Exercise-1

Marked questions are recommended for Revision.

PART - I : SUBJECTIVE QUESTIONS

Section (A) : Ordered pair , Cartesian product, Relation, Domain and Range of Relation

A-1. If A = {2, 4, 5}, B = {7, 8, 9}, then find n(A × B).

A-2. If A = {x : $x^2 - 5x + 6 = 0$}, B = {2, 4}, C = {4, 5} then find A × (B ∩ C).

A-3. A and B are two sets having 3 and 4 elements respectively and having 2 elements in common. Find the number of possible relations which can be defined from A to B.

A-4. If A = {2, 3, 4, 5}, B = {1, 3, 5, 7} and a relation R : A →B such that y = 2x –3, x∈A, y∈B, then find R.

A-5. Let R be a relation defined as R = { (x, y) : y = $\sqrt{(x-1)^2}$, x ∈ Z and $-3 \le x \le 3$} then find

(i) Domain of R (ii) Range of R (iii) Relation R

A-6. The Certesian product A × A has 16 elements S = {(a,b) ∈ A × A| a < b}. (–1,2) and (0,1) are two elements belonging to S. Find the set containing the remaining elements of S.

Section (B) : Types of Relation

B-1. Identify the type of relation among reflexive, symmetric and transitive.

(i) R = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)} on set A = {1, 2, 3}.

(ii) P = {(x, y) | $x^2 + y^2 = 1$, x, y ∈ R}

B-2. Prove that the relation "less than" in the set of natural number is transitive but not reflexive and symmetric.

B-3. Let A = {p, q , r}. Which of the following is an equivalence relation on A ?

(i) R = {(p, q), (q, r), (p, r) (p, p)}

(ii) R = {(p, p), (q, q), (r, r) (q, p)}

(iii) R = {(p, p), (q, q), (r, r)}

(iv) R = {(p, p), (q, q), (r, r), (p, q), (q, r), (p, r)}

(v) R = {(p, p), (q, q), (r, r), (p, q), (q, p)}

B-4. Let R be a relation on the set N be defined by {(x, y)| x, y ∈ N, 2x + y = 41}. Then prove that R is neither reflexive nor symmetric and nor transitive.

B-5. Let n be a fixed positive integer. Define a relation R on the set of integers Z, aRb ⇔ n|(a – b). Then prove that R is equivalence.

B-6. Let S be a set of all square matrices of order 2. If a relation R defined on set S such that AR B ⇒ AB = BA, then identify the type of relation of R (A, B ∈ S) among reflexive, symmetric and transitive.

Section (C) : Definition of function, Domain and Range, Classification of Functions

C-1. Check whether the followings represent function or not

(i) $x^2 + y^2 = 36$, y ∈ [0, 6] (ii) $x^2 + y^2 = 36$, x ∈ [0, 1]

(iii) $x^2 + y^2 = 36$, x ∈ [–6, 6] (iv) $x^2 + y^2 = 36$

C-2. Find the domain of each of the following functions :

(i) $f(x) = \dfrac{x^3 - 5x + 3}{x^2 - 1}$

(ii) $f(x) = \sqrt{\sin(\cos x)}$

(iii) $f(x) = \dfrac{1}{\sqrt{x + |x|}}$

(iv) $f(x) = e^{x + \sin x}$

(v) $f(x) = \dfrac{1}{\log_{10}(1 - x)} + \sqrt{x + 2}$

(vi) $f(x) = \sqrt{\dfrac{\log_2(x-2)}{\log_{1/2}(3x-1)}}$

(vii) $f(x) = \ell n\, [x^2 + x + 1]$, where [.] GIF.

(viii) $f(x) = \dfrac{\sqrt{\cos x - \frac{1}{2}}}{\sqrt{6 + 35x - 6x^2}}$

C-3. Find the domain of definitions of the following functions :

(i) $f(x) = \sqrt{3 - 2^x - 2^{1-x}}$

(ii) $f(x) = \sqrt{1 - \sqrt{1 - x^2}}$

(iii) $f(x) = (x^2 + x + 1)^{-3/2}$

(iv) $f(x) = \sqrt{\dfrac{x-2}{x+2}} + \sqrt{\dfrac{1-x}{1+x}}$

(v) $f(x) = \sqrt{\tan x - \tan^2 x}$

(vi) $f(x) = \dfrac{1}{\sqrt{1 - \cos x}}$

(vii) $f(x) = \sqrt{\ell og_{1/4}\left(\dfrac{5x - x^2}{4}\right)}$

(viii) $f(x) = \ell og_{10}\,(1 - \ell og_{10}(x^2 - 5x + 16))$

C-4. Find the range of each of the following functions :

(i) $f(x) = |x - 3|$

(ii) $f(x) = \dfrac{x}{1 + x^2}$

(iii) $f(x) = \sqrt{16 - x^2}$

(iv) $f(x) = \dfrac{|x - 4|}{x - 4}$

C-5. Find the domain and the range of each of the following functions :

(i) $f(x) = \dfrac{1}{\sqrt{4 + 3\sin x}}$

(ii) $f(x) = x\,!$

(iii) $f(x) = \dfrac{x^2 - 9}{x - 3}$

(iv) $f(x) = \sin^2(x^3) + \cos^2(x^3)$

C-6. Find the range of each of the following functions : (where {.} and [.] represent fractional part and greatest integer part functions respectively)

(i) $f(x) = 5 + 3\sin x + 4\cos x$

(ii) $f(x) = \dfrac{1}{1 + \sqrt{x}}$

(iii) $f(x) = 2 - 3x - 5x^2$

(iv) $f(x) = 3\,|\sin x| - 4\,|\cos x|$

(v) $f(x) = \dfrac{\sin x}{\sqrt{1 + \tan^2 x}} + \dfrac{\cos x}{\sqrt{1 + \cot^2 x}}$

(vi) $f(x) = \ell n\left(\dfrac{\sqrt{8 - x^2}}{x - 2}\right)$

(vii) $f(x) = \left[\dfrac{1}{\sin\{x\}}\right]$

C-7. Find the range of the following functions : (where {.} and [.] represent fractional part and greatest integer part functions respectively)

(i) $f(x) = 1 - |x - 2|$

(ii). $f(x) = \dfrac{1}{\sqrt{16 - 4^{x^2 - x}}}$

(iii) $f(x) = \dfrac{1}{2 - \cos 3x}$

(iv) $f(x) = \dfrac{x + 2}{x^2 - 8x - 4}$

(v) $f(x) = \dfrac{x^2 - 2x + 4}{x^2 + 2x + 4}$

(vi) $f(x) = 3\sin\sqrt{\dfrac{\pi^2}{16} - x^2}$

(vii) $f(x) = x^4 - 2x^2 + 5$

(viii) $f(x) = x^3 - 12x$, where $x \in [-3, 1]$

(ix) $f(x) = \sin^2 x + \cos^4 x$

(x) $f(x) = [\sin x + [\cos x + [\tan x + [\sec x]]]]$ Here $x \in (0, \pi/4)$

(xi) $f(x) = \sec^2 x - \tan^2 x + \sin(\sin x + \cos x)$

C-8. Find whether the following functions are one-one or many-one & into or onto if f : D $\to$ R where D is its domain.

(i) $f(x) = |x^2 + 5x + 6|$ (ii) $f(x) = |\ell n x|$

(iii) $f(x) = \sin 4x : \left(-\frac{\pi}{8}, \frac{\pi}{8}\right) \to (-1, 1)$ (iv) $f(x) = x + \frac{1}{x}$, $x \in (0, \infty)$

(v) $f(x) = \sqrt{1 - e^{\left(\frac{1}{x} - 1\right)}}$ (vi) $f(x) = \frac{3x^2}{4\pi} - \cos \pi x$

(vii) $f(x) = \frac{1 + x^6}{x^3}$ (viii) $f(x) = x \cos x$ (ix) $f(x) = \frac{1}{\sin\sqrt{|x|}}$

C-9. Classify the following functions f(x) defined in R $\to$ R as injective, surjective, both or none.

(i) $f(x) = x|x|$ (ii) $f(x) = \frac{x^2}{1 + x^2}$ (iii) $f(x) = x^3 - 6x^2 + 11x - 6$

C-10. Check whether the following functions is/are many-one or one-one & into or onto

(i) f(x) = tan (2 sin x) (ii) f(x) = tan (sin x)

C-11. Let f : A $\to$ A where A = {x : $-1 \le x \le 1$}. Find whether the following functions are bijective.

(i) x – sin x (ii) x |x| (iii) $\tan \frac{\pi x}{4}$ (iv) x^4

C-12. Let A be a set of n distinct elements. Then find the total number of distinct functions from A to A ? How many of them are onto functions ?

Section (D) : Identical functions, Composite functions

D-1. Check whether following pairs of functions are identical or not ?

(i) $f(x) = \sqrt{x^2}$ and $g(x) = \left(\sqrt{x}\right)^2$ (ii) f(x) = tanx and $g(x) = \frac{1}{\cot x}$

(iii) $f(x) = \sqrt{\frac{1 + \cos 2x}{2}}$ and $g(x) = \cos x$ (iv) f(x) = x and $g(x) = e^{\ell n x}$

D-2. Find for what values of x, the following functions would be identical.

f(x) = log (x – 1) – log (x – 2) and $g(x) = \log\left(\frac{x-1}{x-2}\right)$

D-3. Let $f(x) = x^2 + x + 1$ and g(x) = sin x. Show that fog $\neq$ gof

D-4. Let $f(x) = x^2$, g(x) = sin x, $h(x) = \sqrt{x}$, then verify that [fo (goh)] (x) and [(fog) oh] (x) are equal.

D-5. Find fog and gof, if

(i) $f(x) = e^x$; $g(x) = \ell n x$ (ii) f(x) = |x| ; g(x) = sin x

(iii) f(x) = sin x ; $g(x) = x^2$ (iv) $f(x) = x^2 + 2$; $g(x) = 1 - \frac{1}{1 - x}$, $x \neq 1$

D-6. If $f(x) = \ell n(x^2 - x + 2)$; $R^+ \to R$ and

g(x) = {x} + 1 ; [1, 2] $\to$ [1, 2] , where {x} denotes fractional part of x.

Find the domain and range of f(g(x)) when defined.

D-7. If $f(x) = \begin{cases} 1 + x^2 ; & x \le 1 \\ x + 1 ; & 1 < x \le 2 \end{cases}$ and g(x) = 1 – x ; $-2 \le x \le 1$, then define the function fog(x).

D-8. If $f(x) = \frac{x+2}{x+1}$ and $g(x) = \frac{x-2}{x}$, then find the domain of

(i) fog(x) (ii) gof(x) (iii) fof(x) (iv) fogof(x)

D-9. If $f(x) = \begin{cases} \sqrt{2}x & x \in Q - \{0\} \\ 3x & x \in Q^c \end{cases}$, then define fof(x) and hence define fofof.....f(x) where f is 'n' times.

D-10. Let $f(x) \begin{cases} x+1 & x \le 4 \\ 2x+1 & 4 < x \le 9 \\ -x+7 & x > 9 \end{cases}$ and $g(x) = \begin{cases} x^2 & -1 \le x < 3 \\ x+2 & 3 \le x \le 5 \end{cases}$ then, find f(g(x)).

D-11. If $f(x) = \frac{4^x}{4^x + 2}$, then show that f(x) + f(1 – x) = 1

Section (E) : Even/Odd Functions & Periodic Functions

E-1. Determine whether the following functions are even or odd or neither even nor odd :

(i) $\sin(x^2 + 1)$ (ii) $x + x^2$ (iii) $f(x) = x\left(\frac{a^x - 1}{a^x + 1}\right)$

(iv) f(x) = sin x + cos x (v) $f(x) = (x^2 - 1)\,|x|$

(vi) $f(x) = \begin{cases} |\ell ne^x| & ; \quad x \le -1 \\ [2+x]+[2-x] & ; \quad -1 < x < 1 \\ e^{\ell nx} & ; \quad x \ge 1 \end{cases}$, where [.] is GIF.

E-2. Examine whether the following functions are even or odd or neither even nor odd, where [ ] denotes greatest integer function.

(i) $f(x) = \frac{(1+2^x)^7}{2^x}$ (ii) $f(x) = \frac{\sec x + x^2 - 9}{x \sin x}$

(iii) $f(x) = \sqrt{1+x+x^2} - \sqrt{1-x+x^2}$ (iv) $f(x) = \begin{cases} x\,|x|, & x \le -1 \\ [1+x]+[1-x], & -1 < x < 1 \\ -x\,|x|, & x \ge 1 \end{cases}$

E-3. Which of the following functions are not periodic (where [.] denotes greatest integer function) :

(i) $f(x) = \sin\sqrt{x}$ (ii) f(x) = x + sin x

(iii) f(x) = [sin 3x] + |cos 6x|

E-4. Find the fundamental period of the following functions :

(i) f (x) = 2 + 3cos (x – 2) (ii) $f(x) = \sin 3x + \cos^2 x + |\tan x|$

(iii) $f(x) = \sin\frac{\pi x}{4} + \sin\frac{\pi x}{3}$ (iv) $f(x) = \cos\frac{3}{5}x - \sin\frac{2}{7}x$.

(v) $f(x) = \frac{1}{1-\cos x}$ (vi) $f(x) = \frac{\sin 12x}{1+\cos^2 6x}$

(vii) $f(x) = \sec^3 x + \mathrm{cosec}^3 x$

Section (F) : Inverse of a function

F-1. Let f : D → R, where D is the domain of f. Find the inverse of f, if it exists

(i) $f(x) = 1 - 2^{-x}$ (ii) $f(x) = \left(4 - (x-7)^3\right)^{1/5}$

(iii) $f(x) = \ln(x + \sqrt{1+x^2})$

(iv) Let f : [0, 3] → [1, 13] is defined by $f(x) = x^2 + x + 1$, then find $f^{-1}(x)$.

F-2. Let f : R → R be defined by $f(x) = \dfrac{e^{2x} - e^{-2x}}{2}$. Is f(x) invertible ? If yes, then find its inverse.

F-3. (a) If $f(x) = -x|x|$, then find $f^{-1}(x)$ and hence find the number of solutions of $f(x) = f^{-1}(x)$.

(b) Solve $2x^2 - 5x + 2 = \dfrac{5 - \sqrt{9+8x}}{4}$, where $x < \dfrac{5}{4}$

F-4. If g is inverse of $f(x) = x^3 + x + \cos x$, then find the value of $g'(1)$.

F-5. If $f(x) = \begin{cases} (\alpha-1)x & x \in Q^c \\ -\alpha^2 x + \alpha + 3x - 1 & x \in Q \end{cases}$ and $g(x) = \begin{cases} x & x \in Q^c \\ 1 - x & x \in Q \end{cases}$ are inverse to each other then find all possible values of α.

