Exercise-1

✍ Marked questions are recommended for Revision.

PART - I : SUBJECTIVE QUESTIONS

Section (A) : Measurement of Angles & Allied angles

A-1. Find the radian measures corresponding to the following degree measures

(i) 15° (ii) 240° (iii) 530°

A-2. Find the degree measures corresponding to the following radian measures

(i) $\frac{3\pi}{4}$ (ii) -4π (iii) $\frac{5\pi}{3}$ (iv) $\frac{7\pi}{6}$

A-3. Prove that :

(i) $\sin^2\frac{\pi}{6} + \cos^2\frac{\pi}{3} - \tan^2\frac{\pi}{4} = -\frac{1}{2}$ (ii) $2\sin^2\frac{\pi}{6} + \operatorname{cosec}\frac{7\pi}{6}\cos^2\frac{\pi}{3} = 0$

(iii) $3\cos^2\frac{\pi}{4} + \sec\frac{2\pi}{3} + 5\tan^2\frac{\pi}{3} = \frac{29}{2}$ (iv) $\cot^2\frac{\pi}{6} + \operatorname{cosec}\frac{5\pi}{6} + 3\tan^2\frac{\pi}{6} = 6$

A-4. Find the value of :

(i) cos 210° (ii) sin 225° (iii) tan 330° (iv) cot (– 315°)

A-5. Prove that

(i) $\frac{\cos(\pi+\theta)\cos(-\theta)}{\sin(\pi-\theta)\cos\left(\frac{\pi}{2}+\theta\right)} = \cot^2\theta.$

(ii) $\cos\theta + \sin(270° + \theta) - \sin(270° - \theta) + \cos(180° + \theta) = 0.$

(iii) $\cos\left(\frac{3\pi}{2}+\theta\right)\cos(2\pi + \theta)\left[\cot\left(\frac{3\pi}{2}-\theta\right)+\cot(2\pi+\theta)\right] = 1.$

A-6. If $\tan\theta = -5/12$, θ is not in the second quadrant, then show that

$$\frac{\sin(360^0-\theta)+\tan(90^0+\theta)}{-\sec(270^0+\theta)+\operatorname{cosec}(-\theta)} = \frac{181}{338}$$

Section (B) : Graphs and Basic Identites (sin(A±B), cos(A±B), tan(A±B))

B-1. Sketch the following graphs :

(i) $y = 3\sin 2x$ (ii) $y = 2\tan x$ (iii) $y = \cos\pi x$

B-2. Find number of solutions of equation $\sin x = -4x + 1$

B-3.✍ If $\tan\theta + \sec\theta = \frac{2}{3}$ then $\sec\theta$ is

B-4. Show that :

(i) sin 20° . cos 40° + cos 20° . sin 40° = $\sqrt{3}/2$

(ii) cos 100° . cos 40° + sin 100° . sin 40° = 1/2

B-5. Show that : $\cos 2\theta\cos\frac{\theta}{2} - \cos 3\theta\cos\frac{9\theta}{2} = \sin 5\theta\sin\frac{5\theta}{2}$.

B-6. If A + B = 45^0, prove that (1 + tanA)(1 + tan B) = 2 and hence deduce that tan $22\frac{1}{2}^0 = \sqrt{2} - 1$

B-7. Eliminate θ from the relations a sec θ = 1 – b tan θ , $a^2 \sec^2\theta = 5 + b^2 \tan^2\theta$

Section (C) : $\sin^2A - \sin^2B$, Multiple angles upto 3A, 2sinA cosB, sinC – sinD

C-1. Show that :
(i) $\sin^2 75^0 - \sin^2 15^0 = \sqrt{3}/2$ (ii) $\sin^2 45^0 - \sin^2 15^0 = \sqrt{3}/4$

C-2. Find the value of
(i) 4 sin 18° cos 36° (ii) $\cos^2 72° - \sin^2 54°$
(iii) $\cos^2 48° - \sin^2 12°$

C-3. If α and β are the solution of a cosθ + b sinθ = c, then show that $\cos(\alpha + \beta) = \frac{a^2 - b^2}{a^2 + b^2}$

C-4. Show that : $\sin^2\left(\frac{\pi}{8} + \frac{A}{2}\right) - \sin^2\left(\frac{\pi}{8} - \frac{A}{2}\right) = \left(\frac{1}{\sqrt{2}}\right)\sin A$

C-5. Show that : $\cos^2\alpha + \cos^2(\alpha + \beta) - 2\cos\alpha\cos\beta\cos(\alpha + \beta) = \sin^2\beta$.

C-6. Prove that
(i) $\frac{\sin^2 A - \sin^2 B}{\sin A\cos A - \sin B\cos B} = \tan(A + B)$
(ii) $\cot(A + 15^o) - \tan(A - 15^o) = \frac{4\cos 2A}{1 + 2\sin 2A}$

C-7. If $0 < \theta < \pi/4$, then show that $\sqrt{2 + \sqrt{2(1 + \cos 4\theta)}} = 2\cos\theta$

C-8. Prove that $\frac{\cos^3 A - \cos 3A}{\cos A} + \frac{\sin^3 A + \sin 3A}{\sin A} = 3$

C-9. Prove that
(i) $\left\{\frac{1 - \cot^2\left(\frac{\alpha - \pi}{4}\right)}{1 + \cot^2\left(\frac{\alpha - \pi}{4}\right)} + \cos\frac{\alpha}{2}\cot 4\alpha\right\}\sec\frac{9\alpha}{2} = \text{cosec } 4\alpha.$
(ii) $\frac{1}{\tan 3\alpha - \tan\alpha} - \frac{1}{\cot 3\alpha - \cot\alpha} = \cot 2\alpha.$ (iii) $\frac{\sec 8A - 1}{\sec 4A - 1} = \frac{\tan 8A}{\tan 2A}$
(iv) $\frac{\cos A + \sin A}{\cos A - \sin A} - \frac{\cos A - \sin A}{\cos A + \sin A} = 2\tan 2A$

C-10. Prove that $\sin\theta = \frac{\sin 3\theta}{1 + 2\cos 2\theta}$ and hence deduce the value of sin 15°.

C-11. Prove that $4(\cos^3 20° + \cos^3 40°) = 3(\cos 20° + \cos 40°)$

C-12. Prove that :

(i) $\dfrac{\tan 3x}{\tan x} = \dfrac{2\cos 2x + 1}{2\cos 2x - 1}$ (ii) $\dfrac{2\sin x}{\sin 3x} + \dfrac{\tan x}{\tan 3x} = 1$

C-13. Prove that :
$\tan\theta \tan(60^0 + \theta)\tan(60^0 - \theta) = \tan 3\theta$ and hence deduce that $\tan 20^0 \tan 40^0 \tan 60^0 \tan 80^0 = 3$.

C-14. Prove that :

(i) $(\text{cosec}\,\theta - \sin\theta)(\sec\theta - \cos\theta)(\tan\theta + \cot\theta) = 1$

(ii) $\dfrac{2\sin\theta \quad \tan\theta \quad (1-\tan\theta) + 2\sin\theta \quad \sec^2\theta}{(1+\tan\theta)^2} = \dfrac{2\sin\theta}{(1+\tan\theta)}$

(iii) $\sqrt{\dfrac{1-\sin A}{1+\sin A}} = \pm(\sec A - \tan A)$

(iv) $\dfrac{\cos A\,\text{cosec}A - \sin A \sec A}{\cos A + \sin A} = \text{cosec}\,A - \sec A$

(v) $\dfrac{1}{\sec\alpha - \tan\alpha} - \dfrac{1}{\cos\alpha} = \dfrac{1}{\cos\alpha} - \dfrac{1}{\sec\alpha + \tan\alpha}$

(vi) $\dfrac{\cos^3 A + \sin^3 A}{\cos A + \sin A} + \dfrac{\cos^3 A - \sin^3 A}{\cos A - \sin A} = 2$

Section (D) : Conditional Identities & Trigonometric Series

D-1. For all values of α, β, γ prove that,
$\cos\alpha + \cos\beta + \cos\gamma + \cos(\alpha+\beta+\gamma) = 4\cos\dfrac{\alpha+\beta}{2}.\cos\dfrac{\beta+\gamma}{2}.\cos\dfrac{\gamma+\alpha}{2}$.

D-2. If $x + y + z = \dfrac{\pi}{2}$ show that, $\sin 2x + \sin 2y + \sin 2z = 4\cos x \cos y \cos z$.

D-3. If $x + y = \pi + z$, then prove that $\sin^2 x + \sin^2 y - \sin^2 z = 2\sin x \sin y \cos z$.

D-4. If $A + B + C = 2S$ then prove that
$\cos(S - A) + \cos(S - B) + \cos(S - C) + \cos S = 4\cos\dfrac{A}{2}\cos\dfrac{B}{2}\cos\dfrac{C}{2}$

D-5. If $A + B + C = 0°$ then prove that $\sin 2A + \sin 2B + \sin 2C = -4\sin A \sin B \sin C$

D-6. If ϕ is the exterior angle of a regular polygon of n sides and θ is any constant, then prove that
$\sin\theta + \sin(\theta + \phi) + \sin(\theta + 2\phi) + \ldots\ldots\ldots\ldots$ up to n terms $= 0$

D-7. Prove that $\sin^2\theta + \sin^2 2\theta + \sin^2 3\theta + \ldots\ldots + \sin^2 n\theta = \dfrac{n}{2} - \dfrac{\sin n\theta\cos(n+1)\theta}{2\sin\theta}$

D-8. Prove that :

(i) $\cos\dfrac{2\pi}{7}\cos\dfrac{4\pi}{7}\cos\dfrac{6\pi}{7} = \dfrac{1}{8}$

(ii) $\cos\dfrac{\pi}{11}\cos\dfrac{2\pi}{11}\cos\dfrac{3\pi}{11}\cos\dfrac{4\pi}{11}\cos\dfrac{5\pi}{11} = \dfrac{1}{32}$

Section (E) : Range of Trigonometric Expressions

E-1. Find the extreme values of $\cos x \cos\left(\frac{2\pi}{3}+x\right)\cos\left(\frac{2\pi}{3}-x\right)$

E-2. Find the maximum and minimum values of following trigonometric functions

(i) $\cos 2x + \cos^2 x$ (ii) $\cos^2\left(\frac{\pi}{4}+x\right) + (\sin x - \cos x)^2$

E-3. Find the greatest and least value of y

(i) $y = 10\cos^2 x - 6\sin x\cos x + 2\sin^2 x$ (ii) $y = 3\cos\left(\theta + \frac{\pi}{3}\right) + 5\cos\theta + 3$

Section (F) : Trigonometric Equations

F-1. What are the most general values of θ which satisfy the equations :

(i) $\sin\theta = \frac{1}{\sqrt{2}}$ (ii) $\tan(x-1) = \sqrt{3}$

(iii) $\tan\theta = -1$ (iv) $\text{cosec}\,\theta = \frac{2}{\sqrt{3}}$.

(v) $2\cot^2\theta = \text{cosec}^2\theta$

F-2. Solve %

(i) $\sin 9\theta = \sin\theta$
(ii) $\cot\theta + \tan\theta = 2\,\text{cosec}\,\theta$
(iii) $\sin 2\theta = \cos 3\theta$
(iv) $\cot\theta = \tan 8\theta$
(v) $\cot\theta - \tan\theta = 2$.
(vi) $\text{cosec}\,\theta = \cot\theta + \sqrt{3}$
(vii) $\tan 2\theta \tan\theta = 1$
(viii) $\tan\theta + \tan 2\theta + \sqrt{3}\tan\theta\tan 2\theta = \sqrt{3}$.

F-3. Solve

(i) $\sin\theta + \sin 3\theta + \sin 5\theta = 0$.
(ii) $\cos\theta + \sin\theta = \cos 2\theta + \sin 2\theta$.
(iii) $\cos^2 x + \cos^2 2x + \cos^2 3x = 1$.
(iv) $\sin^2 n\theta - \sin^2(n-1)\theta = \sin^2\theta$, where n is constant and $n \neq 0, 1$

F-4. Solve

(i) $\tan^2\theta - (1+\sqrt{3})\tan\theta + \sqrt{3} = 0$
(ii) $4\cos\theta - 3\sec\theta = 2\tan\theta$
(iii) $\tan x \,.\, \tan\left(x+\frac{\pi}{3}\right) . \tan\left(x+\frac{2\pi}{3}\right) = \sqrt{3}$.

