Exercise-1

✍ **Marked questions are recommended for Revision.**

PART - I : SUBJECTIVE QUESTIONS

Section (A) : Elementary concepts of Parabola

A-1. Find the value of λ for which the equation $\lambda x^2 + 4xy + y^2 + \lambda x + 3y + 2 = 0$ represents a parabola

A-2. Find
(i) The vertex, axis, focus, directrix, length of latusrectum of the parabola $x^2 + 2y - 3x + 5 = 0$.
(ii) The equation of the parabola whose focus is (1, 1) and the directrix is $x + y + 1 = 0$.
(iii) The equation to the parabola whose focus is (1, –1) and vertex is (2, 1).
(iv) The equation of the directrix of the parabola $x^2 - 4x - 3y + 10 = 0$.

A-3. Find the equation of the parabola the extremities of whose latus rectum are (1, 2) and (1, –4).

A-4. Find the axis, vertex, focus, directrix and equation of latus rectum of the parabola $9y^2 - 16x - 12y - 57 = 0$

A-5. Find the locus of a point whose sum of the distances from the origin and the line x = 2 is 4 units.

A-6. Find the value of α for which point $(\alpha, 2\alpha + 1)$ doesn't lie outside the parabola $y = x^2 + x + 1$.

A-7. Find the set of values of α in the interval $[\pi/2, 3\pi/2]$, for which the point $(\sin\alpha, \cos\alpha)$ does not lie outside the parabola $2y^2 + x - 2 = 0$.

A-8. If a circle be drawn so as always to touch a given straight line and also a given circle externally then prove that the locus of its centre is a parabola.(given line and given circle are non intersecting)

Section (B) : Elementary concepts of Ellipse & Hyperbola

B-1. Find the eccentricity of an ellipse of which distance between the focii is 10 and that of focus and corresponding directrix is 15.

B-2 If focus and corresponding directrix of an ellipse are (3, 4) and $x + y - 1 = 0$ respectively and eccentricity is $\frac{1}{2}$ then find the co-ordinates of extremities of major axis.

B-3 Find the set of those value(s) of 'α' for which the point $\left(7 - \frac{5}{4}\alpha, \alpha\right)$ lies inside the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$.

B-4. Write the parametric equation of ellipse $\frac{(x-3)^2}{25} + \frac{(y+2)^2}{16} = 1$.

B-5. Find the set of possible value of α for which point $P(\alpha, 3\alpha)$ lies on the smaller region of the ellipse $9x^2 + 16y^2 = 144$ divided by the line $3x + 4y = 12$.

B-6. Find the equation of the ellipse having its centre at the point (2, –3), one focus at (3, –3) and one vertex at (4, –3).

B-7.✍ Find the equation of the ellipse whose foci are (2, 3), (–2, 3) and whose semi-minor axis is $\sqrt{5}$.

B-8. Find
(i) The centre, eccentricity, foci and directrices of the hyperbola $16x^2 - 9y^2 + 32x + 36y - 164 = 0$.
(ii) The equation of the hyperbola whose directrix is $2x + y = 1$, focus $(1, 2)$ and eccentricity $\sqrt{3}$.

B-9. For the hyperbola $x^2/100 - y^2/25 = 1$, prove that
(i) eccentricity = $\sqrt{5}/2$
(ii) $SA \cdot S'A = 25$, where S & S′ are the foci & A is the vertex .

B-10. The foci of a hyperbola coincide with the foci of the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$. Find the equation of the hyperbola if its eccentricity is 2.

B-11. Find
(i) The foci of the hyperbola $9x^2 - 16y^2 + 18x + 32y - 151 = 0$
(ii) Equation of the hyperbola if vertex and focus of hyperbola are (2, 3) and (6, 3) respectively and eccentricity e of the hyperbola is 2

B-12. Find the position of the point (2, 5) relative to the hyperbola $9x^2 - y^2 = 1$.

B-13. Find the equation of auxiliary circle, of conic which passes through (1, 1) & is having foci (4, 5) & (2, 3).

B-14. Find the eccentricity of the hyperbola with its principal axes along the co-ordinate axes and which passes through (3, 0) & $\left(3\sqrt{2}, 2\right)$.

B-15. If m is a variable, then prove that the locus of the point of intersection of the lines $\frac{x}{3} - \frac{y}{2} = m$ and $\frac{x}{3} + \frac{y}{2} = \frac{1}{m}$ is a hyperbola.

B-16. Given the base of a triangle and the ratio of the tangent of half the base angles. Show that the vertex moves on a hyperbola whose foci are the extremities of the base.

B-17. Show that for rectangular hyperbola $xy = c^2$, length of transverse axis, length of conjugate axis and length of latus rectum are equal to $2\sqrt{2}\,c$

B-18. Prove that the distance of the point $(\sqrt{6}\cos\theta, \sqrt{2}\sin\theta)$ on the ellipse $x^2/6 + y^2/2 = 1$ from the centre of the ellipse is 2, if $\theta = 5\pi/4$

B-19. Find the eccentricity of the ellipse which meets the straight line $2x - 3y = 6$ on the x-axis and the straight line $4x + 5y = 20$ on the y-axis and whose axes lie along the coordinates axes.

B-20. If the foci of the ellipse $\frac{x^2}{25} + \frac{y^2}{b^2} = 1$ & the hyperbola $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$ coincide then find the value of b^2.

Section (C) : Position of line, Equation of chord and various forms of tangents of parabola

C-1. A line $y = x + 5$ intersect the parabola $(y - 3)^2 = 8(x + 2)$ at A & B. Find the length of chord AB.

C-2. Chord joining two distinct points $P(\alpha^2, k_1)$ and $Q\left(k_2, -\frac{16}{\alpha}\right)$ on the parabola $y^2 = 16x$ always passes through a fixed point. Find the co-ordinate of fixed point.

C-3. Find the locus of the mid-points of the chords of the parabola $y^2 = 4ax$ which subtend a right angle at the vertex of the parabola.

C-4. Two perpendicular chords are drawn from the origin 'O' to the parabola $y = x^2$, which meet the parabola at P and Q Rectangle POQR is completed. Find the locus of vertex R.

C-5. Prove that the straight line □x + my + n = 0 touches the parabola $y^2 = 4ax$ if □n = am^2.

C-6. Find the range of c for which the line $y = mx + c$ touches the parabola $y^2 = 8 (x + 2)$.

C-7. Find the equation of that tangent to the parabola $y^2 = 7x$ which is parallel to the straight line $4y – x + 3 = 0$. Find also its point of contact.

C-8. A parabola $y = ax^2 + bx + c$ crosses the x – axis at $(\alpha, 0)$ $(\beta, 0)$ both to the right of the origin. A circle also passes through these two points. Find the length of a tangent from the origin to the circle.

C-9. If tangent at P and Q to the parabola $y^2 = 4ax$ intersect at R then prove that mid point of R and M lies on the parabola, where M is the mid point of P and Q.

Section (D) : Position of line, Equation of chord and various forms of tangents of Ellipse & Hyperbola

D-1. Find the length of chord $x – 2y – 2 = 0$ of the ellipse $4x^2 + 16y^2 = 64$.

D-2. Find the locus of the middle points of chords of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ which are drawn through the positive end of the minor axis.

D-3. Check whether the line $4x + 5y = 40$ touches the ellipse $\frac{x^2}{50} + \frac{y^2}{32} = 1$ or not. If yes, then also find its point of contact.

D-4. An ellipse passes through the point (4, – 1) and touches the line x + 4 y – 10 = 0. Find its equation if its axes coincide with co–ordinate axes.

D-5. Find the equation of the tangents at the ends of the latus rectum of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and also show that they pass through the points of intersection of the major axis and directrices.

D-6. Any tangent to an ellipse is cut by the tangents at the ends of major axis in the points T and T ′. Prove that the circle, whose diameter is T T ′ will pass through the focii of the ellipse.

D-7. If 'P' be a moving point on the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ in such a way that tangent at 'P' intersect $x = \frac{25}{3}$ at Q then circle on PQ as diameter passes through a fixed point. Find that fixed point.

D-8. AB is a chord to the curve $S \equiv \frac{x^2}{9} + \frac{y^2}{16} - 1 = 0$ with A (3, 0) and C is a point on line AB such that AC : AB = 2 : 1 then find the locus of C.

D-9. Find the length of chord $x – 3y – 3 = 0$ of hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$.

D-10. For what value of λ, does the line $y = 3x + \lambda$ touch the hyperbola $9x^2 – 5y^2 = 45$?

D-11. If the straight line $2x + \sqrt{2}\,y + n = 0$ touches the hyperbola $\frac{x^2}{9} - \frac{y^2}{16} = 1$, then find the value of n.

D-12. Find the equation of the tangent to the hyperbola $x^2 – 4y^2 = 36$ which is perpendicular to the line $x – y + 4 = 0$.

D-13. AB is a chord to the curve $S \equiv x^2 - y^2 - 16 = 0$ with A (4, 0) and C is a point on line segment AB such that AC : AB = 1 : 2 then find the locus of C.

D-14. The curve $xy = c(c > 0)$ and the circle $x^2 + y^2 = 25$ touch at two points, then find the distance between the points of contact.

D-15. If the tangent on the point ($3 \sec \phi$, $4 \tan \phi$) (which is in first quadrant) of the hyperbola $\frac{x^2}{9} - \frac{y^2}{16} = 1$ is perpendicular to $3x + 8y - 12 = 0$, then find the value of ϕ is (in degree).

Section (E) : Pair of tangents, Director circle, chord of contact and chord with given middle point of Parabola

E-1. Find the equation of tangents to the parabola $y^2 = 9x$, which pass through the point (4, 10).

E-2. If two tangents to the parabola $y^2 = 4ax$ from a point P make angles θ_1 and θ_2 with the axis of the parabola, then find the locus of P in each of the following cases.

(i) $\theta_1 + \theta_2 = \alpha$ (a constant)

(ii) $\theta_1 + \theta_2 = \frac{\pi}{2}$

(iii) $\tan \theta_1 + \tan\theta_2 = \lambda$ (is constant)

E-3. The equation of a tangent to the parabola $y^2 = 8x$ is $y = x + 2$. Find the point on this line from which the other tangents to the parabola is perpendicular to the given tangent.

E-4. From the point (α, β) two perpendicular tangents are drawn to the parabola $(x - 7)^2 = 8y$. Then find the value of β.

E-5. Find the locus of the middle point of the focal chord of the parabola $y^2 = 4x$.

Section (F): Pair of tangents, Director circle, chord of contact and chord with given middle point of Ellipse & Hyperbola

F-1. Find the equation of tangents to the ellipse $\frac{x^2}{50} + \frac{y^2}{32} = 1$ which passes through a point (15, – 4).

F-2. If $3x + 4y = 12$ intersect the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ at P and Q, then find the point of intersection of tangents at P and Q.

F-3. Find the equation of chord of ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ whose mid point is (3, 1).

F-4. If m_1 & m_2 are the slopes of the tangents to the hyperbola $x^2/25 - y^2/16 = 1$ which passes through the point (4, 2), find the value of (i) $m_1 + m_2$ & (ii) $m_1 m_2$.

F-5. Find the equations of the tangents to the hyperbola $x^2 - 9y^2 = 9$ that are drawn from (3, 2) . Find the area of the triangle that these tangents form with their chord of contact.

F-6. Find the locus of the mid points of the chords of the circle $x^2 + y^2 = 16$, which are tangent to the hyperbola $9x^2 - 16y^2 = 144$.

F-7. Chords of the hyperbola, $x^2 - y^2 = a^2$ touch the parabola, $y^2 = 4ax$. Prove that the locus of their middle points is the curve, $y^2 (x - a) = x^3$.

F-8. Find the condition so that the line px + qy = r intersects the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}$ = 1 in points whose eccentric angles differ by $\frac{\pi}{4}$.

Section (G) : Equation of normal, co-normal points of parabola

G-1. Find equation of all possible normals to the parabola $x^2 = 4y$ drawn from point (1, 2).

G-2. If ax + by = 1 is a normal to the parabola $y^2 = 4Px$, then prove that $Pa^3 + 2aPb^2 = b^2$.

G-3. Find the equation of normal to the parabola $x^2 = 4y$ at (6, 9).

G-4. The normal at the point $P(ap^2, 2ap)$ meets the parabola $y^2 = 4ax$ again at $Q(aq^2, 2aq)$ such that the lines joining the origin to P and Q are at right angle. Then prove that $p^2 = 2$.

G-5. If a line x + y = 1 cut the parabola $y^2 = 4ax$ in points A and B and normals drawn at A and B meet at C (C does not lies on parabola). The normal to the parabola from C other, than above two meet the parabola in D, then find D

G-6. If normal of circle $x^2 + y^2 + 6x + 8y + 9 = 0$ intersect the parabola $y^2 = 4x$ at P and Q then find the locus of point of intersection of tangent's at P and Q.

Section (H) : Equation of normal, co-normal points of Ellipse & Hyperbola

H-1. If the normal at an end of a latus-rectum of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2}$ = 1 passes through one extremity of the minor axis, show that the eccentricity of the ellipse is given by $e^4 + e^2 - 1 = 0$

H-2. A ray emanating from the point (– 4, 0) is incident on the ellipse $9x^2 + 25y^2 = 225$ at the point P with abscissa 3. Find the equation of the reflected ray after first reflection.

H-3. The tangent & normal at a point on $x^2/a^2 - y^2/b^2 = 1$ cut the y – axis respectively at A & B. Prove that the circle on AB as diameter passes through the focii of the hyperbola .

H-4. The normal at P to a hyperbola of eccentricity e, intersects its transverse and conjugate axes at L and M respectively. Show that the locus of the middle point of LM is a hyperbola of eccentricity $\frac{e}{\sqrt{(e^2-1)}}$.

Section (I) : Miscelleneous problems

I-1. Find the equation of a circle touching the parabola $y^2 = 8x$ at (2, 4) and passes through (0, 4).

I-2. An ellipse and a hyperbola have the same centre origin, the same foci and the minor-axis of the one is the same as the conjugate axis of the other. If e_1, e_2 be their eccentricities respectively, then find value of $\frac{1}{e_1^2}+\frac{1}{e_2^2}$.

I-3. x – 2y + 4 = 0 is a common tangent to $y^2 = 4x$ & $\frac{x^2}{4}+\frac{y^2}{b^2}$ = 1. Then find the value of 'b' and the other common tangent.

I-4. The line y = x intersects the hyperbola $\frac{x^2}{9}-\frac{y^2}{25}$ = 1 at the points P and Q. Then find eccentricity of ellipse with PQ as major axis and miror axis of length $\frac{5}{\sqrt{2}}$.

I-5. Find the equation of common tangent to circle $x^2 + y^2 = 5$ and ellipse $x^2 + 9y^2 = 9$.

I-6. If latus rectum of ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ is double ordinate of parabola $y^2 = 4ax$, then find the value of a.

