, then the value of z is, where c is real.

(A) $z = c + 1\, i(-1 \pm \sqrt{1-2c-c^2})$, where $c \in [-1-\sqrt{2}, -1+\sqrt{2}]$

(B) $z = c - 1\, i(-1 \pm \sqrt{1-2c-c^2})$, where $c \in [-1-\sqrt{2}, -1+\sqrt{2}]$

(C) $z = 2c + 1\, i(-1 \pm \sqrt{1-2c-c^2})$, where $c \in [-1-\sqrt{2}, -1+\sqrt{2}]$

(D) $z = c + 1\, i(-1 \pm)\sqrt{1-2c-c^2}$, where $c \in [-1-\sqrt{2}, 1+\sqrt{2}]$

3.✍ If $(a + ib)^5 = \alpha + i\beta$, then $(b + ia)^5$ is equal to

(A) $\beta + i\alpha$ (B) $\alpha - i\beta$ (C) $\beta - i\alpha$ (D) $-\alpha - i\beta$

4. Let z be non real number such that $\frac{1+z+z^2}{1-z+z^2} \in R$, then value of 7|z| is

(A) 1 (B) 3 (C) 5 (D) 7

5.✍ If $|z_1| = 2$, $|z_2| = 3$, $|z_3| = 4$ and $|z_1 + z_2 + z_3| = 2$, then the value of $|4z_2z_3 + 9z_3z_1 + 16z_1z_2|$

(A) 24 (B) 48 (C) 96 (D) 120

6. The minimum value of |3z–3| + |2z–4| equal to

(A) 1 (B) 2 (C) 3 (D) 4

7. If $|z_1 - 1| < 1$, $|z_2 - 2| < 2$, $|z_3 - 3| < 3$, then $|z_1 + z_2 + z_3|$

(A) is less than 6 (B) is more than 3
(C) is less than 12 (D) lies between 6 and 12

8. Let O = (0, 0) ; A=(3, 0) ; B = (0, –1) and C=(3, 2),then minimum value of |z|+|z–3|+|z+i|+|z–3–2i| occur at

(A) intersection point of AB and CO (B) intersection point of AC and BO
(C) intersection point of CB and AO (D) mean of O, A, B, C

9. Given z is a complex number with modulus 1. Then the equation $[(1 + ia)/(1 - ia)]^4 = z$ in 'a' has

(A) all roots real and distinct (B) two real and two imaginary
(C) three roots real and one imaginary (D) one root real and three imaginary

10.✍ The real values of the parameter 'a' for which at least one complex number z = x + iy satisfies both the equality $|z - ai| = a + 4$ and the inequality $|z - 2| < 1$.

(A) $\left(-\frac{21}{10}, -\frac{5}{6}\right)$ (B) $\left(-\frac{7}{2}, -\frac{5}{6}\right)$ (C) $\left(\frac{5}{6}, \frac{7}{2}\right)$ (D) $\left(-\frac{21}{10}, \frac{7}{2}\right)$

11. The points of intersection of the two curves $|z - 3| = 2$ and $|z| = 2$ in an argand plane are:

(A) $\frac{1}{2}\left(7 \pm i\sqrt{3}\right)$ (B) $\frac{1}{2}\left(3 \pm i\sqrt{7}\right)$ (C) $\frac{3}{2} \pm i\sqrt{\frac{7}{2}}$ (D) $\frac{7}{2} \pm i\sqrt{\frac{3}{2}}$

12. The equation of the radical axis of the two circles represented by the equations,
$|z - 2| = 3$ and $|z - 2 - 3i| = 4$ on the complex plane is :
(A) $3iz - 3i\bar{z} - 2 = 0$ (B) $3iz - 3i\bar{z} + 2 = 0$ (C) $iz - i\bar{z} + 1 = 0$ (D) $2iz - 2i\bar{z} + 3 = 0$

13. If $\log_{1/2}\left(\frac{|z-1| + 4}{3|z-1| - 2}\right) > 1$, then the locus of z is
(A) Exterior to circle with center 1 + i0 and radius 10
(B) Interior to circle with center 1 + i0 and radius 10
(C) Circle with center 1 + i0 and radius 10
(D) Circle with center 2 + i0 and radius 10

14. Points z_1 & z_2 are adjacent vertices of a regular octagon. The vertex z_3 adjacent to z_2 ($z_3 \neq z_1$) is represented by :

(A) $z_2 + \frac{1}{\sqrt{2}}(1 \pm i)(z_1 + z_2)$ (B) $z_2 + \frac{1}{\sqrt{2}}(1 \pm i)(z_1 - z_2)$

(C) $z_2 + \frac{1}{\sqrt{2}}(1 \pm i)(z_2 - z_1)$ (D) none of these

15. If $p = a + b\omega + c\omega^2$; $q = b + c\omega + a\omega^2$ and $r = c + a\omega + b\omega^2$ where a, b, c ≠ 0 and ω is the non-real complex cube root of unity, then :
(A) $p + q + r = a + b + c$ (B) $p^2 + q^2 + r^2 = a^2 + b^2 + c^2$
(C) $p^2 + q^2 + r^2 = 2(pq + qr + rp)$ (D) None of these

16. The points $z_1 = 3 + \sqrt{3}\,i$ and $z_2 = 2\sqrt{3} + 6i$ are given on a complex plane. The complex number lying on the bisector of the angle formed by the vectors z_1 and z_2 is :

(A) $z = \frac{(3+2\sqrt{3})}{2} + \frac{\sqrt{3}+2}{2}i$ (B) $z = 5 + 5i$

(C) $z = -1 - i$ (D) none

17. Let ω be the non real cube root of unity which satisfy the equation h(x) = 0 where $h(x) = x\,f(x^3) + x^2 g(x^3)$. If h(x) is polynomial with real coefficient then which statement is incorrect.
(A) f(1) = 0 (B) g(1) = 0 (C) h(1) = 0 (D) g(1) ≠ f(1)

18. If 1, $\alpha_1, \alpha_2, \alpha_3, \ldots\ldots, \alpha_{n-1}$ be the n^{th} roots of unity, then the value of

$\sin\frac{\pi}{n} \cdot \sin\frac{2\pi}{n} \cdot \sin\frac{3\pi}{n} \ldots\ldots \sin\frac{(n-1)\pi}{n}$ equals

(A) $\frac{n}{2^n}$ (B) $\frac{n}{2^{n-1}}$ (C) $\frac{n+1}{2^{n-1}}$ (D) $\frac{n}{2^{n+1}}$

PART - II : SINGLE AND DOUBLE VALUE INTEGER TYPE

1. If a and b are positive integer such that $N = (a + ib)^3 - 107i$ is a positive integer then find the value of $\frac{N}{2}$

2. Let z,w be complex numbers such that $\bar{z} + i\bar{w} = 0$ and arg zw = π. If Re(z) < 0 and principal arg $z = \frac{a\pi}{b}$ then find the value of a + b. (where a & b are co-prime natural numbers)

3. If $x = 9^{1/3}\, 9^{1/9}\, 9^{1/27} \ldots\infty$, $y = 4^{1/3}\, 4^{-1/9}\, 4^{1/27} \ldots \infty$, and $z = \sum_{r=1}^{\infty}(1+i)^{-r}$ and principal argument of $P = (x + yz)$ is $-\tan^{-1}\left(\frac{\sqrt{a}}{b}\right)$ then determine $a^2 + b^2$. (where a & b are co-prime natural numbers)

4. $z_1, z_2 \in c$ and $z_1^2 + z_2^2 \in R$,
$z_1(z_1^2 - 3z_2^2) = 2$, $z_2(3z_1^2 - z_2^2) = 11$
If $z_1^2 + z_2^2 = \lambda$ then determine λ^2

5. Let $|z| = 2$ and $w = \frac{z+1}{z-1}$ where $z, w \in C$ (where C is the set of complex numbers), then find product of maximum and minimum value of $|w|$.

6. A function 'f' is defined by $f(z) = (4 + i)z^2 + \alpha z + \gamma$ for all complex number z, where α and γ are complex numbers if f(1) and f(i) are both real and the smallest possible values of $|\alpha| + |\gamma|$ is p then determine p^2.

