Exercise-1

✍ **Marked questions are recommended for Revision.**

PART - I : SUBJECTIVE QUESTIONS

Section (A) : Integration using Standard Integral :

A-1. Integrate with respect to x :

(i) $(2x + 3)^5$ (ii) $\sin 2x$ (iii) $\sec^2 (4x + 5)$

(iv) $\sec (3x + 2)$ (v) $\tan (2x + 1)$ (vi) 2^{3x+4}

(vii) $\dfrac{1}{2x+1}$ (viii) e^{4x+5}

A-2. Integrate with respect to x :

(i) $\sin^2 x$ (ii) $\cos^3 x$ (iii) $\sin 2x \cos 3x$

(iv) $4\sin x \cos \dfrac{x}{2} \cos \dfrac{3x}{2}$ (v) $\dfrac{1}{\sqrt{x+3}-\sqrt{x+2}}$

Section (B) : Integration using Substitution :

B-1. Integrate with respect to x :

(i) $x \sin x^2$ (ii) $\dfrac{x}{x^2+1}$ (iii) $\sec^2 x \tan x$ (iv) $\dfrac{e^x+1}{e^x+x}$

(v) $\dfrac{1-\sin x}{x+\cos x}$ (vi) $\dfrac{e^{2x}}{e^{2x}-2}$ (vii) $\dfrac{\cos 2x + x + 1}{x^2+\sin 2x + 2x}$ (viii) $\dfrac{\sec x}{\ell n \ (\sec x + \tan x)}$

(ix) $\dfrac{x}{\sqrt{x+2}}$ (x) $\left(e^x + \dfrac{1}{e^x}\right)^2$ (xi) $(e^x + 1)^2 e^x$ (xii)✍ $\dfrac{1}{x\left(x^5+1\right)}$

(xiii)✍ $\dfrac{1}{x^5(1+x^5)^{\frac{1}{5}}}$ (xiv)✍ $\dfrac{\sqrt{x^2-8}}{x^4}$

B-2. Find the value of $\displaystyle\int \frac{d(x^2+1)}{\sqrt{(x^2+2)}}$.

B-3.✍ Evaluate the following :

(i) $\displaystyle\int\left(\frac{x\cos x - \sin x}{x \sin x}\right)dx$ (ii) $\displaystyle\int\left(\frac{\frac{x}{x+1}-\ln(x+1)}{x(\ln(x+1))}\right)dx$

Section (C) : Integration by parts :

C-1. Integrate with respect to x :

(i) $x \ \ell n \ x$ (ii) $x \sin^2 x$ (iii) $x \tan^{-1} x$ (iv) $\ell n \ x$

(v)✍ $\sec^3 x$ (vi) $2x^3 \ e^{x^2}$ (vii) $\sin^{-1}\sqrt{x}$ (viii) $\dfrac{x^2 \tan^{-1} x}{1+x^2}$

(ix) $e^x \sin x$ (x) $e^x (\sec^2 x + \tan x)$

C-2. Find the antiderivative of $f(x) = \ln (\ln x) + (\ln x)^{-2}$ whose graph passes through (e, e).

Section (D) : Algebraic integral :

D-1. Integrate with respect to x :

(i) $\dfrac{1}{x^2+4}$ (ii) $\dfrac{1}{x^2+5}$ (iii) $\dfrac{1}{x^2+2x+5}$

(iv) $\dfrac{2x+1}{x^2+3x+4}$ (v) $\dfrac{x^3-1}{x^3+x}$ (vi) $\dfrac{1}{\sqrt{x^2-4}}$

(vii) $\sqrt{x^2+4}$ (viii) $\sqrt{x^2+2x+5}$ (ix) $(x-1)\sqrt{1-x-x^2}$

(x) $x^5\sqrt{a^3+x^3}$

D-2. Integrate with respect to x :