PART - II : ONLY ONE OPTION CORRECT TYPE

Section (A) : Elementary concepts of Parabola

A-1. The equation of the parabola whose focus is $(-3, 0)$ and the directrix is $x + 5 = 0$ is:
(A) $y^2 = 4(x-4)$ (B) $y^2 = 2(x+4)$ (C) $y^2 = 4(x-3)$ (D) $y^2 = 4(x+4)$

A-2. If (2, 0) is the vertex & y – axis is the directrix of a parabola, then its focus is:
(A) (2, 0) (B) (–2, 0) (C) (4, 0) (D) (–4, 0)

A-3. Length of the latus rectum of the parabola $25[(x-2)^2 + (y-3)^2] = (3x - 4y + 7)^2$ is:
(A) 4 (B) 2 (C) 1/5 (D) 2/5

A-4. A parabola is drawn with its focus at (3, 4) and vertex at the focus of the parabola $y^2 - 12x - 4y + 4 = 0$. The equation of the parabola is:
(A) $x^2 - 6x - 8y + 25 = 0$ (B) $y^2 - 8x - 6y + 25 = 0$
(C) $x^2 - 6x + 8y - 25 = 0$ (D) $x^2 + 6x - 8y - 25 = 0$

A-5. Which one of the following equations parametrically represents equation to a parabolic profile?
(A) $x = 3\cos t;\ y = 4\sin t$ (B) $x^2 - 2 = -2\cos t;\ y = 4\cos^2\frac{t}{2}$
(C) $\sqrt{x} = \tan t;\ \sqrt{y} = \sec t$ (D) $x = \sqrt{1-\sin t}\ ;\ y = \sin\frac{t}{2} + \cos\frac{t}{2}$

A-6. The points on the parabola $y^2 = 12x$ whose focal distance is 4, are
(A) $(2, \sqrt{3}), (2, -\sqrt{3})$ (B) $(1, 2\sqrt{3}), (1, -2\sqrt{3})$ (C) (1, 2), (2, 1) (D) $(2, 2\sqrt{3}), (3, -2\sqrt{3})$

A-7. Find the all possible values of α such that point $P(\alpha, \alpha)$ is outside the parabola $y = x^2 + x + 1$ and inside the circle $x^2 + y^2 = 50$.
(A) $(-5, \infty)$ (B) $(-\infty, \infty)$ (C) $(-1, 5)$ (D) $(-5, 5)$

A-8. If on a given base, a triangle be described such that the sum of the tangents of the base angles is a constant, then the locus of the vertex is :
(A) a circle (B) a parabola (C) an ellipse (D) a hyperbola

A-9. **Statement-1 :** For triangle whose two vertices are ends of a double ordinate for a parabola and third vertex lies on axis of same parabola incentre, circumcentre, centroid are collinear.
Statement-2 : In isosceles triangle incentre, circumcentre; orthocentre, centroid all lie on same line.
(A) STATEMENT-1 is true, STATEMENT-2 is true and STATEMENT-2 is correct explanation for STATEMENT-1
(B) STATEMENT-1 is true, STATEMENT-2 is true and STATEMENT-2 is not correct explanation for STATEMENT-1
(C) STATEMENT-1 is true, STATEMENT-2 is false
(D) STATEMENT-1 is false, STATEMENT-2 is true
(E) Both STATEMENTS are false

A-10. The length of the latus rectum of the parabola whose focus is $\left(\frac{u^2}{2g}\sin 2\alpha, -\frac{u^2}{2g}\cos 2\alpha\right)$ and directrix is $y = \frac{u^2}{2g}$, is
(A) $\frac{u^2}{g}\cos^2\alpha$ (B) $\frac{u^2}{g}\cos 2\alpha$ (C) $\frac{2u^2}{g}\cos 2\alpha$ (D) $\frac{2u^2}{g}\cos^2\alpha$

A-11. The distance between the focus and directrix of the conic $\left(\sqrt{3}x - y\right)^2 = 48\left(x + \sqrt{3}y\right)$ is :
(A) 24 (B) 48 (C) 6 (D) 12

A-12. If one end of a focal chord of the parabola $y^2 = 4x$ is (1, 2), the other end doesn't lie on
(A) $x^2 y + 2 = 0$ (B) $xy + 2 = 0$ (C) $xy - 2 = 0$ (D) $x^2 + xy - y - 1 = 0$

A-13. The angle made by a double ordinate of length 8a at the vertex of the parabola $y^2 = 4ax$ is :
(A) $\pi/3$ (B) $\pi/2$ (C) $\pi/4$ (D) $\pi/6$

Section (B) : Elementary concepts of Ellipse & Hyperbola

B-1. The equation of the ellipse whose focus is (1, –1), directrix is the line x – y – 3 = 0 and the eccentricity is $\frac{1}{2}$, is
(A) $7x^2 + 2xy + 7y^2 - 10x + 10y + 7 = 0$ (B) $7x^2 + 2xy + 7y^2 + 7 = 0$
(C) $7x^2 + 2xy + 7y^2 + 10x - 10y - 7 = 0$ (D) $7x^2 + 4xy + 7y^2 - 10x + 10y + 7 = 0$

B-2. The eccentricity of the ellipse $4x^2 + 9y^2 + 8x + 36y + 4 = 0$ is
(A) $\frac{5}{6}$ (B) $\frac{3}{5}$ (C) $\frac{\sqrt{2}}{3}$ (D) $\frac{\sqrt{5}}{3}$

B-3. The equation $\frac{x^2}{2-r} + \frac{y^2}{r-5} + 1 = 0$ represents an ellipse, if
(A) r > 2 (B) 2< r < 5 (C) r > 5 (D) $r \in (2, 5) - \{3.5\}$

B-4. The length of the latus rectum of the ellipse $9x^2 + 4y^2 = 1$, is
(A) $\frac{3}{2}$ (B) $\frac{8}{3}$ (C) $\frac{4}{9}$ (D) $\frac{8}{9}$

B-5. The equation of the ellipse with its centre at (1, 2), focus at (6, 2) and passing through the point (4, 6) is
(A) $\frac{(x-1)^2}{45} + \frac{(y-2)^2}{20} = 1$ (B) $\frac{(x-1)^2}{20} + \frac{(y-2)^2}{45} = 1$
(C) $\frac{(x-1)^2}{25} + \frac{(y-2)^2}{16} = 1$ (D) $\frac{(x-1)^2}{16} + \frac{(y-2)^2}{25} = 1$

B-6. The position of the point (1, 3) with respect to the ellipse $4x^2 + 9y^2 - 16x - 54y + 61 = 0$
(A) outside the ellipse (B) on the ellipse
(C) on the major axis (D) on the minor axis

B-7. With respect to the hyperbola $(3x - 3y)^2 - (2x + 2y)^2 = 36$
(A) (3,2) lies on conjugate axis (B) (3,2) lies on tranverse axis
(C) (3,2) lies inside hyperbola (D) (3,2) lies outside hyperbola

B-8. Equation of auxilliary circle of the ellipse $2x^2 + 6xy + 5y^2 = 1$ is
(A) $(x - 1)^2 + y^2 = 7 - 3\sqrt{5}$ (B) $x^2 + y^2 = \frac{7+3\sqrt{5}}{2}$
(C) $x^2 + y^2 = \frac{2}{7+3\sqrt{5}}$ (D) $(x - 1)^2 + y^2 = \frac{4}{7+3\sqrt{5}}$

B-9. **Statement-1 :** Eccentricity of ellipse whose length of latus rectum is same as distance between foci is 2sin18°.

Statement-2 : For $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, eccentricity $e = \sqrt{1 - \frac{b^2}{a^2}}$ `

(A) STATEMENT-1 is true, STATEMENT-2 is true and STATEMENT-2 is correct explanation for STATEMENT-1
(B) STATEMENT-1 is true, STATEMENT-2 is true and STATEMENT-2 is not correct explanation for STATEMENT-1
(C) STATEMENT-1 is true, STATEMENT-2 is false
(D) STATEMENT-1 is false, STATEMENT-2 is true
(E) Both STATEMENTS are false

B-10. The curve represented by x = 3 (cos t + sin t), y = 4 (cos t – sin t), is
(A) ellipse (B) parabola (C) hyperbola (D) circle

B-11. The eccentricity of the conic represented by $x^2 - y^2 - 4x + 4y + 16 = 0$ is
(A) 1 (B) $\sqrt{2}$ (C) 2 (D) 1/2

B-12. Which of the following pair, may represent the eccentricities of two conjugate hyperbolas, for all $\alpha \in (0, \pi/2)$?
(A) $\sin\alpha, \cos\alpha$ (B) $\tan\alpha, \cot\alpha$ (C) $\sec\alpha, \operatorname{cosec}\alpha$ (D) $1 + \sin\alpha, 1 + \cos\alpha$

B-13. For hyperbola represented by $16x^2 - 3y^2 - 32x + 12y - 44 = 0$, which of the following statement is INCORRECT
(A) the length of whose transverse axis is $4\sqrt{3}$ (B) the length of whose conjugate axis is 8
(C) whose centre is (1, 2) (D) whose eccentricity is $\sqrt{\frac{19}{3}}$

B-14. **Statement-1 :** If $\sec\theta$, $\theta \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ represent eccentricity of a hyperbola then eccentricity of its conjugate hyperbola is given by $\operatorname{cosec}\theta$.
Statement-2 : If e_1, e_2 are eccentricities of two hyperbolas which are conjugate to each other then $e_1^{-2} + e_2^{-2} = 1$
(A) STATEMENT-1 is true, STATEMENT-2 is true and STATEMENT-2 is correct explanation for STATEMENT-1
(B) STATEMENT-1 is true, STATEMENT-2 is true and STATEMENT-2 is not correct explanation for STATEMENT-1
(C) STATEMENT-1 is true, STATEMENT-2 is false
(D) STATEMENT-1 is false, STATEMENT-2 is true
(E) Both STATEMENTS are false

B-15. The eccentricity of the hyperbola whose conjugate axis is equal to half the distance between the foci, is:
(A) $\frac{4}{3}$ (B) $\frac{4}{\sqrt{3}}$ (C) $\frac{2}{\sqrt{3}}$ (D) $\frac{5}{\sqrt{3}}$

B-16. Identify the following statements for true/false (T/F) in order
S1 : A latus rectum of an ellipse is a line passing through a focus
S2 : A latus rectum of an ellipse is a line through the centre
S3 : A latus rectum of an ellipse is a line perpendicular to the major axis
S4 : A latus rectum of an ellipse is a line parallel to the minor axis
(A) TFTF (B) TTFF (C) TFTT (D) FFFF

B-17. If P ($\sqrt{2}\sec\theta$, $\sqrt{2}\tan\theta$) is a point on the hyperbola whose distance from the origin is $\sqrt{6}$ where P is in the first quadrant then θ =
(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{15}$

B-18. A rectangular hyperbola circumscribe a triangle ABC, then it will always pass through its
(A) orthocenter (B) circum centre (C) centroid (D) incentre

B-19. If $P(x_1, y_1)$, $Q(x_2, y_2)$, $R(x_3, y_3)$ and $S(x_4, y_4)$ are four concyclic points on the rectangular hyperbola $xy = c^2$, the coordinates of orthocentre of the ΔPQR are
(A) (x_4, y_4) (B) $(x_4, -y_4)$ (C) $(-x_4, -x_4)$ (D) $(-x_4, -y_4)$

B-20. The co-ordinates of a focus of the hyperbola $9x^2 - 16y^2 + 18x + 32y - 151 = 0$ is
(A) $(-1, 1)$ (B) $(6, 1)$ (C) $(4, 1)$ (D) $(-6, -1)$

B-21. The set of values of 'a' for which $(13x - 1)^2 + (13y - 2)^2 = a(5x + 12y - 1)^2$ represents an ellipse is
(A) $1 < a < 2$ (B) $0 < a < 1$ (C) $2 < a < 3$ (D) $3 < a < 4$

B-22. Find the equation of latus rectum of rectangular hyperbola $xy = c^2$
(A) $x - y \pm 2\sqrt{2}c = 0$ (B) $x - y \pm \sqrt{2}c = 0$ (C) $x + y \pm 2\sqrt{2}c = 0$ (D) $x + y \pm \sqrt{2}c = 0$

Section (C) : Position of line, Equation of chord and various forms of tangents of parabola

C-1. The locus of point of trisections of the focal chords of the parabola, $y^2 = 4x$ is:
(A) $y^2 = x - 1$ (B) $9y^2 = 4.(3x - 4)$ (C) $y^2 = 2(1 - x)$ (D) None of these

C-2. The latus rectum of a parabola whose focal chord is PSQ such that SP = 3 and SQ = 2 is given by:
(A) 24/5 (B) 12/5 (C) 6/5 (D) 23/5

C-3. Identify following statements for true/false (T/F) in order
S1 : The circles on focal radii of a parabola as diameter touch the tangent at the vertex
S2 : The circles on focal radii of a parabola as diameter touch the axis
S3 : A circle described on any focal chord of the parabola as its diameter will touch the directrix of the parabola
S4 : A circle described on any focal chord of the parabola as its diameter will touch the axis of the parabola
(A) TTFF (B) TFTF (C) FFTT (D) FTFT

C-4. The length of the chord $y = \sqrt{3}x - 2\sqrt{3}$ intercepted by the parabola $y^2 = 4(x - 1)$ is
(A) $4\sqrt{3}$ (B) $\frac{16}{3}$ (C) $\frac{8}{3}$ (D) $\frac{4}{\sqrt{3}}$

C-5. If $y = 2x - 3$ is a tangent to the parabola $y^2 = 4a\left(x - \frac{1}{3}\right)$, then 'a' is equal to, where $a \neq 0$:
(A) 1 (B) -1 (C) $\frac{14}{3}$ (D) $\frac{-14}{3}$

C-6. An equation of a tangent common to the parabolas $y^2 = 4x$ and $x^2 = 4y$ is
(A) $x - y + 1 = 0$ (B) $x + y - 1 = 0$ (C) $x + y + 1 = 0$ (D) $y = 0$

C-7. Equation of a tangent to the parabola $y^2 = 12x$ which make an angle of 45° with line $y = 3x + 77$ is
(A) $2x - 4y + 3 = 0$ (B) $x - 2y + 12 = 0$ (C) $4x + 2y + 5 = 0$ (D) $2x + y - 12 = 0$

C-8. Identify the following statements for true/false (T/F) in order
S1 : The tangents at the extremities of a focal chord of a parabola are perpendicular
S2 : The tangents at the extremities of a focal chord of a parabola are parallel
S3 : The tangents at the extremities of a focal chord of a parabola intersect on the directrix
S4 : The tangents at the extremities of a focal chord of a parabola intersect at the vertex
(A) TFTF (B) TTFF (C) TTTT (D) FFFF

Section (D) : Position of line, Equation of chord and various forms of tangents of Ellipse & Hyperbola

D-1. If the line y = 2x + c be a tangent to the ellipse $\frac{x^2}{8}+\frac{y^2}{4}$ = 1, then c is equal to

(A) ± 4 (B) ± 6 (C) ± 1 (D) ± 8

D-2. The distance of the point of contact from the origin of the line $y = x - \sqrt{7}$ with the ellipse $3x^2 + 4y^2 = 12$, is

(A) $\sqrt{3}$ (B) 2 (C) $\frac{5}{\sqrt{7}}$ (D) $\frac{5}{7}$

D-3. If $\frac{x}{a}+\frac{y}{b}=\sqrt{2}$ touches the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}$ = 1 at a point P, then eccentric angle of P is

(A) 0 (B) 45° (C) 60° (D) 90°

D-4. The point of intersection of the tangents at the point P on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}$ = 1 and its corresponding point Q on the auxiliary circle, lies on the line :

(A) x = a/e (B) x = 0 (C) y = 0 (D) $y = \frac{b}{e}$

D-5. A chord is drawn to the hyperbola xy = 4 from a point A(2, 2) which cuts it again at point B. The locus of point P such that AP : PB = 2 :1

(A) (3x – 2) (3y – 2) = 16 (B) (2x – 3)(2y – 3) = 16
(C) xy = 2 (D) (3x – 2)(2y – 3) = 16

D-6. The number of possible tangents which can be drawn to the curve $4x^2 - 9y^2 = 36$, which are perpendicular to the straight line 5x + 2y –10 = 0 is :

(A) zero (B) 1 (C) 2 (D) 4

D-7. The tangent at any point $P(x_1, y_1)$ on the hyperbola $xy = c^2$ meets the co-ordinate axes at points Q & R. The circumcentre of ΔOQR has co-ordinates.

(A) (0, 0) (B) (x_1, y_1) (C) $\left(\frac{x_1}{2},\frac{y_1}{2}\right)$ (D) $\left(\frac{2x_1}{3},\frac{2y_1}{3}\right)$

D-8. The equation of the tangent lines to the hyperbola $x^2 - 2y^2 = 18$ which are perpendicular to the line y = x are :

(A) y = – x ± 7 (B) y = – x ± 3 (C) y = – x ± 4 (D) none of these

Section (E) : Pair of tangents, Director circle, chord of contact and chord with given middle point of Parabola

E-1. The angle between the tangents drawn from a point (– a, 2a) to $y^2 = 4ax$ is

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{6}$

E-2. The line 4x – 7y + 10 = 0 intersects the parabola, $y^2 = 4x$ at the points A & B. The co-ordinates of the point of intersection of the tangents drawn at the points A & B are:

(A) $\left(\frac{7}{2},\frac{5}{2}\right)$ (B) $\left(-\frac{5}{2},-\frac{7}{2}\right)$ (C) $\left(\frac{5}{2},\frac{7}{2}\right)$ (D) $\left(-\frac{7}{2},-\frac{5}{2}\right)$

E-3. The locus of the middle points of the focal chords of the parabola, $y^2 = 4x$ is:

(A) $y^2 = x - 1$ (B) $y^2 = 2(x - 1)$ (C) $y^2 = 2(1 - x)$ (D) $y^2 = 2(x + 1)$

Section (F) : Pair of tangents, Director circle, chord of contact and chord with given middle point of Ellipse & Hyperbola

F-1. The equation of the locus of the middle point of the portion of the tangent to the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ included between the co-ordinate axes is the curve:
(A) $9x^2 + 16y^2 = 4x^2y^2$ (B) $16x^2 + 9y^2 = 4x^2y^2$
(C) $3x^2 + 4y^2 = 4x^2y^2$ (D) $9x^2 + 16y^2 = x^2y^2$

F-2. The equation of the chord of the ellipse $2x^2 + 5y^2 = 20$ which is bisected at the point (2, 1) is
(A) $4x + 5y + 13 = 0$ (B) $4x + 5y = 13$ (C) $5x + 4y + 13 = 0$ (D) $5x + 4y = 13$

F-3 Point, from which tangents to the ellipse $5x^2 + 4y^2 = 20$ are not perpendicular, is:
(A) $(1, 2\sqrt{2})$ (B) $(2\sqrt{2}, 1)$ (C) $(2, \sqrt{5})$ (D) $(\sqrt{5}, 3)$

F-4. The locus of the middle points of chords of hyperbola $3x^2 - 2y^2 + 4x - 6y = 0$ parallel to $y = 2x$ is
(A) $3x - 4y = 4$ (B) $3y - 4x + 4 = 0$ (C) $4x - 4y = 3$ (D) $3x - 4y = 2$

F-5. The chords passing through L(2, 1) intersects the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$ at P and Q. If the tangents at P and Q intersects at R then Locus of R is
(A) $x - y = 1$ (B) $9x - 8y = 72$ (C) $x + y = 3$ (D) $9x + 8y = 7$