7. If z and ω are two non-zero complex numbers such that $|z\omega| = 1$, and $\arg(z) - \arg(\omega) = \frac{\pi}{2}$, then find the value of $5\, i\, \bar{z}\, \omega$.

8. Number of complex number satisfying $|z| = \max\{|z - 1|, |z + 1|\}$.

9. If z_1 & z_2 both satisfy the relation, $z + \bar{z} = 2\,|z - 1|$ and $\arg(z_1 - z_2) = \frac{\pi}{4}$, then find the imaginary part of $(z_1 + z_2)$.

10. If $a_1, a_2, a_3 \ldots a_n, A_1, A_2, A_3 \ldots A_n$, k are all real numbers and number of imaginary roots of the equation $\frac{A_1^2}{x - a_1} + \frac{A_2^2}{x - a_2} + \ldots\ldots + \frac{A_n^2}{x - a_n} = k$ is α. Then find the value of $\alpha + 15$.

11. How many complex number z such that $|z| < \frac{1}{3}$ and $\sum_{r=1}^{n} a_r z^r = 1$ where $|a_r| < 2$.

12. If a variable circle S touches $S_1 : |z - z_1| = 7$ internally and $S_2 : |z - z_2| = 4$ externally while the curves S_1 & S_2 touch internally to each other, $(z_1 \neq z_2)$. If the eccentricity of the locus of the centre of the curve S is 'e' find the value of 11e.

13. Given that, $|z - 1| = 1$, where ' z ' is a point on the argand plane. $\frac{z-2}{2z} = \alpha\, i \tan(\arg z)$. Then determine $\frac{1}{\alpha^4}$.

14. Area of the region formed by $|z| \le 4$ & $-\frac{\pi}{2} \le \arg z \le \frac{\pi}{3}$ on the Argand diagram is expressed in the form of $\frac{a\pi}{b}$. Then find the value of ab (where a & b are co-prime natural number)

15. The points A, B, C represent the complex numbers z_1, z_2, z_3 respectively on a complex plane & the angle B & C of the triangle ABC are each equal to $\frac{1}{2}(\pi - \alpha)$. If

$(z_2 - z_3)^2 = \lambda\, (z_3 - z_1)\, (z_1 - z_2) \sin^2\frac{\alpha}{2}$ then determine λ.

16. If ω and ω^2 are the non-real cube roots of unity and a, b, c $\in$ R such that $\frac{1}{a+\omega} + \frac{1}{b+\omega} + \frac{1}{c+\omega} = 2\omega^2$ and $\frac{1}{a+\omega^2} + \frac{1}{b+\omega^2} + \frac{1}{c+\omega^2} = 2\omega$. If $\frac{1}{a+1} + \frac{1}{b+1} + \frac{1}{c+1} = \lambda$ then determine λ^4

17. If $L = \lim_{n\to\infty}\left[\frac{n}{(1-n\omega)(1-n\omega^2)} + \frac{n}{(2-n\omega)(2-n\omega^2)} + ... + \frac{n}{(n-n\omega)(n-n\omega^2)}\right]$

then find the value of $\frac{\pi}{\sqrt{3}L}$ {where ω is non real cube root of unity}.

18. The value of $\sum_{k=1}^{6}\left(\sin\frac{2\pi k}{7} - i\cos\frac{2\pi k}{7}\right) = \alpha$ then find α^4

19. Let $Z_r = \left(e^{i\frac{2\pi}{15}}\right)^r$. If $\arg\left(\frac{1 + Z_1 + Z_2 + Z_3 + ... + Z_7}{1 + Z_8 + Z_9 + Z_{10} + ... + Z_{14}}\right) = \frac{a\pi}{b}$, then b – a equals. (where a & b are co-prime natural number)

20. If $A_1, A_2, ..., A_n$ be the vertices of an n-sided regular polygon such that $\frac{1}{A_1A_2} = \frac{1}{A_1A_3} + \frac{1}{A_1A_4}$, then find the value of n

PART - III : ONE OR MORE THAN ONE OPTIONS CORRECT TYPE

1. If the biquadratic $x^4 + ax^3 + bx^2 + cx + d = 0$ (a, b, c, d $\in$ R) has 4 non real roots, two with sum 3 + 4i and the other two with product 13 + i.

(A) b = 51 (B) a = –6 (C) c = –70 (D) d = 170

2. The quadratic equation $z^2 + (p + ip')\, z + q + iq' = 0$; where p, p′, q, q′ are all real.

(A) if the equation has one real root then $q'^2 - pp'q' + qp'^2 = 0$.
(B) if the equation has two equal roots then $pp' = 2q'$.
(C) if the equation has two equal roots then $p^2 - p'^2 = 4q$
(D) if the equation has one real root then $p'^2 - pp'q' + q'^2 = 0$.

3. The value of $i^n + i^{-n}$, for $i = \sqrt{-1}$ and $n \in I$ is :

(A) $\frac{2^n}{(1-i)^{2n}} + \frac{(1+i)^{2n}}{2^n}$ (B) $\frac{(1+i)^{2n}}{2^n} + \frac{(1-i)^{2n}}{2^n}$ (C) $\frac{(1+i)^{2n}}{2^n} + \frac{2^n}{(1-i)^{2n}}$ (D) $\frac{2^n}{(1+i)^{2n}} + \frac{2^n}{(1-i)^{2n}}$

4. If amp $(z_1z_2) = 0$ and $|z_1| = |z_2| = 1$, then

(A) $z_1 + z_2 = 0$ (B) $z_1z_2 = 1$ (C) $z_1 = \bar{z}_2$ (D) $z_1 = z_2$

5. Let z_1 and z_2 are two complex numbers such that $(1 - i)z_1 = 2z_2$ and $\arg(z_1z_2) = \frac{\pi}{2}$, then $\arg(z_2)$ is equal to

(A) $3\pi/8$ (B) $\pi/8$ (C) $5\pi/8$ (D) $-7\pi/8$

6. If $|z_1 + z_2|^2 = |z_1|^2 + |z_z|^2$ (where z_1 and z_2 are non-zero complex numbers), then

(A) $\frac{z_1}{z_2}$ is purely real (B) $\frac{z_1}{z_2}$ is purely imaginary

(C) $z_1\bar{z}_2 + z_2\bar{z}_1 = 0$ (D) amp $\frac{z_1}{z_2}$ may be equal to $\frac{\pi}{2}$

7. a, b, c are real numbers in the polynomial, $P(z) = 2z^4 + az^3 + bz^2 + cz + 3$. If two roots of the equation $P(z) = 0$ are 2 and i. Then which of the following are true.

(A) $a = -\frac{11}{2}$ (B) $b = 5$ (C) $c = -\frac{11}{2}$ (D) $a = -11$

8. If $Z = \frac{(1+i)(1+2i)(1+3i)......(1+ni)}{(1-i)(2-i)(3-i)......(n-i)}$, $n \in N$ then principal argument of Z can be

(A) 0 (B) $\frac{\pi}{2}$ (C) $-\frac{\pi}{2}$ (D) π

9. For complex numbers z and w, if $|z|^2w - |w|^2 z = z - w$. Which of the following can be true :

(A) $z = w$ (B) $z\bar{w} = 1$ (C) $z = w + 2$ (D) $\bar{z}w = 1$

10. If z satisfies the inequality $|z - 1 - 2i| \le 1$, then which of the following are true.

(A) maximum value of $|z| = \sqrt{5} + 1$ (B) minimum value of $|z| = \sqrt{5} - 1$

(C) maximum value of $\arg(z) = \pi/2$ (D) minimum value of $\arg(z) = \tan^{-1}\left(\frac{3}{4}\right)$

11. The curve represented by $z = \frac{3}{2 + \cos\theta + i\sin\theta}$, $\theta \in [0, 2\pi)$

(A) never meets the imaginary axis (B) meets the real axis in exactly two points

(C) has maximum value of |z| as 3 (D) has minimum value of |z| as 1

12. POQ is a straight line through the origin O . P and Q represent the complex number a + i b and c + i d respectively and OP = OQ. Then which of the follwoing are true :

(A) $|a + i b| = |c + i d|$ (B) $a + c = b + d$

(C) $\arg(a + i b) = \arg(c + i d)$ (D) none of these

13. Let $i = \sqrt{-1}$. Define a sequence of complex number by $z_1 = 0$, $z_{n+1} = z_n^2 + i$ for $n \ge 1$. Then which of the following are true.