F-5. Solve

(i) $\sqrt{3}\sin\theta - \cos\theta = \sqrt{2}$
(ii) $5\sin\theta + 2\cos\theta = 5$

Section (G) : Trigonometric Inquations and Height & Distance

G-1. Solve $\tan^2 x \leq 1$

G-2. Solve $2\sin^2 x - \sin x - 1 > 0$

G-3. Solve $\sqrt{\sqrt{3}\cot\theta} < 1$

G-4. Two pillars of equal height stand on either side of a roadway which is 60 m wide. At a point in the roadway between the pillars, the angle of elevation of the top of pillars are 60º and 30º. Then find height of pillars -

G-5. If the angles of elevation of the top of a tower from two points distance a and b from the base and in the same straight line with it are complementary, then find the height of the tower :

G-6. From the top of a cliff 25 m high the angle of elevation of a tower is found to be equal to the angle of depression of the foot of the tower. Then find height of the tower -

PART - II : ONLY ONE OPTION CORRECT TYPE

Section (A) : Measurement of Angles & Allied angles

A-1. $\cos(540° - \theta) - \sin(630° - \theta)$ is equal to
(A) 0 (B) $2\cos\theta$ (C) $2\sin\theta$ (D) $\sin\theta - \cos\theta$

A-2. The value of tan 1° tan 2° tan 3° ... tan 89° is
(A) 1 (B) 0 (C) ∞ (D) $\frac{1}{2}$

A-3. If $x = y\cos\frac{2\pi}{3} = z\cos\frac{4\pi}{3}$, then xy + yz + zx is equal to
(A) – 1 (B) 0 (C) 1 (D) 2

A-4. If $0° < x < 90°$ & $\cos x = \frac{3}{\sqrt{10}}$, then the value of
$\log_{10}\sin x + \log_{10}\cos x + \log_{10}\tan x$ is
(A) 0 (B) 1 (C) – 1 (D) 2

A-5. If $\tan\alpha + \cot\alpha = a$ then the value of $\tan^4\alpha + \cot^4\alpha =$
(A) $a^4 + 4a^2 + 2$ (B) $a^4 - 4a^2 + 2$ (C) $a^4 - 4a^2 - 2$ (D) $a^4 - 2a^2 + 2$

Section (B) : Graphs and Basic Identites (sin(A±B), cos(A±B), tan(A±B))

B-1. **STATEMENT-1 :** sin 2 > sin 3
STATEMENT-2 : If $x, y \in \left(\frac{\pi}{2}, \pi\right)$, x < y, then sin x > sin y
(A) STATEMENT-1 is true, STATEMENT-2 is true and STATEMENT-2 is correct explanation for STATEMENT-1
(B) STATEMENT-1 is true, STATEMENT-2 is true and STATEMENT-2 is not correct explanation for STATEMENT-1
(C) STATEMENT-1 is true, STATEMENT-2 is false
(D) STATEMENT-1 is false, STATEMENT-2 is true

B-2. If $\mathrm{cosec}\theta - \cot\theta = \alpha$, then $\cot\theta$ is :
(A) $\frac{1}{2}\left(\frac{1}{\alpha} + \alpha\right)$ (B) $\frac{1}{2}\left(\frac{1}{\alpha} - \alpha\right)$ (C) $\left(\frac{1}{\alpha} + \alpha\right)$ (D) $\left(\frac{1}{\alpha} - \alpha\right)$

B-3. If $a\cos\theta + b\sin\theta = 3$ & $a\sin\theta - b\cos\theta = 4$ then $a^2 + b^2$ has the value =
(A) 25 (B) 14 (C) 7 (D) 10

B-4. $\dfrac{\tan\left(x-\frac{\pi}{2}\right).\cos\left(\frac{3\pi}{2}+x\right)-\sin^3\left(\frac{7\pi}{2}-x\right)}{\cos\left(x-\frac{\pi}{2}\right).\tan\left(\frac{3\pi}{2}+x\right)}$ when simplified reduces to:

(A) $\sin x\cos x$ (B) $-\sin^2 x$ (C) $-\sin x\cos x$ (D) $\sin^2 x$

B-5. The expression $3\left[\sin^4\left(\frac{3\pi}{2}-\alpha\right)+\sin^4(3\pi+\alpha)\right]-2\left[\sin^6\left(\frac{\pi}{2}+\alpha\right)+\sin^6(5\pi+\alpha)\right]$ is equal to

(A) 0 (B) 1 (C) 3 (D) $\sin 4\alpha + \sin 6\alpha$

B-6. The value of the expression

$$\left(1+\cos\frac{\pi}{10}\right)\left(1+\cos\frac{3\pi}{10}\right)\left(1+\cos\frac{7\pi}{10}\right)\left(1+\cos\frac{9\pi}{10}\right) \text{ is}$$

(A) $\frac{1}{8}$ (B) $\frac{1}{16}$ (C) $\frac{1}{4}$ (D) 0

B-7. The value of $\dfrac{\sin 24^\circ\cos 6^\circ-\sin 6^\circ\sin 66^\circ}{\sin 21^\circ\cos 39^\circ-\cos 51^\circ\sin 69^\circ}$ is

(A) –1 (B) 1 (C) 2 (D) 0

B-8. If $\tan A$ and $\tan B$ are the roots of the quadratic equation $x^2 - ax + b = 0$, then the value of $\sin^2(A + B)$.

(A) $\dfrac{a^2}{a^2+(1-b)^2}$ (B) $\dfrac{a^2}{a^2+b^2}$ (C) $\dfrac{a^2}{(b+c)^2}$ (D) $\dfrac{a^2}{b^2(1-a)^2}$

B-9. If $\tan A - \tan B = x$ and $\cot B - \cot A = y$, then $\cot(A - B)$ is equal to

(A) $\frac{1}{y}-\frac{1}{x}$ (B) $\frac{1}{x}-\frac{1}{y}$ (C) $\frac{1}{x}+\frac{1}{y}$ (D) $\frac{1}{x+y}$

B-10. If $\tan 25^\circ = x$, then $\dfrac{\tan 155^\circ-\tan 115^\circ}{1+\tan 155^\circ\tan 115^\circ}$ is equal to

(A) $\dfrac{1-x^2}{2x}$ (B) $\dfrac{1+x^2}{2x}$ (C) $\dfrac{1+x^2}{1-x^2}$ (D) $\dfrac{1-x^2}{1+x^2}$

B-11. If $A + B = 225^\circ$, then the value of $\left(\dfrac{\cot A}{1+\cot A}\right).\left(\dfrac{\cot B}{1+\cot B}\right)$ is

(A) 2 (B) $\frac{1}{2}$ (C) 3 (D) $-\frac{1}{2}$

B-12. The value of $\tan 203^\circ + \tan 22^\circ + \tan 203^\circ\tan 22^\circ$ is

(A) – 1 (B) 0 (C) 1 (D) 2

Section (C) : sin²A – sin²B, Multiple angles upto 3A, 2sinA cosB, sinC – sinD

C-1. The value of $\dfrac{1-\tan^2 15^\circ}{1+\tan^2 15^\circ}$ is

(A) 1 (B) $\sqrt{3}$ (C) $\frac{\sqrt{3}}{2}$ (D) 2

C-2. If A lies in the third quadrant and $3\tan A - 4 = 0$, then $5\sin 2A + 3\sin A + 4\cos A$ is equal to

(A) 0 (B) $-\frac{24}{5}$ (C) $\frac{24}{5}$ (D) $\frac{48}{5}$

C-3. If cos A = 3/4, then the value of $16\cos^2 (A/2) - 32 \sin (A/2) \sin (5A/2)$ is
(A) – 4 (B) – 3 (C) 3 (D) 4

C-4. If $\tan^2 \theta = 2 \tan^2 \phi + 1$, then the value of $\cos 2\theta + \sin^2 \phi$ is
(A) 1 (B) 2 (C) – 1 (D) Independent of ϕ

C-5. If $\alpha \in \left[\frac{\pi}{2}, \pi\right]$ then the value of $\sqrt{1+\sin\alpha} - \sqrt{1-\sin\alpha}$ is equal to:
(A) $2 \cos \frac{\alpha}{2}$ (B) $2 \sin \frac{\alpha}{2}$ (C) 2 (D) none of these

C-6. The value of $\frac{1}{\cos 290°} + \frac{1}{\sqrt{3} \sin 250°}$ is
(A) $\frac{2\sqrt{3}}{3}$ (B) $\frac{4\sqrt{3}}{3}$ (C) $\sqrt{3}$ (D) none

C-7. The value of tan 3A – tan 2A – tan A is equal to
(A) tan 3A tan 2A tan A (B) – tan 3A tan 2A tan A
(C) tan A tan 2A – tan 2A tan 3A – tan 3A tan A (D) none of these

C-8. $\frac{\cos 20° + 8 \sin 70° \sin 50° \sin 10°}{\sin^2 80°}$ is equal to:
(A) 1 (B) 2 (C) 3/4 (D) 0

C-9. The numerical value of sin 12° . sin48° . sin 54° is equal to
(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\frac{1}{16}$ (D) $\frac{1}{8}$

C-10. If A = tan 6° tan 42° and B = cot 66° cot 78°, then
(A) A = 2B (B) A = $\frac{1}{3}$ B (C) A = B (D) 3A = 2B

Section (D) : Conditional Identities & Trigonometric Series

D-1. In a triangle tan A + tan B + tan C = 6 and tan A tan B = 2, then the values of tan A, tan B and tan C are respectively
(A) 1, 2, 3 (B) 2, 3, 1 (C) 1, 2, 0 (D) none of these

D-2. $\tan\alpha + 2 \tan 2\alpha + 4 \tan 4\alpha + 8 \cot 8\alpha =$
(A) $\tan \alpha$ (B) $\cot \alpha$ (C) $\cot 16\alpha$ (D) $16 \cot \alpha$

D-3. The value of $\cos 0 + \cos\frac{\pi}{7} + \cos\frac{2\pi}{7} + \cos\frac{3\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{5\pi}{7} + \cos\frac{6\pi}{7}$ is
(A) 1/2 (B) –1/2 (C) 0 (D) 1

D-4. The value of $\cos\frac{\pi}{10} \cos\frac{2\pi}{10} \cos\frac{4\pi}{10} \cos\frac{8\pi}{10} \cos\frac{16\pi}{10}$ is :
(A) $\frac{\sqrt{10+2\sqrt{5}}}{64}$ (B) $-\frac{\cos(\pi/10)}{16}$ (C) $\frac{\cos(\pi/10)}{16}$ (D) $-\frac{\sqrt{10+2\sqrt{5}}}{16}$

D-5. The value of $\cos\frac{\pi}{19} + \cos\frac{3\pi}{19} + \cos\frac{5\pi}{19} + ... + \cos\frac{17\pi}{19}$ is equal to :
(A) 1/2 (B) 0 (C) 1 (D) 2

Section (E) : Range of Trigonometric Expressions

E-1. If $f(\theta) = \sin^4\theta + \cos^2\theta$, then range of $f(\theta)$ is

(A) $\left[\frac{1}{2}, 1\right]$ (B) $\left[\frac{1}{2}, \frac{3}{4}\right]$ (C) $\left[\frac{3}{4}, 1\right]$ (D) None of these

E-2. Range of function $f(x) = \cos^2x + 4\sec^2x$ is

(A) $[4, \infty)$ (B) $[0, \infty)$ (C) $[5, \infty)$ (D) $(0, \infty)$

E-3. The difference between maximum and minimum value of the expression $y = 1 + 2\sin x + 3\cos^2 x$ is