F-6. The number of points from where a pair of perpendicular tangents can be drawn to the hyperbola, $x^2 \sec^2\alpha - y^2 \operatorname{cosec}^2\alpha = 1$, $\alpha \in (0, \pi/4)$, is :
(A) 0 (B) 1 (C) 2 (D) infinite

F-7. Locus of the middle points of the parallel chords with gradient m of the rectangular hyperbola $xy = c^2$ is:
(A) $y + mx = 0$ (B) $y - mx = 0$ (C) $my - x = 0$ (D) $my + x = 0$

F-8. The tangents from $(1, 2\sqrt{2})$ to the hyperbola $16x^2 - 25y^2 = 400$ include between them an angle equal to:
(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

F-9. The locus of the mid points of the chords passing through a fixed point (α, β) of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is :
(A) a circle with centre $\left(\frac{\alpha}{2}, \frac{\beta}{2}\right)$ (B) an ellipse with centre $\left(\frac{\alpha}{2}, \frac{\beta}{2}\right)$
(C) a hyperbola with centre $\left(\frac{\alpha}{2}, \frac{\beta}{2}\right)$ (D) straight line passing through

Section (G) : Equation of normal, co-normal points of parabola

G-1. The subtangent, ordinate and subnormal to the parabola $y^2 = 4ax$ at a point (different from the origin) are in
(A) AP (B) GP (C) HP (D) none of these

G-2. Equation of the normal to the parabola, $y^2 = 4ax$ at its point $(am^2, 2am)$ is:
(A) $y = -mx + 2am + am^3$ (B) $y = mx - 2am - am^3$
(C) $y = mx + 2am + am^3$ (D) none

G-3. At what point on the parabola $y^2 = 4x$ the normal makes equal angles with the axes?
(A) (4, 4) (B) (9, 6) (C) (4, – 1) (D) (1, 2)

G-4. The line $2x + y + \lambda = 0$ is a normal to the parabola $y^2 = -8x$, then λ is
(A) 12 (B) – 12 (C) 24 (D) – 24

G-5. The equation of the other normal to the parabola $y^2 = 4ax$ which passes through the intersection of those at $(4a, -4a)$ & $(9a, -6a)$ is:
(A) $5x - y + 115a = 0$ (B) $5x + y - 135a = 0$ (C) $5x - y - 115a = 0$ (D) $5x + y + 115 = 0$

G-6. The normal chord of a parabola $y^2 = 4ax$ at the point $P(x_1, x_1)$ does not subtends a right angle at the
(A) focus (B) point (12a, 0)
(C) one of the end of the latus rectum (D) (a, 2a)

G-7. If three normals can be drawn to the curve $y^2 = x$ from point (c, 0) then 'c' can be equal to
(A) 0 (B) $-\frac{1}{2}$ (C) $\frac{1}{2}$ (D) 2

G-8. The locus of the middle points of normal chords of the parabola $y^2 = 4ax$ is
(A) $y^4 - 2a(x - 2a).\ y^2 + 8a^4 = 0$ (B) $y^4 + 2a(x - 2a).\ y^2 + 8a^4 = 0$
(C) $y^4 - 2a(x + 2a).\ y^2 + 8a^4 = 0$ (D) $y^4 - 2a(x - 2a).\ y^2 - 8a^4 = 0$

Section (H) : Equation of normal, co-normal points of Ellipse & Hyperbola

H-1. If the line $x\cos\alpha + y\sin\alpha = p$ be normal to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, then
(A) $p^2(a^2\cos^2\alpha + b^2\sin^2\alpha) = a^2 - b^2$ (B) $p^2(a^2\cos^2\alpha + b^2\sin^2\alpha) = (a^2 - b^2)^2$
(C) $p^2(a^2\sec^2\alpha + b^2\text{cosec}^2\alpha) = a^2 - b^2$ (D) $p^2(a^2\sec^2\alpha + b^2\text{cosec}^2\alpha) = (a^2 - b^2)^2$

H-2. If the normal at $\left(ct, \frac{c}{t}\right)$ on the curve $xy = c^2$ meets the curve again at t', then
(A) $t' = -\frac{1}{t^3}$ (B) $t' = \frac{1}{t}$ (C) $t' = \frac{1}{t^2}$ (D) $t'^2 = -\frac{1}{t^2}$

H-3. If the focal chord of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, (a > b) is normal at $(a\cos\theta, b\sin\theta)$ then eccentricity of the ellipse is (it is given that $\sin\theta \neq 0$)
(A) $|\sec\theta|$ (B) $|\cos\theta|$ (C) $|\sin\theta|$ (D) None of these

H-4. The locus of the foot of perpendicular drawn from the centre of the hyperbola $x^2 - y^2 = 25$ to its normal.
(A) $100x^2y^2 = (x^2 + y^2)^2(y^2 - x^2)$ (B) $10x^2y^2 = (x^2 + y^2)^2(y^2 - x^2)$
(C) $200x^2y^2 = (x^2 - y^2)^2(y^2 + x^2)$ (D) $100x^2y^2 = (x^2 - y^2)^2(y^2 + x^2)$

H-5. The value of $|\lambda|$, for which the line $2x - \frac{8}{3}\lambda y = -3$ is a normal to the conic $x^2 + \frac{y^2}{4} = 1$ is
(A) $\frac{\sqrt{3}}{2}$ (B) $\frac{1}{2}$ (C) $\frac{3}{4}$ (D) $\frac{3}{8}$

Section (I) : Miscelleneous problems

I-1. The feet of the perpendicular drawn from focus upon any tangent to the parabola, $y = x^2 - 2x - 3$ lies on
(A) $y + 4 = 0$ (B) $y = 0$ (C) $y = -2$ (D) $y + 1 = 0$

I-2. If F_1 & F_2 are the feet of the perpendiculars from the focii S_1 & S_2 of an ellipse $\frac{x^2}{5} + \frac{y^2}{3} = 1$ on the tangent at any point P on the ellipse, then $(S_1F_1).\ (S_2F_2)$ is equal to :
(A) 2 (B) 3 (C) 4 (D) 5

I-3. P & Q are corresponding points on the ellipse $\frac{x^2}{16}+\frac{y^2}{9}$ = 1, and the auxiliary circle respectively. The normal at P to the ellipse meets CQ in R where C is centre of the ellipse. Then ℓ(CR) is
(A) 5 units (B) 6 units (C) 7 units (D) 8 units

I-4. The equation of common tangent of $x^2 + y^2 = 2$ and $y^2 = 8x$ is
(A) $x - y + 2 = 0$ (B) $x + y + 1 = 0$ (C) $x - y + 1 = 0$ (D) $x + y - 2 = 0$

I-5. The equation of common normal to the circle $x^2 + y^2 - 12x + 16 = 0$ and parabola $y^2 = 4x$ is
(A) $y = 0$ (B) $2x - y = 12$ (C) $2x + y = 12$ (D) All of the above

I-6. Equation of common tangent to ellipse $5x^2 + 2y^2 = 10$, and hyperbola $11x^2 - 3y^2 = 33$ is
(A) $y = \pm 3x \pm \sqrt{21}$ (B) $y = \pm x \pm 3$
(C) $y = \pm 4x \pm \sqrt{37}$ (D) $3x \pm y = 12$

I-7. The equation of common tangent to the parabola $y^2 = 8x$ and hyperbola $3x^2 - y^2 = 3$ is
(A) $2x \pm y + 1 = 0$ (B) $2x \pm y - 1 = 0$
(C) $x \pm 2y + 1 = 0$ (D) $x \pm 2y - 1 = 0$

I-8. $x - 2y + 4 = 0$ is a common tangent to $y^2 = 4x$ & $\frac{x^2}{4}+\frac{y^2}{b^2}$ = 1. Then the value of 'b' and the other common tangent are given by :
(A) $b = \sqrt{3}$; $x + 2y + 4 = 0$ (B) $b = 3$; $x + 2y + 4 = 0$
(C) $b = \sqrt{3}$; $x + 2y - 4 = 0$ (D) $b = \sqrt{3}$; $x - 2y - 4 = 0$

PART - III : MATCH THE COLUMN

1. **Match the column**

Column – I	**Column – II**
(A) If the mid point of a chord of the ellipse $\frac{x^2}{16}+\frac{y^2}{25}$ = 1 is (0, 3), then length of the chord is $\frac{4k}{5}$, then k is	(p) 6
(B) Eccentric angle of one of the points on the ellipse $x^2 + 3y^2 = 6$ at a distance 2 units from the centre of the ellipse is $\frac{k\pi}{4}$, then k is	(q) 8
(C) If 'e' is eccentricity and ℓ is the length of latus rectum of the ellipse $9x^2 + 5y^2 - 30y = 0$, then $4(e + \ell)$ is	(r) 3
(D) Sum of distances of a point on the ellipse $\frac{x^2}{9}+\frac{y^2}{16}$ = 1 from the focii	(s) 16

2. AB is a chord of the parabola $y^2 = 4ax$ joining $A(at_1^2, 2at_1)$ and $B(at_2^2, 2at_2)$. Match the following

Column – I	**Column – II**
(A) AB is a normal chord	(p) $t_2 = -t_1 + 2$
(B) AB is a focal chord	(q) $t_2 = -\frac{4}{t_1}$
(C) AB subtends 90º at (0, 0)	(r) $t_2 = -\frac{1}{t_1}$
(D) AB is inclined at 45º to the axis of parabola	(s) $t_2 = -t_1 - \frac{2}{t_1}$

3. A tangent having slope $-\frac{4}{3}$ touches the ellipse $\frac{x^2}{18}+\frac{y^2}{32}$ = 1 at point P and intersects the major and minor axes at A & B respectively, O is the centre of the ellipse

Column - I		Column - II	
(A)	Distance between the parallel tangents having slopes $-\frac{4}{3}$, is	(p)	24
(B)	Area of ΔAOB is	(q)	7/24
(C)	If the tangent in first quadrant touches the ellipse at (h, k) then value of hk is	(r)	48/5
(D)	If equation of the tangent intersecting positive axes is □x + my = 1, then □ + m is equal to	(s)	12

4.

Column - I		Column - II	
(A)	Number of positive integral values of b for which tangent parallel to line y = x + 1 can be drawn to hyperbola $\frac{x^2}{5} - \frac{y^2}{b^2} = 1$ is	(p)	16
(B)	The equation of the hyperbola with vertices (3, 0) and (–3, 0) and semi-latusrectum 4, is given by is $4x^2 - 3y^2 = 4k$, then k =	(q)	2
(C)	The product of the lengths of the perpendiculars from the two focii on any tangent to the hyperbola $\frac{x^2}{25} - \frac{y^2}{3} = 1$ is $\sqrt{k}$, then k is	(r)	4
(D)	An equation of a tangent to the hyperbola, $16x^2 - 25y^2 - 96x + 100y - 356 = 0$ which makes an angle $\frac{\pi}{4}$ with the transverse axis is $y = x + \lambda$, $(\lambda > 0)$, then 2λ is	(s)	9

Exercise-2

Marked questions are recommended for Revision.

PART - I : ONLY ONE OPTION CORRECT TYPE

1. The vertex of a parabola is the point (a, b) and latus rectum is of length 1. If the axis of the parabola is along the positive direction of y-axis, then its equation is :

(A) $(x + a)^2 = \frac{1}{2}(2y - 2b)$
(B) $(x - a)^2 = \frac{1}{2}(2y - 2b)$
(C) $(x + a)^2 = \frac{1}{4}(2y - 2b)$
(D) $(x - a)^2 = \frac{1}{8}(2y - 2b)$

2. Length of the focal chord of the parabola $y^2 = 4ax$ at a distance p from the vertex is:

(A) $\frac{2a^2}{p}$ (B) $\frac{a^3}{p^2}$ (C) $\frac{4a^3}{p^2}$ (D) $\frac{p^2}{a}$

3. The triangle PQR of area 'A' is inscribed in the parabola $y^2 = 4ax$ such that the vertex P lies at the vertex of the parabola and the base QR is a focal chord. The modulus of the difference of the ordinates of the points Q and R is:

(A) $\frac{A}{2a}$ (B) $\frac{A}{a}$ (C) $\frac{2A}{a}$ (D) $\frac{4A}{a}$

4. AB is a chord of the parabola $y^2 = 4ax$ with vertex at A. BC is drawn perpendicular to AB meeting the axis at C. The projection of BC on the axis of the parabola is

(A) a (B) 2a (C) 4a (D) 8a

5. If P_1Q_1 and P_2Q_2 are two focal chords of the parabola $y^2 = 4ax$. then the chords P_1P_2 and Q_1Q_2 1intersect on

(A) tangent at the vertex of the parabola
(B) the directrix of the parabola
(C) at x = –2a
(D) y = 2a and x = –2a

6. The vertex of the locus of a point that divides a chord of slope 2 of the parabola $y^2 = 4x$ internally in the ratio 1 : 2 is

(A) $\left(\frac{1}{9}, \frac{2}{9}\right)$ (B) $\left(\frac{8}{9}, \frac{1}{9}\right)$ (C) $\left(\frac{2}{9}, \frac{8}{9}\right)$ (D) $\left(\frac{1}{9}, \frac{1}{9}\right)$

7. AB, AC are tangents to a parabola $y^2 = 4ax$. p_1, p_2 & p_3 are the lengths of the perpendiculars from A, B & C respectively on any tangent to the curve, then p_2, p_1, p_3 are in:
(A) A.P. (B) G.P. (C) H.P. (D) none of these

8. The mirror image of the parabola $y^2 = 4x$ in the tangent to the parabola at the point (1, 2) is
(A) $(x - 1)^2 = 4(y - 2)$ (B) $(x + 3)^2 = 4(y + 2)$
(C) $(x + 1)^2 = 4(y - 1)$ (D) $(x - 1)^2 = 4(y - 1)$

9. A normal chord of the parabola subtending a right angle at the vertex makes an acute angle θ with the x–axis, then θ =
(A) arc tan 2 (B) arc tan $\sqrt{2}$ (C) arc cot $\sqrt{2}$ (D) arc cot2

10. If two normals to a parabola $y^2 = 4ax$ intersect at right angles then the chord joining their feet passes through a fixed point whose co-ordinates are:
(A) (– 2a, 0) (B) (a, 0) (C) (2a, 0) (D) (–a, 0)

11. If a parabola whose length of latus rectum is 4a touches both the coordinate axes then the locus of its focus is
(A) $xy = a^2 (x^2 + y^2)$ (B) $x^2y^2 = a^2 (x^2 + y^2)$
(C) $x^2 - y^2 = a^2 (x^2 + y^2)$ (D) $x^2y^2 = a^2 (x^2 - y^2)$

12. T is a point on the tangent to a parabola $y^2 = 4ax$ at its point P. TL and TN are the perpendiculars on the focal radius SP and the directrix of the parabola respectively. Then:
(A) SL = 2 (TN) (B) 3 (SL) = 2 (TN) (C) SL = TN (D) 2 (SL) = 3 (TN)

13. In the parabola $y^2 = 4ax$, the locus of middle points of all chords of constant length c is
(A) $(4ax - y^2)(y^2 - 4a^2) = a^2c^2$ (B) $(4ax + y^2)(y^2 + 4a^2) = a^2c^2$
(C) $(4ax + y^2)(y^2 - 4a^2) = a^2c^2$ (D) $(4ax - y^2)(y^2 + 4a^2) = a^2c^2$

14. Through the vertex 'O' of the parabola $y^2 = 4ax$, variable chords OP and OQ are drawn at right angles. If the variable chord PQ intersects the axis of x at R, then the distance OR is equal to
(A) 2a (B) 3a (C) 4a (D) 8a

15. From the focus of the parabola, $y^2 = 8x$ as centre, a circle is described so that a common chord of the curves is equidistant from the vertex & focus of the parabola. The equation of the circle is
(A) $(x - 2)^2 + y^2 = 9$ (B) $(x - 2)^2 + y^2 = 3$ (C) $(x - 2)^2 + y^2 = 2$ (D) $(x - 2)^2 + y^2 = 1$

16. If from the vertex of a parabola $y^2 = 4ax$, a pair of chords be drawn at right angles to one another and with these chords as adjacent sides a rectangle be made, locus of the further angle of the rectangle is
(A) $y^2 = 8a (x - 8a)$. (B) $y^2 = 4a (x + 8a)$. (C) $y^2 = - 4a (x - 8a)$. (D) $y^2 = 4a (x - 8a)$.