(A) $|z_{2050}| = \sqrt{3}$ (B) $|z_{2017}| = \sqrt{2}$ (C) $|z_{2016}| = 1$ (D) $|z_{2111}| = \sqrt{2}$

14. If $|z_1| = |z_2| = ... = |z_n| = 1$ then which of the following are true.

(A) $\bar{z}_1 = \frac{1}{z_1}$

(B) $|z_1 + z_2 + ... + z_n| = \left|\frac{1}{z_1} + \frac{1}{z_2} + ... + \frac{1}{z_n}\right|$.

(C) Centroid of polygon with 2n vertices $z_1, z_2, ... z_n$ $\frac{1}{z_1}, \frac{1}{z_2}, ... \frac{1}{z_n}$ (need not be in order) lies on real axis.

(D) Centroid of polygon with 2n vertices $z_1, z_2, ... z_n$ $\frac{1}{z_1}, \frac{1}{z_2}, ... \frac{1}{z_n}$ (need not be in order) lies on imaginary axis.

15. If $2\cos\theta = x + \frac{1}{x}$ and $2\cos\phi = y + \frac{1}{y}$, then

(A) $x^n + \frac{1}{x^n} = 2\cos(n\theta),\ n \in z$

(B) $\frac{x}{y} + \frac{y}{x} = 2\cos(\theta - \phi)$

(C) $xy + \frac{1}{xy} = 2\cos(\theta + \phi)$

(D) $x^m y^n + \frac{1}{x^m y^n} = 2\cos(m\theta + n\phi),\ m, n \in z$

16. If $\left|\frac{z-\alpha}{z-\beta}\right| = k,\ k > 0$ where, $z = x + iy$ and $\alpha = \alpha_1 + i\alpha_2$, $\beta = \beta_1 + i\beta_2$ are fixed complex numbers. Then which of the following are true

(A) if $k \neq 1$ then locus is a circle whose centre is $\left(\frac{k^2\beta - \alpha}{k^2 - 1}\right)$

(B) if $k \neq 1$ then locus is a circle whose radius is $\left|\frac{k(\alpha - \beta)}{1 - k^2}\right|$

(C) if $k = 1$ then locus is perpendicular bisector of line joining $\alpha = \alpha_1 + i\alpha_2$ and $\beta = \beta_1 + i\beta_2$

(D) if $k \neq 1$ then locus is a circle whose centre is $\left(\frac{k^2\alpha - \beta}{k^2 - 1}\right)$

17. The locus of equation $\text{Arg}\left(\frac{z - 1 - 2i}{z + 3 + i}\right) = \frac{\pi}{3}$ represents part of circle in which

(A) centre is $\left[-1 - \frac{\sqrt{3}}{2} + i\left(\frac{1}{2} + \frac{2}{\sqrt{3}}\right)\right]$

(B) radius is $\frac{5}{\sqrt{3}}$

(C) centre is $\left[-1 - \frac{\sqrt{3}}{2} - i\left(\frac{1}{2} + \frac{2}{\sqrt{3}}\right)\right]$

(D) radius is $\frac{\sqrt{5}}{3}$

18. The equation $||z + i| - |z - i|| = k$ represents

(A) a hyperbola if $0 < k < 2$

(B) a pair of ray if $k > 2$

(C) a straight line if $k = 0$

(D) a pair of ray if $k = 2$

19. The equation $|z - i| + |z + i| = k,\ k > 0$, can represent

(A) an ellipse if $k > 2$

(B) line segment if $k = 2$

(C) an ellipse if $k = 5$

(D) no locus if $k = 1$

20. If $|z_1| = |z_2| = |z_3| = 1$ and z_1, z_2, z_3 are represented by the vertices of an equilateral triangle then

(A) $z_1 + z_2 + z_3 = 0$

(B) $z_1z_2z_3 = 1$

(C) $z_1z_2 + z_2z_3 + z_3z_1 = 0$

(D) $z_2^3 + z_3^3 = 2z_1^3$

21. Let z_1, z_2, z_3 be three distinct complex numbers satisfying, $|z_1-1| = |z_2-1| = |z_3-1| = 1$. Let A, B & C be the points representing vertices of equilateral triangle in the Argand plane corresponding to z_1, z_2 and z_3 respectively. Which of the following are true

(A) $z_1 + z_2 + z_3 = 3$ (B) $z_1^2 + z_2^2 + z_3^2 = 3$

(C) area of triangle = $\frac{3\sqrt{3}}{4}$ (D) $z_1z_2 + z_2z_3 + z_3z_1 = 1$

22. If 1, α_1, α_2, α_3,......., α_{n-1} be the n^{th} roots of unity, then which of the following are true

(A) $\frac{1}{1-\alpha_1} + \frac{1}{1-\alpha_2} + ... + \frac{1}{1-\alpha_{n-1}} = \frac{n-1}{2}$

(B) $(1 - \alpha_1)(1 - \alpha_2)(1 - \alpha_3)........(1 - \alpha_{n-1}) = n$.

(C) $(2 - \alpha_1)(2 - \alpha_2)(2 - \alpha_3)........(2 - \alpha_{n-1}) = 2^n - 1$

(D) $\frac{1}{1-\alpha_1} + \frac{1}{1-\alpha_2} + ... + \frac{1}{1-\alpha_{n-1}} = \frac{n}{2}$

23. Which of the following are true.

(A) $\cos x + {}^nC_1 \cos 2x + {}^nC_2 \cos 3x + ... + {}^nC_n \cos (n + 1) x = 2^n . \cos^n \frac{x}{2} . \cos \left(\frac{n+2}{2}\right) x$

(B) $\sin x + {}^nC_1 \sin 2x + {}^nC_2 \sin 3x + ... + {}^nC_n \sin (n + 1) x = 2^n . \cos^n \frac{x}{2} . \sin \left(\frac{n+2}{2}\right) x$

(C) $1 + {}^nC_1 \cos x + {}^nC_2 \cos 2x + ... + {}^nC_n \cos nx = 2^n . \cos^n \frac{x}{2} . \cos\left(\frac{nx}{2}\right)$

(D) ${}^nC_1 \sin x + {}^nC_2 \sin 2x + ... + {}^nC_n \sin n x = 2^n . \cos^n \frac{x}{2} . \sin\left(\frac{nx}{2}\right)$

24. If α, β, γ are distinct roots of $x^3 - 3x^2 + 3x + 7 = 0$ and ω is non-real cube root of unity, then the value of $\frac{\alpha-1}{\beta-1} + \frac{\beta-1}{\gamma-1} + \frac{\gamma-1}{\alpha-1}$ can be equal to :

(A) ω^2 (B) $2\omega^2$ (C) $3\omega^2$ (D) 3ω

25. If z is a complex number then the equation $z^2 + z|z| + |z^2| = 0$ is satisfied by (ω and ω^2 are imaginary cube roots of unity)

(A) $z = k\omega$ where $k \in R$ (B) $z = k\omega^2$ where k is non negative real

(C) $z = k\omega$ where k is positive real (D) $z = k\omega^2$ where $k \in R$.

26. If α is imaginary n^{th} $(n \geq 3)$ root of unity. Which of the following are true.

(A) $\sum_{r=1}^{n-1}(n-r)\,\alpha^r = \frac{n\alpha}{1-\alpha}$ (B) $\sum_{r=1}^{n-1}(n-r)\sin\frac{2r\pi}{n} = \frac{n}{2}\cot\frac{\pi}{n}$.