(A) $\frac{16}{3}$ (B) $\frac{13}{3}$ (C) 7 (D) 8

E-4. The maximum value of $12\sin\theta - 9\sin^2\theta$ is -

(A) 3 (B) 4 (C) 5 (D) None of these

E-5. The greatest and least value of $y = 10\cos^2x - 6\sin x\cos x + 2\sin^2x$ are respectively

(A) 11, 1 (B) 10, 2 (C) 12, – 4 (D) 11, – 1

Section (F) : Trigonometric Equations

F-1. The solution set of the equation $4\sin\theta.\cos\theta - 2\cos\theta - 2\sqrt{3}\sin\theta + \sqrt{3} = 0$ in the interval $(0, 2\pi)$ is

(A) $\left\{\frac{3\pi}{4}, \frac{7\pi}{4}\right\}$ (B) $\left\{\frac{\pi}{3}, \frac{5\pi}{3}\right\}$ (C) $\left\{\frac{3\pi}{4}, \pi, \frac{\pi}{3}, \frac{5\pi}{3}\right\}$ (D) $\left\{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{11\pi}{6}\right\}$

F-2. All solutions of the equation $2\sin\theta + \tan\theta = 0$ are obtained by taking all integral values of m and n in:

(A) $2n\pi + \frac{2\pi}{3}$, $n \in I$ (B) $n\pi$ or $2m\pi \pm \frac{2\pi}{3}$ where $n, m \in I$

(C) $n\pi$ or $m\pi \pm \frac{\pi}{3}$ where $n, m \in I$ (D) $n\pi$ or $2m\pi \pm \frac{\pi}{3}$ where $n, m \in I$

F-3. Total number of solutions of equation sinx . tan4x = cosx belonging to $(-\pi, 2\pi)$ are :

(A) 4 (B) 7 (C) 8 (D) 15

F-4. If $x \in \left[0, \frac{\pi}{2}\right]$, the number of solutions of the equation $\sin 7x + \sin 4x + \sin x = 0$ is:

(A) 3 (B) 5 (C) 6 (D) 4

F-5. The general solution of equation sinx + sin5x = sin2x + sin4x is :

(A) $\frac{n\pi}{2}$; $n \in I$ (B) $\frac{n\pi}{5}$; $n \in I$ (C) $\frac{n\pi}{3}$; $n \in I$ (D) $\frac{2n\pi}{3}$; $n \in I$

F-6. The general solution of the equation $2\cos 2x = 3.2\cos^2x - 4$ is

(A) $x = 2n\pi, n \in I$ (B) $x = n\pi, n \in I$ (C) $x = n\pi/4, n \in I$ (D) $x = n\pi/2, n \in I$

F-7. If $2\cos^2(\pi + x) + 3\sin(\pi + x)$ vanishes then the values of x lying in the interval from 0 to 2π are

(A) $x = \pi/6$ or $5\pi/6$ (B) $x = \pi/3$ or $5\pi/3$ (C) $x = \pi/4$ or $5\pi/4$ (D) $x = \pi/2$ or $5\pi/2$

F-8. $\frac{\cos 3\theta}{2\cos 2\theta - 1} = \frac{1}{2}$ if

(A) $\theta = n\pi + \frac{\pi}{3}$, $n \in I$ (B) $\theta = 2n\pi \pm \frac{\pi}{3}$, $n \in I$ (C) $\theta = 2n\pi \pm \frac{\pi}{6}$, $n \in I$ (D) $\theta = n\pi + \frac{\pi}{6}$, $n \in I$

F-9. If $\cos 2\theta + 3\cos\theta = 0$, then

(A) $\theta = 2n\pi \pm \alpha$ where $\alpha = \cos^{-1}\left(\frac{\sqrt{17}-3}{4}\right)$ (B) $\theta = 2n\pi \pm \alpha$ where $\alpha = \cos^{-1}\left(\frac{-\sqrt{17}-3}{4}\right)$

(C) $\theta = 2n\pi \pm \alpha$ where $\alpha = \cos^{-1}\left(\frac{\pm\sqrt{17}-3}{4}\right)$ (D) $\theta = 2n\pi \pm \alpha$ where $\alpha = \cos^{-1}\left(\frac{\sqrt{17}+3}{4}\right)$

F-10. If $\sin\theta + 7\cos\theta = 5$, then $\tan(\theta/2)$ is a root of the equation

(A) $x^2 - 6x + 1 = 0$ (B) $6x^2 - x - 1 = 0$ (C) $6x^2 + x + 1 = 0$ (D) $x^2 - x + 6 = 0$

F-11. The most general solution of $\tan\theta = -1$ and $\cos\theta = \frac{1}{\sqrt{2}}$ is :

(A) $n\pi + \frac{7\pi}{4}$, $n \in I$ (B) $n\pi + (-1)^n \frac{7\pi}{4}$, $n \in I$ (C) $2n\pi + \frac{7\pi}{4}$, $n \in I$ (D) $2n\pi + \frac{3\pi}{4}$, $n \in I$

F-12. A triangle ABC is such that $\sin(2A + B) = \frac{1}{2}$. If A, B, C are in A.P. then the angle A, B, C are respectively.

(A) $\frac{5\pi}{12}, \frac{\pi}{4}, \frac{\pi}{3}$ (B) $\frac{\pi}{4}, \frac{\pi}{3}, \frac{5\pi}{12}$ (C) $\frac{\pi}{3}, \frac{\pi}{4}, \frac{5\pi}{12}$ (D) $\frac{\pi}{3}, \frac{5\pi}{12}, \frac{\pi}{4}$

Section (G) : Trigonometric Inquations and Height & Distance

G-1. The complete solution of inequality $\sec^2 3x < 2$ is

(A) $x \in \left(\frac{n\pi}{3} - \frac{\pi}{12}, \frac{n\pi}{3} + \frac{\pi}{12}\right)$, $n \in I$ (B) $x \in \left(\frac{n\pi}{3} - \frac{\pi}{12}, \frac{n\pi}{3} + \frac{\pi}{6}\right)$, $n \in I$

(C) $x \in \left(n\pi - \frac{\pi}{12}, n\pi + \frac{\pi}{12}\right)$, $n \in I$ (D) $x \in \left(\frac{n\pi}{3} - \frac{\pi}{6}, \frac{n\pi}{3} + \frac{\pi}{6}\right)$, $n \in I$

G-2. The complete solution of inequality $2\cos^2 x - 7\cos x + 3 < 0$ is

(A) $n\pi - \frac{\pi}{3} < x < \frac{\pi}{3} + n\pi$ (B) $2n\pi - \frac{\pi}{6} < x < \frac{\pi}{6} + 2n\pi$

(C) $2n\pi - \frac{\pi}{3} < x < \frac{\pi}{3} + 2n\pi$ (D) $n\pi - \frac{\pi}{6} < x < \frac{\pi}{6} + n\pi$

G-3. The complete solution of inequality $\cos 2x \le \cos x$ is

(A) $x \in \left[2n\pi - \frac{\pi}{3}, 2n\pi + \frac{\pi}{3}\right]$ (B) $x \in \left[2n\pi - \frac{2\pi}{3}, 2n\pi + \frac{2\pi}{3}\right]$

(C) $x \in \left[2n\pi, 2n\pi + \frac{2\pi}{3}\right]$ (D) $x \in \left[2n\pi - \frac{2\pi}{3}, 2n\pi\right]$

G-4. Which of the following set of values of x satisfy the inequation $\tan^2 x - (1 + \sqrt{3})\tan x + \sqrt{3} < 0$

(A) $\left(\frac{(4n+1)\pi}{4}, \frac{(3n+1)\pi}{3}\right)$, $(n \in Z)$ (B) $\left(\frac{(2n+1)\pi}{4}, \frac{(2n+1)\pi}{3}\right)$, $(n \in Z)$

(C) $\left(\frac{(4n+1)\pi}{4}, \frac{(4n+1)\pi}{3}\right)$, $(n \in Z)$ (D) $x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right]$

G-5. A tree 12 m high, is broken by the wind in such a way that its top touches the ground and makes an angle 60º with the ground. The height from the bottom of the tree from where it is broken by the wind is appoximately

(A) 5.57 m (B) 5.21 (C) 5.36 (D) 5.9

G-6. AB is a vertical pole and C is the middle point. The end A is on the level ground and P is any point on the level ground other than A. The portion CB subtends an angle β at P. If AP : AB = 2 : 1, then β is equal to-

(A) $\tan^{-1}\left(\frac{1}{9}\right)$ (B) $\tan^{-1}\left(\frac{4}{9}\right)$ (C) $\tan^{-1}\left(\frac{5}{9}\right)$ (D) $\tan^{-1}\left(\frac{2}{9}\right)$

G-7. A round ballon of radius r subtends an angle α at the eye of the observer, while the angle of elevation of its centre is β. The height of the centre of ballon is-

(A) $r \operatorname{cosec} \alpha \sin \frac{\beta}{2}$ (B) $r \sin \alpha \operatorname{cosec} \frac{\beta}{2}$ (C) $r \sin \frac{\alpha}{2} \operatorname{cosec} \beta$ (D) $r \operatorname{cosec} \frac{\alpha}{2} \sin \beta$

G-8. If the angle of elevation of a cloud from a point 200 m above a lake is 30º and the angle of depression of its reflection in the lake is 60º, then the height of the cloud above the lake, is

(A) 200 m (B) 500 m (C) 30 m (D) 400 m

G-9. A man on the top of a vertical tower observes a car moving at a uniform speed coming directly towards it. If it takes 12 minutes for the angle of depression to change from 30º to 45º, then the car will reach the tower in

(A) 17 minutes 23 seconds (B) 16 minutes 23 seconds

(C) 16 minutes 18 seconds (D) 18 minutes 22 seconds

PART - III : MATCH THE COLUMN

1.

Column - I	**Column - II**
(A) $\tan 9° - \tan 27° - \tan 63° + \tan 81°$	(p) 1
(B) $\operatorname{cosec} 10° - \sqrt{3} \sec 10°$	(q) 2
(C) $2\sqrt{2} \sin 10° \left[\frac{\sec 5°}{2} + \frac{\cos 40°}{\sin 5°} - 2 \sin 35°\right]$	(r) 3
(D) $\sqrt{3} (\cot 70° + 4 \cos 70°)$	(s) 4

2.

Column – I	**Column – II**
(A) If for some real x, the equation $x + \frac{1}{x} = 2 \cos \theta$ holds, then $\cos \theta$ is equal to	(p) 2
(B) If $\sin \theta + \operatorname{cosec} \theta = 2$, then $\sin^{2008}\theta + \operatorname{cosec}^{2008}\theta$ is equal to	(q) 1
(C) Maximum value of $\sin^4\theta + \cos^4\theta$ is	(r) 0
(D) Least value of $2 \sin^2\theta + 3 \cos^2\theta$ is	(s) – 1

3.

Column – I	**Column – II**
(A) Number of solutions of $\sin^2\theta + 3 \cos \theta = 3$ in $[-\pi, \pi]$	(p) 0
(B) Number of solutions of $\sin x . \tan 4x = \cos x$ in $(0, \pi)$	(q) 1

(C)	Number of solutions of equation $(1 - \tan\theta)(1 + \tan\theta)\sec^2\theta + 2^{\tan^2\theta} = 0$ where $\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$	(r)	4
(D)	If $[\sin x] + [\sqrt{2}\cos x] = -3$, where $x \in [0, 2\pi]$ then $[\sin 2x]$ equals (Here [.] denotes G.I.F.)	(s)	5

Exercise-2

Marked questions are recommended for Revision.