17. A line of fixed length (a + b) moves so that its ends are always on two fixed perpendicular straight lines. The locus of the point which divided this line into portions of lengths a & b, is :
(A) an ellipse (B) an hyperbola (C) a circle (D) a straight line

18. Coordinates of the vertices B and C of a triangle ABC are (2, 0) and (8, 0) respectively. The vertex A is varying in such a way that 4 tan $\frac{B}{2}$. tan $\frac{C}{2}$ = 1. Then locus of A is
(A) $\frac{(x-5)^2}{25} + \frac{y^2}{16} = 1$ (B) $\frac{(x-5)^2}{16} + \frac{y^2}{25} = 1$
(C) $\frac{(x-5)^2}{25} + \frac{y^2}{9} = 1$ (D) $\frac{(x-5)^2}{9} + \frac{y^2}{25} = 1$

19. The locus of point of intersection of tangents to an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}$ = 1 at the points, the sum of whose eccentric angles is constant, is :

(A) a hyperbola (B) an ellipse (C) a circle (D) a straight line

20. An ellipse with major axis 4 and minor axis 2 touches both the coordinate axis, then Locus of its centre is

(A) $x^2 - y^2 = 5$ (B) $x^2.y^2 = 5$ (C) $\frac{x^2}{4} + y^2 = 5$ (D) $x^2 + y^2 = 5$

21. An ellipse with major axis 4 and minor axis 2 touches both the coordinate axes, then locus of its focus is

(A) $(x^2 - y^2)(1 + x^2y^2) = 16 x^2 y^2$ (B) $(x^2 - y^2)(1 - x^2y^2) = 16 x^2 y^2$.
(C) $(x^2 + y^2)(1 + x^2y^2) = 16 x^2 y^2$ (D) $(x^2 + y^2)(1 - x^2y^2) = 16 x^2 y^2$.

22. A series of concentric ellipses E_1, E_2, ..., E_n are drawn such that E_n touches the extremities of the major axis of E_{n-1} and the foci of E_n coincide with the extremities of minor axis of E_{n-1}. If the eccentricity of the ellipses is independent of n, then the value of the eccentricity is

(A) $\frac{\sqrt{5}-1}{4}$ (B) $\frac{\sqrt{5}-1}{16}$ (C) $\frac{\sqrt{5}-1}{2}$ (D) $\frac{\sqrt{5}-1}{8}$

23. If e and e′ are the eccentricities of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}$ = 1 and $\frac{y^2}{b^2}-\frac{x^2}{a^2}$ = 1, then the point $\left(\frac{1}{e},\frac{1}{e'}\right)$ lies on the circle :

(A) $x^2 + y^2 = 1$ (B) $x^2 + y^2 = 2$ (C) $x^2 + y^2 = 3$ (D) $x^2 + y^2 = 4$

24. P is a point on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}$ = 1, N is the foot of the perpendicular from P on the transverse axis. The tangent to the hyperbola at P meets the transverse axis at T. If O is the centre of the hyperbola, then OT. ON is equal to :

(A) e^2 (B) a^2 (C) b^2 (D) b^2/a^2

25. Tangent at any point on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}$ = 1 cut the axes at A and B respectively. If the rectangle OAPB (where O is origin) is completed then locus of point P is given by

(A) $\frac{a^2}{x^2}-\frac{b^2}{y^2}$ = 1 (B) $\frac{a^2}{x^2}+\frac{b^2}{y^2}$ = 1 (C) $\frac{a^2}{y^2}-\frac{b^2}{x^2}$ = 1 (D) none of these

26. If the chord of contact of tangents from two points (x_1, y_1) and (x_2, y_2) to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}$ = 1 are at right angles, then $\frac{x_1\, x_2}{y_1\, y_2}$ is equal to

(A) $-\frac{a^2}{b^2}$ (B) $-\frac{b^2}{a^2}$ (C) $-\frac{b^4}{a^4}$ (D) $-\frac{a^4}{b^4}$

27^. The sides AC and AB of a triangle ABC touch the conjugate hyperbola of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at C and B respectively. If the vertex A lies on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, the side BC

(A) must touch the ellipse
(B) must cut the ellipse at two distinct points
(C) may not touch the ellipse
(D) may cut the ellipse at two distinct points

28. Tangents are drawn from any point on the hyperbola $\frac{x^2}{9} - \frac{y^2}{4}$ = 1 to the circle $x^2 + y^2 = 9$, then the locus of mid-point of the chord of contact is

(A) $\frac{x^2}{9} - \frac{y^2}{4} = \left(\frac{x^2+y^2}{9}\right)^2$ (B) $\frac{x^2}{9} + \frac{y^2}{4} = \left(\frac{x^2+y^2}{9}\right)^2$

(C) $\frac{x^2}{9} + \frac{y^2}{4} = \left(\frac{x^2-y^2}{9}\right)^2$ (D) $\frac{x^2}{9} - \frac{y^2}{4} = \left(\frac{x^2-y^2}{9}\right)^2$

29. If AB is a double ordinate of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2}$ = 1 such that ΔOAB (O is the origin) is an equilateral triangle, then the eccentricity 'e' of the hyperbola

(A) is greater than $\frac{2}{\sqrt{3}}$ (B) is less than $\frac{2}{\sqrt{3}}$

(C) is equal to $\frac{2}{\sqrt{3}}$ (D) is less than $\frac{1}{\sqrt{3}}$

30. Let two variable ellipse E_1 and E_2 touches each other externally at (0, 0). Their common tangent at (0, 0) is y = x. If one of the focus at E_1 & one of the focus of E_2 always lies or line y = 2x then find locus of other focus of E_1 & E_2.
(A) y = 4x (B) y = –2x (C) y = x/2 (D) y = –x/2

PART - II : SINGLE AND DOUBLE VALUE INTEGER TYPE

1. If $(a^2, a - 2)$ be a point interior to the region of the parabola $y^2 = 2x$ bounded by the chord joining the points (2, 2) and (8, – 4), then the number of all possible integral values of a is :

2. If □ is the distance between focus and directrix of the parabola $9x^2 - 24xy + 16y^2 - 20x - 15y - 60 = 0$ then 6□ is :

3. The number of integral values of a for which the point (–2a, a + 1) will be an interior point of the smaller region bounded by the circle $x^2 + y^2 = 4$ and the parabola $y^2 = 4x$, is :

4. A variable chord PQ of the parabola, $y^2 = 4x$ is drawn parallel to the line y = x. If the parameters of the points P & Q on the parabola be p & q respectively, then (p + q) equal to.

5. The parabola whose axis is parallel to the y-axis and which passes through the points (0, 4), (1, 9) and (–2,6), also passes through $(2, \alpha)$ then the value of α is :

6. Through the vertex O of the parabola $y^2 = 8x$, a perpendicular is drawn to any tangent meeting it at P & the parabola at Q, then the value of OP. OQ is

7. The centre of the circle which passes through the focus of the parabola $x^2 = 4y$ & touches it at the point (6, 9) is (α, β) then $|\alpha - \beta|$ is

8. Points A, B & C lie on the parabola $y^2 = 4ax$. The tangents to the parabola at A, B & C, taken in pairs, intersect at points P, Q & R. the ratio of the areas of the triangles ABC & PQR is $\frac{\lambda}{\mu}$ where λ and μ are co-prime number then $\lambda + \mu$ is

9. A normal is drawn to a parabola $y^2 = 4ax$ at any point other than the vertex and it cuts the parabola again at a point whose distance from the vertex is not less than $\lambda\sqrt{6}\,a$, then the value of λ is

10. If three normal are drawn through (c, 0) to $y^2 = 4x$ and two of which of perpendicular then the value of c is

11. P & Q are the points with eccentric angles θ & $\theta + \pi/6$ on the ellipse $\frac{x^2}{16}+\frac{y^2}{4}=1$, then the area of the triangle OPQ is :

12. If P is a variable point on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ whose focii are S and S′ and e_1 is the eccentricity and the locus of the incentre of $\Delta PSS'$ is an ellipse whose eccentricity is e_2 , then the value of $\left(1+\frac{1}{e_1}\right)e_2^2$ is:

13. If $\left(0, 3+\sqrt{5}\right)$ is a point on the ellipse whose foci are (2, 3), (–2, 3) then the length of semimajor axis is :

14. A circle has the same centre as an ellipse & passes through the focii F_1 & F_2 of the ellipse, such that the two curves intersect at 4 points. Let 'P' be any one of their points of intersection. If the major axis of the ellipse is 17 & the area of the triangle PF_1F_2 is 30, then the distance between the focii is :

15. Point 'O' is the centre of the ellipse with major axis AB & minor axis CD. Point F is one focus of the ellipse. If OF = 6 & the diameter of the inscribed circle of triangle OCF is 2, then the product (AB) (CD) is

16. If 'r' be the radius of largest circle with centre (3, 0) that can be inscribed in the ellipse $9x^2 + 25y^2 = 225$, then $4\sqrt{7}$ r is equal to

17. Minimum length of the intercept made by the axes on the tangent to the ellipse $\frac{x^2}{81}+\frac{y^2}{36} = 1$ is equal to

18. If the distance of the centre of the ellipse $4(x - 2y + 1)^2 + 9(2x + y + 2)^2 = 25$ from the origin is λ times its eccentricity, then $5\lambda^2$ is :

19. The radius of the largest circle with centre (1, 0) that can be inscribed in the ellipse $x^2 + 4y^2 =16$ is $\sqrt{\frac{\alpha}{\beta}}$ where α and β are prime number, then $\alpha + \beta$ is

20. Common tangents are drawn to the parabola $y^2 = 4x$ & the ellipse $3x^2 + 8y^2 = 48$ touching the parabola at A & B and the ellipse at C & D, then the area of the quadrilateral ABCD is $\lambda\sqrt{2}$ the λ is equal to

21. A circle of radius r is concentric with the ellipse $\frac{x^2}{4^2}+\frac{y^2}{3^2} = 1$ and the common tangent is inclined to the major axis at an angle of $\tan^{-1}\sqrt{\left(\frac{r^2-\beta^2}{\alpha^2-r^2}\right)}$; $r \in (b, a)$ then the value of $|\alpha| + |\beta|$ is

22. If CF is perpendicular from the centre of the ellipse $\frac{x^2}{4^2} + \frac{y^2}{3^2} = 1$ to the tangent at P, and G is the point where the normal at P meets the major axis, then the product $CF \cdot PG$ is

23. The eccentricity of an ellipse whose focii are (2, 4) & (14, 9) and touches x-axis is $\frac{\lambda}{\sqrt{313}}$ then the value of λ is

24. If two points P & Q on the hyperbola $x^2/a^2 - y^2/b^2 = 1$ whose centre is C be such that CP is perpendicular to CQ & a < b, then $\frac{1}{CP^2} + \frac{1}{CQ^2} = \lambda\left(\frac{1}{a^2} - \frac{1}{b^2}\right)$ where λ is :

25. If $7x^2 + pxy + qy^2 + rx - sy + t = 0$ is the eqaution of the hyperbola whose one focus is (–1, 1), eccentricity = 3 and the equation of the corresponding directrix is $x - y + 3 = 0$, then the value of 't' is :

26. The hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ passes through the point of intersection of the lines, $7x + 13y - 87 = 0$ & $5x - 8y + 7 = 0$ & the latus rectum is $32\sqrt{2}/5$. The value of $2(a^2 + b^2)$ is :

27. If m_1 and m_2 are slopes of the tangents to the hyperbola $\frac{x^2}{25} - \frac{y^2}{16} = 1$ which passes through the point of contact of $3x - 4y = 5$ and $x^2 - 4y^2 = 5$ then $32(m_1 + m_2 - m_1m_2)$ = ...

28. Tangents are drawn from the point $(\alpha, 2)$ to the hyperbola $3x^2 - 2y^2 = 6$ and are inclined at angles θ & ϕ to the x –axis . If $\tan\theta \cdot \tan\phi = 2$, then the value of $2\alpha^2 - 7$ is

29. C the centre of the hyperbola $\frac{x^2}{9} - \frac{y^2}{16} = 1$. The tangents at any point P on this hyperbola meets the striaght lines $4x - 3y = 0$ and $4x + 3y = 0$ in the points Q and R respectively. Then CQ . CR =

30. If radii of director circles of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ are 2r and r respectively and e_e and e_h be the eccentricities of the ellipse and the hyperbola respectively then $4e_h^2 - e_e^2$ is equal to

31. The length of that focal chord of the hyperbola $xy = 8$ which touches the circle $x^2 + y^2 = 8$ is.

32. The sum of lengths of perpendiculars drawn from focii to any real tangent to the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$ is always greater than a, then find maximum value of a.

33. Let tangent at point A, B and vertex (V) of parabola is $x - 2y + 1 = 0$, $3x + y + 4 = 0$ and $y = x$ respectively. If focus of parabola is $\left(\frac{a}{7}, \frac{b}{7}\right)$ then find the value of (a + 5b).

34. If common tangent of $x^2 + y^2 = r^2$ and $\frac{x^2}{16} + \frac{y^2}{9} = 1$ forms square then find its area.

35. Let $x^2 + y^2 = r^2$ and $xy = 1$ intersect at A & B in first quadrant, If AB = $\sqrt{14}$ then find the value of r.

PART - III : ONE OR MORE THAN ONE OPTIONS CORRECT TYPE

1. Let A be the vertex and L the length of the latus rectum of the parabola, $y^2 - 2y - 4x - 7 = 0$. The equation of the parabola with A as vertex, 2L the length of the latus rectum and the axis at right angles to that of the given curve is:

(A) $x^2 + 4x + 8y - 4 = 0$ (B) $x^2 + 4x - 8y + 12 = 0$
(C) $x^2 + 4x + 8y + 12 = 0$ (D) $x^2 + 8x - 4y + 8 = 0$

2. The locus of the mid point of the focal radii of a variable point moving on the parabola, $y^2 = 4ax$ is a parabola whose
(A) Latus rectum is half the latus rectum of the original parabola
(B) Vertex is (a/2, 0)
(C) Directrix is y-axis
(D) Focus has the co-ordinates (a,0)

3. P is a point on the parabola $y^2 = 4ax$ $(a > 0)$ whose vertex is A. PA is produced to meet the directrix in D and M is the foot of the perpendicular from P on the directrix. If a circle is described on MD as a diameter then it intersects the x–axis at a point whose co–ordinates are:
(A) $(-3a, 0)$ (B) $(-a, 0)$ (C) $(-2a, 0)$ (D) $(a, 0)$

4. Let $y^2 = 4ax$ be a parabola and $x^2 + y^2 + 2bx = 0$ be a circle. If parabola and circle touch each other externally then:
(A) $a > 0, b > 0$ (B) $a > 0, b < 0$ (C) $a < 0, b > 0$ (D) $a < 0, b < 0$

5. P is a point on the parabola $y^2 = 4x$ where abscissa and ordinate are equal. Equation of a circle passing through the focus and touching the parabola at P is:
(A) $x^2 + y^2 - 13x + 2y + 12 = 0$ (B) $x^2 + y^2 - 3x - 18y + 2 = 0$
(C) $x^2 + y^2 + 13x - 2y - 14 = 0$ (D) $x^2 + y^2 - x = 0$

6^. Subset of complete set of values of m for which a chord of slope m of the circle $x^2 + y^2 = 4$ touches parabola $y^2 = 4x$, can be
(A) $\left(-\infty, -\sqrt{\frac{\sqrt{2}-1}{2}}\right)$ (B) (0 , 1/2) (C) $\left(\sqrt{\frac{\sqrt{2}-1}{2}}, \infty\right)$ (D) (–1/2, 0)

7. Locus of the centre of the circle passing through the vertex and the mid-points of perpendicular chords from the vertex of the parabola $y^2 = 4ax$ is.
(A) is a parabola with vertex (–a, a) (B) is a parabola with latus rectum a
(C) is a parabola with vertex (2a,0) (D) is a parabola with latus rectum $\frac{a}{2}$

8. The equation, $3x^2 + 4y^2 - 18x + 16y + 43 = C$.
(A) cannot represent a real pair of straight lines for any value of C
(B) represents an ellipse, if $C > 0$
(C) no locus, if $C < 0$
(D) a point, if $C = 0$

9. If P is a point of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, whose focii are S and S′. Let $\angle PSS' = \alpha$ and $\angle PS'S = \beta$, then
(A) $PS + PS' = 2a$, if $a > b$
(B) $PS + PS' = 2b$, if $a < b$
(C) $\tan\frac{\alpha}{2}\tan\frac{\beta}{2} = \frac{1-e}{1+e}$
(D) $\tan\frac{\alpha}{2}\tan\frac{\beta}{2} = \frac{\sqrt{a^2-b^2}}{b^2}\left[a - \sqrt{a^2-b^2}\right]$ when $a > b$

10. Let $A(\alpha)$ and $B(\beta)$ be the extremeties of a chord of an ellipse . If the slope of AB is equal to the slope of the tangent at a point $C(\theta)$ on the ellipse, then the value of θ, is
(A) $\frac{\alpha+\beta}{2}$ (B) $\frac{\alpha-\beta}{2}$ (C) $\frac{\alpha+\beta}{2} + \pi$ (D) $\frac{\alpha-\beta}{2} - \pi$

11. Let F_1, F_2 be two focii of the ellipse and PT and PN be the tangent and the normal respectively to the ellipse at point P then
(A) PN bisects $\angle F_1PF_2$ (B) PT bisects $\angle F_1PF_2$
(C) PT bisects angle $(180° - \angle F_1PF_2)$ (D) None of these

12. If $\square_1$ be the equation of the common tangent in 1st quadrant to the circle $x^2 + y^2 = 16$ and ellipse $\frac{x^2}{25}+\frac{y^2}{4} = 1$ and λ_1 be the length of the intercept of the common tangent between the coordinate axes then

(A) $\lambda_1 = \frac{14}{\sqrt{3}}$
(B) Equation of $\square_1$ is $2x+\sqrt{3}y = 4\sqrt{7}$
(C) $\lambda_1 = \frac{4}{\sqrt{3}}$
(D) Equation of $\square_1$ is $x+\sqrt{3}y = 4\sqrt{7}$