(C) $\sum_{r=1}^{n-1}(n-r)\,\cos\frac{2r\pi}{n} = -\frac{n}{2}$ (D) $\sum_{r=1}^{n-1}(n-r)\,\alpha^r = \frac{n}{1-\alpha}$

27. Which of the following is true
(A) roots of the equation $z^{10} - z^5 - 992 = 0$ with real part positive = 5
(B) roots of the equation $z^{10} - z^5 - 992 = 0$ with real part negative = 5
(C) roots of the equation $z^{10} - z^5 - 992 = 0$ with imaginary part non-negative = 6
(D) roots of the equation $z^{10} - z^5 - 992 = 0$ with imaginary part negative = 4

PART - IV : COMPREHENSION

Comprehension # 1 (Q. No. 1 - 2)

Let $(1 + x)^n = C_0 + C_1x + C_2x^2 + ...+ C_nx^n$. For sum of series $C_0 + C_1 + C_2 + ...$, put x = 1. For sum of series $C_0 + C_2 + C_4 + C_6 + ...$, or $C_1 + C_3 + C_5 + ...$ add or substract equations obtained by putting x = 1 and x = – 1.
For sum of series $C_0 + C_3 + C_6 + ...$ or $C_1 + C_4 + C_7 + ...$ or $C_2 + C_5 + C_8 + ...$ we subsitute x = 1, $x = \omega$, $x = \omega^2$ and add or manipulate results.
Similarly, if suffixes differe by 'p' then we substitute p^{th} roots of unity and add.

1. $C_0 + C_3 + C_6 + C_9 +........ =$
(A) $\frac{1}{3}\left[2^n - 2\cos\frac{n\pi}{3}\right]$ (B) $\frac{1}{3}\left[2^n + 2\cos\frac{n\pi}{3}\right]$ (C) $\frac{1}{3}\left[2^n - 2\sin\frac{n\pi}{3}\right]$ (D) $\frac{1}{3}\left[2^n + 2\sin\frac{n\pi}{3}\right]$

2. $C_1 + C_5 + C_9 +.... =$
(A) $\frac{1}{4}\left[2^n - 2^{n/2}2\cos\frac{n\pi}{4}\right]$ (B) $\frac{1}{4}\left[2^n + 2^{n/2}2\cos\frac{n\pi}{4}\right]$
(C) $\frac{1}{4}\left[2^n - 2^{n/2}2\sin\frac{n\pi}{4}\right]$ (D) $\frac{1}{4}\left[2^n + 2^{n/2}2\sin\frac{n\pi}{4}\right]$

Comprehension # 2 (Q. No. 3 to 6)

As we know $e^{i\theta} = \cos\theta + i\sin\theta$ and $(\cos\theta_1 + i\sin\theta_1).(\cos\theta_2 + i\sin\theta_2) = \cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)$
Let $\alpha, \beta, \gamma \in R$ such that $\cos(\alpha - \beta) + \cos(\beta - \gamma) + \cos(\gamma - \alpha) = -\frac{3}{2}$

3. $\Sigma \sin(\alpha + \beta) = \Sigma \cos(\alpha + \beta) =$
(A) 0 (B) $3\cos\alpha \cos\beta \cos\gamma$ (C) $3\cos(\alpha + \beta + \gamma)$ (D) 3

4. $\Sigma\cos(2\alpha - \beta - \gamma)$
(A) 0 (B) $3\cos\alpha \cos\beta \cos\gamma$ (C) $3\cos(\alpha + \beta + \gamma)$ (D) 3

5. $\Sigma \cos 3\alpha =$
(A) 0 (B) $3\cos\alpha \cos\beta \cos\gamma$ (C) $3\cos(\alpha + \beta + \gamma)$ (D) 3

6. If $\theta \in R$ then $\frac{\Sigma\cos^3(\theta+\alpha)}{\Pi\cos(\theta+\alpha)} =$
(A) 0 (B) $3\cos\alpha \cos\beta \cos\gamma$ (C) $3\cos(\alpha + \beta + \gamma)$ (D) 3

Comprehension # 3(Q. No. 7 to 8)
ABCD is a rhombus. Its diagonals AC and BD intersect at the point M and satisfy BD = 2AC. Let the points D and M represent complex numbers 1 + i and 2 – i respecetively.
If θ is arbitary real, then $z = re^{i\theta}$, $R_1 \le r \le R_2$ lies in annular region formed by concentric circles $|z| = R_1$, $|z| = R_2$.

7. A possible representation of point A is

(A) $3 - \frac{i}{2}$ (B) $3 + \frac{i}{2}$ (C) $1 + \frac{3}{2}i$ (D) $3 - \frac{3}{2}i$

8. If z is any point on segment DM then $w = e^{iz}$ lies in annular region formed by concentric circles.

(A) $|w|_{min} = 1, |w|_{max} = 2$ (B) $|w|_{min} = \frac{1}{e}, |w|_{max} = e$

(C) $|w|_{min} = \frac{1}{e^2}, |w|_{max} = e^2$ (D) $|w|_{min} = \frac{1}{2}, |w|_{max} = 1$

Comprehension # 4 (Q. No. 9 to 10)

Logarithm of a complex number is given by

$\log_e(x + iy) = \log_e(|z|e^{i\theta})$

$= \log_e|z| + \log_e e^{i\theta}$

$= \log_e |z| + i\theta$

$= \log_e \sqrt{(x^2 + y^2)} + i \arg(z)$

$\therefore \quad \log_e(z) = \log_e|z| + i \arg(z)$

In general $\log_e (x + iy) = \frac{1}{2} \log_e (x^2 + y^2) + i \left(2n\pi + \tan^{-1}\frac{y}{x}\right)$ where $n \in I$.

9.✍ Write $\log_e (1 + \sqrt{3}\, i)$ in (a + ib) form

(A) $\log_e 2 + i(2n\pi + \frac{\pi}{3})$ (B) $\log_e 3 + i(n\pi + \frac{\pi}{3})$

(C) $\log_e 2 + i(2n\pi + \frac{\pi}{6})$ (D) $\log_e 2 + i(2n\pi - \frac{\pi}{3})$

10.✍ Find the real part of $(1 - i)^{-i}$.

(A) $e^{\pi/4 + 2n\pi} \cos\left(\frac{1}{2}\log_e 2\right)$ (B) $e^{-\pi/4 + 2n\pi} \cos\left(\frac{1}{2}\log_e 2\right)$

(C) $e^{-\pi/4 + 2n\pi} \cos\left(\log_e 2\right)$ (D) $e^{-\pi/2 + 2n\pi} \cos\left(\frac{1}{2}\log_e 2\right)$

Exercise-3

PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

✍ Marked questions are recommended for Revision.

*** Marked Questions may have more than one correct option.**

1. Let $z = \cos\theta + i \sin\theta$. Then the value of $\sum_{m=1}^{15} \text{Im}\ (z^{2m-1})$ at $\theta = 2^\circ$ is

[IIT-JEE-2009, Paper-I, (3, – 1), 80]

(A) $\frac{1}{\sin 2^\circ}$ (B) $\frac{1}{3\sin 2^\circ}$ (C) $\frac{1}{2\sin 2^\circ}$ (D) $\frac{1}{4\sin 2^\circ}$

2.✍ Let z = x + iy be a complex number where x and y are integers. Then the area of the rectangle whose vertices are the roots of the equation $z\bar{z}^3 + \bar{z}z^3 = 350$ is **[IIT-JEE-2009, Paper-I, (3, – 1), 80]**

(A) 48 (B) 32 (C) 40 (D) 80

3*. Let z_1 and z_2 be two distinct complex numbers and let $z = (1 - t) z_1 + tz_2$ for some real number t with $0 < t < 1$. If Arg(w) denotes the principal argument of a nonzero complex number w, then

(A) $|z - z_1| + |z - z_2| = |z_1 - z_2|$ (B) $\text{Arg}(z - z_1) = \text{Arg}(z - z_2)$

(C) $\begin{vmatrix} z - z_1 & \bar{z} - \bar{z}_1 \\ z_2 - z_1 & \bar{z}_2 - \bar{z}_1 \end{vmatrix} = 0$ (D) $\text{Arg}(z - z_1) = \text{Arg}(z_2 - z_1)$

[IIT-JEE-2010, Paper-1, (3, 0)/84]

4. Let ω be the complex number $\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}$. Then the number of distinct complex numbers z satisfying $\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 0$ is equal to **[IIT-JEE-2010, Paper-1, (3, 0)/84]**

5. Match the statements in **Column-I** with those in **Column-II.** **[IIT-JEE-2010, Paper-2, (8, 0)/79]**

[Note : Here z takes values in the complex plane and Im z and Re z denote, respectively, the imaginary part and the real part of z.]