(i) $\dfrac{1}{(x+1)(x+2)}$ (ii) $\dfrac{1}{(x^2+1)(x+3)}$

(iii) $\dfrac{3x+2}{(x+1)^2(x+2)}$ (iv) $\dfrac{1}{(x+1)(x+2)(x+3)}$

D-3. Integrate with respect to x :

(i) $\dfrac{1}{x^4+x^2+1}$ (ii) $\dfrac{1+x^2}{1+x^4}$ (iii) $\dfrac{1-x^2}{1-x^2+x^4}$

D-4. Integrate with respect to x :

(i) $\dfrac{1}{(x+1)\sqrt{x+2}}$ (ii) $\dfrac{1}{(x^2-4)\sqrt{x+1}}$

(iii) $\dfrac{1}{(x+1)\sqrt{x^2+2}}$ (iv) $\dfrac{1}{(x^2+1)\sqrt{x^2+2}}$

D-5. **Evaluate the following :**

(i) $\displaystyle\int\sqrt{\left(\frac{1+x}{x}\right)}\,dx$ (ii) $\displaystyle\int\sqrt{\left(\frac{x-1}{x+1}\right)}\,dx$ (iii) $\displaystyle\int\left(\frac{x\sqrt{1+x}}{\sqrt{1-x}}\right)dx$

Section (E) : Integration of trigonometric functions :

E-1. Integrate with respect to x :

(i) $\dfrac{1}{2+\cos x}$ (ii) $\dfrac{1}{2-\cos x}$ (iii) $\dfrac{2\sin x+2\cos x}{3\cos x+2\sin x}$

(iv) $\dfrac{1}{1+\sin x+\cos x}$ (v) $\dfrac{1}{2+\sin^2 x}$ (vi) $\dfrac{\operatorname{cosec}^2 x.\sin x}{(\sin x - \cos x)}$

(vii) $\dfrac{\sin^4 x}{\cos^2 x}$

E-2. Evaluate the following

(i) $\displaystyle\int\left(\frac{\sin x+\cos x}{9+16\sin 2x}\right)dx$ (ii) $\displaystyle\int\left(\frac{\cos x-\sin x}{\sqrt{8-\sin 2x}}\right)dx$

E-3. If $\displaystyle\int\sqrt{\frac{\cos^3 x}{\sin^{11} x}}\,dx = -2\left(A\tan^{\frac{-9}{2}}x + B\tan^{\frac{-5}{2}}x\right) + C$, then find A and B.

Section (F) : Reduction formulae

F-1. If $I_n = \int \frac{1}{(x^2+a^2)^n}dx$ then prove that $I_n = \frac{x}{2a^2(n-1)(x^2+a^2)^{n-1}} + \frac{2n-3}{2(n-1)a^2} I_{n-1}$

F-2. If $I_n = \int x^n (a-x)^{1/2}dx$ then prove that $I_n = \frac{2an}{2n+3} I_{n-1} - \frac{2x^n(a-x)^{3/2}}{2n+3}$

PART - II : ONLY ONE OPTION CORRECT TYPE

*** In each question C is arbitrary constant**

Section (A) : Integration using Standard Integral :

A-1. Integrate with respect to x : $\sqrt{x+1}$

(A) $\frac{(x+1)^{3/2}}{2}$ + C (B) $\frac{3(x+1)^{3/2}}{2}$ + C (C) $\frac{(x+1)^{3/2}}{3}$ + C (D) $\frac{2(x+1)^{3/2}}{3}$ + C

A-2 Integrate with respect to x : $\frac{1}{\sqrt{2x+1}}$

(A) $\sqrt{2x+1}$ + C (B) $(2x+1)^{3/2}$ + C (C) $-\sqrt{2x+1}$ + C (D) $\frac{1}{(2x+1)^{3/2}}$ + C

A-3. If $\int \frac{1}{1+\sin x}$ dx = $\tan\left(\frac{x}{2}+a\right)$ + C, then