13. Let E_1 and E_2 be two ellipses $\frac{x^2}{a^2}+y^2 = 1$ and $x^2+\frac{y^2}{a^2} = 1$ (where a is parameter) the locus of points of intersection of the ellipses E_1 and E_2 is a set of curves
(A) $y = x, y = -x, x^2 + y^2 = 1$
(B) $y = 2x, y = -2x, x^2 + y^2 = 4$
(C) $(4x^2 - y^2)(x^2 + y^2 - 4) = 0$
(D) $(x^2 - y^2)(x^2 + y^2 - 1) = 0$

14. If (5, 12) and (24, 7) are the foci of a conic, passing through the origin then the eccentricity of conic is
(A) $\sqrt{386}/12$
(B) $\sqrt{386}/13$
(C) $\sqrt{386}/25$
(D) $\sqrt{386}/38$

15. The equation of a hyperbola with co-ordinate axes as principal axes, if the distances of one of its vertices from the foci are 3 & 1 can be :
(A) $3x^2 - y^2 = 3$
(B) $x^2 - 3y^2 + 3 = 0$
(C) $x^2 - 3y^2 - 3 = 0$
(D) $x^2 - 3y^2 - 6 = 0$

16. A point moves such that the sum of the squares of its distances from the two sides of length 'a' of a rectangle is twice the sum of the squares of its distances from the other two sides of length 'b'. The locus of the point can be :
(A) a circle
(B) an ellipse
(C) a hyperbola
(D) a pair of lines

17. If $(3\sin\alpha, 2\cos\alpha)$ lies on the same side as that of origin w.r.t conic $2x^2 - 3y^2 = 6$, then $\sin\alpha$ may be
(A) $-\sqrt{\frac{4}{5}}$
(B) $\sqrt{\frac{2}{5}}$
(C) $\frac{1}{\sqrt{5}}$
(D) $\frac{2}{15}$

18. Which of the following equations in parametric form can represent a hyperbolic profile, where 't' is a parameter.
(A) $x = \frac{a}{2}\left(t+\frac{1}{t}\right)$ & $y = \frac{b}{2}\left(t-\frac{1}{t}\right)$
(B) $\frac{tx}{a} - \frac{y}{b} + t = 0$ & $\frac{x}{a}+\frac{ty}{b} - 1 = 0$
(C) $x = e^t + e^{-t}$ & $y = e^t - e^{-t}$
(D) $x^2 - 6 = 2\cos t$ & $y^2 + 2 = 4\cos^2\frac{t}{2}$

19. If two distinct tangents can be drawn from the point $(\alpha, 2)$ on different branches of the hyperbola $\frac{x^2}{9}-\frac{y^2}{16} = 1$, then the range of α is subset of
(A) $\left[-\frac{3}{2},\frac{3}{2}\right]$
(B) $[-2,2]$
(C) $[-1,1]$
(D) $\left(-\frac{1}{2},\frac{1}{2}\right)$

20. A rectangular hyperbola whose centre is C is cut by any circle of radius r in four points P, Q, R and S. Then $CP^2 + CQ^2 + CR^2 + CS^2 =$
(A) 16 if $r = \sqrt{2}$
(B) 16 if r = 2
(C) 2 if r = 1
(D) 4 if r = 1

21. $x^2 + y^2 = 16$ is the auxilliary circle of
(A) $9x^2 - 16y^2 - 144 = 0$
(B) $16x^2 - 9y^2 + 144 = 0$
(C) $9(x - y)^2 - 16(x + y)^2 - 288 = 0$
(D) $16(x - y)^2 - 9(x + y)^2 + 288 = 0$

22. If the chord joining the points whose eccentric angles are 'α' and 'β' on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is a focal chord then

(A) $\pm e\cos\left(\frac{\alpha-\beta}{2}\right) = \cos\left(\frac{\alpha+\beta}{2}\right)$

(B) $\pm e\cos\left(\frac{\alpha+\beta}{2}\right) = \cos\left(\frac{\alpha-\beta}{2}\right)$

(C) $\tan(\alpha/2)\tan(\beta/2) + \left(\frac{ke-1}{ke+1}\right) = 0$ where $k = \pm 1$

(D) $\tan(\alpha/2)\tan(\beta/2) + \left(\frac{ke+1}{ke-1}\right) = 0$ where $k = \pm 1$

23. If the normal at P to the rectangular hyperbola $x^2 - y^2 = 4$ meets the axes in G and g and C is the centre of the hyperbola, then

(A) PG = PC (B) Pg = PC (C) PG = Pg (D) Gg = PC

24. Circles are drawn on chords of rectangular hyperbola $xy = c^2$ parallel to the line $y = x$ as diameters. All such circles pass through two fixed points whose co-ordinates are :

(A) (c, c) (B) (c, –c) (C) (–c, c) (D) (–c, –c)

25. If a circle and the rectangular hyperbola $xy = c^2$ meet in the four points t_1, t_2, t_3 & t_4, then

(A) $t_1 t_2 t_3 t_4 = 1$

(B) The arthmetic mean of the four points bisects the distance between the centres of the two curves.

(C) The geometrical mean of the four points bisects the distance between the centres of the two curves.

(D) the centre of the circle through the points t_1, t_2 & t_3 is :

$$\left\{\frac{c}{2}\left(t_1 + t_2 + t_3 + \frac{1}{t_1 t_2 t_3}\right), \frac{c}{2}\left(\frac{1}{t_1} + \frac{1}{t_2} + \frac{1}{t_3} + t_1 t_2 t_3\right)\right\}$$

26. Two confocal parabola intersect at A and B. If their axis are parallel to x-axis and y-axis respectively, then slope of chord AB can be :

(A) 1 (B) –1 (C) 2 (D) –2

PART - IV : COMPREHENSION

Comprehension # 1 (For Q.No. 1 to 3)

Consider three lines y axis, y = 2 and □x + my = 1 where (□, m) lies on $y^2 = 4x$. answer the following :

1. Locus of circum centre of triangle formed by given three lines is a parabola whose vertex is

(A) (–2, 3/2) (B) (2, –3/2) (C) (–2, –3/2) (D) (2, –5/2)

2. Area of triangle formed by vertex and end points of latus rectum of parabola obtained in questions (1) is

(A) $\frac{1}{2^8}$ (unit)^2 (B) $\frac{1}{2^9}$ (unit)^2 (C) $\frac{1}{2^{10}}$ (unit)^2 (D) $\frac{1}{2^7}$ (unit)^2

3. Any point on the parabola obtained in question (1) can be represented as

(A) $\left(2+\frac{1}{32}t^2, \frac{3}{2}+\frac{t}{16}\right)$ (B) $\left(2+\frac{t^2}{32}, \frac{-3}{2}+\frac{1}{16}t^2\right)$ (C) $\left(-2+\frac{1}{32}t^2, \frac{3}{2}+\frac{t}{16}\right)$ (D) $\left(-2+\frac{1}{16}t^2, \frac{3}{2}+\frac{t}{5}\right)$

Comprehension # 2

Let PQ be a variable focal chord of the parabola $y^2 = 4ax$ where vertex is A. Locus of, centroid of triangle APQ is a parabola 'P_1'

4. Latus rectum of parabola P_1 is

(A) $\frac{2a}{3}$ (B) $\frac{4a}{3}$ (C) $\frac{8a}{3}$ (D) $\frac{16a}{3}$

5. Vertex of parabola P_1 is

(A) $\left(\frac{2a}{3}, 0\right)$ (B) $\left(\frac{4a}{3}, 0\right)$ (C) $\left(\frac{8a}{3}, 0\right)$ (D) $\left(\frac{a}{3}, 0\right)$

6. Let Δ_1 is the area of triangle formed by joining points T_1, T_2 and T_3 on parabola P_1 and Δ_2 be the area of triangle T formed by tangents at T_1, T_2 and T_3, then

(A) $\Delta_2 = 2\Delta_1$

(B) $\Delta_1 = 4\Delta_2$

(C) orthocentre of triangle T lies on $x = a/3$.

(D) Both (A) and (C) are correct.

Comprehension # 3

Two tangents PA and PB are drawn from a point P(h, k) to the ellipse E : $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (a > b). Angle of the tangents with the positive x - axis are θ_1 and θ_2. Normals at A and B are intersecting at Q point.
On the basis of above information answer the following questions.

7. Locus of P, if $\tan\theta_1 . \tan\theta_2 = 4$, is

(A) $\frac{y-b}{x-a} = 2(x + a)$ (B) $y^2 - b^2 = 2(x^2 + a^2)$ (C) $\frac{y+b}{x+a} = \frac{4(x-a)}{y-b}$ (D) $\frac{y+b}{x-b} = \frac{x+a}{y-b}$

8. Circumcentre of ΔQAB is

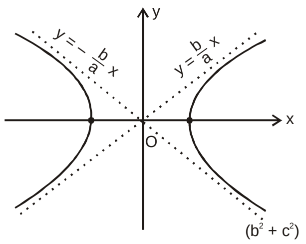
(A) mid point of AB (B) mid point of PQ (C) orthocentre of ΔPAB (D) can't say

9. Locus of P, if $\cot\theta_1 + \cot\theta_2 = \lambda$, is

(A) $2xy = \lambda(y^2 - b^2)$ (B) $2xy - \lambda(b^2 - y^2) = 0$ (C) $xy = \lambda$ (D) $x^2 + xy = \lambda$

Comprehension # 4

Asymptotes are lines whose distance from the curve at infinity tends to zero Let $y = mx + c$ is asymptote of H $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$. Solving the two equations, we have $(b^2 - a^2m^2)x^2 - 2a^2mcx - a^2(b^2 + c^2) = 0$. Both roots of this equation must be infinite so $m = \pm\frac{b}{a}$ and c = 0 which implies that $y = \frac{b}{a} \pm x$ are asymptotes of $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. Note that no real tangent can be drawn to the hyperbola from its centre and only one real tangent can be drawn from a point lying on its asymptote other than centre. Further combined equation of asymptotes is $A = \frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$ and conjugate hyperbola $C = \frac{x^2}{a^2} - \frac{y^2}{b^2} + 1 = 0$. Hence 2A = H + C, as we can see, equation of A, H and C vary only by a constant, for asymptotes which can be evaluated by applying condition of pair of lines.



10. The points of contact of tangents drawn to the hyperbola $\frac{x^2}{3}-\frac{y^2}{2}=1$ from point (2, 1) are

(A) (3, 2), (1, 5) (B) (3, 2), $\left(\frac{9}{5},\frac{2}{5}\right)$ (C) (1, 2), (3, 4) (D) (3, 2), (3, 4)

11.✍ The number of real distinct tangents drawn to hyperbola $4x^2 - y^2 = 4$ from point (1, 2) is
(A) 1 (B) 2 (C) 3 (D) 4

12.✍ The number of real distinct tangents drawn from point (1, 2) to hyperbola $x^2 - y^2 - 2x + 4y - 4 = 0$ is
(A) 1 (B) 2 (C) 3 (D) None of these

13. The asymptotes of $xy - 3y - 2x = 0$ is
(A) $x + 2 = 0$ and $y + 3 = 0$ (B) $x - 2 = 0$ and $y - 3 = 0$
(C) $x - 3 = 0$ and $y - 2 = 0$ (D) $x + 3 = 0$ and $y + 2 = 0$

Comprehension # 5

Equation of the transverse and conjugate axis of a hyperbola are respectively $x + 2y - 3 = 0$, $2x - y + 4 = 0$ and their respectively lengths are $\sqrt{2}$ and $\frac{2}{\sqrt{3}}$ then answer following :

14.✍ If $x^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ is equation of given hyperbola where h, b, g, f, c all are integers then the sum h + b + g + f + c =
(A) 3 (B) 4 (C) 5 (D) 6

15.✍ Equation of one of the directrix is

(A) $2x - y + 4 + \sqrt{\frac{3}{2}} = 0$ (B) $x + 2y + 4 - \sqrt{\frac{2}{3}} = 0$

(C) $2x - y = \sqrt{\frac{3}{2}}$ (D) $2x - y + 4 + \sqrt{\frac{3}{2}} = \sqrt{3}$

16.✍ Coordinates of one of possible focus of hyperbola is

(A) $\left(-1+\frac{2}{\sqrt{6}}, 2-\frac{1}{\sqrt{6}}\right)$ (B) $\left(\left(-1+\frac{2}{\sqrt{5}}\right),\left(2-\frac{1}{\sqrt{5}}\right)\right)$

(C) $\left(\left(-1-\frac{2}{\sqrt{5}}\right),\left(2+\frac{1}{\sqrt{5}}\right)\right)$ (D) $\left(\left(-1-\frac{2}{\sqrt{5}}\right), \left(2-\frac{1}{\sqrt{5}}\right)\right)$

Exercise-3

✍ **Marked questions are recommended for Revision.**

*** Marked Questions may have more than one correct option.**

PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

1*. The tangent PT and the normal PN to the parabola $y^2 = 4ax$ at a point P on it meet its axis at points T and N, respectively. The locus of the centroid of the triangle PTN is a parabola whose

[IIT-JEE 2009, Paper-2, (4, –1), 80]

(A) vertex is $\left(\frac{2a}{3}, 0\right)$ (B) directrix is $x = 0$ (C) latus rectum is $\frac{2a}{3}$ (D) focus is (a, 0)

2. The line passing through the extremity A of the major axis and extremity B of the minor axis of the ellipse
$x^2 + 9y^2 = 9$ meets its auxiliary circle at the point M. Then the area of the triangle with vertices at A, M and the origin O is **[IIT-JEE 2009, Paper-1, (3, –1)/ 80]**

(A) $\frac{31}{10}$ (B) $\frac{29}{10}$ (C) $\frac{21}{10}$ (D) $\frac{27}{10}$

3*. In a triangle ABC with fixed base BC, the vertex A moves such that $\cos B + \cos C = 4 \sin^2 \frac{A}{2}$. If a, b and c denote the lengths of the sides of the triangle opposite to the angles A, B and C respectively, then **[IIT-JEE 2009, Paper-1, (4, –1)/ 80]**

(A) $b + c = 4a$ (B) $b + c = 2a$
(C) locus of points A is an ellipse (D) locus of point A is a pair of straight lines

4. The normal at a point P on the ellipse $x^2 + 4y^2 = 16$ meets the x-axis at Q. If M is the mid point of the line segment PQ, then the locus of M intersects the latus rectum of the given ellipse at the points **[IIT-JEE 2009, Paper-2, (3, –1)/ 80]**

(A) $\left(\pm\frac{3\sqrt{5}}{2}, \pm\frac{2}{7}\right)$ (B) $\left(\pm\frac{3\sqrt{5}}{2}, \pm\frac{\sqrt{19}}{4}\right)$ (C) $\left(\pm 2\sqrt{3}, \pm\frac{1}{7}\right)$ (D) $\left(\pm 2\sqrt{3}, \pm\frac{4\sqrt{3}}{7}\right)$

5. Match the conics in **Column - I** with the statements/expressions in **Column - II.** **[IIT-JEE-2009, Paper-1, (8, 0), 80]**

Column - I	**Column - II**
(A) Circle	(p) The locus of the point (h, k) for which the line $hx + ky = 1$ touches the circle $x^2 + y^2 = 4$
(B) Parabola	(q) Points z in the complex plane satisfying $\|z + 2\| – \|z – 2\| = \pm 3$
(C) Ellipse	(r) Points of the conic have parametric representation $x = \sqrt{3}\left(\frac{1-t^2}{1+t^2}\right)$, $y = \frac{2t}{1+t^2}$
(D) Hyperbola	(s) The eccentricity of the conic lies in the interval $1 \le x < \infty$
	(t) Points z in the complex plane satisfying $\text{Re}(z + 1)^2 = \|z\|^2 + 1$

6*. An ellipse intersects the hyperbola $2x^2 – 2y^2 = 1$ orthogonally. The eccentricity of the ellipse is reciprocal of that of the hyperbola. If the axes of the ellipse are along the coordinate axes, then **[IIT-JEE 2009, Paper-2, (4, –1), 80]**

(A) Equation of ellipse is $x^2 + 2y^2 = 2$ (B) The foci of ellipse are $(\pm 1, 0)$
(C) Equation of ellipse is $x^2 + 2y^2 = 4$ (D) The foci of ellipse are $(\pm, \sqrt{2}\ \ 0)$

7*. Let A and B be two distinct points on the parabola $y^2 = 4x$. If the axis of the parabola touches a circle of radius r having AB as its diameter, then the slope of the line joining A and B can be **[IIT-JEE-2010, Paper-1(3, 0)/84]**

(A) $-\frac{1}{r}$ (B) $\frac{1}{r}$ (C) $\frac{2}{r}$ (D) $-\frac{2}{r}$

Comprehension # 1 (Q.8 - 10)

Tangents are drawn from the point P(3, 4) to the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ touching the ellipse at point A and B.