Column-I	**Column-II**
(A) The set of points z satisfying $|z - i|z|| = |z + i|z||$ is contained in or equal to	(p) an ellipse with eccentricity $\frac{4}{5}$
(B) The set of points z satisfying $|z + 4| + |z - 4| = 10$ is contained in or equal to	(q) the set of points z satisfying Im z = 0
(C) If $|w| = 2$, then the set of points $z = w - \frac{1}{w}$ is contained in or equal to	(r) the set of point z satisfying $|\text{Im } z| \le 1$
(D) If $|w| = 1$, then the set of points $z = w + \frac{1}{w}$ is contained in or equal to	(s) the set of points z satisfying $|\text{Re } z| \le 2$
	(t) the set of points z satisfying $|z| \le 3$

6. If z is any complex number satisfying $|z - 3 - 2i| \le 2$, then the minimum value of $|2z - 6 + 5i|$ is
[IIT-JEE 2011, Paper-1, (4, 0), 80]

7. Let $\omega = e^{\frac{\pi i}{3}}$, and a, b, c, x, y, z be non-zero complex numbers such that

$$a + b + c = x$$
$$a + b\omega + c\omega^2 = y$$
$$a + b\omega^2 + c\omega = z.$$

Then the value of $\frac{|x|^2 + |y|^2 + |z|^2}{|a|^2 + |b|^2 + |c|^2}$ is **[IIT-JEE 2011, Paper-2, (4, 0), 80]**

8. Let z be a complex number such that the imaginary part of z is non zero and $a = z^2 + z + 1$ is real. Then a cannot take the value **[IIT-JEE 2012, Paper-1, (3, –1), 70]**

(A) –1 (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{3}{4}$

9. Let complex numbers α and $\frac{1}{\bar{\alpha}}$ lies on circles $(x - x_0)^2 + (y - y_0)^2 = r^2$ and $(x - x_0)^2 + (y - y_0)^2 = 4r^2$, respectively. If $z_0 = x_0 + iy_0$ satisfies the equation $2|z_0|^2 = r^2 + 2$, then $|\alpha| =$

[JEE (Advanced) 2013, Paper-1, (2, 0)/60]

(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{1}{2}$ (C) $\frac{1}{\sqrt{7}}$ (D) $\frac{1}{3}$

10.* Let $w = \frac{\sqrt{3}+i}{2}$ and $P = \{w^n : n = 1,2,3,....\}$. Further $H_1 = \left\{z \in C : \text{Re } z > \frac{1}{2}\right\}$ and $H_2 = \left\{z \in C : \text{Re } z < -\frac{1}{2}\right\}$, where C is the set of all complex numbers. If $z_1 \in P \cap H_1$, $z_2 \in P \cap H_2$ and O represents the origin, then $\angle z_1 O z_2 =$ **[JEE (Advanced) 2013, Paper-2, (3, –1)/60]**

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{6}$ (C) $\frac{2\pi}{3}$ (D) $\frac{5\pi}{6}$

11.* Let ω be a complex cube root of unity with $\omega \neq 1$ and $P = [p_{ij}]$ be a n × n matrix with $p_{ij} = \omega^{i+j}$. Then $P^2 \neq 0$, when n = **[JEE (Advanced) 2013, Paper-2, (3, –1)/60]**

(A) 57 (B) 55 (C) 58 (D) 56

Paragraph for Question Nos. 12 to 13

Let $S = S_1 \cap S_2 \cap S_3$, where

$S_1 = \{z \in C : |z| < 4\}$, $S_2 = \left\{z \in C : \text{Im}\left[\frac{z-1+\sqrt{3}\, i}{1-\sqrt{3}\, i}\right] > 0\right\}$ and

$S_3 : \{z \in C : \text{Re } z > 0\}$.

12. Area of S = **[JEE (Advanced) 2013, Paper-2, (3, –1)/60]**

(A) $\frac{10\pi}{3}$ (B) $\frac{20\pi}{3}$ (C) $\frac{16\pi}{3}$ (D) $\frac{32\pi}{3}$

13. $\min_{z \in S} |1 - 3i - z| =$ **[JEE (Advanced) 2013, Paper-2, (3, –1)/60]**

(A) $\frac{2-\sqrt{3}}{2}$ (B) $\frac{2+\sqrt{3}}{2}$ (C) $\frac{3-\sqrt{3}}{2}$ (D) $\frac{3+\sqrt{3}}{2}$

14. Let $z_k = \cos\left(\frac{2k\pi}{10}\right) + i\sin\left(\frac{2k\pi}{10}\right)$; $k = 1, 2, \dots 9$. **[JEE (Advanced) 2014, Paper-2, (3, –1)/60]**

List I		**List II**	
P.	For each z_k there exists a z_j such that $z_k . z_j = 1$	**1.**	True
Q.	There exists a $k \in \{1,2,\dots,9\}$ such that $z_1 . z = z_k$ has no solution z in the set of complex numbers.	**2.**	False
R.	$\frac{\|1-z_1\|\|1-z_2\|\dots\|1-z_9\|}{10}$ equals	**3.**	1
S.	$1 - \sum_{k=1}^{9}\cos\left(\frac{2k\pi}{10}\right)$ equals	**4.**	2

	P	Q	R	S
(A)	1	2	4	3
(B)	2	1	3	4
(C)	1	2	3	4
(D)	2	1	4	3

15. For any integer k, let $\alpha_k = \cos\left(\frac{k\pi}{7}\right) + i\sin\left(\frac{k\pi}{7}\right)$, where $i = \sqrt{-1}$. The value of the expression $\frac{\sum_{k=1}^{12}|\alpha_{k+1} - \alpha_k|}{\sum_{k=1}^{3}|\alpha_{4k-1} - \alpha_{4k-2}|}$ is **[JEE (Advanced) 2015, P-2 (4, 0) / 80]**

16. Let $z = \frac{-1+\sqrt{3}\,i}{2}$, where $i = \sqrt{-1}$ and $r, s \in \{1,2,3\}$. Let $P = \begin{bmatrix} (-z)^r & z^{2s} \\ z^{2s} & z^r \end{bmatrix}$ and I be the identity matrix of order 2. Then the total number of ordered pairs (r, s) for which $P^2 = -I$ is
[JEE (Advanced) 2016, Paper-1, (3, 0)/62]

17. Let $a, b \in R$ and $a^2 + b^2 \neq 0$. Suppose $S = \left\{z \in R : z = \frac{1}{a + ibt}, t \in R, t \neq 0\right\}$, where $i = \sqrt{-1}$.
If $z = x + iy$ and $z \in S$, then (x, y) lies on **[JEE (Advanced) 2016, Paper-2, (4, –2)/62]**

(A) the circle with radius $\frac{1}{2a}$ and centre $\left(\frac{1}{2a}, 0\right)$ for $a > 0$, $b \neq 0$

(B) the circle with radius $-\frac{1}{2a}$ and centre $\left(\frac{1}{2a}, 0\right)$ for $a < 0$, $b \neq 0$

(C) the x-axis for $a \neq 0$, $b = 0$

(D) the y-axis for $a = 0$, $b \neq 0$

18. Let a, b, x and y be real numbers such that $a - b = 1$ and $y \neq 0$. If the complex number $z = x + iy$ satisfies $\text{Im}\left(\frac{az+b}{z+1}\right) = y$, then which of the following is(are) possible value(s) of x ?