Section (G) : Definition, graphs and fundamentals & Inverse Trigonometry

G-1. Find the domain of each of the following functions :

(i) $f(x) = \dfrac{\sin^{-1} x}{x}$ (ii) $f(x) = \sqrt{1-2x} + 3\sin^{-1}\left(\dfrac{3x-1}{2}\right)$ (iii) $f(x) = 2^{\sin^{-1} x} + \dfrac{1}{\sqrt{x-2}}$

G-2. Find the range of each of the following functions :

(i) $f(x) = \ln(\sin^{-1} x)$ (ii) $f(x) = \sin^{-1}\left(\dfrac{\sqrt{3x^2+1}}{5x^2+1}\right)$

(iii) $f(x) = \cos^{-1}\left(\dfrac{(x-1)(x+5)}{x(x-2)(x-3)}\right)$

G-3. Find the simplified value of the following expressions :

(i) $\sin\left[\dfrac{\pi}{3} - \sin^{-1}\left(-\dfrac{1}{2}\right)\right]$ (ii) $\tan\left[\cos^{-1}\dfrac{1}{2} + \tan^{-1}\left(-\dfrac{1}{\sqrt{3}}\right)\right]$

(iii) $\sin^{-1}\left[\cos\left\{\sin^{-1}\left(\dfrac{\sqrt{3}}{2}\right)\right\}\right]$

G-4. (i) If $\sum_{i=1}^{n} \cos^{-1} \alpha_i = 0$, then find the value of $\sum_{i=1}^{n} i \cdot \alpha_i$

(ii) If $\sum_{i=1}^{2n} \sin^{-1} x_i = n\pi$, then show that $\sum_{i=1}^{2n} x_i = 2n$

G-5. Solve the following inequalities:

(i) $\cos^{-1} x > \cos^{-1} x^2$ (ii) $\text{arccot}^2 x - 5\,\text{arccot}\, x + 6 > 0$

(iii) $\sin^{-1} x > -1$ (iv) $\cos^{-1} x < 2$ (v) $\cot^{-1} x < -\sqrt{3}$

G–6. Let f : $\left[-\frac{\pi}{3}, \frac{\pi}{6}\right] \to$ B defined by f (x) = 2 $\cos^2 x$ + $\sqrt{3}$ sin2x + 1. Find B such that f^{-1} exists. Also find $f^{-1}(x)$.

Section (H) : Trig (trig⁻¹x), trig⁻¹ (trig x), trig⁻¹ (–x) and Property (π/2)

H-1. Evaluate the following inverse trigonometric expressions :

(i) $\sin^{-1}\left(\sin\frac{7\pi}{6}\right)$ (ii) $\tan^{-1}\left(\tan\frac{2\pi}{3}\right)$

(iii) $\cos^{-1}\left(\cos\frac{5\pi}{4}\right)$ (iv) $\sec^{-1}\left(\sec\frac{7\pi}{4}\right)$

H-2. Find the value of the following inverse trigonometric expressions :

(i) $\sin^{-1}(\sin 4)$ (ii) $\cos^{-1}(\cos 10)$

(iii) $\tan^{-1}(\tan(-6))$ (iv) $\cot^{-1}(\cot(-10))$

(v) $\cos^{-1}\left(\frac{1}{\sqrt{2}}\left(\cos\frac{9\pi}{10} - \sin\frac{9\pi}{10}\right)\right)$

H-3. Find the value of following expressions :

(i) $\cot(\tan^{-1} a + \cot^{-1} a)$ (ii) $\sin(\sin^{-1}x + \cos^{-1}x)$, $|x| \le 1$

H-4. Solve the inequality $\tan^{-1} x > \cot^{-1} x$.

Section (I) : Interconversion/Simplification

I-1. Evaluate the following expressions :

(i) $\sin\left(\cos^{-1}\frac{3}{5}\right)$ (ii) $\tan\left(\cos^{-1}\frac{1}{3}\right)$

(iii) $\text{cosec}\left(\sec^{-1}\frac{\sqrt{41}}{5}\right)$ (iv) $\tan\left(\text{cosec}^{-1}\frac{65}{63}\right)$

(v) $\sin\left(\frac{\pi}{6} + \cos^{-1}\frac{1}{4}\right)$ (vi) $\cos\left(\sin^{-1}\frac{4}{5} + \cos^{-1}\frac{2}{3}\right)$

(vii) $\sec\left(\tan\left\{\tan^{-1}\left(-\frac{\pi}{3}\right)\right\}\right)$ (viii) $\cos\tan^{-1}\sin\cot^{-1}\left(\frac{1}{2}\right)$

(ix) $\tan\left[\cos^{-1}\left(\frac{3}{4}\right) + \sin^{-1}\left(\frac{3}{4}\right) - \sec^{-1}3\right]$

I-2. Find the value of $\sin^{-1}(\cos(\sin^{-1}x)) + \cos^{-1}(\sin(\cos^{-1}x))$

I-3. If $\tan^{-1}x + \cot^{-1}\frac{1}{y} + 2\tan^{-1}z = \pi$, then prove that $x + y + 2z = xz^2 + yz^2 + 2xyz$

I-4. If $\cos^{-1}x + 2\sin^{-1}x + 3\cot^{-1}y + 4\tan^{-1}y = 4\sec^{-1}z + 5\,\text{cosec}^{-1}z$, then prove that $\sqrt{z^2-1} = \frac{\sqrt{1-x^2} - xy}{x + y\sqrt{1-x^2}}$

I-5. Consider, $f(x) = \tan^{-1}\left(\frac{2}{x}\right)$, $g(x) = \sin^{-1}\left(\frac{2}{\sqrt{4+x^2}}\right)$ and $h(x) = \tan(\cos^{-1}(\sin x))$, then show that

$(h(f(x)) + h(g(x)) = \begin{cases} 0 & , \quad x < 0 \\ x & , \quad x > 0 \end{cases}$

I-6. Prove each of the following relations :

(i) $\tan^{-1} x = -\pi + \cot^{-1}\frac{1}{x} = \sin^{-1}\frac{x}{\sqrt{1+x^2}} = -\cos^{-1}\frac{1}{\sqrt{1+x^2}}$ when $x < 0$.

(ii) $\cos^{-1}x = \sec^{-1}\frac{1}{x} = \pi - \sin^{-1}\sqrt{1-x^2} = \pi + \tan^{-1}\frac{\sqrt{1-x^2}}{x} = \cot^{-1}\frac{x}{\sqrt{1-x^2}}$ when $-1<x<0$

I-7. Express in terms of

(i) $\tan^{-1}\frac{2x}{1-x^2}$ to $\tan^{-1} x$ for $x > 1$

(ii) $\sin^{-1}(2x\sqrt{1-x^2})$ to $\sin^{-1} x$ for $1 \geq x > \frac{1}{\sqrt{2}}$

(iii) $\cos^{-1}(2x^2 - 1)$ to $\cos^{-1}x$ for $-1 \leq x < 0$

I-8. Simplify $\tan\left\{\frac{1}{2}\sin^{-1}\left(\frac{2x}{1+x^2}\right)+\frac{1}{2}\cos^{-1}\left(\frac{1-y^2}{1+y^2}\right)\right\}$, if $x > y > 1$.

I-9. Solve for x

(i) $\cos(2\sin^{-1}x) = \frac{1}{3}$

(ii) $\cot^{-1} x + \tan^{-1} 3 = \frac{\pi}{2}$

(iii) $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$

(iv) $\sin^{-1}x + \sin^{-1}2x = \frac{2\pi}{3}$

Section (J) : Addition and Subtraction Rule

J-1. Prove that

(i) $\sin^{-1}\left(\frac{3}{5}\right) + \sin^{-1}\left(\frac{8}{17}\right) = \sin^{-1}\frac{77}{85}$

(ii) $\tan^{-1}\frac{3}{4} + \sin^{-1}\frac{5}{13} = \cos^{-1}\frac{33}{65}$

(iii) $\sin^{-1}\left(\frac{1}{\sqrt{5}}\right) + \cot^{-1} 3 = \frac{\pi}{4}$

(iv) $\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{8}\right) = \frac{\pi}{4}$

J-2. Find the sum of each of the following series :

(i) $\tan^{-1}\frac{1}{x^2+x+1} + \tan^{-1}\frac{1}{x^2+3x+3} + \tan^{-1}\frac{1}{x^2+5x+7} + \tan^{-1}\frac{1}{x^2+7x+13}$ .... upto n terms.

(ii) $\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{2}{9} + ... + \tan^{-1}\frac{2^{n-1}}{1+2^{2n-1}} + $ ....upto infinite terms

(iii) $\sin^{-1}\frac{1}{\sqrt{2}} + \sin^{-1}\frac{\sqrt{2}-1}{\sqrt{6}} + ... + \sin^{-1}\frac{\sqrt{n}-\sqrt{n-1}}{\sqrt{n(n+1)}} + $ .... upto infinite terms

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PART - II : ONLY ONE OPTION CORRECT TYPE

Section (A) : Ordered pair , Cartesian product, Relation, Domain and Range of Relation

A-1. If A = {a, b}, B = {c, d}, C = {d, e}, then {(a, c), (a, d), (a, e), (b, c), (b, d), (b, e)} is equal to
(A) $A \cap (B \cup C)$ (B) $A \cup (B \cap C)$ (C) $A \times (B \cup C)$ (D) $A \times (B \cap C)$

A-2. If A={1, 2, 3} and B = {1, 2} and C = {4, 5, 6}, then what is the number of elements in the set $A \times B \times C$?
(A) 8 (B) 9 (C) 15 (D) 18

A-3. Let A = {a, b, c} and B = {1, 2}. Consider a relation R defined from set A to set B. Then R can equal to set
(A) A (B) B (C) A × B (D) B × A

A-4. Let R be relation from a set A to a set B, then
(A) $R = A \cup B$ (B) $R = A \cap B$ (C) $R \subseteq A \times B$ (D) $R \subseteq B \times A$

A-5. Let X = {1, 2, 3, 4, 5} and Y = {1, 3, 5, 7, 9}. Which of the following is not a relation from X to Y
(A) $R_1 = \{(x, y) \mid y = 2 + x, x \in X, y \in Y\}$ (B) $R_2 = \{(1, 1), (2, 1), (3, 3), (4, 3), (5, 5)\}$
(C) $R_3 = \{(1, 1), (1, 3) (3, 5), (3, 7), (5, 7)\}$ (D) $R_4 = \{(1, 3), (2, 5), (2, 4), (7, 9)\}$

A-6. The relation R defined in A = {1, 2, 3} by a R b if $-5 \le a^2 - b^2 \le 5$. Which of the following is false?
(A) R = {(1, 2), (2, 2), (3, 3), (2, 1), (2, 3), (3, 2)} (B) Co-domain of R = {1, 2, 3}
(C) Domain of R = {1, 2, 3} (D) Range of R = {1, 2, 3}

Section (B) : Types of Relation

B-1. The relation R defined in N as aRb $\Leftrightarrow$ b is divisible by a is
(A) Reflexive but not symmetric (B) Symmetric but not transitive
(C) Symmetric and transitive (D) Equivalence relation

B-2. In the set A = {1, 2, 3, 4, 5} a relation R is defined by R = {(x, y)| x, y $\in$ A and x < y}. Then R is
(A) Reflexive (B) Symmetric (C) Transitive (D) Equivalence relation

B-3. Which one of the following relations on R is equivalence relation-
(A) $x R_1 y \Leftrightarrow x^2 = y^2$ (B) $x R_2 y \Leftrightarrow x \ge y$ (C) $x R_3 y \Leftrightarrow x \mid y$ (x divides y) (D) $x R_4 y \Leftrightarrow x < y$

B-4. Let R_1 be a relation defined by $R_1 = \{(a, b) \mid a \ge b ; a, b \in R\}$. Then R_1 is
(A) An equivalence relation on R (B) Reflexive, transitive but not symmetric
(C) Symmetric, Transitive but not reflexive (D) Neither transitive nor reflexive but symmetric

B-5. Let L denote the set of all straight lines in a plane. Let a relation R be defined by $\alpha R \beta \Leftrightarrow \alpha \perp \beta$, $\alpha, \beta \in L$. The R is
(A) Reflexive (B) Symmetric (C) Transitive (D) equivalence relation

B-6. Let S be the set of all real numbers. Then the relation R =
{(a, b) : 1 + ab > 0} on S is
(A) Reflexive and symmetric but not transitive (B) Reflexive, transitive but not symmetric
(C) Symmetric, transitive but not reflexive (D) Reflexive, transitive and symmetric

B-7. Consider the following :
1. If R = {(a, b) $\in$ N × N : a divides b in N} then the relation R is reflexive and symmetric but not transitive.
2. If A = {1, 2, 3, 4, 5, 6} and $R = \{(S_1, S_2) : S_1, S_2$ are subsets of A, $S_1 \not\subset S_2\}$, then the relation R is not reflexive, not symmetric and not transitive.

Which of the statements is/are correct ?
(A) 1 only (B) 2 only (C) Both 1 and 2 (D) Neither 1 nor 2

B-8. Let R be a relation over the set N × N and it is defined by (a, b) R (c, d) $\Rightarrow$ a + d = b + c. Then R is
(A) Symmetric only (B) Transitive only (C) Reflexive only (D) Equivalence only

B-9. Let L be the set of all straight lines in the Euclidean plane. Two lines $\square_1$ and $\square_2$ are said to be related by the relation R if $\square_1$ is parallel to $\square_2$. Then R is
(A) Symmetric only (B) Transitive only (C) Reflexive only (D) Equivalence only

B-10. Let R = {(x, y) : x, y $\in$ A, x + y = 5} where A = {1, 2, 3, 4, 5} then R is
(A) Reflexive (B) symmetric (C) Transitive (D) Equivalence

B-11. Let S be a set of all square matrices of order 2. If a relation R defined on set S such that
AR B $\Rightarrow$ AB = O, where O is zero square matirx of order 2, then relation R is (A, B $\in$ S)
(A) Reflexive (B) Transitive
(C) Symmetric (D) Not equivalence

Section (C) : Definition of function, Domain and Range, Classification of Functions

C-1. The domain of the function $f(x) = \frac{\sqrt{-\log_{0.3}(x-1)}}{\sqrt{x^2+2x+8}}$ is

(A) (1, 4) (B) (– 2, 4) (C) (2, 4) (D) [2, ∞)

C-2. Range of $f(x) = \ln(3x^2 - 4x + 5)$ is

(A) $\left[\ln\frac{11}{3}, \infty\right)$ (B) $[\ln 10, \infty)$ (C) $\left[\ln\frac{11}{6}, \infty\right)$ (D) $\left[\ln\frac{11}{12}, \infty\right)$

C-3. Range of $f(x) = 4^x + 2^x + 1$ is

(A) (0, ∞) (B) (1, ∞) (C) (2, ∞) (D) (3, ∞)

C-4. Range of $f(x) = \log_{\sqrt{5}}(\sqrt{2}(\sin x - \cos x) + 3)$ is

(A) [0, 1] (B) [0, 2] (C) $\left[0, \frac{3}{2}\right]$ (D) [1, 2]

C-5. Let f : R → R be a function defined by $f(x) = \frac{2x^2 - x + 5}{7x^2 + 2x + 10}$, then f is :

(A) one – one but not onto (B) onto but not one – one
(C) onto as well as one – one (D) neither onto nor one – one

C-6. Let f : R → R be a function defined by $f(x) = x^3 + x^2 + 3x + \sin x$. Then f is:

(A) one – one and onto (B) one – one and into
(C) many one and onto (D) many one and into

C-7. Domain of definition of the function $f(x) = \frac{3}{4 - x^2} + \log_{10}(x^3 - x)$, is :

(A) (1,2) (B) (–1,0) ∪ (1,2)
(C) (1,2) ∪ (2, ∞) (D) (–1,0) ∪ (1,2) ∪ (2, ∞)

C-8. If f : [0, ∞) → [0, ∞), and $f(x) = \frac{x}{1+x}$, then f is:

(A) one-one and onto (B) one-one but not onto
(C) onto but not one-one (D) neither one-one nor onto

C-9. Range of the function $f(x) = \frac{(x-2)^2}{(x-1)(x-3)}$ is

(A) (1, ∞) (B) (–∞, 1) (C) R – (0, 1] (D) (0, 1]

C-10. Range of the function $f(x) = \frac{x-2}{x^2-4x+3}$ is

(A) (–∞, 0) (B) R (C) (0, ∞) (D) R – {0}

C-11. **Statement - 1** If f (x) and g (x) both are one one and f(g (x)) exists, then f(g (x)) is also one one.
Statement - 2 If $f(x_1) = f(x_2) \Leftrightarrow x_1 = x_2$, then f(x) is one-one.