8. The coordinates of A and B are **[IIT-JEE 2010, Paper-2, (3, –1), 79]**

(A) (3, 0) and (0, 2)
(B) $\left(-\frac{8}{5}, \frac{2\sqrt{161}}{15}\right)$ and $\left(-\frac{9}{5}, \frac{8}{5}\right)$
(C) $\left(-\frac{8}{5}, \frac{2\sqrt{161}}{15}\right)$ and (0, 2)
(D) (3, 0) and $\left(-\frac{9}{5}, \frac{8}{5}\right)$

9. The orthocentre of the triangle PAB is **[IIT-JEE 2010, Paper-2, (3, –1), 79]**

(A) $\left(5, \frac{8}{7}\right)$ (B) $\left(\frac{7}{5}, \frac{25}{8}\right)$ (C) $\left(\frac{11}{5}, \frac{8}{5}\right)$ (D) $\left(\frac{8}{25}, \frac{7}{5}\right)$

10. The equation of the locus of the point whose distances from the point P and the line AB are equal, is

(A) $9x^2 + y^2 - 6xy - 54x - 62y + 241 = 0$
(B) $x^2 + 9y^2 + 6xy - 54x + 62y - 241 = 0$
(C) $9x^2 + 9y^2 - 6xy - 54x - 62y - 241 = 0$
(D) $x^2 + y^2 - 2xy + 27x + 31y - 120 = 0$

[IIT-JEE 2010, Paper-2, (3, –1), 79]

Comprehension

The circle $x^2 + y^2 - 8x = 0$ and hyperbola $\frac{x^2}{9}-\frac{y^2}{4}=1$ intersect at the points A and B.

11. Equation of a common tangent with positive slope to the circle as well as to the hyperbola is

(A) $2x - \sqrt{5}\,y - 20 = 0$
(B) $2x - \sqrt{5}\,y + 4 = 0$
(C) $3x - 4y + 8 = 0$
(D) $4x - 3y + 4 = 0$ **[IIT-JEE-2010, Paper-1(3, –1)/84]**

12. Equation of the circle with AB as its diameter is **[IIT-JEE-2010, Paper-1(3, –1)/84]**

(A) $x^2 + y^2 - 12x + 24 = 0$
(B) $x^2 + y^2 + 12x + 24 = 0$
(C) $x^2 + y^2 + 24x - 12 = 0$
(D) $x^2 + y^2 - 24x - 12 = 0$

13. The line $2x + y = 1$ is tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. If this line passes through the point of intersection of the nearest directrix and the x-axis, then find the eccentricity of the hyperbola.
[IIT-JEE-2010, Paper-1(3, 0)/84]

14. Consider the parabola $y^2 = 8x$. Let Δ_1 be the area of the triangle formed by the end points of its latus rectum and the point $P\left(\frac{1}{2}, 2\right)$ on the parabola, and Δ_2 be the area of the triangle formed by drawing tangents at P and at the end points of the latus rectum. Then $\frac{\Delta_1}{\Delta_2}$ is **[IIT-JEE 2011, Paper-1, (4, 0), 80]**

15. Let (x, y) be any point on the parabola $y^2 = 4x$. Let P be the point that divides the line segment from (0, 0) to (x, y) in the ratio 1 : 3. Then the locus of P is **[IIT-JEE 2011, Paper-2, (3, –1), 80]**

(A) $x^2 = y$ (B) $y^2 = 2x$ (C) $y^2 = x$ (D) $x^2 = 2y$

16*. Let L be a normal to the parabola $y^2 = 4x$. If L passes through the point (9, 6), then L is given by

(A) $y - x + 3 = 0$ (B) $y + 3x - 33 = 0$ (C) $y + x - 15 = 0$ (D) $y - 2x + 12 = 0$

[IIT-JEE 2011, Paper-2, (4, 0), 80]

17*. Let the eccentricity of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be reciprocal to that of the ellipse $x^2 + 4y^2 = 4$. If the hyperbola passes through a focus of the ellipse, then **[IIT-JEE 2011, Paper-1, (4, 0), 80]**

(A) the equation of the hyperbola is $\frac{x^2}{3} - \frac{y^2}{2} = 1$

(B) a focus of the hyperbola is (2, 0)

(C) the eccentricity of the hyperbola is $\sqrt{\frac{5}{3}}$

(D) the equation of the hyperbola is $x^2 - 3y^2 = 3$

18. Let P(6, 3) be a point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If the normal at the point P intersects the x-axis at (9, 0), then the eccentricity of the hyperbola is **[IIT-JEE 2011, Paper-2, (3, –1), 80]**

(A) $\sqrt{\frac{5}{2}}$ (B) $\sqrt{\frac{3}{2}}$ (C) $\sqrt{2}$ (D) $\sqrt{3}$

19. The ellipse $E_1 : \frac{x^2}{9} + \frac{y^2}{4} = 1$ is inscribed in a rectangle R whose sides are parallel to the coordinate axes. Another ellipse E_2 passing through the point (0, 4) circumscribes the rectangle R. The eccentricity of the ellipse E_2 is **[IIT-JEE 2012, Paper-1, (3, –1), 70]**

(A) $\frac{\sqrt{2}}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) $\frac{1}{2}$ (D) $\frac{3}{4}$

20. Tangents are drawn to the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$, parallel to the straight line 2x – y = 1. The points of contacts of the tangents on the hyperbola are **[IIT-JEE 2012, Paper-1, (4, 0), 70]**

(A) $\left(\frac{9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ (B) $\left(-\frac{9}{2\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$ (C) $\left(3\sqrt{3}, -2\sqrt{2}\right)$ (D) $\left(-3\sqrt{3}, 2\sqrt{2}\right)$

21. Let S be the focus of the parabola $y^2 = 8x$ and let PQ be the common chord of the circle $x^2 + y^2 - 2x - 4y = 0$ and the given parabola. The area of the triangle PQS is.
[IIT-JEE 2012, Paper-1, (4, 0), 70]

Paragraph for Question Nos. 22 to 23

Let PQ be a focal chord of the parabola $y^2 = 4ax$. The tangents to the parabola at P and Q meet at a point lying on the line y = 2x + a, a > 0. **[IIT-JEE - 2013, Paper-2, (3,–1), 60]**

22. Length of chord PQ is
(A) 7a (B) 5a (C) 2a (D) 3a

23. If chord PQ subtends an angle θ at the vertex of $y^2 = 4ax$, then $\tan \theta =$

(A) $\frac{2}{3}\sqrt{7}$ (B) $\frac{-2}{3}\sqrt{7}$ (C) $\frac{2}{3}\sqrt{5}$ (D) $\frac{-2}{3}\sqrt{5}$

24. The common tangents to the circle $x^2 + y^2 = 2$ and the parabola $y^2 = 8x$ touch the circle at the points P, Q and the parabola at the points R, S. Then the area of the quadrilateral PQRS is
[JEE (Advanced) 2014, Paper-2, (3, –1)/60]
(A) 3 (B) 6 (C) 9 (D) 15

Paragraph For Questions 25 and 26

Let a, r, s, t be nonzero real numbers. Let P(at^2, 2at), Q, R (ar^2, 2ar) and S(as^2, 2as) be distinct points on the parabola $y^2 = 4ax$. Suppose that PQ is the focal chord and lines QR and PK are parallel, where K is the point (2a, 0) **[JEE (Advanced) 2014, Paper-2, (3, –1)/60]**

25. The value of r is

(A) $-\frac{1}{t}$ (B) $\frac{t^2+1}{t}$ (C) $\frac{1}{t}$ (D) $\frac{t^2-1}{t}$

26. If st = 1, then the tangent at P and the normal at S to the parabola meet at a point whose ordinate is

(A) $\frac{(t^2+1)^2}{2t^3}$ (B) $\frac{a(t^2+1)^2}{2t^3}$ (C) $\frac{a(t^2+1)^2}{t^3}$ (D) $\frac{a(t^2+2)^2}{t^3}$

27. Let the curve C be the mirror image of the parabola $y^2 = 4x$ with respect to the line x + y + 4 = 0. If A and B are the points of intersection of C with the line y= – 5, then the distance between A and B is
[JEE (Advanced) 2015, P-1 (4, 0) /88]

28. If the normals of the parabola $y^2 = 4x$ drawn at the end points of its latus rectum are tangents to the circle $(x-3)^2 + (y+2)^2 = r^2$, then the value of r^2 is **[JEE (Advanced) 2015, P-1 (4, 0) /88]**

29*. Let P and Q be distinct points on the parabola $y^2 = 2x$ such that a circle with PQ as diameter passes through the vertex O of the parabola, if P lies in the first quadrant and the area of the triangle ΔOPQ is $3\sqrt{2}$, then which of the following is (are) the coordinates of P ?
[JEE (Advanced) 2015, P-1 (4, –2)/ 88]

(A) $(4,\ 2\sqrt{2})$ (B) $(9,\ 3\sqrt{2})$ (C) $\left(\frac{1}{4}, \frac{1}{\sqrt{2}}\right)$ (D) $(1,\ \sqrt{2})$

30. Suppose that the foci of the ellipse $\frac{x^2}{9}+\frac{y^2}{5} = 1$ are $(f_1, 0)$ and $(f_2, 0)$ where $f_1 > 0$ and $f_2 < 0$. Let P_1 and P_2 be two parabolas with a common vertex at (0, 0) and with foci at $(f_1, 0)$ and $(2f_2, 0)$, respectively. Let T_1 be a tangent to P_1 which passes through $(2f_2, 0)$ and T_2 be a tangent to P_2 which passes through $(f_1,0)$. If m_1 is the slope of T_1 and m_2 is the slope of T_2, then the value of $\left(\frac{1}{m_1^2}+m_2^2\right)$ is.
[JEE (Advanced) 2015, P-2 (4, 0) / 80]

31*. Let E_1 and E_2 be two ellipses whose centers are at the origin. The major axes of E_1 and E_2 lie along the x-axis and the y-axis, respectively. Let S be the circle $x^2 + (y-1)^2 = 2$. The straight line x + y = 3 touches the curves S, E_1 and E_2 at P,Q and R, respectively. Suppose that PQ = PR = $\frac{2\sqrt{2}}{3}$. If e_1 and e_2 are the eccentricities of E_1 and E_2, respectively, then the correct expression(s) is (are)
[JEE (Advanced) 2015, P-2 (4, –2)/ 80]

(A) $e_1^2 + e_2^2 = \frac{43}{40}$ (B) $e_1e_2 = \frac{\sqrt{7}}{2\sqrt{10}}$

(C) $|e_1^2 - e_2^2| = \frac{5}{8}$ (D) $e_1e_2 = \frac{\sqrt{3}}{4}$

32*. Consider the hyperbola H : $x^2 - y^2 = 1$ and a circle S with center $N(x_2, 0)$. Suppose that H and S touch each other at a point $P(x_1, y_1)$ with $x_1 > 1$ and $y_1 > 0$. The common tangent to H and S at P intersects the x-axis at point M. If (l, m) is the centroid of the triangle ΔPMN, then the correct expression(s) is(are)
[JEE (Advanced) 2015, P-2 (4, –2)/ 80]

(A) $\frac{dl}{dx_1} = 1-\frac{1}{3x_1^2}$ for $x_1 > 1$ (B) $\frac{dm}{dx_1} = \frac{x_1}{3\left(\sqrt{x_1^2-1}\right)}$ for $x_1 > 1$

(C) $\frac{dl}{dx_1} = 1+\frac{1}{3x_1^2}$ for $x_1 > 1$ (D) $\frac{dm}{dy_1} = \frac{1}{3}$ for $y_1 > 0$

33. Let P be the point on the parabola $y^2 = 4x$ which is at the shortest distance from the center S of the circle $x^2 + y^2 - 4x - 16y + 64 = 0$. Let Q be the point on the circle dividing the line segment SP internally. Then **[JEE (Advanced) 2016, Paper-2, (3, –1)/60]**

(A) SP = $2\sqrt{5}$

(B) SQ : QP = $\left(\sqrt{5}+1\right) : 2$

(C) the x-intercept of the normal to the parabola at P is 6

(D) the slope of the tangent to the circle at Q is $\frac{1}{2}$

34. If $2x - y + 1 = 0$ is a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{16} = 1$, then which of the following CANNOT be sides of a right angled triangle ? **[JEE(Advanced) 2017, Paper-1,(4, –2)/61]**

(A) a, 4, 1 (B) 2a, 4, 1 (C) a, 4, 2 (D) 2a, 8, 1

35. If a chord, which is not a tangent, of the parabola $y^2 = 16x$ has the equation $2x + y = p$, and midpoint (h, k), then which of the following is(are) possible value(s) of p, h and k?

[JEE(Advanced) 2017, Paper-1,(4, –2)/61]

(A) p = –1, h = 1, k = –3 (B) p = 2, h = 3, k = –4

(C) p = –2, h = 2, k = –4 (D) p = 5, h = 4, k = –3

Answer Q.36, Q.37 and Q.38 by appropriately matching the information given in the three columns of the following table. **[JEE(Advanced) 2017, Paper-1,(3, –1)/61]**

Columns 1, 2 and 3 contain conics, equations of tangents to the conics and points of contact, repectively.

Column-1	Column-2	Column-3
(I) $x^2 + y^2 = a^2$	(i) $my = m^2x + a$	(P) $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$
(II) $x^2 + a^2y^2 = a^2$	(ii) $y = mx + a\sqrt{m^2+1}$	(Q) $\left(\frac{-ma}{\sqrt{m^2+1}}, \frac{a}{\sqrt{m^2+1}}\right)$
(III) $y^2 = 4ax$	(iii) $y = mx + \sqrt{a^2m^2-1}$	(R) $\left(\frac{-a^2m}{\sqrt{a^2m^2+1}}, \frac{1}{\sqrt{a^2m^2+1}}\right)$
(IV) $x^2 - a^2y^2 = a^2$	(iv) $y = mx + \sqrt{a^2m^2+1}$	(S) $\left(\frac{-a^2m}{\sqrt{a^2m^2-1}}, \frac{-1}{\sqrt{a^2m^2-1}}\right)$

36. For $a = \sqrt{2}$, if a tangent is drawn to a suitable conic (Column 1) at the point of contact (–1, 1), then which of the following options is the only CORRECT combination for obtaining its equation ?

[JEE(Advanced) 2017, Paper-1,(3, –1)/61]

(A) (I) (ii) (Q) (B) (I) (i) (P) (C) (III) (i) (P) (D) (II) (ii) (Q)

37. The tangent to a suitable conic (Column 1) at $\left(\sqrt{3}, \frac{1}{2}\right)$ is found to be $\sqrt{3}x + 2y = 4$, then which of the following options is the only CORRECT combination? **[JEE(Advanced) 2017, Paper-1,(3, –1)/61]**

(A) (IV) (iv) (S) (B) (II) (iv) (R) (C) (IV) (iii) (S) (D) (II) (iii) (R)

38. If a tangent to a suitable conic (Column 1) is found to be y = x + 8 and its point of contact is (8, 16), then which of the following options is the only CORRECT combination?

[JEE(Advanced) 2017, Paper-1,(3, –1)/61]

(A) (III) (i) (P) (B) (I) (ii) (Q) (C) (II) (iv) (R) (D) (III) (ii) (Q)

39. Consider two straight lines, each of which is tangent to both the circle $x^2 + y^2 = \frac{1}{2}$ and the parabola $y^2 = 4x$. Let these lines intersect at the point Q. Consider the ellipse whose center is at the origin O(0, 0) and whose semi-major axis is OQ. If the length of the minor axis of this ellipse is $\sqrt{2}$, then which of the following statement(s) is (are) TRUE ? **[JEE(Advanced) 2018, Paper-2,(3, –1)/60]**

(A) For the ellipse, the eccentricity is $\frac{1}{\sqrt{2}}$ and the length of the latus rectum is 1

(B) For the ellipse, the eccentricity is $\frac{1}{2}$ and the length of the latus rectum is $\frac{1}{2}$

(C) The area of the region bounded by the ellipse between the lines $x = \frac{1}{\sqrt{2}}$ and x = 1 is $\frac{1}{4\sqrt{2}}(\pi - 2)$

(D) The area of the region bounded by the ellipse between the lines $x = \frac{1}{\sqrt{2}}$ and x = 1 is $\frac{1}{16}(\pi - 2)$

40. Let H : $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, where a > b > 0, be a hyperbola in the xy-plane whose conjugate axis LM subtends an angle of 60° at one of its vertices N. Let the area of the triangle LMN be $4\sqrt{3}$.