[JEE(Advanced) 2017, Paper-1,(4, –2)/61]

(A) $1 - \sqrt{1+y^2}$ (B) $-1 - \sqrt{1-y^2}$ (C) $1 + \sqrt{1+y^2}$ (D) $-1 + \sqrt{1-y^2}$

19. For a non-zero complex number z, let arg(z) denote the principal argument with $-\pi < \arg(z) \le \pi$. Then, which of the following statement(s) is (are) **FALSE** ?

(A) $\text{Arg}(-1 - i) = \frac{\pi}{4}$, where $i = \sqrt{-1}$ **[JEE(Advanced) 2018, Paper-1,(4, –2)/60]**

(B) The function $f : R \to (-\pi, \pi]$, defined by $f(t) = \arg(-1 + it)$ for all $t \in R$, is continuous at all points of R, where $i = \sqrt{-1}$

(C) For any two non-zero complex numbers z_1 and z_2, $\arg\left(\frac{z_1}{z_2}\right) - \arg(z_1) + \arg(z_2)$ is an integer multiple of 2π

(D) For any three given distinct complex numbers z_1, z_2 and z_3, the locus of the point z satisfying the condition $\arg\left(\frac{(z - z_1)(z_2 - z_3)}{(z - z_3)(z_2 - z_1)}\right) = \pi$, lies on a straight line.

20. Let s, t, r be non-zero complex numbers and L be the set of solutions $z = x + iy$ $(x, y \in R, i = \sqrt{-1})$ of the equation $sz + t\bar{z} + r = 0$, where $\bar{z} = x - iy$. Then,which of the following statement(s) is (are) TRUE ?

(A) If L has exactly one element, then $|s| \ne |t|$ **[JEE(Advanced) 2018, Paper-2,(4, –2)/60]**

(B) If $|s| = |t|$, then L has infinitely many elements

(C) The number of elements in $L \cap \{z : |z - 1 + i| = 5\}$ is at most 2

(D) If L has more than one element, then L has infinitely many elements

PART - II : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. If $\left|z - \frac{4}{z}\right| = 2$, then the maximum value of |z| is equal to : **[AIEEE 2009, (4, –1), 144]**

(1) $\sqrt{5} + 1$ (2) 2 (3) $2 + \sqrt{2}$ (4) $\sqrt{3} + 1$

2. If α and β are the roots of the equation $x^2 - x + 1 = 0$, then $\alpha^{2009} + \beta^{2009} =$ **[AIEEE 2010, (4, –1), 144]**

(1) – 1 (2) 1 (3) 2 (4) –2

3. The number of complex numbers z such that $|z - 1| = |z + 1| = |z - i|$ equals **[AIEEE 2010, (4, –1), 120]**

(1) 1 (2) 2 (3) ∞ (4) 0

4. If $\omega(\ne 1)$ is a cube root of unity, and $(1 + \omega)^7 = A + B\omega$. Then (A, B) equals **[AIEEE 2011, I, (4, –1), 120]**

(1) (0, 1) (2) (1, 1) (3) (1, 0) (4) (–1, 1)

5. Let α, β be real and z be a complex number. If $z^2 + \alpha z + \beta = 0$ has two distinct roots on the line Re z = 1, then it is necessary that : **[AIEEE- 2011, I, (4, –1), 120]**

(1) $\beta \in (0, 1)$ (2) $\beta \in (-1, 0)$ (3) $|\beta| = 1$ (4) $\beta \in (1, \infty)$

6. If z is a complex number of unit modulus and argument θ, then $\arg\left(\frac{1+z}{1+\bar{z}}\right)$ equals :

(1) $-\theta$ (2) $\frac{\pi}{2}-\theta$ (3) θ (4) $\pi-\theta$

[AIEEE - 2013, (4, –1), 120]

7. If z a complex number such that $|z| \geq 2$, then the minimum value of $\left|z+\frac{1}{2}\right|$:

(1) is strictly greater than 5/2 **[JEE(Main) 2014, (4, – 1), 120]**
(2) is strictly greater than 3/2 but less than 5/2
(3) is equal to 5/2
(4) lie in the interval (1, 2)

8. A complex number z is said to be unimodular if $|z| = 1$. Suppose z_1 and z_2 are complex numbers such that $\frac{z_1-2z_2}{2-z_1\bar{z}_2}$ is unimodular and z_2 is not unimodular. Then the point z_1 lies on a :

[JEE(Main) 2015, (4, – 1), 120]

(1) straight line parallel to x-axis (2) straight line parallel to y-axis
(3) circle of radius 2 (4) circle of radius $\sqrt{2}$

9. A value of θ for which $\frac{2+3i\sin\theta}{1-2i\sin\theta}$ is purely imaginary, is : **[JEE(Main) 2016, (4, – 1), 120]**

(1) $\frac{\pi}{6}$ (2) $\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$ (3) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (4) $\frac{\pi}{3}$

10. Let ω be a complex number such that $2\omega + 1 = z$ where $z = \sqrt{-3}$. If $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2-1 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix} = 3k$, then k is equal to : **[JEE(Main) 2017, (4, – 1), 120]**

(1) $-z$ (2) z (3) –1 (4) 1

11. If $\alpha, \beta \in C$ are the distinct roots, of the equation $x^2 - x + 1 = 0$, then $\alpha^{101} + \beta^{107}$ is equal to :
[JEE(Main) 2018, (4, – 1), 120]

(1) 1 (2) 2 (3) –1 (4) 0

12. Let α and β be two roots of the equation $x^2 + 2x + 2 = 0$, then $\alpha^{15} + \beta^{15}$ is equal to :
[JEE(Main) 2019, Online (09-01-19),P-1 (4, – 1), 120]

(1) 512 (2) –256 (3) 256 (4) –512

13. Let z be a complex number such that $|z| + z = 3 + i$, (where $i = \sqrt{-1}$) then $|z|$ is equal to :
[JEE(Main) 2019, Online (11-01-19),P-2 (4, – 1), 120]

(1) $\frac{\sqrt{34}}{3}$ (2) 5/4 (3) 5/3 (4) $\frac{\sqrt{41}}{4}$

14. Let z_1 and z_2 be two complex numbers satisfying $|z_1| = 9$ and $|z_2 - 3 - 4i| = 4$. Then the minimum value of $|z_1 - z_2|$ is : **[JEE(Main) 2019, Online (12-01-19),P-2 (4, – 1), 120]**

(1) 0 (2) 1 (3) $\sqrt{2}$ (4) 2

Answers

EXERCISE - 1

PART – I

A-1. (i) 3, –1 (ii) x = 1 and y = 2 (iii) (1, 1) $\left(0, \frac{5}{2}\right)$ (iv) $x = K, y = \frac{3K}{2}$, $K \in R$

A-2_. (i) 8 (ii) 10π **A-3.** (i) [(–2, 2) ; (–2, –2)] (ii) – (77 + 108 i)

A-4. (i) ± (4 + 3i) (ii) $\pm\sqrt{2}$ + 0i or 0 $\pm\sqrt{2}$ I **A-5.** z = (2 + i) or (1 – 3i)

A-6. (i) $\frac{21}{5} - \frac{12}{5}$ i (ii) 3 + 4 i (iii) $\frac{1}{2\left(1+3\cos^2\frac{\theta}{2}\right)} + i\frac{-\cot\frac{\theta}{2}}{1+3\cos^2\frac{\theta}{2}}$ (iv) 2

A-7. (i) $\pi e^{i\pi}$ (ii) $5e^{i\frac{\pi}{2}}$ (iii) $2e^{-i\frac{5\pi}{6}}$ c (iv) $2e^{-i\frac{4\pi}{5}}$

A-8. (i) $|z| = 2\cos\frac{9\pi}{25}$ Principal Arg $z = \frac{9\pi}{25}$, arg $z = \frac{9\pi}{25} + 2k\pi$, $k \in I$

(ii) Modulus = 2, Arg = $2k\pi - \frac{5\pi}{6}$, $k \in I$, Principal Arg = $-\frac{5\pi}{6}$

(iii) Modulus = $\sec^2 1$, arg = $2k\pi + (2 - \pi)$, Principal Arg = $(2 - \pi)$