(A) STATEMENT-1 is true, STATEMENT-2 is true and STATEMENT-2 is correct explanation for STATEMENT-1
(B) STATEMENT-1 is true, STATEMENT-2 is true and STATEMENT-2 is not correct explanation for STATEMENT-1
(C) STATEMENT-1 is true, STATEMENT-2 is false
(D) STATEMENT-1 is false, STATEMENT-2 is true
(E) Both STATEMENTS are false

C-12. **Statement - 1** If y = f (x) is increasing in [α, β], then its range is [f (α), f (β)]
Statement - 2 Every increasing function need not to be continuous.
(A) STATEMENT-1 is true, STATEMENT-2 is true and STATEMENT-2 is correct explanation for STATEMENT-1
(B) STATEMENT-1 is true, STATEMENT-2 is true and STATEMENT-2 is not correct explanation for STATEMENT-1
(C) STATEMENT-1 is true, STATEMENT-2 is false
(D) STATEMENT-1 is false, STATEMENT-2 is true
(E) Both STATEMENTS are false

C-13. If the functions f(x) and g(x) are defined on R$\rightarrow$R such that f(x) = $\begin{cases} 0 & , \quad x \in \text{rational} \\ x & , \quad x \in \text{irrational} \end{cases}$,

g(x) = $\begin{cases} 0 & , \quad x \in \text{irrational} \\ x & , \quad x \in \text{rational} \end{cases}$, then (f – g) (x) is
(A) one-one and onto
(B) neither one-one nor onto
(C) one-one but not onto
(D) onto but not one-one

Section (D) : Identical functions, Composite functions

D-1. Which of the following pair of functions are identical –
(A) f(x) = $\sin^2x + \cos^2x$ and g(x) = 1
(B) f(x) = $\sec^2x - \tan^2x$ and g(x) = 1
(C) f(x) = $\text{cosec}^2x - \cot^2 x$ and g(x) = 1
(D) f(x) = $\ell n x^2$ and g(x) = $2\ell n x$

D-2. Let f(x) be a function whose domain is [– 5, 7]. Let g(x) = |2x + 5|, then domain of (fog) (x) is
(A) [– 4, 1]
(B) [– 5, 1]
(C) [– 6, 1]
(D) [– 5, 7]

D-3. Let g (x) = 1 + x – [x] and f (x) = $\begin{cases} -1 & , \quad x < 0 \\ 0 & , \quad x = 0 \\ 1 & , \quad x > 0 \end{cases}$. Then for all x, f (g (x)) is equal to (where [.] denotes greatest integer function)
(A) x
(B) 1
(C) f (x)
(D) g (x)

Section (E) : Even/Odd Functions & Periodic Functions

E-1. The function f(x) = log $\left(\frac{1+\sin x}{1-\sin x}\right)$ is
(A) even
(B) odd
(C) neither even nor odd
(D) both even and odd

E-2. The function f(x) = [x] + $\frac{1}{2}$, x $\notin$ I is a/an (where [.] denotes greatest integer function)
(A) Even
(B) odd
(C) neither even nor odd
(D) Even as well as odd

E-3. The graph of the function y = f(x) is symmetrical about the line x = 2, then :
(A) f(x + 2) = f(x – 2)
(B) f(2 + x) = f(2 – x)
(C) f(x) = f(–x)
(D) f(x) = –f(–x)

E-4. Fundamental period of f(x) = sec (sin x) is
(A) $\frac{\pi}{2}$
(B) 2π
(C) π
(D) aperiodic

E-5. If f (x) = $\sin\left(\sqrt{[a]}\ x\right)$ (where [.] denotes the greatest integer function) has π as its fundamental period, then
(A) a = 1
(B) a = 9
(C) a $\in$ [1, 2)
(D) a $\in$ [4, 5)

E-6. Find the area below the curve y = $\left[\sqrt{2+2\cos 2x}\right]$ but above the x-axis in [–3π, 6π] is (where [.] denotes the greatest integer function) :
(A) 2π square units (B) π square units (C) 6π square units (D) 8π square units

Section (F) : Inverse of a function

F-1. The inverse of the function f(x) = $\frac{e^x - e^{-x}}{e^x + e^{-x}}$ is
(A) $\frac{1}{2}\ln\frac{1+x}{1-x}$ (B) $\frac{1}{2}\ln\frac{2+x}{2-x}$ (C) $\frac{1}{2}\ln\frac{1-x}{1+x}$ (D) 2 ln (1 + x)

F-2. If f: [1, ∞) → [2, ∞) is given by f (x) = x + $\frac{1}{x}$, then f^{-1} (x) equals:
(A) $\frac{x+\sqrt{x^2-4}}{2}$ (B) $\frac{x}{1+x^2}$ (C) $\frac{x-\sqrt{x^2-4}}{2}$ (D) $1-\sqrt{x^2-4}$

F-3. If f : R → R is an invertible function such that f(x) and f^{-1}(x) are also mirror image to each other about the line y = –x, then
(A) f(x) is odd
(B) f(x) and f^{-1}(x) may not be mirror image about the line y = x
(C) f(x) may not be odd
(D) f(x) is even

F-4. If f(x) = $\frac{ax+b}{cx+d}$, then (fof) (x) = x, provided that
(A) d + a = 0 (B) d – a = 0 (C) a = b = c = d = 1 (D) a = b = 1

F-5. Let f(x) = $\begin{cases} x & -1 \le x \le 1 \\ x^2 & 1 < x \le 2 \end{cases}$ the range of h^{-1}(x), where h(x) = fof(x) is
(A) $\left[-1, \sqrt{2}\right]$ (B) [–1, 2] (C) [–1, 4] (D) [–2, 2]

F-6. **Statement – 1** All points of intersection of y = f (x) and y = f^{-1}(x) lies on y = x only.
Statement – 2 If point P (α, β) lies on y = f (x), then Q (β, α) lies on y = f^{-1}(x).
Statement – 3 Inverse of invertible function is unique and its range is equal to the function domain.
Which of the following option is correct for above statements in order
(A) T T F (B) F T T (C) T T T (D) T F T

Section (G) : Definition, graphs and fundamentals of Inverse Trigonometric functions

G-1. The domain of definition of f(x) = $\sin^{-1}(|x-1| - 2)$ is:
(A) [– 2, 0] ∪ [2, 4] (B) (– 2, 0) ∪ (2, 4) (C) [– 2, 0] ∪ [1, 3] (D) (– 2, 0) ∪ (1, 3)

G-2. The function f(x) = $\cot^{-1}\sqrt{(x+3)x} + \cos^{-1}\sqrt{x^2+3x+1}$ is defined on the set S, where S is equal to:
(A) {0, 3} (B) (0, 3) (C) {0, – 3} (D) [– 3, 0]

G-3. Domain of f(x) = $\cos^{-1}x + \cot^{-1}x + \mathrm{cosec}^{-1}x$ is
(A) [– 1, 1] (B) R (C) (– ∞, – 1] ∪ [1, ∞) (D) {– 1, 1}

G-4. Range of f(x) = $\sin^{-1}x + \tan^{-1}x + \sec^{-1}x$ is
(A) $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$ (B) $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$ (C) $\left\{\frac{\pi}{4}, \frac{3\pi}{4}\right\}$ (D) $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$

G-5. $\text{cosec}^{-1}(\cos x)$ is real if

(A) $x \in [-1, 1]$ (B) $x \in R$

(C) x is an odd multiple of $\frac{\pi}{2}$ (D) x is a multiple of π

G-6. Domain of definition of the function $f(x) = \sqrt{\sin^{-1}(2x) + \frac{\pi}{6}}$ for real valued 'x' is:

(A) $\left[-\frac{1}{4}, \frac{1}{2}\right]$ (B) $\left[-\frac{1}{2}, \frac{1}{2}\right]$ (C) $\left(-\frac{1}{2}, \frac{1}{9}\right)$ (D) $\left[-\frac{1}{4}, \frac{1}{4}\right]$

G-7. The solution of the equation $\sin^{-1}\left(\tan\frac{\pi}{4}\right) - \sin^{-1}\left(\sqrt{\frac{3}{x}}\right) - \frac{\pi}{6} = 0$ is

(A) x = 2 (B) x = – 4 (C) x = 4 (D) x = 3

G-8. Number of solutions of the equation $\cot^{-1}\sqrt{4-x^2} + \cos^{-1}(x^2-5) = \frac{3\pi}{2}$ is :

(A) 2 (B) 4 (C) 6 (D) 8

Section (H) : Trig (trig⁻¹x), trig⁻¹ (trig x), trig⁻¹ (–x) and Property (π/2)

H-1. If $\pi \le x \le 2\pi$, then $\cos^{-1}(\cos x)$ is equal to

(A) x (B) $\pi - x$ (C) $2\pi + x$ (D) $2\pi - x$

H-2. If $\sin^{-1}x + \sin^{-1}y = \frac{2\pi}{3}$, then $\cos^{-1}x + \cos^{-1}y$ is equal to

(A) $\frac{2\pi}{3}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{6}$ (D) π

H-3. If $x \ge 0$ and $\theta = \sin^{-1}x + \cos^{-1}x - \tan^{-1}x$, then

(A) $\frac{\pi}{2} \le \theta \le \frac{3\pi}{4}$ (B) $0 \le \theta \le \frac{\pi}{4}$ (C) $0 \le \theta < \frac{\pi}{2}$ (D) $\frac{\pi}{4} \le \theta \le \frac{\pi}{2}$

H-4. Number of solutions of equation $\tan^{-1}(e^{-x}) + \cot^{-1}(|\ell nx|) = \pi/2$ is :

(A) 0 (B) 1 (C) 3 (D) 2

Section (I) : Interconversion/Simplification

I-1. The numerical value of $\cot\left(2\sin^{-1}\frac{3}{5} + \cos^{-1}\frac{3}{5}\right)$ is

(A) $\frac{-4}{3}$ (B) $\frac{-3}{4}$ (C) $\frac{3}{4}$ (D) $\frac{4}{3}$

I-2. **STATEMENT-1 :** $\tan^2(\sec^{-1} 2) + \cot^2(\text{cosec}^{-1} 3) = 11$.

STATEMENT-2 : $\tan^2\theta + \sec^2\theta = 1 = \cot^2\theta + \text{cosec}^2\theta$.

(A) STATEMENT-1 is true, STATEMENT-2 is true and STATEMENT-2 is correct explanation for STATEMENT-1

(B) STATEMENT-1 is true, STATEMENT-2 is true and STATEMENT-2 is not correct explanation for STATEMENT-1

(C) STATEMENT-1 is true, STATEMENT-2 is false

(D) STATEMENT-1 is false, STATEMENT-2 is true

(E) Both STATEMENTS are false

I-3. If α is a real root of the equation $x^3 + 3x - \tan 2 = 0$, then $\cot^{-1}\alpha + \cot^{-1}\frac{1}{\alpha} - \frac{\pi}{2}$ can be equal to

(A) 0 (B) $\frac{\pi}{2}$ (C) π (D) $\frac{3\pi}{2}$

I-4. If $\sin^{-1}\left(\frac{\sqrt{x}}{2}\right) + \sin^{-1}\left(\sqrt{1-\frac{x}{4}}\right) + \tan^{-1}y = \frac{2\pi}{3}$, then :

(A) maximum value of $x^2 + y^2$ is $\frac{49}{3}$
(B) maximum value of $x^2 + y^2$ is 4
(C) minimum value of $x^2 + y^2$ is $\frac{1}{2}$
(D) minimum value of $x^2 + y^2$ is 3

I-5. If x < 0, then value of $\tan^{-1}(x) + \tan^{-1}\left(\frac{1}{x}\right)$ is equal to

(A) $\frac{\pi}{2}$ (B) $-\frac{\pi}{2}$ (C) 0 (D) $-\pi$

I-6. If $\sin^{-1}x + \cot^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{2}$, then x is equal to

(A) 0 (B) $\frac{1}{\sqrt{5}}$ (C) $\frac{2}{\sqrt{5}}$ (D) $\frac{\sqrt{3}}{2}$

I-7. The numerical value of $\tan\left(2\tan^{-1}\frac{1}{5}-\frac{\pi}{4}\right)$ is

(A) $\frac{-7}{17}$ (B) $\frac{7}{17}$ (C) $\frac{17}{7}$ (D) $-\frac{2}{3}$

Section (J) : Addition and Subtraction Rule

J-1. If $f(x) = \tan^{-1}\left(\frac{\sqrt{3}x-3x}{3\sqrt{3}+x^2}\right) + \tan^{-1}\left(\frac{x}{\sqrt{3}}\right)$, $0 \le x \le 3$, then range of f(x) is

(A) $\left[0, \frac{\pi}{2}\right)$ (B) $\left[0, \frac{\pi}{4}\right]$ (C) $\left[\frac{\pi}{6}, \frac{\pi}{3}\right]$ (D) $\left[0, \frac{\pi}{3}\right]$

J-2. **STATEMENT-1 :** If a > 0, b > 0, $\tan^{-1}\left(\frac{a}{x}\right) + \tan^{-1}\left(\frac{b}{x}\right) = \frac{\pi}{2}$. $\Rightarrow$ $x = \sqrt{ab}$.

STATEMENT-2 : If m, n $\in$ N, n $\ge$ m, then $\tan^{-1}\left(\frac{m}{n}\right) + \tan^{-1}\left(\frac{n-m}{n+m}\right) = \frac{\pi}{4}$.

(A) STATEMENT-1 is true, STATEMENT-2 is true and STATEMENT-2 is correct explanation for STATEMENT-1
(B) STATEMENT-1 is true, STATEMENT-2 is true and STATEMENT-2 is not correct explanation for STATEMENT-1
(C) STATEMENT-1 is true, STATEMENT-2 is false
(D) STATEMENT-1 is false, STATEMENT-2 is true
(E) Both STATEMENTS are false

J-3. If $\cos^{-1}x - \cos^{-1}\frac{y}{2} = \alpha$, then $4x^2 - 4xy\cos\alpha + y^2$ is equal to-

(A) $2\sin 2\alpha$ (B) 4 (C) $4\sin^2\alpha$ (D) $-4\sin^2\alpha$

PART - III : MATCH THE COLUMN

1. Match the relation defined on set A = {a,b,c} in column I with the corresponding type in column II

Column I		**Column II**	
(A)	{a,b), (b,a)	(p)	symmetric but not reflexive and transitive
(B)	{(a,b), (b,a), (a,a), (b,b)}	(q)	equivalence
(C)	{(a,b), (b,c), (a,c)}	(r)	symmetric and transitive but not reflexive
(D)	{(a,a), (b,b), (c,c)}	(s)	transitive but not reflexive and symmetric

2.