PART - I : ONLY ONE OPTION CORRECT TYPE

1. In a triangle ABC if $\tan A < 0$ then:
(A) $\tan B.\ \tan C > 1$ (B) $\tan B.\ \tan C < 1$ (C) $\tan B.\ \tan C = 1$ (D) Data insufficient

2. If $\sin\alpha = 1/2$ and $\cos\theta = 1/3$, then the values of $\alpha + \theta$ (if θ, α are both acute) will lie in the interval
(A) $\left[\frac{\pi}{3}, \frac{\pi}{2}\right]$ (B) $\left[\frac{\pi}{2}, \frac{2\pi}{3}\right]$ (C) $\left[\frac{2\pi}{3}, \frac{5\pi}{6}\right]$ (D) $\left[\frac{5\pi}{6}, \pi\right]$

3. If $\frac{\sin A}{\sin B} = \frac{\sqrt{3}}{2}$ and $\frac{\cos A}{\cos B} = \frac{\sqrt{5}}{2}$, $0 < A, B < \pi/2$, then $\tan A + \tan B$ is equal to
(A) $\sqrt{3}/\sqrt{5}$ (B) $\sqrt{5}/\sqrt{3}$ (C) 1 (D) $(\sqrt{5}+\sqrt{3})/\sqrt{5}$

4. In a right angled triangle the hypotenuse is $2\sqrt{2}$ times the perpendicular drawn from the opposite vertex. Then the other acute angles of the triangle are
(A) $\frac{\pi}{3}$ & $\frac{\pi}{6}$ (B) $\frac{\pi}{8}$ & $\frac{3\pi}{8}$ (C) $\frac{\pi}{4}$ & $\frac{\pi}{4}$ (D) $\frac{\pi}{5}$ & $\frac{3\pi}{10}$

5. If $3\cos x + 2\cos 3x = \cos y$, $3\sin x + 2\sin 3x = \sin y$, then the value of $\cos 2x$ is
(A) -1 (B) $\frac{1}{8}$ (C) $-\frac{1}{8}$ (D) $\frac{7}{8}$

6. If $\cos\alpha + \cos\beta = a$, $\sin\alpha + \sin\beta = b$ and $\alpha - \beta = 2\theta$, then $\frac{\cos 3\theta}{\cos\theta} =$
(A) $a^2 + b^2 - 2$ (B) $a^2 + b^2 - 3$ (C) $3 - a^2 - b^2$ (D) $(a^2 + b^2)/4$

7. If $\frac{3\pi}{4} < \alpha < \pi$, then $\sqrt{2\cot\alpha + \frac{1}{\sin^2\alpha}}$ is equal to
(A) $1 + \cot\alpha$ (B) $-1 - \cot\alpha$ (C) $1 - \cot\alpha$ (D) $-1 + \cot\alpha$

8. For $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$, $\frac{\sin\theta + \sin 2\theta}{1 + \cos\theta + \cos 2\theta}$ lies in the interval
(A) $(-\infty, \infty)$ (B) $(-2, 2)$ (C) $(0, \infty)$ (D) $(-1, 1)$

9. The number of all possible triplets (a_1, a_2, a_3) such that $a_1 + a_2\cos 2x + a_3\sin^2 x = 0$ for all x is
(A) 0 (B) 1 (C) 2 (D) infinite

10. If $A + B + C = \frac{3\pi}{2}$, then cos 2A + cos2B + cos2C is equal to

(A) 1 – 4cos A cosB cosC
(B) 4 sin A sin B sin C
(C) 1 + 2 cos A cos B cos C
(D) 1– 4 sin A sin B sin C

11. If A + B + C = π & cosA = cosB. cosC then tanB. tanC has the value equal to:

(A) 1 (B) 1/2 (C) 2 (D) 3

12. The general solution of the equation $\tan^2\alpha + 2\sqrt{3}\tan\alpha = 1$ is given by:

(A) $\alpha = \frac{n\pi}{2}$, $n \in I$
(B) $\alpha = (2n+1)\frac{\pi}{2}$, $n \in I$
(C) $\alpha = (6n+1)\frac{\pi}{12}$, $n \in I$
(D) $\alpha = \frac{n\pi}{12}$, $n \in I$

13. The general solution of the equation $\tan x + \tan\left(x + \frac{\pi}{3}\right) + \tan\left(x + \frac{2\pi}{3}\right) = 3$ is

(A) $\frac{n\pi}{4} + \frac{\pi}{12}$, $n \in I$
(B) $\frac{n\pi}{3} + \frac{\pi}{6}$, $n \in I$
(C) $\frac{n\pi}{3} + \frac{\pi}{12}$, $n \in I$
(D) $\frac{n\pi}{3} + \frac{\pi}{4}$, $n \in I$

14. The complete solution set of the equation $1 + 2\,\text{cosec}\, x = -\frac{\sec^2\frac{x}{2}}{2}$ is

(A) $2n\pi - \frac{\pi}{2}$, $n \in I$
(B) $n\pi - \frac{\pi}{2}$, $n \in I$
(C) $2n\pi + \frac{\pi}{2}$, $n \in I$
(D) $n\pi + \frac{\pi}{2}$, $n \in I$

15. The principal solution set of the equation $2\cos x = \sqrt{2 + 2\sin 2x}$ is

(A) $\left\{\frac{\pi}{8}, \frac{13\pi}{8}\right\}$
(B) $\left\{\frac{\pi}{4}, \frac{13\pi}{8}\right\}$
(C) $\left\{\frac{\pi}{4}, \frac{13\pi}{10}\right\}$
(D) $\left\{\frac{\pi}{8}, \frac{13\pi}{10}\right\}$

16. The solution of |cosx| = cosx – 2sinx is

(A) $x = n\pi$, $n \in I$
(B) $x = n\pi + \frac{\pi}{4}$, $n \in I$
(C) $x = n\pi + (-1)^n\frac{\pi}{4}$, $n \in I$
(D) $x = (2n + 1)\pi + \frac{\pi}{4}$, $n \in I$

17. The solution of inequality $4^{\tan x} - 3.2^{\tan x} + 2 \le 0$ is

(A) $x \in \left[n\pi,\ n\pi + \frac{\pi}{4}\right]$; $n \in I$
(B) $x \in \left[n\pi,\ n\pi - \frac{\pi}{4}\right]$; $n \in I$
(C) $x \in \left[n\pi,\ n\pi + \frac{\pi}{6}\right]$; $n \in I$
(D) $x \in \left[n\pi,\ n\pi - \frac{\pi}{6}\right]$; $n \in I$

18. The solution of inequality $\sqrt{5 - 2\sin x} \ge 6\sin x - 1$ is

(A) $[\pi(12n - 7)/6, \pi(12n + 7)/6]$ $(n \in Z)$
(B) $[\pi(12n - 7)/6, \pi(12n + 1)/6]$ $(n \in Z)$
(C) $[\pi(2n - 7)/6, \pi(2n + 1)/6]$ $(n \in Z)$
(D) $[\pi(12n - 7)/3, \pi(12n + 1)/3]$ $(n \in Z)$

PART - II : SINGLE AND DOUBLE VALUE INTEGER TYPE

1. If 3 sinα =5 sinβ, then find the value of $\dfrac{\tan\frac{\alpha+\beta}{2}}{\tan\frac{\alpha-\beta}{2}}$.

2. If α, β ($\alpha - \beta \neq 2n\pi$, n $\in$ I) are different values of θ satisfying the equation 5 cos θ – 12 sin θ = 11. If the value of sin ($\alpha + \beta$) = $-\dfrac{5k}{169}$, then find the value of k.

3. If x $\in \left(\pi, \dfrac{3\pi}{2}\right)$, then $4\cos^2\left(\dfrac{\pi}{4}-\dfrac{x}{2}\right) + \sqrt{4\sin^4 x + \sin^2 2x}$ is always equal to

4. If three angles A, B, C are such that
cos A + cos B + cos C = 0 and if
λ cos A cos B cos C = cos 3A + cos 3B + cos 3C, then value of λ is :

5. Find the number of integral values of λ, for which equation 4cos x + 3 sin x = 2λ + 1 has a solution.

6. If a $\cos^3\alpha$ + 3a $\cos\alpha \sin^2\alpha$ = m and a $\sin^3\alpha$ + 3a $\cos^2\alpha \sin\alpha$ = n. if $(m+n)^{2/3} + (m-n)^{2/3} = 2\lambda a^{2/3}$, then find value of λ.

7. If 2 cos x + sin x = 1, then find the sum of all possible values of 7 cos x + 6 sin x.

8. The number of roots of the equation cot x = $\dfrac{\pi}{2}$ + x in $\left[-\pi, \dfrac{3\pi}{2}\right]$ is,

9. If $2\tan^2 x$ – 5 secx – 1 = 0 has 7 different roots in $\left[0, \dfrac{n\pi}{2}\right]$, n $\in$ N, then find the greatest value of n.

10. Find the number of integral values of a for which the equation cos 2x + a sin x = 2a – 7 possesses a solution.

11. The number of solutions of the equation |sinx| = | cos3x| in [–2π, 2π] is

12. In any triangle ABC, which is not right angled $\sum$cosA .cosecB.cosecC is equal to

13. If A + B + C = π, then find value of tan B tan C + tan C tan A + tan A tan B – sec A sec B sec C.

14. If the arithmetic mean of the roots of the equation $4\cos^3x - 4\cos^2x - \cos(\pi + x) - 1 = 0$ in the interval [0, 315] is equal to kπ , then find the value of k

15. cos ($\alpha - \beta$) = 1 and cos ($\alpha + \beta$) = $\dfrac{1}{e}$, where α, $\beta \in [-\pi, \pi]$. Then number of ordered pairs (α, β) which satisfy both the equations.

16. Number of values of θ between 0° and 90° which satisfy the equation $\sec^2\theta .\text{cosec}^2\theta + 2\,\text{cosec}^2\theta = 8$

17. Find the number of all values of $\theta \in$ [0, 10.5] satisfying the equation
$\cos 6\theta + \cos 4\theta + \cos 2\theta + 1 = 0$.

18. In (0, 6π), find the number of solutions of the equation tanθ + tan 2θ + tan 3θ = tan θ.tan2θ.tan3θ

19. If $0 \le x \le 3\pi$, $0 \le y \le 3\pi$ and cos x . sin y = 1, then find the possible number of values of the ordered pair (x, y)

20. Find the number of values of θ satisfying the equation $\sin 3\theta = 4\sin\theta . \sin 2\theta . \sin 4\theta$ in $0 \le \theta \le 2\pi$

21. Find the number of solutions of the equation $\cos 6x + \tan^2 x + \cos 6x . \tan^2 x = 1$ in the interval $[0, 2\pi]$.

22. Consider $\tan\theta + \sin\phi = \frac{3}{2}$ & $\tan^2\theta + \cos^2\phi = \frac{7}{4}$, find maximum value of $\frac{12(\theta+\phi)}{\pi}$ if $\theta + \phi \in (0, 2\pi)$.

23. Find the number of integral values of n so that sinx(sinx + cosx) = n has at least one solution.

24. Find the number of values of x in $(0, 2\pi)$ satisfying the equation $\cot x - 2\sin 2x = 1$.

25. Find the number of solutions of $\sin\theta + 2\sin 2\theta + 3\sin 3\theta + 4\sin 4\theta = 10$ in$(0, \pi)$.

26. Find the values of x satisfying the equation $2\sin x = 3x^2 + 2x + 3$.

27. Find the number of solution of sinx cosx – 3 cosx + 4 sinx – 13 > 0 in $[0,2\pi]$.

PART - III : ONE OR MORE THAN ONE OPTIONS CORRECT TYPE

1. The value of $\frac{(\cos 11^\circ + \sin 11^\circ)}{(\cos 11^\circ - \sin 11^\circ)}$ is

(A) –tan 304° (B) tan 56° (C) cot 214° (D) cot 34°

2. If $\sin t + \cos t = \frac{1}{5}$ then $\tan\frac{t}{2}$ can be

(A) -1 (B) $-\frac{1}{3}$ (C) 2 (D) $-\frac{1}{6}$

3. The value of $\frac{\sin x + \cos x}{\cos^3 x}$ =

(A) $1 + \tan x + \tan^2 x - \tan^3 x$ (B) $1 + \tan x + \tan^2 x + \tan^3 x$

(C) $1 - \tan x + \tan^2 x + \tan^3 x$ (D) $(1 + \tan x)\sec^2 x$

4. If (sec A + tan A) (sec B + tan B) (sec C + tan C) = (sec A – tan A) (sec B – tan B) (sec C – tan C) then each side can be

(A) 1 (B) –1 (C) 0 (D) none

5. Which of the following is correct ?

(A) sin 1° > sin 1 (B) sin 1° < sin 1 (C) cos 1° > cos 1 (D) cos 1° < cos 1

6. If sin x + sin y = a & cos x + cos y = b, then which of the following may be true.

(A) $\sin(x + y) = \frac{2ab}{a^2 + b^2}$ (B) $\tan\frac{x-y}{2} = \sqrt{\frac{4-a^2-b^2}{a^2+b^2}}$

(C) $\tan\frac{x-y}{2} = -\sqrt{\frac{4-a^2-b^2}{a^2+b^2}}$ (D) $\cos(x + y) = \frac{2ab}{a^2+b^2}$

7. If $\cos(A - B) = \frac{3}{5}$ and tan A tan B = 2, then which of the following is/are correct

(A) $\cos A \cos B = -\frac{1}{5}$ (B) $\sin A \sin B = \frac{2}{5}$ (C) $\cos(A + B) = -\frac{1}{5}$ (D) $\sin A \cos B = \frac{4}{5}$

8. If $P_n = \cos^n\theta + \sin^n\theta$ and $Q_n = \cos^n\theta - \sin^n\theta$, then which of the following is/are true.