[JEE(Advanced) 2018, Paper-2,(3, –1)/60]

LIST-I	**LIST-II**
(P) The length of the conjugate axis of H is	(1) 8
(Q) The eccentricity of H is	(2) $\frac{4}{\sqrt{3}}$
(R) The distance between the foci of H is	(3) $\frac{2}{\sqrt{3}}$
(S) The length of the latus rectum of H is	(4) 4

The correct option is:
(A) P → 4; Q → 2; R → 1; S → 3
(B) P → 4; Q → 3; R → 1; S → 2
(C) P → 4; Q → 1; R → 3; S → 2
(D) P → 3; Q → 4; R → 2; S → 1

PART - II : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. The ellipse $x^2 + 4y^2 = 4$ is inscribed in a rectangle alingent with the coordinate axes, which in turn is inscribed in another ellipse that passes through the point (4, 0). Then the equation of the ellipse is : **[AIEEE 2009 (4, –1), 144]**

(1) $x^2 + 12y^2 = 16$ (2) $4x^2 + 48y^2 = 48$ (3) $4x^2 + 64y^2 = 48$ (4) $x^2 + 16y^2 = 16$

2*. Equation of the ellipse whose axes are the axes of coordinates and which passes through the point (–3, 1) and has eccentricity $\sqrt{\frac{2}{5}}$ is : **[AIEEE 2011, I, (4, –1), 120]**

(1) $3x^2 + 5y^2 - 32 = 0$ (2) $5x^2 + 3y^2 - 48 = 0$ (3) $3x^2 + 5y^2 - 15 = 0$ (4) $5x^2 + 3y^2 - 32 = 0$

3. The equation of the hyperbola whose foci are (–2, 0) and (2, 0) and eccentricity is 2 is given by : **[AIEEE 2011, II, (4, –1), 120]**

(1) $x^2 - 3y^2 = 3$ (2) $3x^2 - y^2 = 3$ (3) $-x^2 + 3y^2 = 3$ (4) $-3x^2 + y^2 = 3$

4. **Statement-1 :** An equation of a common tangent to the parabola $y^2 = 16\sqrt{3}x$ and the ellipse $2x^2 + y^2 = 4$ is $y = 2x + 2\sqrt{3}$. **[AIEEE - 2013, (4, – 1) 120]**

Statement-2 : If the line $y = mx + \frac{4\sqrt{3}}{m}$, $(m \neq 0)$ is a common tangent to the parabola $y^2 = 16\sqrt{3}x$ and the ellipse $2x^2 + y^2 = 4$, then m satisfies $m^4 + 2m^2 = 24$.

(1) Statement-1 is false, Statement-2 is true.
(2) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for Statement-1.
(3) Statement-1 is true, statement-2 is true; statement-2 is **not** a correct explanation for Statement-1.
(4) Statement-1 is true, statement-2 is false.

5. An ellipse is drawn by taking a diameter of the circle $(x - 1)^2 + y^2 = 1$ as its semi-minor axis and a diameter of the circle $x^2 + (y - 2)^2 = 4$ is semi-major axis. If the centre of the ellipse is at the origin and its axes are the coordinate axes, then the equation of the ellipse is : **[AIEEE-2012, (4, –1)/120]**

(1) $4x^2 + y^2 = 4$ (2) $x^2 + 4y^2 = 8$ (3) $4x^2 + y^2 = 8$ (4) $x^2 + 4y^2 = 16$

6. The equation of the circle passing through the foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$, and having centre at (0, 3) is **[AIEEE - 2013, (4, – 1)]**

(1) $x^2 + y^2 - 6y - 7 = 0$ (2) $x^2 + y^2 - 6y + 7 = 0$
(3) $x^2 + y^2 - 6y - 5 = 0$ (4) $x^2 + y^2 - 6y + 5 = 0$

7. The locus of the foot of perpendicular drawn from the centre of the ellipse $x^2 + 3y^2 = 6$ on any tangent to it is: **[JEE(Main) 2014, (4, – 1), 120]**

(1) $(x^2 + y^2)^2 = 6x^2 + 2y^2$ (2) $(x^2 + y^2)^2 = 6x^2 - 2y^2$ (3) $(x^2 - y^2)^2 = 6x^2 + 2y^2$ (4) $(x^2 - y^2)^2 = 6x^2 - 2y^2$

8. The slope of the line touching both the parabolas $y^2 = 4x$ and $x^2 = -32y$ is : **[JEE(Main) 2014, (4, – 1), 120]**

(1) $\frac{1}{8}$ (2) $\frac{2}{3}$ (3) $\frac{1}{2}$ (4) $\frac{3}{2}$

9. The area (in sq.units) of the quadrilateral formed by the tangents at the end points of the latera recta to the ellipse , $\frac{x^2}{9} + \frac{y^2}{5} = 1$ is **[JEE(Main) 2015, (4, – 1), 120]**

(1) $\frac{27}{4}$ (2) 18 (3) $\frac{27}{2}$ (4) 27

10. Let O be the vertex and Q be any point on the parabola, $x^2 = 8y$. If the point P divides the line segment OQ internally in the ratio 1 : 3, then the locus of P is **[JEE(Main) 2015, (4, – 1), 120]**

(1) $x^2 = y$ (2) $y^2 = x$ (3) $y^2 = 2x$ (4) $x^2 = 2y$

11. Let P be the point on the parabola, $y^2 = 8x$ which is at a minimum distance from the centre C of the circle, $x^2 + (y + 6)^2 = 1$. Then the equation of the circle, passing through C and having its centre at P is : **[JEE(Main) 2016, (4, – 1), 120]**

(1) $x^2 + y^2 - x + 4y - 12 = 0$ (2) $x^2 + y^2 - \frac{x}{4} + 2y - 24 = 0$
(3) $x^2 + y^2 - 4x + 9y + 18 = 0$ (4) $x^2 + y^2 - 4x + 8y + 12 = 0$

12. The eccentricity of the hyperbola whose length of the latus rectum is equal to 8 and the length of its conjugate axis is equal to half of the distance between its foci, is : **[JEE(Main) 2016, (4, – 1), 120]**

(1) $\frac{4}{\sqrt{3}}$ (2) $\frac{2}{\sqrt{3}}$ (3) $\sqrt{3}$ (4) $\frac{4}{3}$

13. The eccentricity of an ellipse whose centre is at the origin is $\frac{1}{2}$. If one of its directrices is $x = -4$, then the equation of the normal to it at $\left(1, \frac{3}{2}\right)$ is **[JEE(Main) 2017, (4, – 1), 120]**

(1) $2y - x = 2$ (2) $4x - 2y = 1$ (3) $4x + 2y = 7$ (4) $x + 2y = 4$

14. A hyperbola passes through the point P($\sqrt{2}$, $\sqrt{3}$) and has foci at (±2, 0). Then the tangent to this hyperbola at P also passes through the point : **[JEE(Main) 2017, (4, – 1), 120]**

(1) $(3\sqrt{2}, 2\sqrt{3})$ (2) $(2\sqrt{2}, 3\sqrt{3})$ (3) $(\sqrt{3}, \sqrt{2})$ (4) $(-\sqrt{2}, -\sqrt{3})$

15. If the tangent at (1, 7) to the curve $x^2 = y - 6$ touches the circle $x^2 + y^2 + 16x + 12y + c = 0$ then the value of c is : **[JEE(Main) 2018, (4, – 1), 120]**

(1) 85 (2) 95 (3) 195 (4) 185

16. Tangents are drawn to the hyperbola $4x^2 - y^2 = 36$ at the points P and Q. If these tangents intersect at the point T(0, 3) then the area (in sq. units) of ΔPTQ is : **[JEE(Main) 2018, (4, – 1), 120]**

(1) $60\sqrt{3}$ (2) $36\sqrt{5}$ (3) $45\sqrt{5}$ (4) $54\sqrt{3}$

17. Tangent and normal are drawn at P(16,16) on the parabola $y^2 = 16x$, which intersect the axis of the parabola at A and B, respectively. If C is the centre of the circle through the points P, A and B and $\angle CPB = \theta$, then a value of $\tan\theta$ is : **[JEE(Main) 2018, (4, – 1), 120]**

(1) 3 (2) $\frac{4}{3}$ (3) $\frac{1}{2}$ (4) 2

18. Equation of a common tangent to the parabola $y^2 = 4x$ and the hyperbola xy = 2 is :
[JEE(Main) 2019, Online (11-01-19),P-1 (4, – 1), 120]

(1) x + 2y + 4 = 0 (2) x – 2y + 4 = 0 (3) 4x + 2y + 1 = 0 (4) x + y + 1 = 0

19. If the parabolas $y^2 = 4b(x - c)$ and $y^2 = 8ax$ have a common normal, then which one of the following is a valid choice for the ordered triad (a, b, c) ?
[JEE(Main) 2019, Online (10-01-19),P-1 (4, – 1), 120]

(1) (1, 1, 3) (2) $\left(\frac{1}{2}, 2, 3\right)$ (3) $\left(\frac{1}{2}, 2, 0\right)$ (4) (1, 1, 0)

20. The length of the chord of the parabola $x^2 = 4y$ having equation $x - \sqrt{2}y + 4\sqrt{2} = 0$ is :
[JEE(Main) 2019, Online (10-01-19),P-2 (4, – 1), 120]

(1) $6\sqrt{3}$ (2) $8\sqrt{2}$ (3) $3\sqrt{2}$ (4) $2\sqrt{11}$

21. Let S = $\left\{(x,y) \in R^2 : \frac{y^2}{1+r} - \frac{x^2}{1-r} = 1\right\}$, where r ≠ ± 1. Then S represents :

(1) a hyperbola whose eccentricity $\frac{2}{\sqrt{1-r}}$, when 0 < r < 1.

(2) an ellipse whose eccentricity is $\sqrt{\frac{2}{r+1}}$, when r > 1.

(3) a hyperbola whose eccentricity is $\frac{2}{\sqrt{r+1}}$, when 0 < r < 1.

(4) an ellipse whose eccentricity is $\frac{1}{\sqrt{r+1}}$, when r > 1.

[JEE(Main) 2019, Online (10-01-19),P-2 (4, – 1), 120]

22. If tangents are drawn to the ellipse $x^2 + 2y^2 = 2$ at all points on the ellipse other than its four vertices then the mid points of the tangents intercepted between the coordinate axes lie on the curve :
[JEE(Main) 2019, Online (11-01-19),P-1 (4, – 1), 120]

(1) $\frac{x^2}{4} + \frac{y^2}{2} = 1$ (2) $\frac{1}{2x^2} + \frac{1}{4y^2} = 1$ (3) $\frac{1}{4x^2} + \frac{1}{2y^2} = 1$ (4) $\frac{x^2}{2} + \frac{y^2}{4} = 1$

23. Let S and S' be the foci of an ellipse and B be any one of the extremities of its minor axis. If ΔS'BS is a right angled triangle with right angle at B and are (ΔS'BS) = 8 sq. units, then the length of a latus
[JEE(Main) 2019, Online (12-01-19),P-2 (4, – 1), 120]

(1) 2 (2) $4\sqrt{2}$ (3) 4 (4) $2\sqrt{2}$

Answers

EXERCISE # 1

PART-I

Section (A) :

A-1. 4

A-2. (i) vertex $\equiv \left(\frac{3}{2}, -\frac{11}{8}\right)$, focus $\left(\frac{3}{2}, -\frac{15}{8}\right)$

axis x = $\frac{3}{2}$, directrix y = $-\frac{7}{8}$, length of latus rectum = 2.

(ii) $x^2 - 2xy + y^2 - 6x - 6y + 3 = 0$

(iii) $4x^2 - 4xy + y^2 + 8x + 46y - 71 = 0$

(iv) $y = 5/4$

A-3. $(y + 1)^2 = 3(2x + 1)$ & $(y + 1)^2 = -3(2x - 5)$

A-4. $y = \frac{2}{3}$, $\left(-\frac{61}{16}, \frac{2}{3}\right)$, $\left(-\frac{485}{144}, \frac{2}{3}\right)$, $x = \frac{-613}{144}, x = -\frac{485}{144}$

A-5. $\sqrt{x^2 + y^2} + |x - 2| = 4$

A-6. $\alpha \in [0, 1]$

A-7. $\alpha \in [\pi/2, 5\pi/6] \cup [\pi, 3\pi/2]$

Section (B) :

B-1. $\left(e = \frac{1}{2}\right)$

B-2 ((2, 3) & (6, 7))

B-3 $\left(\frac{12}{5}, \frac{16}{5}\right)$

B-4. $(x = 3 + 5\cos\theta, y = -2 + 4\sin\theta)$

B-5. $\frac{4}{5} < \alpha < \frac{4}{\sqrt{17}}$

B-6. $3x^2 + 4y^2 - 12x + 24y + 36 = 0$

B-7. $5x^2 + 9y^2 - 54y + 36 = 0$

B-8. (i) Centre (–1, 2), $e = \frac{5}{3}$, foci = (4, 2), (–6, 2), $x = -\frac{14}{5}$ and $x = \frac{4}{5}$

(ii) $7x^2 - 2y^2 + 12xy - 2x + 14y - 22 = 0$

B-10. $3x^2 - y^2 - 12 = 0$

B-11. (i) (4, 1), (–6, 1) (ii) $\frac{(x+2)^2}{16} - \frac{(y-3)^2}{48} = 1$

B-12. Inside

B-13. $(x - 3)^2 + (y - 4)^2 = \left(\frac{5 \pm \sqrt{5}}{2}\right)^2$

B-14. $\frac{\sqrt{13}}{3}$

B-19. $\frac{\sqrt{7}}{4}$

B-20. 16

Section (C) :

C-1. $8\sqrt{2}$

C-2. (4, 0), (–4, 0)

C-3. $y^2 - 2ax + 8a^2 = 0$

C-4. $y = x^2 + 2$

C-6. $(-\infty, -4] \cup [4, \infty)$

C-7. $x - 4y + 28 = 0$ at (28, 14)

C-8. $\sqrt{\frac{c}{a}}$

Section (D) :

D-1. $\sqrt{35}$ **D-2.** $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{y}{b}$ **D-3.** (yes, (5, 4))

D-4. $x^2 + 64y^2 = 80$ & $x^2 + 4y^2 = 20$ **D-5.** $ex \pm y = a, -ex \pm y = a$

D-7. (3, 0) **D-8.** $16x^2 + 9y^2 + 96x = 432$ or $16x^2 + 9y^2 - 288x + 720 = 0$

D-9. $\frac{8}{3}\sqrt{10}$ **D-10.** $\lambda = \pm 6$ **D-11.** $n = \pm 2$ **D-12.** $x + y \pm 3\sqrt{3} = 0$

D-13. $x^2 - y^2 - 4x = 0$ **D-14.** 10 **D-15.** 30

Section (E) :

E-1. $4y = 9x + 4, 4y = x + 36$ **E-2.** (i) $y = (x - a) \tan \alpha$, (ii) $x = a$, (iii) $y = \lambda x$

E-3. (–2, 0) **E-4.** –2 **E-5.** $y^2 = 2x - 2$

Section (F):

F-1. $4x + 5y = 40, 4x - 35y = 200.$ **F-2.** $\left(\frac{25}{4}, \frac{16}{3}\right)$ **F-3.** $48x + 25y - 169 = 0$

F-4. **(i)** –16/9 **(ii)** –20/9 **F-5.** $y = \frac{5}{12}x + \frac{3}{4}$; $x - 3 = 0$; 8 sq. unit

F-6. $(x^2 + y^2)^2 = 16x^2 - 9y^2$ **F-8.** $a^2p^2 + b^2q^2 = r^2\sec^2 \frac{\pi}{8} = (4 - 2\sqrt{2})\, r^2$

Section (G) :

G-1. $x + y = 3$ **G-3.** $x + 3y = 33$ **G-5.** (4a, 4a) **G-6.** $x + 2y - 3 = 0$

Section (H) :

H-2. $12x + 5y = 48$; $12x - 5y = 48$

Section (I) :

I-1. $(x - 2)^2 + (y - 4)^2 + 2(x - y + 2) = 0$ **I-2.** 2 **I-3.** $b = \sqrt{3}$; $x + 2y + 4 = 0$

I-4. $\frac{2\sqrt{2}}{3}$ **I-5.** $y = x \pm \sqrt{10}$, $y = -x \pm \sqrt{10}$ **I-6.** $\frac{64}{75}$

PART - II

Section (A) :

A-1. (D) **A-2.** (C) **A-3.** (D) **A-4.** (A) **A-5.** (B) **A-6.** (B) **A-7.** (D)

A-8. (B) **A-9.** (A) **A-10.** (D) **A-11.** (D) **A-12.** (C) **A-13.** (B)

Section (B) :

B-1.	(A)	**B-2.**	(B)	**B-3.**	(D)	**B-4.**	(C)	**B-5.**	(A)	**B-6.**	(C)	**B-7.**	(D)
B-8.	(B)	**B-9.**	(C)	**B-10.**	(A)	**B-11.**	(B)	**B-12.**	(C)	**B-13.**	(A)	**B-14.**	(D)
B-15.	(C)	**B-16.**	(C)	**B-17.**	(A)	**B-18.**	(A)	**B-19.**	(D)	**B-20.**	(C)	**B-21.**	(B)
B-22.	(C)												

Section (C) :

C-1.	(D)	**C-2.**	(A)	**C-3.**	(B)	**C-4.**	(B)	**C-5.**	(D)	**C-6.**	(C)	**C-7.**	(B)
C-8.	(A)												

Section (D) :

D-1.	(B)	**D-2.**	(C)	**D-3.**	(B)	**D-4.**	(C)	**D-5.**	(A)	**D-6.**	(A)	**D-7.**	(B)
D-8.	(B)												

Section (E) :

E-1.	(B)	**E-2.**	(C)	**E-3.**	(B)

Section (F) :

F-1.	(A)	**F-2.**	(B)	**F-3**	(D)	**F-4.**	(A)	**F-5.**	(B)	**F-6.**	(D)	**F-7.**	(A)
F-8.	(D)	**F-9.**	(C)										

Section (G) :

G-1.	(B)	**G-2.**	(A)	**G-3.**	(D)	**G-4.**	(C)	**G-5.**	(B)	**G-6.**	(D)	**G-7.**	(D)
G-8.	(A)												