Column – I		**Column – II**	
(A)	If S be set of all triangles and $f : S \to R^+$, $f(\Delta)$ = Area of Δ, then f is	(p)	one-one
(B)	$f : R \to \left[\frac{3\pi}{4}, \pi\right)$ and $f(x) = \cot^{-1}(2x - x^2 - 2)$, then f(x) is	(q)	many one
(C)	If $f : R \to R$ such that $f(x) = \frac{2x^2 - x + 1}{7x^2 - 4x + 4}$, then f(x) is	(r)	onto function
(D)	$f : R \to R$ and $f(x) = e^{px} \sin qx$ where $p, q \in R^+$, then f(x) is	(s)	into function

3. **Match The column**

(A)	If f(x) is even & g(x) is odd	(p)	then fog must be odd
(B)	If g(x) is periodic	(q)	then fog must be manyone
(C)	If f(x) & g(x) are bijective	(r)	then fog is periodic
(D)	If f(x) is into	(s)	then fog is injective
		(t)	then fog is into

4. Let $f(x) = \sin^{-1} x$, $g(x) = \cos^{-1} x$ and $h(x) = \tan^{-1} x$. For what complete interval of variation of x the following are true.

Column – I		**Column – II**	
(A)	$f\left(\sqrt{x}\right) + g\left(\sqrt{x}\right) = \pi/2$	(p)	$[0, \infty)$
(B)	$f(x) + g\left(\sqrt{1 - x^2}\right) = 0$	(q)	[0, 1]
(C)	$g\left(\frac{1 - x^2}{1 + x^2}\right) = 2h(x)$	(r)	$(-\infty, 1)$
(D)	$h(x) + h(1) = h\left(\frac{1 + x}{1 - x}\right)$	(s)	[– 1, 0]

5. **Match the column**

	Column - I		**Column - II**
(A)	Let a, b, c be three positive real numbers $\theta = \tan^{-1}\sqrt{\frac{a(a+b+c)}{bc}} + \tan^{-1}\sqrt{\frac{b(a+b+c)}{ca}} + \tan^{-1}\sqrt{\frac{c(a+b+c)}{ab}}$, then θ is equal to	(p)	π
(B)	The value of the expression $\tan^{-1}\left(\frac{1}{2}\tan 2A\right) + \tan^{-1}(\cot A) + \tan^{-1}(\cot^3 A)$ for $0 < A < (\pi/4)$ is equal to	(q)	$-\frac{\pi}{2}$
(C)	If x < 0, then $\frac{1}{2}\{\cos^{-1}(2x^2 - 1) + 2\cos^{-1} x\}$ is equal to	(r)	$-\pi$
(D)	The value of $\sin^{-1}\left(\frac{3}{5}\right) - \cos^{-1}\left(\frac{12}{13}\right) + \cos^{-1}\left(\frac{16}{65}\right)$ is equal to	(s)	$\frac{\pi}{2}$

Exercise-2

✍ **Marked questions are recommended for Revision.**

PART - I : ONLY ONE OPTION CORRECT TYPE

1.✍ For real numbers x and y, we write x R y $\Rightarrow$ x – y + $\sqrt{2}$ is an irrational number. Then the relation R is-
(A) Reflexive (B) Symmetric (C) Transitive (D) Equivalence relation

2. Let A = N × N be the Certesian product of N and N. Let
S = {((m, n), (p, q)) $\in$ A × A : m + q = n + p}
Consider the following statements:
I.If ((m,n), (p , q)) $\in$ S, and ((p,q), (r, s)) $\in$ S then ((r,s), (m,n)) $\in$ S
II.There exists at least one element ((m,n), (p, q)) $\in$ S such that ((p , q), (m, n)) $\notin$ S
Which of the statements given above is / are correct ?
(A) I only (B) II only (C) Both I and II (D) Neither I nor II.

3. Let A = Z, the set of integers. Let R_1 = {(m, n) $\in$ Z × Z : (m + 4n) is divisible by 5 in Z}.
Let R_2 = {(m, n) $\in$ Z × Z : (m + 9n) is divisible by 5 in Z}.
Which one of the following is correct ?
(A) R_1 is a proper subset of R_2 (B) R_2 is a proper subset of R_1
(C) $R_1 = R_2$ (D) R_1 is not a symmetric relation on Z

4. Let X be the set of all persons living in a state. Elements x, y in X are said to be related if 'x < y', whenever y is 5 years older than x. Which one of the following is correct?
(A) The relation is an equivalence relation
(B) The relation is transitive only
(C) The relation is transitive and symmetric, but not reflexive
(D) The relation is neither reflexive, nor symmetric, nor transitive

5. The domain of the function f (x) = $\log_{1/2}\left(-\log_2\left(1+\frac{1}{\sqrt[4]{x}}\right)-1\right)$ is:
(A) 0 < x < 1 (B) 0 < x ≤ 1 (C) x ≥ 1 (D) null set

6. If $q^2 - 4pr = 0$, p > 0, then the domain of the function f (x) = log (p x^3 + (p + q) x^2 + (q + r) x + r) is:
(A) $R - \left\{-\frac{q}{2p}\right\}$ (B) $R - \left[(-\infty, -1] \cup \left\{-\frac{q}{2p}\right\}\right]$
(C) $R - \left[(-\infty, -1) \cap \left\{-\frac{q}{2p}\right\}\right]$ (D) R

7. Let f (x) = $\frac{x-[x]}{1+x-[x]}$, R $\to$ A is onto then find set A. (where {.} and [.] represent fractional part and greatest integer part functions respectively)
(A) $\left(0, \frac{1}{2}\right]$ (B) $\left[0, \frac{1}{2}\right]$ (C) $\left[0, \frac{1}{2}\right)$ (D) $\left(0, \frac{1}{2}\right)$

8.✍ Let f be a real valued function defined by $f(x) = \frac{e^x - e^{-|x|}}{e^x + e^{|x|}}$, then the range of f(x) is :
(A) R (B) [0, 1] (C) [0, 1) (D) $\left[0, \frac{1}{2}\right)$

9.✍ The range of the function f (x) = $\log_{\sqrt{2}}\left(2 - \log_2\left(16\sin^2 x + 1\right)\right)$ is
(A) (–∞, 1) (B) (–∞, 2) (C) (–∞, 1] (D) (–∞, 2]

10. Which of the following pair of functions are identical ?

(A) $\sqrt{1+\sin x}$, $\sin\frac{x}{2}+\cos\frac{x}{2}$ (B) $x, \frac{x^2}{x}$

(C) $\sqrt{x^2}$, $\left(\sqrt{x}\right)^2$ (D) $\ell n\, x^3 + \ell n\, x^2$, $5\, \ell n\, x$

11. If domain of f(x) is $(-\infty , 0]$, then domain of $f(6\{x\}^2 - 5\{x\} + 1)$ is (where {·} represents fractional part function).

(A) $\bigcup\limits_{n\in I}\left[n+\frac{1}{3}\ ,\ n+\frac{1}{2}\right]$ (B) $(-\infty, 0)$ (C) $\bigcup\limits_{n\in I}\left[n+\frac{1}{6}\ ,\ n+1\right]$ (D) $\bigcup\limits_{n\in I}\left[n-\frac{1}{2}, n-\frac{1}{3}\right]$

12. Let f: $(e, \infty) \to R$ be defined by $f(x) = \ell n\,(\ell n(\ell n\, x))$, then

(A) f is one one but not onto (B) f is onto but not one - one

(C) f is one-one and onto (D) f is neither one-one nor onto

13. If f (x) = 2 [x] + cos x, then f: $R \to R$ is: (where [ ] denotes greatest integer function)

(A) one–one and onto (B) one–one and into

(C) many–one and into (D) many–one and onto

14. If f : $R \to R$ be a function such that $f(x) = \begin{cases} x\,|x|-4 & ; x \in Q \\ x\,|x|-\sqrt{3} & ; x \notin Q \end{cases}$, then f(x) is

(A) one-one, onto (B) many one, onto (C) one-one, into (D) many one, into

15. $f(x) = |x-1|$, f: $R^+ \to R$, $g(x) = e^x$, g: $[-1, \infty) \to R$. If the function fog (x) is defined, then its domain and range respectively are:

(A) $(0, \infty)$ and $[0, \infty)$ (B) $[-1, \infty)$ and $[0, \infty)$

(C) $[-1, \infty)$ and $\left[1-\frac{1}{e}, \infty\right)$ (D) $[-1, \infty)$ and $\left[\frac{1}{e}-1, \infty\right)$

16. Let f : (2, 4) $\to$ (1, 3) be a function defined by $f(x) = x - \left[\frac{x}{2}\right]$ (where [.] denotes the greatest integer function), then $f^{-1}(x)$ is equal to :

(A) 2x (B) $x + \left[\frac{x}{2}\right]$ (C) x + 1 (D) x – 1

17. The mapping f : $R \to R$ given by $f(x) = x^3 + ax^2 + bx + c$ is a bijection if

(A) $b^2 \le 3a$ (B) $a^2 \le 3b$ (C) $a^2 \ge 3b$ (D) $b^2 \ge 3a$

18. If the function f: $[1, \infty) \to [1, \infty)$ is defined by $f(x) = 2^{x(x-1)}$ then f^{-1} is

(A) $(1/2)^{x(x-1)}$ (B) $\frac{1}{2}\left(1+\sqrt{1+4\log_2 x}\right)$

(C) $\frac{1}{2}\left(1-\sqrt{1+4\log_2 x}\right)$ (D) Not defined

19. Let f : $N \to N$, where $f(x) = x + (-1)^{x-1}$, then the inverse of f is.

(A) $f^{-1}(x) = x + (-1)^{x-1}$, $x \in N$ (B) $f^{-1}(x) = 3x + (-1)^{x-1}$, $x \in N$

(C) $f^{-1}(x) = x$, $x \in N$ (D) $f^{-1}(x) = (-1)^{x-1}$, $x \in N$

20. $\tan\left(\frac{\pi}{4}+\frac{1}{2}\cos^{-1}x\right)+\tan\left(\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x\right)$, $x \ne 0$ is equal to

(A) x (B) 2x (C) $\frac{2}{x}$ (D) $\frac{x}{2}$

21. The value of $\cot^{-1}\left\{\dfrac{\sqrt{1-\sin x}+\sqrt{1+\sin x}}{\sqrt{1-\sin x}-\sqrt{1+\sin x}}\right\}$, where $\dfrac{\pi}{2} < x < \pi$, is:

(A) $\pi - \dfrac{x}{2}$ (B) $\dfrac{\pi}{2} + \dfrac{x}{2}$ (C) $\dfrac{x}{2}$ (D) $2\pi - \dfrac{x}{2}$

22. The domain of the function $f(x) = \sin^{-1}\left(\dfrac{1+x^3}{2x^{3/2}}\right) + \sqrt{\sin(\sin x)} + \log_{(3\{x\}+1)}(x^2+1)$,

where {.} represents fractional part function, is:

(A) $x \in \{1\}$ (B) $x \in R - \{1, -1\}$ (C) $x > 3, x \neq I$ (D) $x \in \phi$

23. A function g(x) satisfies the following conditions

(i) Domain of g is $(-\infty, \infty)$ (ii) Range of g is $[-1, 7]$

(iii) g has a period π and (iv) $g(2) = 3$

Then which of the following may be possible.

(A) $g(x) = 3 + 4\sin(n\pi + 2x - 4), n \in I$ (B) $g(x) = \begin{cases} 3 & ; \ x = n\pi \\ 3 + 4\sin x & ; \ x \neq n\pi \end{cases}$

(C) $g(x) = 3 + 4\cos(n\pi + 2x - 4), n \in I$ (D) $g(x) = 3 - 8\sin(n\pi + 2x - 4), n \in I$

24. The complete solution set of the inequality $[\cot^{-1}x]^2 - 6[\cot^{-1}x] + 9 \le 0$, where [.] denotes greatest integer function, is

(A) $(-\infty, \cot 3]$ (B) $[\cot 3, \cot 2]$ (C) $[\cot 3, \infty)$ (D) $(-\infty, \cot 2]$

25. The inequality $\sin^{-1}(\sin 5) > x^2 - 4x$ holds for

(A) $x \in \left(2 - \sqrt{9-2\pi}, 2 + \sqrt{9-2\pi}\right)$ (B) $x > 2 + \sqrt{9-2\pi}$

(C) $x < 2 - \sqrt{9-2\pi}$ (D) $x \in \phi$

26. If $\sin^{-1}\left(x - \dfrac{x^2}{2} + \dfrac{x^3}{4} - ....\right) + \cos^{-1}\left(x^2 - \dfrac{x^4}{2} + \dfrac{x^6}{4} - ....\right) = \dfrac{\pi}{2}$ for $0 < |x| < \sqrt{2}$, then x equals

(A) 1/2 (B) 1 (C) – 1/2 (D) –1

27. $\cot^{-1}\left(\sqrt{\cos\alpha}\right) - \tan^{-1}\left(\sqrt{\cos\alpha}\right) = x$. then sin x is equal to -

(A) $\tan^2\left(\dfrac{\alpha}{2}\right)$ (B) $\cot^2\left(\dfrac{\alpha}{2}\right)$ (C) $\tan\alpha$ (D) $\cot\left(\dfrac{\alpha}{2}\right)$

28. The Inverse trigonometric equation $\sin^{-1}x = 2\sin^{-1}\alpha$, has a solution for

(A) $-\dfrac{\sqrt{3}}{2} < \alpha < \dfrac{\sqrt{3}}{2}$ (B) all real values of α (C) $|\alpha| \le \dfrac{1}{\sqrt{2}}$ (D) $|\alpha| \ge \dfrac{1}{\sqrt{2}}$

29. If $f(x) = \cot^{-1}x$: $R^+ \to \left(0, \dfrac{\pi}{2}\right)$

and $g(x) = 2x - x^2$: $R \to R$. Then the range of the function f(g(x)) wherever define is

(A) $\left(0, \dfrac{\pi}{2}\right)$ (B) $\left(0, \dfrac{\pi}{4}\right)$ (C) $\left[\dfrac{\pi}{4}, \dfrac{\pi}{2}\right)$ (D) $\left\{\dfrac{\pi}{4}\right\}$

30. Given the functions $f(x) = e^{\cos^{-1}\left(\sin\left(x + \frac{\pi}{3}\right)\right)}$, $g(x) = \operatorname{cosec}^{-1}\left(\dfrac{4 - 2\cos x}{3}\right)$ and the function

h(x) = f(x) defined only for those values of x, which are common to the domains of the functions f(x) and g(x). The range of the function h(x) is :

(A) $[e^{\frac{\pi}{6}}, e^{\pi}]$ (B) $[e^{-\frac{\pi}{6}}, e^{\pi}]$ (C) $(e^{\frac{\pi}{6}}, e^{\pi})$ (D) $[e^{-\frac{\pi}{6}}, e^{\frac{\pi}{6}}]$