(iv) Modulus = $\frac{1}{\sqrt{2}}$ cosec $\frac{\pi}{5}$, arg z = $2k\pi + \frac{11\pi}{20}$, Principal Arg = $\frac{11\pi}{20}$

A-9. $\frac{iz}{2} + \frac{1}{2} + i$ **A-10.** (i) 4 (ii) $\sqrt{3}$

Section (B) :

B-1. (i) $(a - ib)^2$ **B-2.** π **B-3.** 1 **B-7.** 0 **B-8.** (i) $\pm\frac{\pi}{2}$ (ii) 1

B-10. arg $z \in \left[-\frac{\pi}{2}, -\frac{\pi}{4}\right]$

Section (C) :

C-1. $\sqrt{5} + 2$ & $\sqrt{5} - 2$ **C-2.** (i) 0, 6 (ii) 1, 7 (iii) 0, 5

C-3. (i) The region between the concentric circles with centre at (0, 2) & radii 1 and 3 units

(ii) The part of the complex plane on or above the line y = 1

(iii) a ray emanating from the point (3 + 4i) directed away from the origin & having equation, $\sqrt{3}x - y + 4 - 3\sqrt{3} = 0$, $x > 3$

(iv) Region outside or on the circle with centre $\frac{1}{2}$ + 2i and radius $\frac{1}{2}$

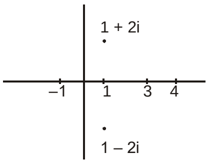
C-4. (i) |z| = 20 (ii) OP = OQ = PR = QR = 20 **C-7.** $9\frac{\pi}{\sqrt{2}}$ **C-8.** 5 **C-9.** –4 – 3 i, $2\sqrt{5}$

C-10. a rhombous but not a square

Section (D) :

D-1. 3 **D-2.** –5 **D-3.** 4^n **D-4.** (i) 1 (ii) 1

D-6. z = – 1, 3, 1 – 2 i, 1 + 2 i



Sum = 4
centroid = 1

D-7. (i) – 1 (ii) $e^{(6n+1)\frac{\pi}{4}i}$, n = 0, 1, 2, 3. Continued product = 1

D-8. ω **D-9.** $x^2 + x + 2 = 0$

PART - II

Section (A) :

A-1. (A) **A-2.** (D) **A.3.** (B) **A-4.** (A) **A-5.** (D) **A-6.** (D) **A-7.** (D)
A-8. (B) **A-9.** (A) **A-10.** (A) **A-11.** (C) **A-12.** (D)

Section (B) :

B-1. (A) **B-2.** (C) **B-3.** (A) **B-4.** (D) **B-5.** (D) **B-6.** (A) **B-7.** (D)
B-8. (D) **B-9.** (C) **B-10.** (C) **B-11.** (A) **B-12.** (B) **B-13.** (D)

Section (C) :

C-1. (A) **C-2.** (D) **C-3.** (D) **C-4.** (A) **C-5.** (B) **C-6.** (A) **C-7.** (C)
C-8. (B) **C-9.** (C) **C-10.** (A) **C-11.** (B) **C-12.** (B) **C-13.** (C)

Section (D) :

D-1. (B) **D-2.** (C) **D-3.** (A) **D-4.** (A) **D-5.** (A) **D-6.** (C) **D-7.** (A)
D-8. (C)

PART - III

1. (A) → (s), (B) → (r), (C) → (q), (D) → (p) **2.** A → s; B → r; C → p; D → q.

3. a → p, r; b → p,q,r, t; c → p,r,s; d → p,q,r,s, t. **4.** (A) → (p), (B) → (q), (C) → (r), (D) → (s)

EXERCISE - 2

PART - I

1. (B) **2.** (A) **3.** (A) **4.** (D) **5.** (B) **6.** (B) **7.** (C) **8.** (C)
9. (A) **10.** (A) **11.** (B) **12.** (B) **13.** (A) **14.** (C) **15.** (C) **16.** (B)
17. (D) **18.** (B)

PART - II

1.	99	**2.**	7	**3.**	13	**4.**	25	**5.**	1	**6.**	2	**7.**	5
8.	0	**9.**	2	**10.**	15	**11.**	0	**12.**	3	**13.**	16	**14.**	60
15.	4	**16.**	16	**17.**	3	**18.**	11	**9.**	1	**20.**	7		

PART - III

1.	(ABCD)	**2.**	(ABC)	**3.**	(BD)	**4.**	(BC)	**5.**	(BD)	**6.**	(BCD)
7.	(ABC)	**8.**	(ABCD)	**9.**	(ABD)	**10.**	(ABCD)	**11.**	(ABCD)	**12.**	(AB)
13.	(BCD)	**14.**	(ABC)	**15.**	(ABCD)	**16.**	(ABC)	**17.**	(AB)	**18.**	(ACD)
19.	(ABCD)	**20.**	(ACD)	**21.**	(ABC)	**22.**	(ABC)	**23.**	(ABCD)	**24.**	(CD)
25.	(BC)	**26.**	(ABC)	**27.**	(ABCD)						

PART - IV

1.	(B)	**2.**	(D)	**3.**	(A)	**4.**	(D)	**5.**	(C)	**6.**	(D)	**7.**	(A)
8.	(B)	**9.**	(A)	**10.**	(B)	**9.**	(A)	**10.**	(B)				

EXERCISE - 3

PART - I

1.	(D)	**2.**	(A)	**3*.**	(ACD)	**4.**	1

5. (A) - (q,r), (B)-(p), (C) - (p,s,t), (D) - (q,r,s,t) **6.** 5

7. Bonus (w = $e^{i\pi/3}$ is a typographical error, because of this the answer cannot be an integer.)

(if w = $e^{i\frac{2\pi}{3}}$ then answer comes out to be 3)

8.	(D)	**9.**	(C)	**10.***	(CD)	**11.***	(BCD)	**11.**	(B)	**13.**	(C)	**14.**	(C)
15.	4	**16.**	1	**17.**	(ACD)	**18.**	(BD)	**19.**	(ABD)	**20.**	(ACD)		

PART - II

1.	(1)	**2.**	(2)	**3.**	(1)	**4.**	(2)	**5.**	(4)	**6.**	(3)		
7.	(4)	**8.**	(3)	**9.**	(3)	**10.**	(1)	**11.**	(1)	**12.**	(2)	**13.**	(3)
14.	(1)												

Advance Level Problems (ALP)

1. If the equation $z^4 + a_1z^3 + a_2z^2 + a_3z + a_4 = 0$ where a_1, a_2, a_3, a_4 are real coefficient different from zero, has a purely imaginary root, then find the value of $\frac{a_3}{a_1a_2} + \frac{a_1a_4}{a_2a_3}$.

2. If $|z_1| = 2$, $|z_2| = 3$, $|z_3| = 4$ and $|2z_1 + 3z_2 + 4z_3| = 4$, then find the value of $|8z_2z_3 + 27z_3z_1 + 64z_1z_2|$

3. If $|z|^2 + \bar{A}\ z^2 + A\bar{z}^2 + B\bar{z} + \bar{B}z + c = 0$ represents a pair of intersecting lines with angle of intersection 'θ' then find the value of |A|.

4. If $z^2 + \alpha z + \beta = 0$ (α, β are complex numbers) has a real root then prove that $(\bar{\alpha} - \alpha)(\alpha\bar{\beta} - \bar{\alpha}\beta) = (\beta - \bar{\beta})^2$.

5. If z_1, z_2, z_3 be three complex number such that

$|z_1| = |z_2| = |z_3| = 1$ and $\frac{z_1^2}{z_2z_3} + \frac{z_2^2}{z_1z_3} + \frac{z_3^2}{z_1z_2} + 1 = 0$

then sum of all the possible values of $|z_1 + z_2 + z_3|$.

6. Number of complex number (z) satisfying $|z|^2 = |z|^{n-2}z^2 + |z|^{n-2}z + 1$ such that $\text{Re}(z) \neq -\frac{1}{2}$ and $n = 2\lambda + 1$, $\lambda \in N$.