(A) $P_n - P_{n-2} = -\sin^2\theta\cos^2\theta\, P_{n-4}$ (B) $Q_n - Q_{n-2} = -\sin^2\theta\cos^2\theta\, Q_{n-4}$

(C) $P_4 = 1 - 2\sin^2\theta\cos^2\theta$ (D) $Q_4 = \cos^2\theta - \sin^2\theta$

9. $\tan^2\alpha + 2\tan\alpha.\tan2\beta = \tan^2\beta + 2\tan\beta.\tan2\alpha$, if

(A) $\tan^2\alpha + 2\tan\alpha.\tan2\beta = 0$ (B) $\tan\alpha + \tan\beta = 0$

(C) $\tan^2\beta + 2\tan\beta.\tan2\alpha = 1$ (D) $\tan\alpha = \tan\beta$

10. If the sides of a right angled triangle are $\{\cos2\alpha + \cos2\beta + 2\cos(\alpha+\beta)\}$ and $\{\sin2\alpha + \sin2\beta + 2\sin(\alpha+\beta)\}$, then the length of the hypotenuse is :

(A) $2[1+\cos(\alpha-\beta)]$ (B) $2[1-\cos(\alpha+\beta)]$ (C) $4\cos^2\dfrac{\alpha-\beta}{2}$ (D) $4\sin^2\dfrac{\alpha+\beta}{2}$

11. For $0 < \theta < \pi/2$, $\tan\theta + \tan2\theta + \tan3\theta = 0$ if

(A) $\tan\theta = 0$ (B) $\tan2\theta = 0$ (C) $\tan3\theta = 0$ (D) $\tan\theta\tan2\theta = 2$

12. $(a+2)\sin\alpha + (2a-1)\cos\alpha = (2a+1)$ if $\tan\alpha =$

(A) $\dfrac{3}{4}$ (B) $\dfrac{4}{3}$ (C) $\dfrac{2a}{a^2+1}$ (D) $\dfrac{2a}{a^2-1}$

13. If $\tan x = \dfrac{2b}{a-c}$, $(a \neq c)$

$y = a\cos^2x + 2b\sin x\cos x + c\sin^2x$

$z = a\sin^2x - 2b\sin x\cos x + c\cos^2x$, then

(A) $y = z$ (B) $y + z = a + c$ (C) $y - z = a - c$ (D) $y - z = (a-c)^2 + 4b^2$

14. The value of $\left(\dfrac{\cos A + \cos B}{\sin A - \sin B}\right)^n + \left(\dfrac{\sin A + \sin B}{\cos A - \cos B}\right)^n$ is

(A) $2\tan^n\dfrac{A-B}{2}$ (B) $2\cot^n\dfrac{A-B}{2}$: n is even

(C) 0 : n is odd (D) 0 : n is even

15. The equation $\sin^6x + \cos^6x = a^2$ has real solution if

(A) $a \in (-1, 1)$ (B) $a \in \left(-1, -\dfrac{1}{2}\right)$ (C) $a \in \left(-\dfrac{1}{2}, \dfrac{1}{2}\right)$ (D) $a \in \left(\dfrac{1}{2}, 1\right)$

16. If $\sin(x-y) = \cos(x+y) = 1/2$ then the values of x & y lying between 0 and π are given by:

(A) $x = \pi/4$, $y = 3\pi/4$ (B) $x = \pi/4$, $y = \pi/12$

(C) $x = 5\pi/4$, $y = 5\pi/12$ (D) $x = 11\pi/12$, $y = 3\pi/4$

17. If $2\sec^2\alpha - \sec^4\alpha - 2\operatorname{cosec}^2\alpha + \operatorname{cosec}^4\alpha = 15/4$, then $\tan\alpha$ can be

(A) $1/\sqrt{2}$ (B) $1/2$ (C) $1/2\sqrt{2}$ (D) $-1/\sqrt{2}$

18. If $3\sin\beta = \sin(2\alpha+\beta)$, then $\tan(\alpha+\beta) - 2\tan\alpha$ is

(A) independent of α (B) independent of β

(C) dependent of both α and β (D) independent of α but dependent of β

19. If $\alpha + \beta + \gamma = 2\pi$, then

(A) $\tan\dfrac{\alpha}{2} + \tan\dfrac{\beta}{2} + \tan\dfrac{\gamma}{2} = \tan\dfrac{\alpha}{2}\tan\dfrac{\beta}{2}\tan\dfrac{\gamma}{2}$

(B) $\tan\dfrac{\alpha}{2}\tan\dfrac{\beta}{2} + \tan\dfrac{\beta}{2}\tan\dfrac{\gamma}{2} + \tan\dfrac{\gamma}{2}\tan\dfrac{\alpha}{2} = 1$

(C) $\tan\dfrac{\alpha}{2} + \tan\dfrac{\beta}{2} + \tan\dfrac{\gamma}{2} = -\tan\dfrac{\alpha}{2}\tan\dfrac{\beta}{2}\tan\dfrac{\gamma}{2}$

(D) $\tan\frac{\alpha}{4}\tan\frac{\beta}{4}+\tan\frac{\beta}{4}\tan\frac{\gamma}{4}+\tan\frac{\gamma}{4}\tan\frac{\alpha}{4}=1$

20. If x + y = z, then $\cos^2 x + \cos^2 y + \cos^2 z - 2\cos x\cos y\cos z$ is equal to
(A) $\cos^2 z$ (B) $\sin^2 z$ (C) cos (x + y – z) (D) 1

21. If tanA + tan B + tan C = tan A. tan B. tan C, then
(A) A, B, C may be angles of a triangle
(B) A + B + C is an integral multiple of π
(C) sum of any two of A, B, C is equal to third
(D) none of these

22. Which of the following values of ' t ' may satisfy the condition $2\sin t=\frac{1-2x+5x^2}{3x^2-2x-1}$, $t\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$.
(A) $\left[-\frac{\pi}{2},-\frac{\pi}{10}\right]$ (B) $\left[0,\frac{\pi}{2}\right]$ (C) $\left[\frac{3\pi}{10},\frac{\pi}{2}\right]$ (D) $\left[\frac{-\pi}{10},\frac{3\pi}{10}\right]$

23. sinx, sin2x, sin3x are in A.P if
(A) $x = n\pi/2, n\in I$ (B) $x = n\pi, n\in I$ (C) $x = 2n\pi, n\in I$ (D) $x = (2n+1)\pi, n\in I$

24. sin x + sin2x + sin 3x = 0 if
(A) sin x = 1/2 (B) sin 2x = 0 (C) sin 3x = $\sqrt{3}/2$ (D) cos x = – 1/2

25. cos4x cos8x – cos5x cos9x = 0 if
(A) cos12x = cos 14 x
(B) sin13 x = 0
(C) sinx = 0
(D) cosx = 0

26. $\sin x - \cos^2 x - 1$ assumes the least value for the set of values of x given by:
(A) $x = n\pi + (-1)^{n+1}(\pi/6)$, $n\in I$
(B) $x = n\pi + (-1)^n(\pi/6)$, $n\in I$
(C) $x = n\pi + (-1)^n(\pi/3)$, $n\in I$
(D) $x = n\pi - (-1)^n(\pi/6)$, $n\in I$

27. Let $0\le\theta\le\frac{\pi}{2}$ and $x = X\cos\theta + Y\sin\theta$, $y = X\sin\theta - Y\cos\theta$ such that $x^2 + 4xy + y^2 = aX^2 + bY^2$, where a, b are constants then
(A) a = –1, b = 3 (B) $\theta = \pi/4$ (C) a = 3, b = – 1 (D) $\theta=\frac{\pi}{3}$

28. If the equation $\sin(\pi x^2) - \sin(\pi x^2 + 2\pi x) = 0$ is solved for positive roots, then in the increasing sequence of positive root
(A) first term is $\frac{-1+\sqrt{7}}{2}$
(B) first term is $\frac{-1+\sqrt{3}}{2}$
(C) third term is 1
(D) third term is $\frac{-1+\sqrt{11}}{2}$

29. The general solution of the equation cosx . cos6x = – 1, is :
(A) $x = (2n + 1)\pi, n\in I$
(B) $x = 2n\pi, n\in I$
(C) $x = (2n - 1)\pi, n\in I$
(D) none of these

30. Which of the following set of values of x satisfy the inequation sin 3x < sin x.
(A) $\left(\frac{(8n-1)\pi}{4},2n\pi\right)$, $n\in I$
(B) $\left(\frac{(8n-1)\pi}{4},\frac{(8n+1)\pi}{4}\right)$, $n\in I$
(C) $\left(\frac{(8n+1)\pi}{4},\frac{(8n+3)\pi}{4}\right)$, $n\in I$
(D) $\left((2n+1)\pi,\frac{(8n+5)\pi}{4}\right)$, $n\in I$

31. The equation $2\sin\frac{x}{2} . \cos^2x + \sin^2x = 2\sin\frac{x}{2} . \sin^2x + \cos^2x$ has a root for which

(A) $\sin 2x = 1$ (B) $\sin 2x = -1$ (C) $\cos x = \frac{1}{2}$ (D) $\cos 2x = -\frac{1}{2}$

32. $\cos 15x = \sin 5x$ if

(A) $x = -\frac{\pi}{20} + \frac{n\pi}{5}, n \in I$ (B) $x = \frac{\pi}{40} + \frac{n\pi}{10}, n \in I$

(C) $x = \frac{3\pi}{20} + \frac{n\pi}{5}, n \in I$ (D) $x = -\frac{3\pi}{40} + \frac{n\pi}{10}, n \in I$

33. $5\sin^2 x + \sqrt{3}\sin x\cos x + 6\cos^2 x = 5$ if

(A) $\tan x = -1/\sqrt{3}$ (B) $\sin x = 0$

(C) $x = n\pi + \pi/2, n \in I$ (D) $x = n\pi + \pi/6, n \in I$

34. $\sin^2x + 2\sin x\cos x - 3\cos^2x = 0$ if

(A) $\tan x = 3$ (B) $\tan x = -1$

(C) $x = n\pi + \pi/4, n \in I$ (D) $x = n\pi + \tan^{-1}(-3), n \in I$

35. Solution set of inequality $\sin^3 x\cos x > \cos^3 x\sin x$, where $x \in (0, \pi)$, is

(A) $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ (B) $\left(\frac{3\pi}{4}, \pi\right)$ (C) $\left(0, \frac{\pi}{4}\right)$ (D) $\left(\frac{\pi}{2}, \frac{3\pi}{4}\right)$

36. $4\sin^4x + \cos^4x = 1$ if

(A) $x = n\pi$; $(n \in I)$ (B) $x = n\pi \pm \frac{1}{2}\cos^{-1}\left(\frac{1}{5}\right)$; $(n \in I)$

(C) $x = \frac{n\pi}{2}$; $(n \in I)$ (D) $x = -n\pi$; $(n \in I)$

37. $\sin x + \sin 2x + \sin 3x = \cos x + \cos 2x + \cos 3x$ if

(A) $\cos x = -\frac{1}{2}$ (B) $\sin 2x = \cos 2x$

(C) $x = \frac{n\pi}{2} + \frac{\pi}{8}$ (D) $x = 2n\pi \pm \frac{2\pi}{3}$, $(n \in I)$

PART - IV : COMPREHENSION

Comprehenssion # 1

Let p be the product of the sines of the angles of a triangle ABC and q is the product of the cosines of the angles.