PART - II : SINGLE AND DOUBLE VALUE INTEGER TYPE

1. The domain of the function y = $\sqrt{\sin x + \cos x}$ + $\sqrt{7x - x^2 - 6}$ is $\left[p, \frac{q\pi}{4}\right] \cup \left[\frac{r\pi}{4}, s\right]$ then value of p + q + r + s is

2. The domain of f(x) such that the f(x) = $\frac{\left[x + \frac{1}{2}\right]}{\left[x - \frac{1}{2}\right]}$ is prime is $[x_1, x_2)$, then the value of $2(x_1^{\,2} + x_2^{\,2})$. [Where [.] denotes greatest integer function less than or equal to x]

3. Number of integers in the range of the function f(x) = $\frac{x^3 + 2x^2 + 3x + 2}{x^3 + 2x^2 + 2x + 1}$; $x \in R - \{0\}$ is :

4. Range of the function f(x) = |sin x |cos x| + cos x |sin x|| is [a, b] then (a + b) is equal to

5. If f and g are two distinct linear functions defined on R such that they map[–1, 1] onto [0, 2] and h : R – {–1, 0, 1} → R defined by h(x) = $\frac{f(x)}{g(x)}$, then |h(h(x)) + h(h(1/x))| > n. Then maximum integral value of n is :

6. If f(x) = $\frac{1}{1-x}$, g(x) = f (f(x)), h(x) = f(f(f(x))), then the absolute value of f(x) . g(x) . h(x), where $x \neq 0, 1$, is

7. If f(x) = $ax^7 + bx^3 + cx - 5$; a, b, c are real constants and f(–7) = 7 then maximum value of |f(7)+17cosx| is

8. If f(x) = $\frac{4a-7}{3}x^3 + (a-3)x^2 + x + 5$ is a one-one function, then number of possible integral values of a is

9. Number of solutions of the equation $e^{-\sin^2 x} = \tan 2x$ in [0, 10π] is

10. Let f(x)= $([a]^2 - 5[a] + 4)x^3 - (6\{a\}^2 - 5\{a\} + 1)x - (\tan x)\,\text{sgn}(x)$ be an even function ∀ x ∈ R, then the sum of all possible values of '3a' is
(where [·] denotes G.I. F and {·} fractional part functional part function)

11. Let f be a one–one function with domain {21, 22, 23} and range {x,y,z}. It is given that exactly one of the following statements is true and the remaining two are false. f(21) = x; f(22) ≠ x ; f(23) ≠ y. Then $f^{-1}(x)$ is :

12. Let f : $[-\sqrt{2} + 1, \sqrt{2} + 1] \to \left[\frac{-\sqrt{2}+1}{2}, \frac{\sqrt{2}+1}{2}\right]$ be a function defined by f(x) = $\frac{1-x}{1+x^2}$.

If $f^{-1}(x) = \begin{cases} \frac{-1+\lambda\left(\sqrt{4x-4x^2+1}\right)}{2x}, & x \neq 0 \\ \mu, & x = 0 \end{cases}$, then $\lambda + \mu$ is.

13. The number of real solutions of the equation $x^3 + 1 = 2\sqrt[3]{2x-1}$, is :

14. If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$, where $-1 \le x, y, z \le 1$, then find the value of $x^2 + y^2 + z^2 + 2xyz$

15. The sum of absolute value of all possible values of x for which $\cos \tan^{-1} \sin \cot^{-1} x = \sqrt{\dfrac{226}{227}}$.

16. If $\cot^{-1} \dfrac{n}{\pi} > \dfrac{\pi}{6}$, $n \in N$, then the maximum value of ' n ' is:

17. If $x \in (0, 1)$ and $f(x) = \sec\left\{\tan^{-1}\left(\dfrac{\sin(\cos^{-1}x) + \cos(\sin^{-1}x)}{\cos(\cos^{-1}x) + \sin(\sin^{-1}x)}\right)\right\}$, then $\sum_{r=2}^{10} f\left(\dfrac{1}{r}\right)$ is

18. If $\dfrac{1}{2}\sin^{-1}\left(\dfrac{3\sin 2\theta}{5 + 4\cos 2\theta}\right) = \dfrac{\pi}{4}$, then $\tan\theta$ is equal to

19. The number of real solutions of equation $\sqrt{1 + \cos 2x} = \sqrt{2}\,\sin^{-1}(\sin x)$, $-10\pi \le x \le 10\pi$, is/are

20. The number of solution(s) of the equation, $\sin^{-1}x + \cos^{-1}(1 - x) = \sin^{-1}(-x)$, is/are

21. Find the value of $3\sum_{n=1}^{\infty}\left\{\dfrac{1}{\pi}\sum_{k=1}^{\infty}\cot^{-1}\left(1 + 2\sqrt{\sum_{r=1}^{k} r^3}\right)\right\}^n$

PART - III : ONE OR MORE THAN ONE OPTIONS CORRECT TYPE

1. Let A = {1, 2, 3, 4} and R be a relation in A given by R = {(1, 1), (2, 2), (3, 3), (4, 4), (1, 2), (2, 1), (3, 1), (1, 3)}, then relation R is
(A) Reflexive (B) Symmetric (C) Equivalence (D) Reflexive and Symmetric

2. For $n, m \in N$, n | m means that n is a factor of m, then relation | is
(A) Reflexive (B) symmetric (C) Transitive (D) Equivalence

3. If $f(x) = \sin \ell n\left(\dfrac{\sqrt{4 - x^2}}{1 - x}\right)$, then
(A) domain of f(x) is (– 2, 1) (B) domain of f(x) is [–1, 1]
(C) range of f(x) is [–1, 1] (D) range of f(x) is [–1, 1)

4. D is domain and R is range of $f(x) = \sqrt{x - 1} + 2\sqrt{3 - x}$, then
(A) D : [1 , 3] ; (B) D : $(-\infty, 1] \cup [3, \infty)$,
(C) R : $\left[1, \sqrt{3}\right]$ (D) R : $\left[\sqrt{2}, \sqrt{10}\right]$

5. If [2 cos x] + [sin x] = – 3, then the range of the function, $f(x) = \sin x + \sqrt{3}\cos x$ in [0, 2π] lies in (where [ ] denotes greatest integer function)
(A) $[-\sqrt{3}, \sqrt{3})$ (B) $[-2, -\sqrt{3}]$ (C) [–3, –1] (D) $[-2, -\sqrt{3})$

6. Let D ≡ [–1, 1] is the domain of the following functions, state which of them are injective.
(A) $f(x) = \begin{cases} \tan^{-1}\dfrac{1}{x} & x \ne 0 \\ 1 & x = 0 \end{cases}$ (B) $g(x) = x^3$
(C) h(x) = sin 2x (D) k(x) = sin (πx/2)

7. Let $f(x) = x^{135} + x^{125} - x^{115} + x^5 + 1$. If f(x) divided by $x^3 - x$, then the remainder is some function of x say g(x). Then g(x) is an :
(A) one-one function (B) many one function (C) into function (D) onto function

8. The function $f : X \to Y$, defined by $f(x) = x^2 - 4x + 5$ is both one–one and onto if
(A) $X = [2, \infty)$ & $Y = [1, \infty)$ (B) $X = (-\infty, 2]$ & $Y = [1, \infty)$
(C) $X = [3, \infty)$ & $Y = [2, \infty)$ (D) $X = (-\infty, 2]$ & $Y = (1, \infty)$

9. $f : N \to N$ where $f(x) = x - (-1)^x$ then f is :
(A) one-one (B) many-one (C) onto (D) into

10. Which one of the following pair of functions are **NOT** identical ?
(A) $e^{(\ell nx)/2}$ and $\sqrt{x}$
(B) tan (tanx) and cot (cotx)
(C) $\cos^2 x + \sin^4 x$ and $\sin^2 x + \cos^4 x$
(D) $\frac{|x|}{x}$ and sgn (x), where sgn(x) stands for signum function.

11. If the graph of the function $f(x) = \frac{a^x - 1}{x^n (a^x + 1)}$ is symmetric about y-axis, then n is equal to:
(A) 1/5 (B) 1/3 (C) 1 / 4 (D) – 1 / 3

12. If $f(x) = \begin{cases} x^2 & x \le 1 \\ 1 - x & x > 1 \end{cases}$ & composite function h(x) = |f(x)| + f(x + 2), then
(A) $h(x) = 2x^2 + 4x + 4 \quad \forall\, x \le -1$
(B) $h(x) = x^2 + x + 1 \quad \forall\, -1 < x \le 1$
(C) $h(x) = x^2 - x - 1 \quad \forall\, -1 < x \le 1$
(D) $h(x) = -2 \quad \forall\, x > 1$

13. Let $f(x) = \begin{cases} 0 & \text{for } x = 0 \\ x^2 \sin\left(\frac{\pi}{x}\right) & \text{for } -1 < x < 1 \ (x \ne 0) \\ x\ |x| & \text{for } x > 1 \text{ or } x < -1 \end{cases}$, then:
(A) f(x) is an odd function (B) f(x) is an even function
(C) f(x) is neither odd nor even (D) $f'(x)$ is an even function

14. If $f : [-2, 2] \to R$ where $f(x) = x^3 + \tan x + \left[\frac{x^2 + 1}{P}\right]$ is a odd function, then the value of parametric P, where [.] denotes the greatest integer function, can be
(A) 5 < P < 10 (B) P < 5 (C) P > 5 (D) P = 15

15. If f: $R \to [-1, 1]$, where $f(x) = \sin\left(\frac{\pi}{2}[x]\right)$, (where [.] denotes the greatest integer function), then
(A) f (x) is onto (B) f (x) is into (C) f (x) is periodic (D) f (x) is many one

16. If $f(x) = \frac{2x\ (\sin x + \tan x)}{2\left[\frac{x + 2\pi}{\pi}\right] - 3}$ then it is, (where [.] denotes the greatest integer function)
(A) odd (B) Even (C) many one (D) one-one

17. Identify the statement(s) which is/are incorrect ?
(A) the function f(x) = sinx + cosx is neither odd nor even
(B) the fundamental period of f(x) = cos (sin x) + cos (cos x) is π
(C) the range of the function f(x) = cos (3 sin x) is [–1, 1]
(D) f(x) = 0 is a periodic function with period 2

18.✍ If $F(x) = \dfrac{\sin \pi [x]}{\{x\}}$, then F (x) is: (where { . } denotes fractional part function and [ ] denotes greatest integer function and sgn (x) is a signum function)

(A) periodic with fundamental period 1
(B) even
(C) range is singleton
(D) identical to $\text{sgn}\left(\text{sgn}\dfrac{\{x\}}{\sqrt{\{x\}}}\right) - 1$

19. Let f : R → R and g : R → R be two one-one and onto functions such that they are mirror images of each other about the line y = a. If h(x) = f(x) + g(x), then h(x) is

(A) one-one
(B) into
(C) onto
(D) many-one

20. Which of following pairs of functions are identical.

(A) $f(x) = e^{\ell n \sec^{-1} x}$ and $g(x) = \sec^{-1} x$
(B) $f(x) = \tan(\tan^{-1} x)$ and $g(x) = \cot(\cot^{-1} x)$
(C) f(x) = sgn (x) and g(x) = sgn (sgn (x))
(D) $f(x) = \cot^2 x . \cos^2 x$ and $g(x) = \cot^2 x - \cos^2 x$

21. If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \dfrac{3\pi}{2}$, then

(A) $x^{100} + y^{100} + z^{100} - \dfrac{9}{x^{101} + y^{101} + z^{101}} = 0$
(B) $x^{22} + y^{42} + z^{62} - x^{220} - y^{420} - z^{620} = 0$
(C) $x^{50} + y^{25} + z^{5} = 0$
(D $\dfrac{x^{2008} + y^{2008} + z^{2008}}{(xyz)^{2009}} = 0$)

22. If X = cosec $\tan^{-1}$ cos $\cot^{-1}$ sec $\sin^{-1}$ a and Y = sec $\cot^{-1}$ sin $\tan^{-1}$ cosec $\cos^{-1}$ a; where $0 \le a < 1$. Find the relation between X and Y. Then

(A) X = Y
(B) $Y = \sqrt{3 - a^2}$
(C) $X \neq Y$
(D) X = 2Y

23. If α satisfies the inequation $x^2 - x - 2 > 0$, then a value exists for

(A) $\sin^{-1} \alpha$
(B) $\cos^{-1} \alpha$
(C) $\sec^{-1} \alpha$
(D) $\text{cosec}^{-1} \alpha$

24. For the function $f(x) = \ell n (\sin^{-1} \ell og_2 x)$,

(A) Domain is $\left[\dfrac{1}{2}, 2\right]$
(B) Range is $\left(-\infty, \ell n\dfrac{\pi}{2}\right]$
(C) Domain is (1, 2]
(D) Range is R

25. In the following functions defined from [–1, 1] to [–1, 1] , then functions which are not bijective are

(A) $\sin(\sin^{-1}x)$
(B) $\dfrac{2}{\pi}\sin^{-1}(\sin x)$
(C) $(\text{sgn } x)\ \ell n\ e^x$
(D) x^3 sgn x

26. The expression $\dfrac{1}{\sqrt{2}}\left\{\dfrac{\sin\cot^{-1}\cos\tan^{-1} t}{\cos\tan^{-1}\sin\cot^{-1}\sqrt{2}\, t}\right\} . \left\{\sqrt{\dfrac{1+2t^2}{2+t^2}}\right\}$ can take the value

(A) 1/2
(B) –5
(C) 1
(D) 3/4

27.✍ If 0 < x < 1, then $\tan^{-1}\dfrac{\sqrt{1-x^2}}{1+x}$ is equal to:

(A) $\dfrac{1}{2}\cos^{-1} x$
(B) $\cos^{-1}\sqrt{\dfrac{1+x}{2}}$
(C) $\cos^{-1}\sqrt{\dfrac{1-x}{2}}$
(D) $\dfrac{1}{2}\sin^{-1}x$

28. If $f(x) = \cos^{-1}x + \cos^{-1}\left\{\frac{x}{2} + \frac{1}{2}\sqrt{3-3x^2}\right\}$, then

(A) $f\left(\frac{2}{3}\right) = \frac{\pi}{3}$ (B) $f\left(\frac{2}{3}\right) = \frac{\pi}{2}$

(C) $f\left(\frac{1}{3}\right) = \frac{\pi}{3}$ (D) $f\left(\frac{1}{3}\right) = 2\cos^{-1}\frac{1}{3} - \frac{\pi}{3}$

29. $\sum_{n=1}^{\infty} \tan^{-1}\frac{4n}{n^4 - 2n^2 + 2}$ is equal to:

(A) $\tan^{-1} 2 + \tan^{-1} 3$ (B) $4\tan^{-1} 1$ (C) $\pi/2$ (D) $\sec^{-1}\left(-\sqrt{2}\right)$

30. If $\sin^2 (2\cos^{-1} (\tan x)) = 1$ then x may be

(A) $x = \pi + \tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$ (B) $x = \pi - \tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$

(C) $x = -\pi + \tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$ (D) $x = -\pi - \tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$