Section (H) :

H-1.	(D)	**H-2.**	(A)	**H-3.**	(D)	**H-4.** (A)	**H-5.**	(A)

Section (I) :

I-1.	(A)	**I-2.**	(B)	**I-3.**	(C)	**I-4.**	(A)	**I-5.**	(D)	**I-6.**	(C)	**I-7.**	(A)
I-8.	(A)												

PART - III

1.	(A) $\rightarrow$ (q),	(B) $\rightarrow$ (r),	(C) $\rightarrow$ (s),	(D) $\rightarrow$ (q)
2.	(A) $\rightarrow$ (s),	(B) $\rightarrow$ (r),	(C) $\rightarrow$ (q),	(D) $\rightarrow$ (p)
3.	(A) $\rightarrow$ (r),	(B) $\rightarrow$ (p),	(C) $\rightarrow$ (s),	(D) $\rightarrow$ (q)
4.	(A) $\rightarrow$ (q),	(B) $\rightarrow$ (s),	(C) $\rightarrow$ (s),	(D) $\rightarrow$ (r)

EXERCISE # 2

PART-I

1.	(B)	**2.**	(C)	**3.**	(C)	**4.**	(C)	**5.**	(B)	**6.**	(C)	**7.**	(B)
8.	(C)	**9.**	(B)	**10.**	(B)	**11.**	(B)	**12.**	(C)	**13.**	(D)	**14.**	(C)
15.	(A)	**16.**	(D)	**17.**	(A)	**18.**	(A)	**19.**	(D)	**20.**	(D)	**21.**	(C)
22.	(C)	**23.**	(A)	**24.**	(B)	**25.**	(A)	**26.**	(D)	**27.**	(A)	**28.**	(A)

29.	(A)	**30.**	(C)

PART-II

1.	1	**2.**	3	**3.**	0	**4.**	2	**5.**	18	**6.**	16	**7.**	23
8.	3	**9.**	4	**10.**	3	**11.**	2	**12.**	2	**13.**	3	**14.**	13
15.	65	**16.**	21	**17.**	15	**18.**	9	**19.**	14	**20.**	55	**21.**	7
22.	9	**23.**	13	**24.**	1	**25.**	77	**26.**	57	**27.**	22	**28.**	4
29.	0025	**30.**	6	**31.**	16	**32.**	6	**33.**	6	**34.**	50	**35.**	3

PART-III

1.	(AB)	**2.**	(ABCD)	**3.**	(AD)	**4.**	(AD)	**5.**	(AD)	**6.**	(AC)	**7.**	(BC)
8.	(ABCD)	**9.**	(ABC)	**10.**	(AC)	**11.**	(AC)	**12.**	(AB)	**13.**	(AD)	**14.**	(AD)
15.	(AB)	**16.**	(CD)	**17.**	(BCD)	**18.**	(ACD)	**19.**	(AB)	**20.**	(BD)		
21.	(ABCD)	**22.**	(ACD)	**23.**	(ABC)	**24.**	(AD)	**25.**	(ABD)	**26.**	(AB)		

PART - IV

1.	(A)	**2.**	(B)	**3.**	(C)	**4.**	(B)	**5.**	(A)	**6.**	(C)	**7.**	(C)
8.	(B)	**9.**	(A)	**10.**	(B)	**11.**	(A)	**12.**	(D)	**13.**	(C)	**14.**	(A)
15.	(A)	**16.**	(A)										

EXERCISE # 3

PART-I

1*.	(AD)	**2.**	(D)	**3*.**	(BC)	**4.**	(C)						
5.	(A) $\rightarrow$ (p),		(B) $\rightarrow$ (s, t),		(C) $\rightarrow$ (r),		(D) $\rightarrow$ (q, s)			**6*.**	(AB)	**7*.**	(CD)
8.	(D)	**9.**	(C)	**10.**	(A)	**11.**	(B)	**12.**	(A)	**13.**	2	**14.**	(2)
15.	(C)	**16*.**	(ABD)	**17*.**	(BD)	**18.**	(B)	**19.**	(C)	**20.**	(AB)	**21.**	(4)
22.	(B)	**23.**	(D)	**24.**	(D)	**25.**	(D)	**26.**	(B)	**27.**	4	**28.**	2
29*.	(AD)	**30.**	4	**31*.**	(AB)	**32*.**	(ABD)	**33.**	(ACD)	**34.**	(ACD)	**35.**	(B)
36.	(A)	**37.**	(B)	**38.**	(A)	**39.**	(AC)	**40.**	(B)				

PART - II

1.	(1)	**2*.**	(1,2)	**3.**	(2)	**4.**	(2)	**5.**	(4)	**6.**	(1)	**7.**	(1)
8.	(3)	**9.**	(4)	**10.**	(4)	**11.**	(4)	**12.**	(2)	**13.**	(2)	**14.**	(2)
15.	(2)	**16.**	(3)	**17.**	(4)	**18.**	(1)	**19.**	(1)	**20.**	(1)	**21.**	(2)
22.	(2)	**23.**	(3)										

Advance Level Problems (ALP)

SUBJECTIVE QUESTIONS

1. Prove that in a parabola the angle θ that the latus rectum subtends at the vertex of the parabola isindependent of the latus rectum and lies between $\frac{2\pi}{3}$ & $\frac{3\pi}{4}$

2. A parabola is drawn to pass through A and B, the ends of a diameter of a given circle of radius a, and to have as directrix a tangent to a concentric circle of radius b; then axes being AB and a perpendicular diameter, prove that the locus of the focus of the parabola is $\frac{x^2}{b^2} + \frac{y^2}{b^2 - a^2} = 1$

3. Find the points of intersection of the curves whose parametric equations are $x = t^2 + 1$, $y = 2t$ and $x = 2s$, $y = 2/s$.

4. If r_1, r_2 be the length of the perpendicular chords of the parabola $y^2 = 4ax$ drawn through the vertex, then show that $(r_1 r_2)^{4/3} = 16a^2(r_1^{2/3} + r_2^{2/3})$.

5. Prove that the circle circumscribing the triangle formed by any three tangents to a parabola passes through the focus.

6. A chord is a normal to a parabola and is inclined at an angle θ to the axis; prove that the area of the triangle formed by it and the tangents at its extremities is $4a^2 \sec^3\theta \operatorname{cosec}^3\theta$

7. From an external point P, pair of tangent lines are drawn to the parabola, $y^2 = 4x$. If θ_1 & θ_2 are the inclinations of these tangents with the axis of x such that, $\theta_1 + \theta_2 = \frac{\pi}{4}$, then find the locus of P .

8. TP and TQ are tangents to the parabola and the normals at P and Q meet at a point R on the curve ; prove that the centre of the circle circumscribing the triangle TPQ lies on the parabola $2y^2 = a(x - a)$.

9. From an external point P, tangents are drawn to the parabola; find the equation of the locus of P when these tangents make angles θ_1 and θ_2 with the axis, such that $\cos\theta_1 \cos\theta_2 = \mu$, which is constant.

10. A pair of tangents are drawn to the parabola which are equally inclined to a straight line whose inclination to the axis is α ; prove that the locus of their point of intersection is the straight line $y = (x - a)\tan 2\alpha$.

11. Prove that the normals at the points, where the straight line $\ell x + my = 1$ meets the parabola $y^2 = 4ax$, meet on the normal at the point $\left(\frac{4am^2}{\ell^2}, \frac{4am}{\ell}\right)$ on the parabola.

12. Prove that the equation to the circle, which passes through the focus and touches the parabola $y^2 = 4ax$ at the point $(at^2, 2at)$, is $x^2 + y^2 - ax(3t^2 + 1) - ay(3t - t^3) + 3a^2t^2 = 0$.
Prove also that the locus of its centre is the curve $27ay^2 = (2x - a)(x - 5a)^2$.

13. Two tangents to the parabola $y^2 = 8x$ meet the tangent at its vertex in the points P & Q. If PQ = 4 units, prove that the locus of the point of the intersection of the two tangents is $y^2 = 8(x + 2)$.

14. Find locus of a point P if the three normals drawn from it to the parabola $y^2 = 4ax$ are such that two of them make complementry angles with the axis of the parabola

15. Prove that the orthocentre of any triangle formed by three tangents to a parabola lies on the directrix.

16. If tangent drawn at a point $(t^2, 2t)$ on the parabola $y^2 = 4x$ is same as the normal drawn at a point $(\sqrt{5}\cos\phi, 2\sin\phi)$ on the ellipse $4x^2 + 5y^2 = 20$. Find the values of t & ϕ.

17. Find the locus of centre of a family of circles passing through the vertex of the parabola $y^2 = 4ax$, and cutting the parabola orthogonally at the other point of intersection.

18. Let A, B, C be three points on the parabola $y^2 = 4ax$. If the orthocentre of the triangle ABC is at the focus then show that the circumcircle of ΔABC touches the y-axis.

19. If α, β are eccentric angles of the extremities of a focal chord of an ellipse, then eccentricity of the ellipse is

20. If circumcentre of an equilateral triangle inscribed in $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, with vertices having eccentric angles α, β, γ respectively is (x_1, y_1), then find $\Sigma \cos\alpha \cos\beta + \Sigma \sin\alpha \sin\beta$.

21. Find the locus of extremities of latus rectum of the family of ellipse $b^2x^2 + y^2 = a^2b^2$ where b is a parameter $(b^2 < 1)$.

22. A point moves such that the sum of the square of the distances from two fixed straight lines intersecting at angle 2α is a constant. Prove that the locus is an ellipse of eccentricity
$\frac{\sqrt{\cos 2\alpha}}{\cos\alpha}$ if $\alpha < \frac{\pi}{4}$ and $\frac{\sqrt{-\cos 2\alpha}}{\sin\alpha}$ if $\alpha > \frac{\pi}{4}$

23. A straight line PQ touches the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the circle $x^2 + y^2 = r^2 (b < r < a)$. RS is a focal chord of the ellipse. If RS is parallel to PQ and meets the circle at points R and S. Find the length of RS.

24. Prove that the sum of the eccentric angles of the extremities of a chord of an ellipse, which is drawn in a given direction is constant and is equal to twice the eccentric angle of the point at which the tangent is parallel to the given direction.

25. If the normals at $\alpha, \beta, \gamma, \delta$ on an ellipse are concurrent, prove that $(\Sigma\cos\alpha)(\Sigma\sec\alpha) = 4$
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26. Show that the equation of the pair of tangents to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at the points of intersection with the line, $px + qy + 1 = 0$ is $\left(\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1\right) \cdot (p^2a^2 + q^2b^2 - 1) = (px + qy + 1)^2$.

27. A tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ meets the ellipse $\frac{x^2}{a} + \frac{y^2}{b} = a + b$
at the points P and Q; prove that the tangents at P and Q are at right angles.

28. Find the locus of the point, the chord of contact of the tangents drawn from which to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ touches the circle $x^2 + y^2 = c^2$, where $c < b < a$.

29. A chord of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ whose eccentric angles of extremities are α and β, intersects its director circle at point A and B. Tangents at A and B intersect at point P. Find the equation of circumcircle of triangle ABP.

30. A tangent is drawn at any fixed point P on the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ and if chord of contact of the ellipse $\frac{x^2}{9}+\frac{y^2}{16}=1$ with respect to any point on this tangent passes through a fixed point, then prove that the line joining this fixed point to the point P never subtends right angle at the origin.

31. If the parabola $y^2 = 4ax$ cuts the ellipse $\frac{(x-a)^2}{a^2}+\frac{y^2}{b^2}=1$ in three distinct points then show that the eccentricity of the ellipse e belongs to $\left(\frac{1}{\sqrt{2}},\ 1\right)$.

32. Find the number of integral points lying on or inside the ellipse $2x^2 + 6xy + 6y^2 - 1 = 0$.

33. The equations of the transverse and conjugate axes of a hyperbola are respectively $x + 2y - 3 = 0$, $2x - y + 4 = 0$, and their respective lengths are $\sqrt{2}$ and $2/\sqrt{3}$. Then find the equation of the hyperbola .

34. If P is any point common to the hyperbola $\frac{x^2}{16}-\frac{y^2}{25}=1$ and the circle having line segment joining its focii as diameter then find the sum of focal distances of point P.

35. The transverse axis of a hyperbola is of length 2a and a vertex divides the segment of the axis between the centre and the corresponding focus in the ratio 2 : 1. Find the equation of the hyperbola.

36. If $x \cos \alpha + y \sin \alpha = p$, a variable chord of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{2a^2} = 1$ subtends a right angle at the centre of the hyperbola, then the chords touch a fixed circle, find the radius of the circle.

37. If the distance between the centres of the hyperbolas :
$x^2 - 16xy - 11y^2 - 12x + 6y + 21 = 0$ ..... (i)
$9x^2 - 16y^2 - 18x - 32y - 151 = 0$ ..... (ii) is d then $125\, d^2 =$ ....

38. Find an equation of the hyperbola whose directrix is the normal to circle $x^2 + y^2 - 4x - 6y + 9 = 0$ having slope is 2 and eccentricity is equal to radius of given circle where focus of hyperbola is point of contact of given circle with y-axis.

39. PQ is the chord joining the points whose eccentric angles are ϕ_1 and ϕ_2 on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2} = 1$, If $\phi_1 - \phi_2 = 2\alpha$, where α is constant, prove that PQ touches the hyperbola $\cos^2\alpha\ \frac{x^2}{a^2}-\frac{y^2}{b^2} = 1$.

40. Find the locus of the mid-points of the chord of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2} = 1$ which subtend a right angle at the origin is

41. If a chord joining the points P $(a \sec \theta, a \tan \theta)$ & Q $(a \sec \phi, a \tan \phi)$ on the hyperbola $x^2 - y^2 = a^2$ is a normal to it at P, then show that $\tan \phi = \tan \theta\,(4 \sec^2 \theta - 1)$.

42. Chords of the hyperbola $x^2/a^2 - y^2/b^2 = 1$ are tangents to the circle drawn on the line joining the foci as diameter . Find the locus of the point of intersection of tangents at the extremities of the chords .

43. From any point on the hyperbola $H_1 : \frac{x^2}{a^2}-\frac{y^2}{b^2} = 1$ tangents are drawn to the hyperbola $H_2 : \frac{x^2}{a^2}-\frac{y^2}{b^2} = 2$. Then find the area cut-off by the chord of contact on the asymptotes of H_2 .

44. The chord PQ of the rectangular hyperbola $xy = a^2$ meets the x-axis at A; C is the mid point of PQ & 'O' is the origin. Then prove that the Δ ACO is isosceles.

45. If the normals at (x_i, y_i), i = 1, 2, 3, 4 on the rectangular hyperbola, $xy = c^2$, meet at the point (α, β) show that

(i) $\Sigma x_i = \alpha$ (ii) $\Sigma y_i = \beta$ (iii) $\Pi x_i = \Pi y_i = -c^4$,

(iv) $\Sigma x_i^2 = \alpha^2$ (v) $\Sigma y_i^2 = \beta^2$

46. If α, β, γ & δ be the eccentric angles of feet of four co–normal points of a hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ from any point in its plane then prove that $\alpha + \beta + \gamma + \delta$ is odd integral multiple of π.

47. Prove that a normal to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ cannot be normal to its conjugate hyperbola.

48. Let P be a point from where perpendicular tangents are drawn to the circle $2x^2 + 2y^2 - a^2 = 0$. Let a line from P perpendicular to OP is drawn which intersect hyperbola $x^2 - y^2 = a^2$ at Q and R. Find number of all possible positions of P such that product of ordinates of points Q and R is.

(i) $\frac{3}{2}a^2$ (ii) a^2 (iii) $\frac{a^2}{2}$

Answers

3. (2, 2) **7.** $x - y - 1 = 0$ **9.** $x^2 = \mu^2 \{(x - a)^2 + y^2\}$ **14.** $y^2 = a (x - a)$

16. $\phi = \pi - \tan^{-1}2,\ t = -\frac{1}{\sqrt{5}}$; $\phi = \pi + \tan^{-1}2,\ t = \frac{1}{\sqrt{5}}$; $\phi = \pm\frac{\pi}{2}$, t = 0

17. $2y^2(2y^2 + x^2 - 12ax) = ax(3x - 4a)^2$ **19.** $\frac{\sin\alpha + \sin\beta}{\sin(\alpha + \beta)}$ **20.** $\frac{9x_1^2}{2a^2} + \frac{9y_1^2}{2b^2} - \frac{3}{2}$

21. $x^2 \pm ay = a^2$ **23.** RS = 2b **28.** $\frac{x^2}{a^4} + \frac{y^2}{b^4} = \frac{1}{c^2}$

33. $\frac{2}{5}(2x - y + 4)^2 - \frac{3}{5}(x + 2y - 3)^2 = 1$ **34.** $2\sqrt{66}$ **35.** $5x^2 - 4y^2 = 5a^2$

36. $\sqrt{2}\,a$ **37.** 0025 **38.** $11x^2 - y^2 - 16xy - 16x + 38y - 41 = 0$

42. $\frac{x^2}{a^4} + \frac{y^2}{b^4} = \frac{1}{a^2 + b^2}$ **43.** $4ab$

48. (i) 4 (ii) 2 (iii) 0