1. In this triangle $\tan A + \tan B + \tan C$ is equal to

(A) $p + q$ (B) $p - q$ (C) $\frac{p}{q}$ (D) none of these

2. $\tan A \tan B + \tan B \tan C + \tan C \tan A$ is equal to

(A) $1 + q$ (B) $\frac{1+q}{q}$ (C) $1 + p$ (D) $\frac{1+p}{p}$

3. The value of $\tan^3 A + \tan^3 B + \tan^3 C$ is

(A) $\frac{p^3 - 3pq^2}{q^3}$ (B) $\frac{q^3}{p^3}$ (C) $\frac{p^3}{q^3}$ (D) $\frac{p^3 - 3pq}{q^3}$

Comprehension # 2

Let a, b, c, d $\in$ R. Then the cubic equation of the type $ax^3 + bx^2 + cx + d = 0$ has either one root real or all three roots are real. But in case of trigonometric equations of the type $a \sin^3 x + b \sin^2 x + c \sin x + d = 0$ can possess several solutions depending upon the domain of x.

To solve an equation of the type $a \cos\theta + b \sin\theta = c$. The equation can be written as $\cos(\theta - \alpha) = c/\sqrt{(a^2 + b^2)}$.

The solution is $\theta = 2n\pi + \alpha \pm \beta$, where $\tan \alpha = b/a$, $\cos \beta = c/\sqrt{(a^2 + b^2)}$.

4. On the domain $[-\pi, \pi]$ the equation $4\sin^3 x + 2 \sin^2 x - 2\sin x - 1 = 0$ possess

(A) only one real root (B) three real roots

(C) four real roots (D) six real roots

5. In the interval $[-\pi/4, \pi/2]$, the equation, $\cos 4x + \frac{10 \tan x}{1 + \tan^2 x} = 3$ has

(A) no solution (B) one solution (C) two solutions (D) three solutions

6. $|\tan x| = \tan x + \frac{1}{\cos x}$ $(0 \le x \le 2\pi)$ has

(A) no solution (B) one solution (C) two solutions (D) three solutions

Comprehension # 3

To solve a trigonometric inequation of the type $\sin x \ge a$ where $|a| \le 1$, we take a hill of length 2π in the sine curve and write the solution within that hill. For the general solution, we add $2n\pi$. For instance, to solve $\sin x \ge -\frac{1}{2}$, we take the hill $\left[-\frac{\pi}{2}, \frac{3\pi}{2}\right]$ over which solution is $-\frac{\pi}{6} < x < \frac{7\pi}{6}$. The general solution is $2n\pi - \frac{\pi}{6} < x < 2n\pi + \frac{7\pi}{6}$, n is any integer. Again to solve an inequation of the type $\sin x \le a$, where $|a| \le 1$, we take a hollow of length 2π in the sine curve. (since on a hill, $\sin x \le a$ is satisfied over two intervals). Similarly $\cos x \ge a$ or $\cos x \le a$, $|a| \le 1$ are solved.

7. Solution to the inequation $\sin^6 x + \cos^6 x < \frac{7}{16}$ must be

(A) $n\pi + \frac{\pi}{3} < x < n\pi + \frac{\pi}{2}$ (B) $2n\pi + \frac{\pi}{3} < x < 2n\pi + \frac{\pi}{2}$

(C) $\frac{n\pi}{2} + \frac{\pi}{6} < x < \frac{n\pi}{2} + \frac{\pi}{3}$ (D) none of these

8. Solution to inequality $\cos 2x + 5 \cos x + 3 \ge 0$ over $[-\pi, \pi]$ is

(A) $[-\pi, \pi]$ (B) $\left[\frac{-5\pi}{6}, \frac{5\pi}{6}\right]$ (C) $[0, \pi]$ (D) $\left[\frac{-2\pi}{3}, \frac{2\pi}{3}\right]$

9. Over $[-\pi, \pi]$, the solution of $2 \sin^2\left(x + \frac{\pi}{4}\right) + \sqrt{3} \cos 2x \ge 0$ is

(A) $[-\pi, \pi]$ (B) $\left[-\frac{5\pi}{6}, \frac{5\pi}{6}\right]$

(C) $[0, \pi]$ (D) $\left[-\pi, \frac{-7\pi}{12}\right] \cup \left[-\frac{\pi}{4}, \frac{5\pi}{12}\right] \cup \left[\frac{3\pi}{4}, \pi\right]$

Exercise-3

✍ **Marked questions are recommended for Revision.**
*** Marked Questions may have more than one correct option.**

PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

1*.✍ If $\frac{\sin^4 x}{2} + \frac{\cos^4 x}{3} = \frac{1}{5}$, then **[IIT-JEE - 2009 ,Paper-1, (4, –1), 80]**

(A) $\tan^2 x = \frac{2}{3}$
(B) $\frac{\sin^8 x}{8} + \frac{\cos^8 x}{27} = \frac{1}{125}$
(C) $\tan^2 x = \frac{1}{3}$
(D) $\frac{\sin^8 x}{8} + \frac{\cos^8 x}{27} = \frac{1}{125}$

2*.✍ For $0 < \theta < \frac{\pi}{2}$, the solution(s) of $\sum_{m=1}^{6} \operatorname{cosec}\left(\theta + \frac{(m-1)\pi}{4}\right) \operatorname{cosec}\left(\theta + \frac{m\pi}{4}\right) = 4\sqrt{2}$ is(are)

[IIT-JEE - 2009, Paper-2, (4, –1), 80]

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{12}$ (D) $\frac{5\pi}{12}$

3. The maximum value of the expression $\frac{1}{\sin^2\theta + 3\sin\theta\cos\theta + 5\cos^2\theta}$ is

[IIT-JEE-2010, Paper-1, (3, 0)/84]

4.✍ The positive integer value of n > 3 satisfying the equation

$\frac{1}{\sin\left(\frac{\pi}{n}\right)} = \frac{1}{\sin\left(\frac{2\pi}{n}\right)} + \frac{1}{\sin\left(\frac{3\pi}{n}\right)}$ is **[IIT-JEE 2011, Paper-1, (4, 0), 80]**

5.✍ The number of values of θ in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ such that $\theta \neq \frac{n\pi}{5}$ for n = 0, ±1, ± 2 and $\tan\theta = \cot 5\theta$ as well as $\sin 2\theta = \cos 4\theta$ is **[IIT-JEE-2010, Paper-1, (3, 0)/84]**

6. Let P = {θ : $\sin\theta - \cos\theta = \sqrt{2}\cos\theta$} and Q = {$\theta$: $\sin\theta + \cos\theta = \sqrt{2}\sin\theta$} be two sets. Then

(A) $P \subset Q$ and $Q - P \neq \varnothing$
(B) $Q \not\subset P$
(C) $P \not\subset Q$
(D) P = Q **[IIT-JEE 2011, Paper-1, (3, –1), 80]**

7*. Let $\theta, \phi \in [0, 2\pi]$ be such that $2\cos\theta(1 - \sin\phi) = \sin^2\theta\left(\tan\frac{\theta}{2} + \cot\frac{\theta}{2}\right)\cos\phi - 1$, $\tan(2\pi - \theta) > 0$ and $-1 < \sin\theta < -\frac{\sqrt{3}}{2}$. Then ϕ cannot satisfy **[IIT-JEE 2012, Paper-1, (4, 0), 70]**

(A) $0 < \phi < \frac{\pi}{2}$ (B) $\frac{\pi}{2} < \phi < \frac{4\pi}{3}$ (C) $\frac{4\pi}{3} < \phi < \frac{3\pi}{2}$ (D) $\frac{3\pi}{2} < \phi < 2\pi$

8. For $x \in (0, \pi)$, the equation $\sin x + 2\sin 2x - \sin 3x = 3$ has

[JEE (Advanced) 2014, Paper-2, (3, –1)/60]

(A) infinitely many solutions
(B) three solutions
(C) one solution
(D) no solution

9.✍ The number of distinct solutions of the equation

$\frac{5}{4}\cos^2 2x + \cos^4 x + \sin^4 x + \cos^6 x + \sin^6 x = 2$ in the interval $[0, 2\pi]$ is

[JEE (Advanced) 2015, P-1 (4, 0) /88]

10. The value of $\sum_{k=1}^{13}\frac{1}{\sin\left(\frac{\pi}{4}+\frac{(k-1)\pi}{6}\right)\sin\left(\frac{\pi}{4}+\frac{k\pi}{6}\right)}$ is equal to **[JEE (Advanced) 2016, Paper-2, (3, –1)/62]**

(A) $3-\sqrt{3}$ (B) $2\left(3-\sqrt{3}\right)$ (C) $2\left(\sqrt{3}-1\right)$ (D) $2\left(2+\sqrt{3}\right)$

11. Let S = $\left\{x \in (-\pi,\pi) : x \neq 0, \pm\frac{\pi}{2}\right\}$. The sum of all distinct solutions of the equation $\sqrt{3}$ sec x + cosec x + 2(tan x – cot x) = 0 in the set S is equal to

[JEE (Advanced) 2016, Paper-1, (3, –1)/62]

(A) $-\frac{7\pi}{9}$ (B) $-\frac{2\pi}{9}$ (C) 0 (D) $\frac{5\pi}{9}$

12. Let α and β be nonzero real numbers such that 2(cos β – cos α) + cos α cos β = 1. Then which of the following is/are true? **[JEE(Advanced) 2017, Paper-2,(4, –2)/61]**

(A) $\sqrt{3}\tan\left(\frac{\alpha}{2}\right)-\tan\left(\frac{\beta}{2}\right)=0$ (B) $\tan\left(\frac{\alpha}{2}\right)-\sqrt{3}\tan\left(\frac{\beta}{2}\right)=0$

(C) $\tan\left(\frac{\alpha}{2}\right)+\sqrt{3}\tan\left(\frac{\beta}{2}\right)=0$ (D) $\sqrt{3}\tan\left(\frac{\alpha}{2}\right)+\tan\left(\frac{\beta}{2}\right)=0$

13. Let a, b, c be three non-zero real numbers such that the equation $\sqrt{3}$ a cos x + 2b sinx = c, $x \in \left[-\frac{\pi}{2},\frac{\pi}{2}\right]$, has two distinct real roots α and β with $\alpha + \beta = \frac{\pi}{3}$. Then, the value of $\frac{b}{a}$ is ______.