31. If $\sin^{-1} x + 2\cot^{-1} (y^2 - 2y) = 2\pi$, then

(A) $x + y = y^2$ (B) $x^2 = x + y$ (C) $y = y^2$ (D) $x^2 - x + y = y^2$

PART - IV : COMPREHENSION

Comprehension # 1

Given a function f : A → B ; where A = {1, 2, 3, 4, 5} and B = {6, 7, 8}

1. Find number of all such functions y = f(x) which are one-one ?

(A) 0 (B) 3^5 (C) ${}^5P_3$ (D) 5^3

2. Find number of all such functions y = f(x) which are onto

(A) 243 (B) 93 (C) 150 (D) none of these

3. The number of mappings of g(x) : B → A such that $g(i) \le g(j)$ whenever $i < j$ is

(A) 60 (B) 140 (C) 10 (D) 35

Comprehension # 2

Let the domain and range of inverse circular functions are defined as follows

	Domain	**Range**
$\sin^{-1}x$	$[-1, 1]$	$\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$
$\cos^{-1}x$	$[-1, 1]$	$[0, \pi]$
$\tan^{-1}x$	R	$\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$
$\cot^{-1}x$	R	$(0, \pi)$
$\text{cosec}^{-1}x$	$(-\infty, -1] \cup [1, \infty)$	$\left[\frac{\pi}{2}, \frac{3\pi}{2}\right] - \{\pi\}$
$\sec^{-1}x$	$(-\infty, -1] \cup [1, \infty)$	$[0, \pi] - \left\{\frac{\pi}{2}\right\}$

4. ✍ $\sin^{-1}x < \frac{3\pi}{4}$ then solution set of x is

(A) $\left(\frac{1}{\sqrt{2}}, 1\right]$ (B) $\left(-\frac{1}{\sqrt{2}}, -1\right]$ (C) $\left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$ (D) none of these

5. ✍ If $x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$, cosec^{-1} cosec x is

(A) $2\pi - x$ (B) $\pi + x$ (C) $\pi - x$ (D) $-\pi - x$

6. ✍ If $x \in [-1, 1]$, then range of $\tan^{-1}(-x)$ is

(A) $\left[\frac{3\pi}{4}, \frac{7\pi}{4}\right]$ (B) $\left[\frac{3\pi}{4}, \frac{5\pi}{4}\right]$ (C) $[-\pi, 0]$ (D) $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

Exercise-3

PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

*** Marked Questions may have more than one correct option.**

✍ Marked questions are recommended for Revision.

1. The maximum value of the function $f(x) = 2x^3 - 15x^2 + 36x - 48$ on the set $A = \{x \mid x^2 + 20 \le 9x\}$ is

[IIT-JEE 2009,P-2,(4, –1), 80]

2. ✍ If the function $f(x) = x^3 + e^{\frac{x}{2}}$ and $g(x) = f^{-1}(x)$, then the value of $g'(1)$ is **[IIT-JEE 2009,P-2,(4, –1), 80]**

3. Let $f(x) = x^2$ and $g(x) = \sin x$ for all $x \in R$. Then the set of all x satisfying (f o g o g o f) (x) = (g o g o f) (x), where (f o g) (x) = f(g(x)), is **[IIT-JEE 2011, Paper-2, (3, –1), 80]**

(A) $\pm\sqrt{n\pi}$, $n \in \{0, 1, 2, ...\}$ (B) $\pm\sqrt{n\pi}$, $n \in \{1, 2, ...\}$

(C) $\frac{\pi}{2} + 2n\pi$, $n \in \{.....-2, -1, 0, 1, 2, ...\}$ (D) $2n\pi$, $n \in \{...., -2, -1, 0, 1, 2, ...\}$

4. Let $f(\theta) = \sin\left(\tan^{-1}\left(\frac{\sin\theta}{\sqrt{\cos 2\theta}}\right)\right)$, where $-\frac{\pi}{4} < \theta < \frac{\pi}{4}$. Then the value of $\frac{d}{d(\tan\theta)}(f(\theta))$ is

[IIT-JEE 2011, Paper-1, (4, 0), 80]

5. The function $f : [0, 3] \to [1, 29]$, defined by $f(x) = 2x^3 - 15x^2 + 36x + 1$, is

(A) one-one and onto (B) onto but not one-one

(C) one-one but not onto (D) neither one-one nor onto

[IIT-JEE 2012, Paper-1, (3, –1), 70]

6*. Let $f : (-1, 1) \to IR$ be such that $f(\cos 4\theta) = \frac{2}{2 - \sec^2\theta}$ for $\theta \in \left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$. Then the value(s) of $f\left(\frac{1}{3}\right)$ is (are) **[IIT-JEE 2012, Paper-2, (4, 0), 66]**

(A) $1 - \sqrt{\frac{3}{2}}$ (B) $1 + \sqrt{\frac{3}{2}}$ (C) $1 - \sqrt{\frac{2}{3}}$ (D) $1 + \sqrt{\frac{2}{3}}$

7. The value of $\cot\left(\sum_{n=1}^{23} \cot^{-1}\left(1 + \sum_{k=1}^{n} 2k\right)\right)$ is **[JEE (Advanced) 2013, Paper-1, (2, 0)/60]**

(A) $\frac{23}{25}$ (B) $\frac{25}{23}$ (C) $\frac{23}{24}$ (D) $\frac{24}{23}$

8. Match List I with List II and select the correct answer using the code given below the lists :

List - I		List - II	
P.	$\left(\frac{1}{y^2}\left(\frac{\cos(\tan^{-1}y)+y\sin(\tan^{-1}y)}{\cot(\sin^{-1}y)+\tan(\sin^{-1}y)}\right)^2+y^4\right)^{1/2}$ takes value	1.	$\frac{1}{2}\sqrt{\frac{5}{3}}$
Q.	If cos x + cos y + cox z = 0 = sin x + sin y + sin z then possible value of cos $\frac{x-y}{2}$ is	2.	$\sqrt{2}$
R.	If cos $\left(\frac{\pi}{4}-x\right)$cos 2x + sin x sin 2x sec x = cos xsin 2x sec x + cos$\left(\frac{\pi}{4}+x\right)$ cos 2x then possible value of sec x is	3.	$\frac{1}{2}$
S.	If cot $\left(\sin^{-1}\sqrt{1-x^2}\right)$ = sin $\left(\tan^{-1}\left(x\sqrt{6}\right)\right)$, $x \neq 0$, then possible value of x is	4.	1

[JEE (Advanced) 2013, Paper-2, (3, –1)/60]

Codes :

	P	Q	R	S
(A)	4	3	1	2
(B)	4	3	2	1
(C)	3	4	2	1
(D)	3	4	1	2

9*. Let f: $\left(-\frac{\pi}{2},\frac{\pi}{2}\right) \to R$ be given by f(x) = $(\log(\sec x + \tan x))^3$. Then

[JEE (Advanced) 2014, Paper-1, (3, 0)/60]

(A) f(x) is an odd function
(B) f(x) is a one-one function
(C) f(x) is an onto function
(D) f(x) is an even function

10. Let f : [0, 4π] → [0, π] be defined by f(x) = $\cos^{-1}$ (cos x). The number of points x ∈ [0, 4π] satisfying the equation f(x) = $\frac{10-x}{10}$ is **[JEE (Advanced) 2014, Paper-1, (3, 0)/60]**

11*. If $\alpha = 3\sin^{-1}\left(\frac{6}{11}\right)$ and $\beta = 3\cos^{-1}\left(\frac{4}{9}\right)$, where the inverse trigonometric functions take only the principal values, then the correct option(s) is(are) **[JEE (Advanced) 2015, P-2 (4, –2)/ 80]**

(A) cos β > 0
(B) sin β < 0
(C) cos(α + β) > 0
(D) cos α < 0

12. The number of real solutions of the equation

$$\sin^{-1}\left(\sum_{i=1}^{\infty} x^{i+1} - x\sum_{i=1}^{\infty}\left(\frac{x}{2}\right)^i\right) = \frac{\pi}{2} - \cos^{-1}\left(\sum_{i=1}^{\infty}\left(-\frac{x}{2}\right)^i - \sum_{i=1}^{\infty}(-x)^i\right)$$

lying in the interval $\left(-\frac{1}{2},\frac{1}{2}\right)$ is ___________. **[JEE(Advanced) 2018, Paper-1,(3, 0)/60]**

(Here, the inverse trigonometric functions $\sin^{-1}x$ and $\cos^{-1}x$ assume values in $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ and [0, π], respectively).

13. Let X be a set with exactly 5 elements and Y be a set with exactly 7 elements. If α is the number of one-one functions from X to Y and β is the number of onto function form Y to X, then the value of $\frac{1}{5!}(\beta-\alpha)$ is ______ . **[JEE(Advanced) 2018, Paper-2,(3, 0)/60]**

14. Let $E_1 = \left\{x \in R : x \neq 1 \text{ and } \frac{x}{x-1} > 0\right\}$ and $E_2 = \left\{x \in E_1 : \sin^{-1}\left(\log_e\left(\frac{x}{x-1}\right)\right) \text{is a real number}\right\}$.

(Here, the inverse trigonometric function $\sin^{-1}x$ assumes values in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.)

Let $f : E_1 \rightarrow R$ be the function defined by $f(x) = \log_e\left(\frac{x}{x-1}\right)$

and $g : E_2 \rightarrow R$ be the function defined by $g(x) = \sin^{-1}\left(\log_e\left(\frac{x}{x-1}\right)\right)$

[JEE(Advanced) 2018, Paper-2,(3, –1)/60]

LIST-I	**LIST-II**
(P) The range of f is	(1) $\left(-\infty, \frac{1}{1-e}\right] \cup \left[\frac{e}{e-1}, \infty\right)$
(Q) The range of g contains	(2) (0, 1)
(R) The domain of f contains	(3) $\left[-\frac{1}{2}, \frac{1}{2}\right]$
(S) The domain of g is	(4) $(-\infty, 0) \cup (0, \infty)$
	(5) $\left(-\infty, \frac{e}{e-1}\right]$
	(6) $(-\infty, 0) \cup \left(\frac{1}{2}, \frac{e}{e-1}\right]$

The correct option is
(A) P → 4; Q → 2; R → 1; S → 1
(B) P → 3; Q → 3; R → 6; S → 5
(C) P → 4; Q → 2; R → 1; S → 6
(D) P → 4; Q → 3; R → 6; S → 5

PART - II : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. Consider the following relations : **[AIEEE-2010, (4, – 1), 144]**
R : {(x, y)| x ,y are real numbers and x = wy for some rational number w}
$S = \left\{\left(\frac{m}{n}, \frac{p}{q}\right) \mid m, n, p \text{ and } q \text{ are integers such that } n, q \neq 0 \text{ and } qm = pn\right\}$
Then
(1) neither R nor S is an equivalence relation
(2) S is an equivalence relation but R is not an equivalence relation
(3) R and S both are equivalence relations
(4) R is an equivalence relation but S is not an equivalence relation

2. Let R be the set of real numbers. **[AIEEE-2011(Part-I), (4, – 1), 120]**
Statement-1 : A = {(x, y) ∈ R × R : y – x is an integer} is an equivalence relation on R.
Statement-2 : B = {(x, y) ∈ R × R : x = αy for some rational number α} is an equivalence relation on R.
(1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is true, Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1.
(3) Statement-1 is true, Statement-2 is false.
(4) Statement-1 is false, Statement-2 is true.

3. Consider the following relation R on the set of real square matrices of order 3.
R = {(A, B)|A = P^{-1} BP for some invertible matrix P}. **[AIEEE-2011(Part-II), (3, – 1), 120]**
Statement -1 : R is equivalence relation.
Statement - 2 : For any two invertible 3 × 3 matrices M and N, $(MN)^{-1} = N^{-1}M^{-1}$.
(1) Statement-1 is true, statement-2 is a correct explanation for statement-1.
(2) Statement-1 is true, statement-2 is true; statement-2 is not a correct explanation for statement-1.
(3) Statement-1 is true, statement-2 is false.
(4) Statement-1 is false, statement-2 is true.

4. The domain of the function $f(x) = \frac{1}{\sqrt{|x| - x}}$ is : **[AIEEE 2011, I, (4, –1), 120]**

(1) $(-\infty, \infty)$ (2) $(0, \infty)$ (3) $(-\infty, 0)$ (4) $(-\infty, \infty) - \{0\}$

5. Let f be a function defined by $f(x) = (x-1)^2 + 1$, $(x \geq 1)$. **[AIEEE 2011, II, (4, –1), 120]**
Statement - 1 : The set $\{x : f(x) = f^{-1}(x)\} = \{1, 2\}$.
Statement - 2 : f is a bijection and $f^{-1}(x) = 1 + \sqrt{x-1}$, $x \geq 1$.
(1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is true, Statement-2 is true; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is true, Statement-2 is false
(4) Statement-1 is false, Statement-2 is true .

6. If x, y, z are in A.P. and $\tan^{-1}x$, $\tan^{-1}y$ and $\tan^{-1}z$ are also in A.P., then **[AIEEE - 2013, (4, –1),120]**
(1) $x = y = z$ (2) $2x = 3y = 6z$ (3) $6x = 3y = 2z$ (4) $6x = 4y = 3z$

7. If g is the inverse of a function f and $f'(x) = \frac{1}{1+x^5}$, then g'(x) equal to :
[JEE(Main)2014,(4, – 1), 120]

(1) $\frac{1}{1+\{g(x)\}^5}$ (2) $1 + \{g(x)\}^5$ (3) $1 + x^5$ (4) $5x^4$

8. Let $\tan^{-1}y = \tan^{-1}x + \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, where $|x| < \frac{1}{\sqrt{3}}$. Then a value of y is
[JEE(Main)2015,(4, – 1), 120]

(1) $\frac{3x-x^3}{1-3x^2}$ (2) $\frac{3x+x^3}{1-3x^2}$ (3) $\frac{3x-x^3}{1+3x^2}$ (4) $\frac{3x+x^3}{1+3x^2}$

9. If $f(x) + 2f\left(\frac{1}{x}\right) = 3x$, $x \neq 0$, and $S = \{x \in R : f(x) = f(-x)\}$; then S : **[JEE(Main)2016,(4, – 1), 120]**
(1) contains exactly one element (2) contains exactly two elements.
(3) contains more than two elements. (4) is an empty set.