(A) $a = -\frac{\pi}{4}$, $C \in R$ (B) $a = \frac{\pi}{4}$, $C \in R$ (C) $a = \frac{5\pi}{4}$, $C \in R$ (D) $a = \frac{\pi}{3}$, $C \in R$

A-4. If $\int (\sin 2x - \cos 2x)$ dx = $\frac{1}{\sqrt{2}}$ sin (2x – a) + C, then

(A) $a = \frac{5\pi}{4}$, $C \in R$ (B) $a = -\frac{5\pi}{4}$, $C \in R$ (C) $a = \frac{\pi}{4}$, $C \in R$ (D) $a = \frac{\pi}{2}$, $C \in R$

A-5. The value of $\int \frac{\cos 2x}{\cos x}$ dx is equal to

(A) 2 sin x – ℓn |sec x + tan x| + C (B) 2 sin x – ℓn |sec x – tan x| + C
(C) 2 sin x + ℓn |sec x + tan x| + C (D) sin x – ℓn |sec x – tan x| + C

A-6. If $\int \frac{\cos 4x + 1}{\cot x - \tan x}$ dx = A cos 4x + B; where A & B are constants, then

(A) A = – 1/4 & B may have any value (B) A = – 1/8 & B may have any value
(C) A = – 1/2 & B = – 1/4 (D) A = B = 1/2

Section (B) : Integration using Substitution :

B-1. The value of $\int \frac{a^{\sqrt{x}}}{\sqrt{x}}$ dx is equal to

(A) $\frac{a^{\sqrt{x}}}{\sqrt{x}}$ + C (B) $\frac{2a^{\sqrt{x}}}{\ell n\ a}$ + C (C) $2a^{\sqrt{x}} . \ell n\ a$ + C (D) $2a^{\sqrt{x}}$ + C

B-2. The value of $\int 5^{5^{5^x}} . \; 5^{5^x} . \; 5^x \; dx$ is equal to

(A) $\dfrac{5^{5^{5^x}}}{(\ell n \; 5)^3} + C$ (B) $5^{5^{5^x}} \; (\ell n\ 5)^3 + C$ (C) $\dfrac{5^{5^{5^x}}}{(\ell n \; 5)^3} + C$ (D) $\dfrac{5^{5^{5^x}}}{(\ell n\ 5)^2} + C$

B-3. The value of $\int \dfrac{\sqrt{\tan x}}{\sin x \; \cos x} \; dx$ is equal to

(A) $2\sqrt{\tan x} + C$ (B) $2\sqrt{\cot x} + C$ (C) $\dfrac{\sqrt{\tan x}}{2} + C$ (D) $\sqrt{\tan x} + C$

B-4. If $\int \dfrac{2^x}{\sqrt{1-4^x}} dx = K \sin^{-1} (2^x) + C$, then the value of K is equal to

(A) $\ell n\ 2$ (B) $\dfrac{1}{2} \ell n\ 2$ (C) $\dfrac{1}{2}$ (D) $\dfrac{1}{\ell n \; 2}$

B-5. If $y = \int \dfrac{dx}{\left(1+x^2\right)^{3/2}}$ and y = 0 when x = 0, then value of y when x = 1, is:

(A) $\sqrt{\dfrac{2}{3}}$ (B) $\sqrt{2}$ (C) $3\sqrt{2}$ (D) $\dfrac{1}{\sqrt{2}}$

B-6. The value of $\int \tan^3 2x \sec 2x \, dx$ is equal to :

(A) $\dfrac{1}{3}\sec^3 2x - \dfrac{1}{2}\sec 2x + C$ (B) $-\dfrac{1}{6}\sec^3 2x - \dfrac{1}{2}\sec 2x + C$

(C) $\dfrac{1}{6}\sec^3 2x - \dfrac{1}{2}\sec 2x + C$ (D) $\dfrac{1}{3}\sec^3 2x + \dfrac{1}{2}\sec 2x + C$