[JEE(Advanced) 2018, Paper-1,(4, –2)/60]

PART - II : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. Let A and B denote the statements **[AIEEE 2009 (4, –1), 144]**

A : cos α + cos β + cos γ = 0

B : sin α + sin β + sin γ = 0

If cos ($\beta - \gamma$) + cos ($\gamma - \alpha$) + cos ($\alpha - \beta$) = $-\frac{3}{2}$, then :

(1) A is false and B is true (2) both A and B are true

(3) both A and B are false (4) A is true and B is false

2. Let $\cos(\alpha+\beta)=\frac{4}{5}$ and let $\sin(\alpha-\beta)=\frac{5}{13}$, where $0 \le \alpha, \beta \le \frac{\pi}{4}$. Then tan 2$\alpha$ =

[AIEEE 2010 (4, –1), 144]

(1) $\frac{56}{33}$ (2) $\frac{19}{12}$ (3) $\frac{20}{7}$ (4) $\frac{25}{16}$

3. If A = $\sin^2 x + \cos^4 x$, then for all real x : **[AIEEE 2011 (4, –1), 120]**

(1) $\frac{3}{4} \le A \le 1$ (2) $\frac{13}{16} \le A \le 1$ (3) $1 \le A \le 2$ (4) $\frac{3}{4} \le A \le \frac{13}{16}$

4. In a ΔPQR, if 3 sin P + 4 cos Q = 6 and 4 sin Q + 3 cos P = 1, then the angle R is equal to :

[AIEEE-2012, (4, –1)/120]

(1) $\frac{5\pi}{6}$ (2) $\frac{\pi}{6}$ (3) $\frac{\pi}{4}$ (4) $\frac{3\pi}{4}$

5. The expression $\frac{\tan A}{1-\cot A}+\frac{\cot A}{1-\tan A}$ can be written as : **[AIEEE - 2013, (4, –1),360]**

(1) sinA cosA + 1 (2) secA cosecA + 1 (3) tanA + cotA (4) secA + cosecA

6. Let $f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x)$ where $x \in R$ and $k \geq 1$. Then $f_4(x) - f_6(x)$ equals

[JEE(Main)2014,(4, – 1), 120]

(1) $\frac{1}{4}$ (2) $\frac{1}{12}$ (3) $\frac{1}{6}$ (4) $\frac{1}{3}$

7. If the angles of elevation of the top of a tower from three collinear points A, B and C, on a line leading to the foot of the tower, are 30º, 45º and 60º respectively, then the ratio, AB : BC , is

[JEE(Main)2015,(4, – 1), 120]

(1) $\sqrt{3}:1$ (2) $\sqrt{3}:\sqrt{2}$ (3) $1 : \sqrt{3}$ (4) 2 : 3

8. A man is walking towards a vertical pillar in a straight path, at a uniform speed. At a certain point A on the path, he observes that the angle of elevation of the top of the pillar is 30°, After walking for 10 minutes from A in the same direction, at a point B, he observes that the angle of elevation of the top of the pillar is 60°. Then the time taken (in minutes) by him, from B to reach the pillar, is :

[JEE(Main)2016,(4, – 1), 120]

(1) 10 (2) 20 (3) 5 (4) 6

9. If $0 \leq x < 2\pi$, then the number of real values of x, which satisfy the equation
cosx + cos2x + cos3x + cos4x = 0, is **[JEE(Main)2016,(4, – 1), 120]**

(1) 5 (2) 7 (3) 9 (4) 3

10. If $5(\tan^2 x - \cos^2 x) = 2\cos 2x + 9$, then the value of cos4x is : **[JEE(Main)2017,(4, – 1), 120]**

(1) $\frac{-3}{5}$ (2) $\frac{1}{3}$ (3) $\frac{2}{9}$ (4) $-\frac{7}{9}$

11. Let a vertical tower AB have its end A on the level ground. Let C be the mid-point of AB and P be a point on the ground such that AP = 2AB. If $\angle BPC = \beta$, then $\tan\beta$ is equal to

[JEE(Main)2017,(4, – 1), 120]

(1) $\frac{6}{7}$ (2) $\frac{1}{4}$ (3) $\frac{2}{9}$ (4) $\frac{4}{9}$

12. If sum of all the solutions of the equation $8\cos x.\left(\cos\left(\frac{\pi}{6}+x\right).\cos\left(\frac{\pi}{6}-x\right)-\frac{1}{2}\right) = 1$ in $[0, \pi]$ is $k\pi$, then k is equal to : **[JEE(Main)2018,(4, – 1), 120]**

(1) $\frac{8}{9}$ (2) $\frac{20}{9}$ (3) $\frac{2}{3}$ (4) $\frac{13}{9}$

13. PQR is a triangular park with PQ = PR = 200 m. A T.V. tower stands at the mid-point of QR. If the angles of elevation of the top of the tower at P, Q and R are respectively 45º, 30º and 30º, then the height of the tower (in m) is : **[JEE(Main)2018,(4, – 1), 120]**

(1) $100\sqrt{3}$ (2) $50\sqrt{2}$ (3) 100 (4) 50

14. The sum of all values of $\theta \in \left(0, \frac{\pi}{2}\right)$ satisfying $\sin^2 2\theta + \cos^4 2\theta = \frac{3}{4}$ is :

[JEE(Main) 2019, Online (10-01-19),P-1 (4, – 1), 120]

(1) π (2) $\frac{\pi}{2}$ (3) $\frac{3\pi}{8}$ (4) $\frac{5\pi}{4}$

15. The value of $\cos\frac{\pi}{2^2}.\cos\frac{\pi}{2^3}.\ldots..\cos\frac{\pi}{2^{10}}.\sin\frac{\pi}{2^{10}}$ is :

[JEE(Main) 2019, Online (10-01-19),P-2 (4, – 1), 120]

(1) $\frac{1}{1024}$ (2) $\frac{1}{2}$ (3) $\frac{1}{512}$ (4) $\frac{1}{256}$

16. If $\sin^4\theta + 4\cos^4\beta + 2 = 4\sqrt{2}\sin\alpha\cos\beta$; $\alpha, \beta \in [0, \pi]$, then $\cos(\alpha+\beta) - \cos(\alpha-\beta)$ is equal to

[JEE(Main) 2019, Online (12-01-19),P-2 (4, – 1), 120]

(1) $-\sqrt{2}$ (2) 0 (3) $\sqrt{2}$ (4) –1

Answers

EXERCISE - 1

PART - I

Section (A) :

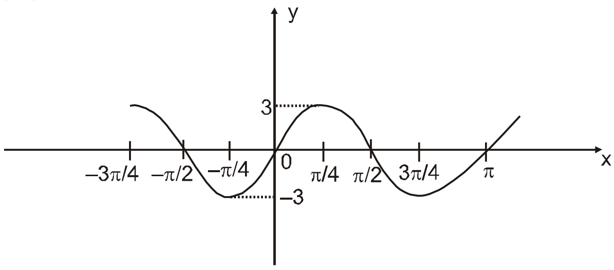
A-1. (i) $\frac{\pi}{12}$ (ii) $\frac{4\pi}{3}$ (iii) $\frac{53\pi}{18}$

A-2. (i) 135° (ii) – 720° (iii) 300° (iv) 210°

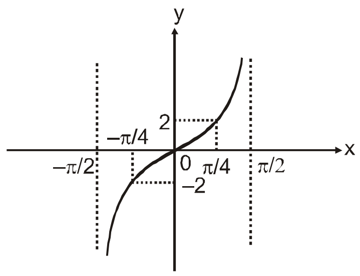
A-4. (i) $\left(\frac{-\sqrt{3}}{2}\right)$ (ii) $-\frac{1}{\sqrt{2}}$ (iii) $-\frac{1}{\sqrt{3}}$ (iv) 1

Section (B) :

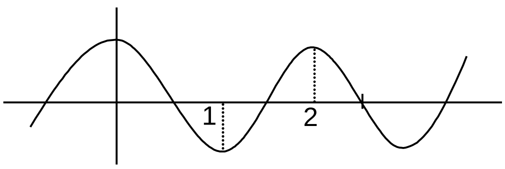
B-1. (i)



(ii)



(iii)



B-2. 1 **B-3.** $\frac{13}{12}$ **B-7.** $a^2b^2 + 4a^2 = 9b^2$

Section (C) :

C-2. (i) 1 (ii) $-\sqrt{5}/4$ (iii) $\frac{\sqrt{5}+1}{8}$ **C-10.** $\frac{\sqrt{3}-1}{2\sqrt{2}}$

Section (E) :

E-1. $-\frac{1}{4}, \frac{1}{4}$ **E-2.** (i) 2, –1 (ii) 3, 0 **E-3.** (i) $y_{max} = 11; y_{min} = 1$ (ii) $y_{max} = 10; y_{min} = -4$

Section (F) :

F-1. (i) $n\pi + (-1)^n \frac{\pi}{4}, n \in I$ (ii) $n\pi + \frac{\pi}{3} + 1, n \in I$

(iii) $n\pi - \frac{\pi}{4}, n \in I$ (iv) $n\pi + (-1)^n \frac{\pi}{3}, n \in I$ (v) $n\pi \pm \frac{\pi}{4}, n \in I$

F-2. (i) $\frac{m\pi}{4}, m \in I$ or $\frac{(2m+1)\pi}{10}$, $m \in I$ (ii) $2n\pi \pm \frac{\pi}{3}$, $n \in I$

(iii) $\left(2n+\frac{1}{2}\right)\frac{\pi}{5}, n \in I$ or $2n\pi - \frac{\pi}{2}, n \in I$ (iv) $\left(n+\frac{1}{2}\right)\frac{\pi}{9}, n \in I$

(v) $\left(n+\frac{1}{4}\right)\frac{\pi}{2}, n \in I$ (vi) $2n\pi + \frac{2\pi}{3}, n \in I$ (vii) $n\pi \pm \frac{\pi}{6}$

(viii) $\left(n+\frac{1}{3}\right)\frac{\pi}{3}, n \in I$

F-3. (i) $\frac{n\pi}{3}$, $n \in I$ or $\left(n \pm \frac{1}{3}\right)\pi$, $n \in I$ (ii) $2n\pi$, $n \in I$ or $\frac{2n\pi}{3} + \frac{\pi}{6}$, $n \in I$

(iii) $x = (2n+1)\frac{\pi}{4}$, $n \in I$ or $x = (2n+1)\frac{\pi}{2}$, $n \in I$ or $x = n\pi \pm \frac{\pi}{6}$, $n \in I$

(iv) $m\pi$, $m \in I$ or $\frac{m\pi}{n-1}$, $m \in I$ or $\left(m + \frac{1}{2}\right)\frac{\pi}{n}$, $m \in I$

F-4. (i) $n\pi + \frac{\pi}{3}$, $n \in I$ or $n\pi + \frac{\pi}{4}$, $n \in I$ **(ii)** $n\pi + (-1)^n \frac{\pi}{10}$, $n \in I$ or ;k $n\pi - (-1)^n \frac{3\pi}{10}$, $n \in I$

(iii) $x = \frac{n\pi}{3} - \frac{\pi}{9}$, $n \in I$ **F-5.** (i) $n\pi + \frac{\pi}{6} + (-1)^n \frac{\pi}{4}$, $n \in I$

(ii) $2n\pi + \frac{\pi}{2}$, $n \in I$ or $2n\pi + 2\alpha$ where $\alpha = \tan^{-1}\frac{3}{7}$, $n \in I$

Section (G) :

G-1. $x \in \left[n\pi - \frac{\pi}{4}, n\pi + \frac{\pi}{4}\right] : n \in I$ **G-2.** $\left(2n\pi + \frac{7\pi}{6},\ 2n\pi + \frac{11\pi}{6}\right)$ **G-3.** $\theta \in (n\pi + \pi/3, n\pi + \pi/2]$

G-4. $15\sqrt{3}$ m **G-5.** $\sqrt{ab}$ **G-6.** 50 m

PART - II

Section (A) :

A-1. (A) **A-2.** (A) **A-3.** (B) **A-4.** (C) **A-5.** (B)

Section (B) :

B-1. (A) **B-2.** (B) **B-3.** (A) **B-4.** (D) **B-5.** (B) **B-6.** (B) **B-7.** (A)
B-8. (A) **B-9.** (C) **B-10.** (A) **B-11.** (B) **B-12.** (C)

Section (C) :

C-1. (C) **C-2.** (A) **C-3.** (C) **C-4.** (D) **C-5.** (A) **C-6.** (B) **C-7.** (A)
C-8. (B) **C-9.** (D) **C-10.** (C)

Section (D) :

D-1. (A) **D-2.** (B) **D-3.** (D) **D-4.** (B) **D-5.** (A)

Section (E) :

E-1. (C) **E-2.** (C) **E-3.** (A) **E-4.** (B) **E-5.** (A)

Section (F) :

F-1. (D) **F-2.** (B) **F-3.** (D) **F-4.** (B) **F-5.** (C) **F-6.** (B) **F-7.** (A)
F-8. (B) **F-9.** (A) **F-10.** (B) **F-11.** (C) **F-12.** (B)

Section (G) :

G-1. (A) **G-2.** (C) **G-3.** (B) **G-4.** (A) **G-5.** (A) **G-6.** (D) **G-7.** (D)
G-8. (D) **G-9.** (B)