10. Two sets A and B are as under : $A = \{(a, b) \in R \times R : |a - 5| < 1 \text{ and } |b - 5| < 1\}$;
$B = \{(a, b) \in R \times R : 4(a-6)^2 + 9(b-5)^2 \leq 36\}$. Then; **[JEE(Main)2018,(4, – 1), 120]**
(1) $A \cap B = \phi$ (an empty set) (2) Neither $A \subset B$ nor $B \subset A$
(3) $B \subset A$ (4) $A \subset B$

11. If $\cos^{-1}\left(\frac{2}{3x}\right) + \cos^{-1}\left(\frac{3}{4x}\right) = \frac{\pi}{2}\left(x > \frac{3}{4}\right)$ then x is equal to :

[JEE(Main) 2019, Online (09-01-19),P-1 (4, – 1), 120]

(1) $\frac{\sqrt{145}}{12}$ (2) $\frac{\sqrt{145}}{10}$ (3) $\frac{\sqrt{146}}{12}$ (4) $\frac{\sqrt{145}}{11}$

12. For $x \in R - \{0, 1\}$, let $f_1(x) = \frac{1}{x}$, $f_2(x) = 1 - x$ and $f_3(x) = \frac{1}{1-x}$ be three given functions. If a function, J(x) satisfies $(f_2 \circ J \circ f_1)(x) = f_3(x)$ is equal to : **[JEE(Main) 2019, Online (09-01-19),P-1 (4, – 1), 120]**

(1) $\frac{1}{x}f_3(x)$ (2) $f_1(x)$ (3) $f_3(x)$ (4) $f_2(x)$

13. The value of $\cot\left(\sum_{n=1}^{19}\cot^{-1}\left(1+\sum_{p=1}^{n}2p\right)\right)$ is : **[JEE(Main) 2019, Online (10-01-19),P-2 (4, – 1), 120]**

(1) $\frac{19}{21}$ (2) $\frac{21}{19}$ (3) $\frac{22}{23}$ (4) $\frac{23}{22}$

14. The number of functions f from {1, 2, 3, …., 20}, onto {1,2,3,………..,20} such that f(k) is a multiple of 3, wherever k is a multiple of 4, is : **[JEE(Main) 2019, Online (11-01-19),P-2 (4, – 1), 120]**

(1) $5! \times 6!$ (2) $(15)! \times 6!$ (3) $6^5 \times (15)!$ (4) $5^6 \times 15$

15. Let Z be the set of integers. If $A = \{x \in Z : 2^{(x+2)(x^2-5x+6)} = 1\}$ and $B = \{x \in Z : -3 < 2x - 1 < 9\}$ then the number of subsets of the set A × B, is – **[JEE(Main) 2019, Online (12-01-19),P-2 (4, – 1), 120]**

(1) 2^{18} (2) 2^{12} (3) 2^{15} (4) 2^{10}

Answers

EXERCISE - 1

PART - I

Section (A) :

A-1. 9 **A-2.** {(2, 4), (3, 4)} **A-3.** 2^{12} **A-4.** R = { (2, 1), (3, 3), (4, 5), (5, 7)}

A-5. (i) {– 3, – 2, – 1, 0, 1, 2, 3} (ii) {0, 1, 2, 3, 4}
(iii) {(–3, 4), (–2, 3), (–1, 2), (0, 1), (1, 0), (2, 1), (3, 2)}

A-6. {(–1,0), (–1,1), (0,2), (1,2)}

Section (B) :

B-1. (i) Reflexive and transitive but not symmetric.
(ii) neither reflexive nor transitive but it is symmetric

B-3. (iii) & (v) **B-6.** Reflexive and symmetric but not transitive

Section (C) :

C-1. (i) yes (ii) no (iii) no (iv) no

C-2. (i) R – {–1, 1} (ii) $2n\pi - \frac{\pi}{2} \le x \le 2n\pi + \frac{\pi}{2}$, $n \in I$ (iii) $(0, \infty)$ (iv) R (v) $[-2, 0) \cup (0, 1)$
(vi) (2, 3] (vii) $(-\infty, -1] \cup [0, \infty)$ (viii) $\left(-\frac{1}{6}, \frac{\pi}{3}\right] \cup \left[\frac{5\pi}{3}, 6\right)$

C-3. (i) [0, 1] (ii) [– 1, 1] (iii) R (iv) ϕ
(v) $\bigcup_{n\in I}\left[n\pi, n\pi + \frac{\pi}{4}\right]$ (vi) R – {2nπ}, $n \in I$ (vii) $(0, 1] \cup [4,5)$ (viii) (2, 3)

C-4. (i) $[0, \infty)$ (ii) $\left[-\frac{1}{2}, \frac{1}{2}\right]$ (iii) [0, 4] (iv) {–1, 1}

C-5. (i) Domain : R, Range : $\frac{1}{\sqrt{7}} \le y \le 1$ (ii) Domain : $N \cup \{0\}$, Range : (n! : n = 0, 1, 2,...}
(iii) Domain R – {3}, Range : R – {6} (iv) Domain : R, Range : {1}

C-6. (i) [0, 10] (ii) (0, 1] (iii) $(-\infty\ \frac{49}{20},]$ (iv) [–4, 3] (v) [–1, 1] (vi) R
(vii) $n \in N$

C-7. (i) $(-\infty, 1]$ (ii) $\left[\frac{1}{\sqrt{16-1/\sqrt{2}}}, \infty\right)$ (iii) $\left[\frac{1}{3}, 1\right]$
(iv) $\left(-\infty, -\frac{1}{4}\right] \cup \left[-\frac{1}{20}, \infty\right)$ (v) $\left[\frac{1}{3}, 3\right]$ (vi) $\left[0, \frac{3}{\sqrt{2}}\right]$
(vii) $[4, \infty)$ (viii) [–11, 16] (ix) $\left[\frac{3}{4}, 1\right]$
(x) 1 (xi) $\left[1-\sin\sqrt{2}, 1+\sin\sqrt{2}\right]$

C-8. (i) many-one & into (ii) many-one & into (iii) one-one & onto (iv) many-one & into
(v) one – one & into (vi) many-one & into (vii) many-one & into (viii) many-one & onto
(ix) many-one & into

C-9. (i) bijective (injective as well as surjective) (ii) neither surjective nor injective
(iii) surjective but not injective

C-10. (i) many-one & onto (ii) many-one & into

C-11. (i) No (ii) Yes (iii) Yes (iv) No **C-12.** n^n , n!

Section (D) :

D-1. (i) No (ii) No (iii) No (iv) No **D-2.** $(2, \infty)$

D-4. $[fo(goh)](x) = [(fog)oh](x) = \sin^2\sqrt{x}$

D-5. (i) $fog = x, x > 0$; $gof = x, x \in R$ (ii) $|\sin x|, \sin|x|$

(iii) $\sin(x^2), (\sin x)^2$ (iv) $\dfrac{3x^2-4x+2}{(x-1)^2}, \dfrac{x^2+2}{x^2+1}$

D-6. Domain : [1 , 2] ; Range : $[\ell n2 , \ell n4)$ **D-7.** $f(g(x)) = \begin{cases} 2-2x+x^2, & 0 \le x \le 1 \\ 2-x, & -1 \le x < 0 \end{cases}$

D-8. **(i)** $x \in R - \{0, 1\}$ **(ii)** $x \in R - \{-2, -1\}$

(iii) $x \in R - \left\{-\dfrac{3}{2}, -1\right\}$ **(iv)** $x \in R - \{-2, -1\}$

D-9. $fof(x) = \begin{cases} 3\sqrt{2}\ x & x \in Q - \{0\} \\ 3^2\ x & x \in Q^c \end{cases}$, $fofof.....f(x) = \begin{cases} 3^{n-1}\sqrt{2}x & x \in Q - \{0\} \\ 3^n x & x \in Q^c \end{cases}$

D-10. $f(g(x)) = \begin{cases} x^2+1 & x \in [-1, 2] \\ 2x^2+1 & x \in (2, 3) \\ 2x+5 & x \in [3, 5] \end{cases}$

Section (E) :

E-1. (i) even, (ii) neither even nor odd (iii) even, (iv) neither even nor odd
(v) even (vi) even

E-2. (i) neither even nor odd (ii) even (iii) odd (iv) even

E-4. (i) 2π (ii) 2π (iii) 24 (iv) 70π (v) 2π (vi) $\pi/6$ (vii) 2π

Section (F) :

F-1. **(i)** f^{-1} Does not exists **(ii)** $f^{-1} : R \to R$; $f^{-1} = 7 + (4 - x^5)^{1/3}$

(iii) $f^{-1} : R \to R$; $f^{-1} = \dfrac{e^x - e^{-x}}{2}$ **(iv)** $f^{-1}(x) = \dfrac{-1+\sqrt{4x-3}}{2}$

F-2. $f^{-1} : R \to R$, $f^{-1}(x) = \dfrac{1}{2}\ \ell n\left(x + \sqrt{x^2+1}\right)$ **F-3. (a)** $f^{-1}(x) = \begin{cases} \sqrt{-x} & x \le 0 \\ -\sqrt{x} & x > 0 \end{cases}$, 3 **(b)** $x = \dfrac{3-\sqrt{5}}{2}$

F-4. **1** **F-5.** $\alpha = 2$

Section (G) :

G-1. (i) $[-1, 1] - \{0\}$ (ii) $\left[-\dfrac{1}{3}, \dfrac{1}{2}\right]$ (iii) ϕ **G-2.** (i) $(-\infty, \ell n\ \pi/2]$(ii) $(0, \pi/2]$ (iii) $[0, \pi]$

G-3. (i) 1 (ii) $\dfrac{1}{\sqrt{3}}$ (iii) $\dfrac{\pi}{6}$ **G-4.** (i) $n\left(\dfrac{n+1}{2}\right)$

G-5. (i) $[-1, 0)$ (ii) $(-\infty, \cot 3) \cup (\cot 2, \infty)$
(iii) $-\sin 1 < x \le 1$ (iv) $\cos 2 < x \le 1$ (v) no solution

G–6. $B = [0, 4]$; $f^{-1}(x) = \dfrac{1}{2}\left(\sin^{-1}\left(\dfrac{x-2}{2}\right) - \dfrac{\pi}{6}\right)$

Section (H) :

H-1. (i) $-\dfrac{\pi}{6}$ (ii) $-\dfrac{\pi}{3}$ (iii) $\dfrac{3\pi}{4}$ (iv) $\dfrac{\pi}{4}$

H-2. (i) $\pi - 4$ (ii) $4\pi - 10$ (iii) $2\pi - 6$ (iv) $4\pi - 10$ (v) $\dfrac{17\pi}{20}$

H-3. (i) 0 (ii) 1 **H-4.** $x > 1$

Section (I) :

I-1. (i) $\frac{4}{5}$ (ii) $2\sqrt{2}$ (iii) $\frac{\sqrt{41}}{4}$ (iv) $\frac{63}{16}$ (v) $\frac{1+3\sqrt{5}}{8}$ (vi) $\frac{6-4\sqrt{5}}{15}$ (vii) 2 (viii) $\frac{\sqrt{5}}{3}$ (ix) $\frac{1}{2\sqrt{2}}$

I-2. $\frac{\pi}{2}$ **I-7.** (i) $2\tan^{-1}x - \pi$ (ii) $\pi - 2\sin^{-1}x$ (iii) $2\pi - 2\cos^{-1}x$

I-8. $\frac{1+xy}{x-y}$

I-9 (i) $\pm\frac{1}{\sqrt{3}}$ (ii) $x = 3$ (iii) $\pm\frac{1}{\sqrt{2}}$ (iv) $x = \frac{1}{2}$

Section (J) :

J-2. (i) $\tan^{-1}(x + n) - \tan^{-1} x$ (ii) $\frac{\pi}{4}$ (iii) $\frac{\pi}{2}$

PART - II

Section (A) :

A-1. (C) **A-2.** (D) **A-3.** (C) **A-4.** (C) **A-5.** (D) **A-6.** (A)

Section (B) :

B-1. (A) **B-2.** (C) **B-3.** (A) **B-4.** (B) **B-5.** (B) **B-6.** (A)
B-7. (B) **B-8.** (D) **B-9.** (D) **B-10.** (B) **B-11.** (D)

Section (C) :

C-1. (D) **C-2.** (A) **C-3.** (B) **C-4.** (B) **C-5.** (D) **C-6.** (A) **C-7.** (D)
C-8. (B) **C-9.** (C) **C-10.** (B) **C-11.** (A) **C-12.** (D) **C-13.** (A)

Section (D) :

D-1. (A) **D-2.** (C) **D-3.** (B)

Section (E) :

E-1. (B) **E-2.** (B) **E-3.** (B) **E-4.** (C) **E-5.** (D) **E-6.** (C)

Section (F) :

F-1. (A) **F-2.** (A) **F-3.** (A) **F-4.** (A) **F-5.** (A) **F-6.** (B)

Section (G) :

G-1. (A) **G-2.** (C) **G-3.** (D) **G-4.** (C) **G-5.** (D) **G-6.** (A) **G-7.** (C)
G-8. (A)

Section (H) :

H-1. (D) **H-2.** (B) **H-3.** (D) **H-4.** (D)

Section (I) :

I-1. (B) **I-2.** (C) **I-3.** (C) **I-4.** (A) **I-5.** (B) **I-6.** (B) **I-7.** (A)

Section (J) :

J-1. (B) **J-2.** (B) **J-3.** (C)

PART – III

1. (1) $\rightarrow$ (p), (2) $\rightarrow$ (r), (3) $\rightarrow$ (s), (4) $\rightarrow$ (q)

2. (A) $\rightarrow$ (q,r), (B) $\rightarrow$ (q,r), (C) $\rightarrow$ (q,s), (D) $\rightarrow$ (q,r),

3. (A$\rightarrow$ q ; B$\rightarrow$ r,q ; C$\rightarrow$ s ; D$\rightarrow$ t)

4. (A) $\rightarrow$ (q), (B) $\rightarrow$ (s), (C) $\rightarrow$ (p), (D) $\rightarrow$ (r),

5. (A) $\rightarrow$ (p), (B) $\rightarrow$ (p), (C) $\rightarrow$ (p), (D) $\rightarrow$ (s)

EXERCISE - 2

PART - I

1.	(A)	**2.**	(A)	**3.**	(C)	**4.**	(D)	**5.**	(D)	**6.**	(B)	**7.**	(C)
8.	(D)	**9.**	(D)	**10.**	(D)	**11.**	(A)	**12.**	(C)	**13.**	(C)	**14.**	(D)
15.	(B)	**16.**	(C)	**17.**	(B)	**18.**	(B)	**19.**	(A)	**20.**	(C)	**21.**	(B)
22.	(D)	**23.**	(A)	**24.**	(A)	**25.**	(A)	**26.**	(B)	**27.**	(A)	**28.**	(C)
29.	(C)	**30.**	(A)										

PART - II

1.	17	**2.**	17	**3.**	0	**4.**	1	**5.**	2	**6.**	1	**7.**	34
8.	7	**9.**	20	**10.**	35	**11.**	22	**12.**	2	**13.**	3	**14.**	1
15.	30	**16.**	5	**17.**	54	**18.**	3	**19.**	20	**20.**	1	**21.**	1

PART – III

1.	(ABD)	**2.**	(AC)	**3.**	(AC)	**4.**	(AD)	**5.**	(BCD)	**6.**	(BD)	**7.**	(AD)
8.	(ABC)	**9.**	(AC)	**10.**	(ABD)	**11.**	(ABD)	**12.**	(ACD)	**13.**	(AD)	**14.**	(ACD)
15.	(BCD)	**16.**	(AC)	**17.**	(BC)	**18.**	(ABCD)	**19.**	(BD)	**20.**	(BCD)	**21.**	(AB)
22.	(AB)	**23.**	(CD)	**24.**	(BC)	**25.**	(BCD)	**26.**	(AD)	**27.**	(AB)	**28.**	(AD)
29.	(AD)	**30.**	(ABCD)	**31.**	(CD)								

PART - IV

1.	(A)	**2.**	(C)	**3.**	(D)	**4.**	(A)	**5.**	(C)	**6.**	(B)

EXERCISE - 3

PART - I

1.	7	**2.**	2	**3.**	(A)	**4.**	**1**	**5.**	(B)	**6*.**	(AB)	**7.**	(B)
8.	(B)	**9.**	(ABC)	**10.**	**3**	**11.**	(BCD)	**12.**	2	**13.**	119	**14.**	(A)

PART – II

1.	(2)	**2.**	(3)	**3.**	(2)	**4.**	(3)	**5.**	(1)	**6.**	(1)	**7.**	(2)
8.	(1)	**9.**	(2)	**10.**	(4)	**11.**	(1)	**12.**	(3)	**13.**	(2)	**14.**	(2)
15.	(3)												

Advance Level Problems (ALP)

SUBJECTIVE QUESTIONS

✍ Marked Questions may have for Revision Questions.