7. Let z_1 & z_2 be any two arbitrary complex numbers then prove that

(i) $|z_1 + z_2| = \left|\frac{z_1}{|z_1|}\ |z_2| + \frac{z_2}{|z_2|}\ |z_1|\right|$ (ii) $|z_1 + z_2| \geq \frac{1}{2}\left(|z_1| + |z_2|\right)\left|\frac{z_1}{|z_1|} + \frac{z_2}{|z_2|}\right|$.

8. Prove that

(i) $\left|\frac{z}{|z|} - 1\right| \leq |\arg z|$. (ii) $|z - 1| \leq ||z| - 1| + |z|\ |\arg z|$.

9. Prove that

$|\text{Img}(z^n)| \leq n\ |\text{Img}(z)||z|^{n-1}$, $n \in I^{\oplus}$

10. If $|z-1| + |z+3| \leq 8$ then find the range of values of $|z-4|$.

11. Show that all the roots of the equation $a_1z^3 + a_2z^2 + a_3z + a_4 = 3$, where $|a_i| \leq 1$, i = 1, 2, 3, 4 lie outside the circle with centre origin and radius 2/3.

12. Consider the locus of the complex number z in the Argand plane is given by $\text{Re}(z) - 2 = |z - 7 + 2i|$. Let $P(z_1)$ and $Q(z_2)$ be two complex number satisfying the given locus and also satisfying $\arg\left(\frac{z_1 - (2+\alpha i)}{z_2 - (2+\alpha i)}\right) = \frac{\pi}{2} (\alpha \in R)$ then find the minimum value of PQ

13. Find the mirror image of the curve $\left|\frac{z - z_1}{z - z_2}\right| = a$, $a \in R^+$ $a \neq 1$ about the line $|z - z_1| = |z - z_2|$.

14. Let z_1 and z_2 are the two compelx numbers satisfying $|z - 3 - 4i| = 3$. Such that $\text{Arg}\left(\frac{z_1}{z_2}\right)$ is maximum then find the value of $|z_1 - z_2|$.

15. If z_1 and z_2 are the two complex numbers satisfying $|z - 3 - 4i| = 8$ and $\text{Arg}\left(\frac{z_1}{z_2}\right) = \frac{\pi}{2}$ then find the range of the values of $|z_1 - z_2|$.

16. If $|z - z_1| = |z_1|$ and $|z - z_2| = |z_2|$ be the two circles and the two circles touch each other then prove that $\text{Img}\left(\frac{z_1}{z_2}\right) = 0$

17. If $\begin{vmatrix} p & q & r \\ q & r & p \\ r & p & q \end{vmatrix} = 0$; where p, q, r are the modulus of non-zero complex numbers u, v, w respectively, prove that, $\arg \frac{w}{v} = \arg \left(\frac{w-u}{v-u}\right)^2$.

18. If $|z_2 + iz_1| = |z_1| + |z_2|$ and $|z_1| = 3$ & $|z_2| = 4$, if affix of A, B, C are z_1, z_2, $\left(\frac{z_2 - iz_1}{1-i}\right)$ respectively. Then find the area of ΔABC

19. Find the locus of mid-point of line segment intercepted between real and imaginary axes, by the line $a\bar{z} + \bar{a}z + b = 0$, where 'b' is real parameter and 'a' is a fixed complex number such that $\text{Re}(a) \neq 0$, $\text{Im}(a) \neq 0$.

20. Given $z_1 + z_2 + z_3 = A$, $z_1 + z_2\,\omega + z_3\,\omega^2 = B$, $z_1 + z_2\,\omega^2 + z_3\,\omega = C$, where ω is cube root of unity,

(a) express z_1, z_2, z_3 in terms of A, B, C.

(b) prove that, $|A|^2 + |B|^2 + |C|^2 = 3\left(|z_1|^2 + |z_2|^2 + |z_3|^2\right)$.

(c) prove that $A^3 + B^3 + C^3 - 3ABC = 27z_1 z_2 z_3$

21. If $w \neq 1$ is n^{th} root of unity, then find the value of $\sum_{k=0}^{n-1} |z_1 + w^k \quad z_2|^2$

22. Let a, b, c be distinct complex numbers such that $\frac{a}{1-b} = \frac{b}{1-c} = \frac{c}{1-a} = k$, $(a, b, c \neq 1)$. Find the value of k.

23. If $\alpha = e^{\frac{2\pi i}{7}}$ and $f(x) = A_0 + \sum_{k=1}^{20} A_k x^k$, then find the value of,
$f(x) + f(\alpha x) + \ldots\ldots + f(\alpha^6 x)$ independent of α.

24. Given, $z = \cos\frac{2\pi}{2n+1} + i\sin\frac{2\pi}{2n+1}$, 'n' a positive integer, find the equation whose roots are,
$\alpha = z + z^3 + \ldots\ldots + z^{2n-1}$ and $\beta = z^2 + z^4 + \ldots\ldots + z^{2n}$.

25. Prove that $\cos\left(\frac{2\pi}{2n+1}\right) + \cos\left(\frac{4\pi}{2n+1}\right) + \cos\left(\frac{6\pi}{2n+1}\right) + \ldots\ldots + \cos\left(\frac{2n\pi}{2n+1}\right) = -\frac{1}{2}$ When tc $n \in N$.

26. Proof that

(i) $\sin\frac{\pi}{2k+1}\sin\frac{2\pi}{2k+1}\ldots\ldots\ldots\sin\frac{k\pi}{2k+1} = \frac{\sqrt{2k+1}}{2^k}$

(ii) $\cos\frac{\pi}{2k+1}\cos\frac{2\pi}{2k+1}\ldots\ldots\ldots\cos\frac{k\pi}{2k+1} = \frac{1}{2^k}$

27. If Z_r , r = 1, 2, 3, ..., 2m, m $\in$ N are the roots of the equation

$Z^{2m} + Z^{2m-1} + Z^{2m-2} + ... + Z + 1 = 0$, then prove that $\sum_{r=1}^{2m} \frac{1}{Z_r - 1} = -m$

28. The points represented by the complex numbers a, b, c lie on a circle with centre O and radius r. The tangent at c cuts the chord joining the points a, b at z. Show that $z = \frac{a^{-1} + b^{-1} - 2c^{-1}}{a^{-1}b^{-1} - c^{-2}}$

29. Show that for the given complex numbers z_1 and z_2 and for a real constant c the equation

$$(z_1 + \lambda z_2)\bar{z} + (\bar{z}_1 + \lambda \bar{z}_2)z + c = 0$$

represents a family of concurrent lines and and also find the fixed point of the family.
(where λ is a real parameter)

30. Let z_1 , z_2 , z_3 are three pair wise distinct complex numbers and t_1, t_2, t_3 are non-negative real numbers such that $t_1 + t_2 + t_3 = 1$. Prove that the complex number $z = t_1z_1 + t_2z_2 + t_3z_3$ lies inside a triangle with vertices z_1, z_2, z_3 or on its boundary.

Answers

1. 1 **2.** 96 **3.** $\frac{\sec\theta}{2}$ **5.** 3 **6.** 2 **10.** [1, 9] **12.** 10

13. $\left|\frac{z - z_2}{z - z_1}\right| = a$ **14.** $\frac{24}{5}$ **15.** $\left[\sqrt{103} - 5, \sqrt{103} + 5\right]$ **18.** $\frac{25}{4}$ **19.** $\bar{a}\bar{z} + az = 0$

20. (a) $z_1 = \frac{A + B + C}{3}$, $z_2 = \frac{A + B\omega^2 + C\omega}{3}$, $z_3 = \frac{A + B\omega + C\omega^2}{3}$

21. $n(|z_1|^2 + |z_2|^2)$ **22.** $-\omega$ or $-\omega^2$ **23.** $7A_0 + 7A_7x^7 + 7A_{14}x^{14}$

24. $z^2 + z + \frac{\sin^2 n\theta}{\sin^2\theta} = 0$, where $\theta = \frac{2\pi}{2n + 1}$ **29.** $z = \frac{cz_2}{z_1\bar{z}_2 - z_2\bar{z}_1}$