PART - III

1. (A) → (s), (B) → (s), (C) → (s), (D) → (r)
2. (A) → (q, s), (B) → (p), (C) → (q), (D) → (p)
3. (A) → (q), (B) → (s), (C) → (r), (D) → (p)

EXERCISE - 2

PART – I

1.	(B)	**2.**	(B)	**3.**	(D)	**4.**	(B)	**5.**	(A)	**6.**	(B)	**7.**	(B)
8.	(A)	**9.**	(D)	**10.**	(D)	**11.**	(C)	**12.**	(C)	**13.**	(C)	**14.**	(A)
15.	(A)	**16.**	(D)	**17.**	(A)	**18.**	(B)						

PART - II

1.	4	**2.**	24	**3.**	2	**4.**	12	**5.**	6	**6.**	1	**7.**	8
8.	3	**9.**	15	**10.**	5	**11.**	24	**12.**	2	**13.**	1	**14.**	50
15.	4	**16.**	2	**17.**	17	**18.**	17	**19.**	6	**20.**	15	**21.**	07
22.	17	**23.**	2	**24.**	6	**25.**	0	**26.**	0	**27.**	0		

PART - III

1.	(ABCD)	**2.**	(BC)	**3.**	(BD)	**4.**	(AB)	**5.**	(BC)	**6.**	(ABC)	**7.**	(BC)
8.	(ABCD)	**9.**	(BCD)	**10.**	(AC)	**11.**	(CD)	**12.**	(BD)	**13.**	(BC)	**14.**	(BC)
15.	(BD)	**16.**	(BD)	**17.**	(AD)	**18.**	(AB)	**19.**	(AD)	**20.**	(CD)	**21.**	(AB)
22.	(AC)	**23.**	(ABCD)	**24.**	(BD)	**25.**	(ABC)	**26.**	(AD)	**27.**	(BC)	**28.**	(BC)
29.	(AC)	**30.**	(ACD)	**31.**	(ABCD)	**32.**	(ABCD)	**33.**	(AC)	**34.**	(CD)	**35.**	(AB)
36.	(ABD)	**37.**	(ABCD)										

PART - IV

1.	(C)	**2.**	(B)	**3.**	(D)	**4.**	(D)	**5.**	(C)	**6.**	(B)	**7.**	(C)
8.	(D)	**9.**	(D)										

EXERCISE - 3

PART – I

1.*	(AB)	**2*.**	(CD)	**3.**	2	**4.**	7	**5.**	3	**6.**	(D)
7.*	(ACD)	**8.**	(D)	**9.**	8	**10.**	(C)	**11.**	(C)	**12.**	Bonus
13.	(0.5)										

PART – II

1.	(2)	**2.**	(1)	**3.**	(1)	**4.**	(2)	**5.**	(2)	**6.**	(2)	**7.**	(1)
8.	(3)	**9.**	(2)	**10.**	(4)	**11.**	(3)	**12.**	(4)	**13.**	(3)	**14.**	(2)
15.	(3)	**16.**	(1)										

Advanced Level Problems

1. Prove that :

(i) $\sec^4 A\,(1 - \sin^4 A) - 2\tan^2 A = 1$ (ii) $\dfrac{\cot^2\theta\ \ (\sec\theta - 1)}{1+\sin\theta} = \sec^2\theta\ \dfrac{1-\sin\theta}{1+\sec\theta}$.

2. Simplify the expression

$$\sqrt{\sin^4 x + 4\cos^2 x} - \sqrt{\cos^4 x + 4\sin^2 x}$$

3. Let a, b, c, d be numbers in the interval $[0, \pi]$ such that

$\sin a + 7\sin b = 4(\sin c + 2\sin d)$,

$\cos a + 7\cos b = 4(\cos c + 2\cos d)$

Prove that $2\cos(a - d) = 7\cos(b - c)$.

4. Prove that $(4\cos^2 9^\circ - 3)(4\cos^2 27^\circ - 3) = \tan 9^\circ$

5. If $\cos(\alpha + \beta) = \dfrac{4}{5}$; $\sin(\alpha - \beta) = \dfrac{5}{13}$ & α, β lie between 0 & $\dfrac{\pi}{4}$, then find the value of $\tan 2\alpha$.

6. If α & β are two distinct roots of the equation $a\tan\theta + b\sec\theta = c$, then prove that $\tan(\alpha + \beta) = \dfrac{2ac}{a^2 - c^2}$.

7. If $\tan\alpha = \dfrac{p}{q}$ where $\alpha = 6\beta$, α being an acute angle, prove that;

$\dfrac{1}{2}(p\,\text{cosec}\,2\beta - q\sec 2\beta) = \sqrt{p^2 + q^2}$.

8. If $\sin(\theta + \alpha) = a$ & $\sin(\theta + \beta) = b$ $(0 < \alpha, \beta, \theta < \pi/2)$ then find the value of $\cos 2(\alpha - \beta) - 4ab\cos(\alpha - \beta)$

9. Show that:

(i) $\cot 7\dfrac{1^\circ}{2}$ or $\tan 82\dfrac{1^\circ}{2} = \left(\sqrt{3} + \sqrt{2}\right)\left(\sqrt{2} + 1\right)$ or $\sqrt{2} + \sqrt{3} + \sqrt{4} + \sqrt{6}$

(ii) $\tan 142\dfrac{1^\circ}{2} = 2 + \sqrt{2} - \sqrt{3} - \sqrt{6}$.

10. If $\tan\beta = \dfrac{\tan\alpha + \tan\gamma}{1 + \tan\alpha\,.\,\tan\gamma}$, prove that $\sin 2\beta = \dfrac{\sin 2\alpha + \sin 2\gamma}{1 + \sin 2\alpha\,.\sin 2\gamma}$.

11. If α & β satisfy the equation $a\cos 2\theta + b\sin 2\theta = c$ then prove that: $\cos^2\alpha + \cos^2\beta = \dfrac{a^2 + ac + b^2}{a^2 + b^2}$.

12. Show that : $4\sin 27^\circ = (5 + \sqrt{5})^{1/2} - (3 - \sqrt{5})^{1/2}$

13. If $xy + yz + xz = 1$, then prove that

$$\frac{x}{1-x^2} + \frac{y}{1-y^2} + \frac{z}{1-z^2} = \frac{4xyz}{(1-x^2)(1-y^2)(1-z^2)}.$$

14. Let $a = \dfrac{\pi}{7}$

(a) Show that $\sin^2 3a - \sin^2 a = \sin 2a \sin 3a$
(b) Show that $\text{cosec}\, a = \text{cosec}\, 2a + \text{cosec}\, 4a$
(c) Evaluate $\cos a - \cos 2a + \cos 3a$
(d) Prove that $\cos a$ is a root of the equation $8x^3 - 4x^2 - 4x + 1 = 0$
(e) Evaluate $\tan a \tan 2a \tan 3a$
(f) Evaluate $\tan^2 a + \tan^2 2a + \tan^2 3a$
(g) Evaluate $\tan^2 a \tan^2 2a + \tan^2 2a \tan^2 3a + \tan^2 3a\ \tan^2 a$
(h) Evaluate $\cot^2 a + \cot^2 2a + \cot^2 3a$

15. In a ΔABC, prove that $\sin\frac{A}{2} + \sin\frac{B}{2} + \sin\frac{C}{2} = 1 + 4\sin\left(\frac{\pi - A}{4}\right)\sin\left(\frac{\pi - B}{4}\right)\sin\left(\frac{\pi - C}{4}\right)$

16. Evaluate

$\cos a \cos 2a \cos 3a \ldots\ldots\ldots \cos 999a$, where $a = \frac{2\pi}{1999}$

17. Prove that the average of the numbers
$2\sin 2^\circ, 4\sin 4^\circ, 6\sin 6^\circ, \ldots\ldots\ldots 180\sin 180^\circ$ is $\cot 1^\circ$

18. Solve $\tan 2\theta = \tan\frac{2}{\theta}$.

19. Find the general values of x and y satisfying the equations
$5\sin x\cos y = 1$, $4\tan x = \tan y$

20. Solve $\dfrac{\sin^3\frac{x}{2} - \cos^3\frac{x}{2}}{2 + \sin x} = \dfrac{\cos x}{3}$.

21. Solve the system of equations :

$x + y = \frac{2\pi}{3}$, $\sin x + \sin y = \frac{3}{2}$ and $x, y \in \left[0, \frac{\pi}{2}\right]$

22. Solve the following system of simultaneous equations for x and y:
$4^{\sin x} + 3^{1/\cos y} = 11$
$5.16^{\sin x} - 2.3^{1/\cos y} = 2$

23. Solve : $\cos\theta + \sin\theta = \cos 2\theta + \sin 2\theta$.

24. Solve $8\sin x = \dfrac{\sqrt{3}}{\cos x} + \dfrac{1}{\sin x}$

25. Solve the equation $\sin^3 x\cos 3x + \cos^3 x\sin 3x + 0.375 = 0$.

26. Solve the equation $\frac{\sqrt{3}}{2}\sin x - \cos x = \cos^2 x$.

27. Solve the equation $\sin^4 x + \cos^4 x - 2\sin^2 x + \frac{3}{4}\sin^2 2x = 0$

28. Solve for x, the equation $\sqrt{13 - 18\tan x} = 6\tan x - 3$, where $-2\pi < x < 2\pi$.

29. Solve the equation $3 - 2\cos\theta - 4\sin\theta - \cos 2\theta + \sin 2\theta = 0$

30. Solve the equation $\sin^2 4x + \cos^2 x = 2\sin 4x \cdot \cos^4 x$

31. Prove that : $\cos 5A = 16\cos^5 A - 20\cos^3 A + 5\cos A$

32. If $\cos\theta = \frac{1}{2}\left(a + \frac{1}{a}\right)$ and $\cos 3\theta = \frac{1}{2}\left(a^k + \frac{1}{a^k}\right)$ then number of natural numbers 'k' less than 50 is (given $a \in R$)

33. Consider the equation for $0 \le \theta \le 2\pi$; $\left(\sin 2\theta + \sqrt{3}\cos 2\theta\right)^2 - 5 = \cos\left(\frac{\pi}{6} - 2\theta\right)$. If greatest value of θ is $\frac{k\pi}{p}$ (k, p are coprime), then find the value of (k + p).

Answer Key (ALP)

2. $\cos^2 x - \sin^2 x = \cos 2x$

5. $\frac{56}{33}$

8. $1 - 2a^2 - 2b^2$

14. (c) $\frac{1}{2}$ (e) $\sqrt{7}$ (f) 21 (g) 35 (h) 5

16. $\frac{1}{2^{999}}$

18. $\frac{n\pi}{4} \pm \sqrt{1 + \frac{n^2\pi^2}{16}}$, $n \in I$

20. $x = (4n + 1)\frac{\pi}{2}$, $n \in I$

21. $x = \frac{\pi}{2}$, $y = \frac{\pi}{6}$ or $x = \frac{\pi}{6}$, $y = \frac{\pi}{2}$

22. $x = n\pi + (-1)^n \frac{\pi}{6}$, $y = 2n\pi \pm \frac{\pi}{3}$

23. $2n\pi$, $n \in I$ or $\frac{2n\pi}{3} + \frac{\pi}{6}$, $n \in I$

24. $x = n\pi + \frac{\pi}{6}$, $n \in I$, $x = \frac{n\pi}{2} - \frac{\pi}{12}$, $n \in I$

25. $x = \frac{n\pi}{4} + (-1)^{n+1} \cdot \frac{\pi}{6}$, $n \in I$

26. $x = (2n + 1)\pi : n \in I$, $x = 2n\pi \pm \frac{\pi}{3}$, $n \in I$

7. $x = n\pi \pm \frac{1}{2}\cos^{-1}\left(2 - \sqrt{5}\right)$, $n \in I$

28. $\alpha - 2\pi;\ \alpha - \pi,\ \alpha,\ \alpha + \pi$, where $\tan\alpha = \frac{2}{3}$

29. $\theta = (4n + 1)\pi/2$, $\theta = 2n\pi$, $n \in I$

30. $x = (2n + 1)\frac{\pi}{2}$, $n \in I$

32. 25

33. 31