1. Find the domain of the function f(x) = $\sqrt{-\log_{\frac{x+4}{2}}\left(\log_2 \frac{2x-1}{3+x}\right)}$

2. Let f (x) = $(x^{12} - x^9 + x^4 - x + 1)^{-1/2}$. The domain of the function is :

3. Find the values of 'a' in the domain of the definition of the function , f (a) = $\sqrt{2a^2 - a}$ for which the roots of the equation , $x^2 + (a + 1)x + (a - 1) = 0$ lie between −2 & 1 .

4. The domain of the function f (x) = $\sqrt{\frac{1}{(|x| - 1)\cos^{-1}(2x + 1) \cdot \tan 3x}}$ is:

5. Find domain of the following functions

(i) f(x) = $\sqrt{\log_{1/3}\log_4([x]^2 - 5)}$, where [.] denotes greatest integer function.

(ii) f (x) = $\frac{1}{[|x-1|] + [|12-x|] - 11}$, where [x] denotes the greatest integer not greater than x.

(iii) f(x) = $(x + 0.5)^{\log_{(0.5+x)}\frac{x^2+2x-3}{4x^2-4x-3}}$

(iv) f (x) = $\frac{5}{\left[\frac{x-1}{2}\right]} - 3^{\sin^{-1}x^2} + \frac{(7x+1)!}{\sqrt{x+1}}$, where [.] denotes greatest integer function.

(v) $3^y + 2^{x^4} = 2^{4x^2-1}$

6. The range of the function f (x) = $\sin^{-1}\left[x^2 + \frac{1}{2}\right] + \cos^{-1}\left[x^2 - \frac{1}{2}\right]$, where [ ] is the greatest integer function, is:

7. Find the range of f(x) = $\frac{1}{2\{-x\}} - \{x\}$, (where {.} represents fractional part of x)

8. If $f : R \to R$; $f(x) = \frac{\sqrt{x^2+1}-3x}{\sqrt{x^2+1}+x}$ then find the range of f(x).

9. If a function is defined as $f(x) = \sqrt{\log_{h(x)} g(x)}$, where g(x) = |sinx| + sinx , h(x) = sinx + cosx , $0 \le x \le \pi$ .Then find the doman of f(x).

10. Find the domain and range of the following functions.

(i) $f(x) = \cos^{-1}\sqrt{\log_{[x]}\frac{|x|}{x}}$, where [.] denotes the greatest integer function .

(ii) $f(x) = \sqrt{\log_{1/2}\log_2\left[x^2+4x+5\right]}$ where [.] denotes the greatest integer function

(iii) $f(x) = \sin^{-1}\left[\log_2\left(\frac{x^2}{2}\right)\right]$, where [.] denotes greatest integer function .

(iv) $f(x) = \log_{[x-1]}\sin x$, where [ ] denotes greatest integer function .

(v) $f(x) = \tan^{-1}\left(\sqrt{[x]+[-x]}\right) + \sqrt{2-|x|} + \frac{1}{x^2}$, (where [ ] denotes greatest integer function)

11. If $f(x) = \frac{\sin^2 x + 4\sin x + 5}{2\sin^2 x + 8\sin x + 8}$, then range of f(x) is

12. Find range of the function $f(x) = \log_2\left[3x-\left[x+\left[x+\left[x\right]\right]\right]\right]$ (where [·] is greatest integer function)

13. If $f(x) = \frac{1}{x^2+1}$ and $g(x) = \sin\pi x + 8\left\{\frac{x}{2}\right\}$ where {·} denotes fractional part function then the find range of f(g(x))

14. If the range of the function $f(x) = \left\{\frac{x}{4}\right\} + \cos\pi\left(\frac{(1-2[x])}{2}\right) + \sin\left(\frac{\pi[x]}{2}\right)$ is $\left[\frac{\alpha}{4},\frac{\beta}{4}\right) \cup \left[\frac{\gamma}{4},\frac{\delta}{4}\right) \cup \left[\frac{2\gamma+1}{4},\frac{\delta}{2}\right)$, (where {.} and [.] represent fractional part and greatest integer part functions respectively), then $\alpha^2+\beta^2+\gamma^2+\delta^2$ is

15. The fundamental period of $\sin\frac{\pi}{4}[x] + \cos\frac{\pi x}{2} + \cos\frac{\pi}{3}[x]$, where [.] denotes the integral part of x, is.

16. Consider the function g(x) defined as g(x), $\left(x^{\left(2^{2011}-1\right)}-1\right) = (x+1)(x^2+1)(x^4+1)..........(x^{2^{2010}}+1)-1$

$(|x| \neq 1)$. Then the value of g(2) is equal to

17. It is given that f(x) is a function defined on N, satisfying f(1) = 1 and for any $x \in N$

$f(x + 5) \geq f(x) + 5$ and $f(x + 1) \leq f(x) + 1$

If g(x) = f(x) + 1 – x, then g(2016) equals

18. Find the integral solutions to the equation [x] [y] = x + y. Show that all the non-integral solutions lie on exactly two lines. Determine these lines. Here [.] denotes greatest integer function.

19. Let $f(x) = Ax^2 + Bx + C$, where A, B, C are real numbers. Prove that if f(x) is an integer whenever x is integer, then the numbers 2A, A + B and C are all integers. Conversely, prove that if the numbers 2A, A + B and C are all integer then f(x) is an integer whenever x is an integer.

20. Suppose X and Y are two sets and $f : X \to Y$ is a function. For a subset A of X, define f(A) to be the subset $\{f(a) : a \in A\}$ of Y. For a subset B of Y, define $f^{-1}(B)$ to be the subset $\{x \in X : f(x) \in B\}$ of X. Then prove the followings

(i) Statement "$f^{-1}(f(A)) = A$ for every $A \subset X$" is false

(ii) Statement "$f^{-1}(f(A)) = A$ for every $A \subset X$ if only if f(X) = Y" is false

(iii) Statement "$f(f^{-1}(B)) = B$ for every $B \subset Y$" is false

(iv) Statement "$f(f^{-1}(B)) = B$ for every $B \subset Y$ if only if f(X) = Y" is true

21. Let $g : R \to (0, \pi/3]$ is defined by $g(x) = \cos^{-1}\left(\frac{x^2-k}{1+x^2}\right)$. Then find the possible values of 'k' for which g is surjective.

22. Let $0 < \alpha, \beta, \gamma < \frac{\pi}{2}$ are the solutions of the equations cosx = x, cos(sinx) = x and sin(cosx) = x respectively, then show that $\gamma < \alpha < \beta$

23. Let $f(x) = \log_2\log_3\log_4\log_5(\sin x + a^2)$. Find the set of values of a for which domain of f(x) is R.

24. $\tan^{-1}(\tan\theta) = \begin{cases} \pi+\theta & , \quad -\dfrac{3\pi}{2} < \theta < -\dfrac{\pi}{2} \\ \theta & , \quad -\dfrac{\pi}{2} < \theta < \dfrac{\pi}{2} \\ -\pi+\theta & , \quad \dfrac{\pi}{2} < \theta < \dfrac{3\pi}{2} \end{cases}$, $\sin^{-1}(\sin\theta) = \begin{cases} -\pi-\theta & , \quad -\dfrac{3\pi}{2} \le \theta < -\dfrac{\pi}{2} \\ \theta & , \quad -\dfrac{\pi}{2} \le \theta \le \dfrac{\pi}{2} \\ \pi-\theta & , \quad \dfrac{\pi}{2} < \theta \le \dfrac{3\pi}{2} \end{cases}$,

$\cos^{-1}(\cos\theta) = \begin{cases} -\theta & , \quad -\pi \le \theta < 0 \\ \theta & , \quad 0 \le \theta \le \pi \\ 2\pi-\theta & , \quad \pi < \theta \le 2\pi \end{cases}$

Based on the above results, prove each of the following :

(i) $\cos^{-1} x = \sin^{-1}\sqrt{1-x^2}$ if $0 < x < 1$

(ii) $\sin^{-1} x = \cos^{-1}\sqrt{1-x^2}$ if $0 < x < 1$

(iii) $\cos^{-1} x = \pi + \tan^{-1}\dfrac{\sqrt{1-x^2}}{x}$ if $-1 < x < 0$

25. Express cot ($\text{cosec}^{-1}x$) as an algebraic function of x.

26. Express $\sin^{-1}x$ in terms of (i) $\cos^{-1}\sqrt{1-x^2}$ (ii) $\tan^{-1}\dfrac{x}{\sqrt{1-x^2}}$ (iii) $\cot^{-1}\dfrac{\sqrt{1-x^2}}{x}$

27. If $f(x) = \begin{bmatrix} x & , & x<1 \\ x^2 & , & 1\le x\le 4 \\ 8\sqrt{x} & , & x>4 \end{bmatrix}$, then find $f^{-1}(x)$.

28. $\sin^{-1}\left(\dfrac{x^2}{4}+\dfrac{y^2}{9}\right) + \cos^{-1}\left(\dfrac{x}{2\sqrt{2}}+\dfrac{y}{3\sqrt{2}}-2\right)$ equals to :

29. If $\alpha = 2\tan^{-1}\left(\dfrac{1+x}{1-x}\right)$ & $\beta = \sin^{-1}\left(\dfrac{1-x^2}{1+x^2}\right)$ for $0 < x < 1$, then prove that $\alpha+\beta = \pi$. What the value of $\alpha+\beta$ will be if $x > 1$?

30. Solve $\{\cos^{-1} x\} + [\tan^{-1} x] = 0$ for real values of x. Where { . } and [.] are fractional part and greatest integer functions respectively.

31. Find the set of all real values of x satisfying the inequality $\sec^{-1}x > \tan^{-1}x$.

32. Find the solution of $\sin^{-1}\sqrt{\dfrac{x}{1+x}} - \sin^{-1}\dfrac{x-1}{x+1} = \sin^{-1}\dfrac{1}{\sqrt{1+x}}$.

33. (i) Find all positive integral solutions of the equation, $\tan^{-1} x + \cot^{-1} y = \tan^{-1} 3$.
(ii) If 'k' be a positive integer, then show that the equation:
$\tan^{-1} x + \tan^{-1} y = \tan^{-1} k$ has no non–zero integral solution.

34. Determine the integral values of 'k' for which the system, $(\tan^{-1} x)^2 + (\cos^{-1} y)^2 = \pi^2 k$ and $\tan^{-1} x + \cos^{-1} y = \dfrac{\pi}{2}$ possess solution and find all the solutions.

Answer Key (ALP)

1. $(-4, -3) \cup (4, \infty)$ **2.** $(-\infty, \infty)$ **3.** $a \in \left(-\frac{1}{2}, 0\right] \cup \left[\frac{1}{2}, 1\right)$ **4.** $\left(-\frac{\pi}{6}, 0\right)$

5. **(i)** $[-3, -2) \cup [3, 4)$ **(ii)** $R - \{(0, 1) \cup \{1, 2, \ldots\ldots, 12\} \cup (12, 13)\}$

(iii) $\left(-\frac{1}{2}, \frac{1}{2}\right) \cup \left(\frac{1}{2}, 1\right) \cup \left(\frac{3}{2}, \infty\right)$ **(iv)** $\left(\frac{n}{7}, n \in I, -1 \le n \le 6\right)$

(v). $\left(\frac{-\sqrt{3}-1}{\sqrt{2}}, \frac{-\sqrt{3}+1}{\sqrt{2}}\right) \cup \left(\frac{\sqrt{3}-1}{\sqrt{2}}, \frac{\sqrt{3}+1}{\sqrt{2}}\right)$

6. π **7.** $[\sqrt{2} - 1, \infty)$ **8.** $(-1, \infty)$ **9.** $\left[\frac{\pi}{6}, \frac{\pi}{2}\right)$

10. **(i)** **D : $[2, \infty)$; R : $\{\pi/2\}$** **(ii)** **D :** $\left(-2-\sqrt{2}, -3\right]$ **U ,** $\left[-1, -2+\sqrt{2}\right)$ **; R {0}]**

(iii) **D :** $\left(-\sqrt{8}, -1\right] \cup \left[1, \sqrt{8}\right)$ **; R :** $\left\{-\frac{\pi}{2}, 0, \frac{\pi}{2}\right\}$ **]**

(iv) **D : $[3, \pi) \cup \bigcup_{n=1}^{\infty} (2n\pi, 2n\pi + \pi)$; R : $(-\infty, 0]$ }**

(v) **D :** $\{-2, -1, 1, 2\}$ **; R :** $\left\{\frac{1}{4}, 2\right\}$

11. $\left[\frac{5}{9}, 1\right]$ **12.** **{0, 1}** **13.** $\left(\frac{1}{65}, 1\right]$ **14.** 15 **15.** 24

16. 2 **17.** 1 **18.** Integral solution (0, 0); (2, 2). x + y = 6, x + y = 0

21. $k = -\frac{1}{2}$ **23.** **a**$\in (-\infty, -\sqrt{626}) \cup (\sqrt{626}, \infty)$ **25.** $\cot(\operatorname{cosec}^{-1}x) = \begin{cases} -\sqrt{x^2-1} & \text{if } x \le -1 \\ \sqrt{x^2-1} & \text{if } x \ge 1 \end{cases}$

26. (i) $\sin^{-1}x = \begin{cases} -\cos^{-1}\sqrt{1-x^2}, & \text{if } -1 \le x < 0 \\ \cos^{-1}\sqrt{1-x^2} & \text{if } 0 \le x \le 1 \end{cases}$ (ii) $\sin^{-1}x = \tan^{-1}\frac{x}{\sqrt{1-x^2}}$, for all $x \in (-1, 1)$

(iii) $\sin^{-1}x = \begin{cases} \cot^{-1}\frac{\sqrt{1-x^2}}{x} - \pi & \text{if } -1 \le x < 0 \\ \cot^{-1}\frac{\sqrt{1-x^2}}{x} & \text{if } 0 < x \le 1 \end{cases}$

27. $f^{-1}(x) = \begin{cases} x, & x < 1 \\ \sqrt{x}, & 1 \le x \le 16 \\ \frac{x^2}{64}, & x > 16 \end{cases}$

28. $\frac{3\pi}{2}$ **29.** $-\pi$ **30.** $\{1, \cos 1\}$ **31.** $\{x : x \in (-\infty, -1]\}$ **32.** $x \ge 0$

33. (i) Two solutions (1, 2) (2, 7) **34.** $k = 1, x = \tan(1 - \sqrt{7})\frac{\pi}{4}, y = \cos(\sqrt{7} + 1)\frac{\pi}{4}$