B-7. If $\int x^{13/2} . \left(1+x^{5/2}\right)^{1/2} dx = P\,(1 + x^{5/2})^{7/2} + Q\,(1 + x^{5/2})^{5/2} + R\,(1 + x^{5/2})^{3/2} + C$, then P,Q and R are

(A) $P = \dfrac{4}{35}$, $Q = -\dfrac{8}{25}$, $R = \dfrac{4}{15}$ (B) $P = \dfrac{4}{35}$, $Q = \dfrac{8}{25}$, $R = \dfrac{4}{15}$

(C) $P = -\dfrac{4}{35}$, $Q = -\dfrac{8}{25}$, $R = \dfrac{4}{15}$ (D) $P = \dfrac{4}{35}$, $Q = -\dfrac{8}{25}$, $R = -\dfrac{4}{15}$

B-8. The value of $\int \dfrac{1-x^7}{x\left(1+x^7\right)} dx$ is equal to

(A) $\ell n|x| + \dfrac{2}{7}\ell n|1 + x^7| + C$ (B) $\ell n|x| - \dfrac{2}{7}\ell n|1 - x^7| + C$

(C) $\ell n|x| - \dfrac{2}{7}\ell n|1 + x^7| + C$ (D) $\ell n|x| + \dfrac{2}{7}\ell n|1 - x^7| + C$

Section (C) : Integration by parts :

C-1. The value of $\int (x-1) \; e^{-x} \; dx$ is equal to

(A) $-xe^x + C$ (B) $xe^x + C$ (C) $-xe^{-x} + C$ (D) $xe^{-x} + C$

C-2. The value of $\int e^{\tan^{-1}x}\left(\frac{1+x+x^2}{1+x^2}\right)dx$ is equal to

(A) $x\,e^{\tan^{-1}x}$ + C (B) $x^2\ e^{\tan^{-1}x}$ + C (C) $\frac{1}{x}\ e^{\tan^{-1}x}$ + C (D) $x\,e^{\cot^{-1}x}$ + C

C-3. The value of $\int [f(x)g''(x) - f''(x)g(x)]\ dx$ is equal to

(A) $\frac{f(x)}{g'(x)}$ + C

(B) $f'(x)\ g(x) - f(x)\ g'(x)$ + C

(C) $f(x)\ g'(x) - f'(x)\ g(x)$ + C

(D) $f(x)\ g'(x) + f'(x)\ g'(x)$ + C

C-4. $\int \frac{x\ \ell n\, x}{\left(x^2-1\right)^{3/2}}$ dx equals

(A) $\text{arc}\sec x - \frac{\ell n\ x}{\sqrt{x^2-1}} + C$

(B) $\sec^{-1}x + \frac{\ell n\ x}{\sqrt{x^2-1}} + C$

(C) $\cos^{-1}x - \frac{\ell n\ x}{\sqrt{x^2-1}} + C$

(D) $\sec x - \frac{\ell n\ x}{\sqrt{x^2-1}} + C$

C-5. The value of $\int (x\ e^{\ell n \sin x} - \cos x)\ dx$ is equal to:

(A) x cos x + C (B) sin x – x cos x + C (C) $-\ e^{\ell n\, x} \cos x$ + C (D) sin x + x cos x + C

Section (D) : Algebraic integral :

D-1. The value of $\int \frac{dx}{x^2+x+1}$ is equal to

(A) $\frac{\sqrt{3}}{2}\ \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$ + C

(B) $\frac{2}{\sqrt{3}}\ \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$ + C

(C) $\frac{1}{\sqrt{3}}\ \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$ + C

(D) $\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)$ + C

D-2. The value of $\int \frac{1}{x^2(x^4+1)^{3/4}}$ dx is equal to

(A) $\left(1+\frac{1}{x^4}\right)^{1/4}$ + C (B) $(x^4 + 1)^{1/4}$ + C (C) $\left(1-\frac{1}{x^4}\right)^{1/4}$ + C (D) $-\left(1+\frac{1}{x^4}\right)^{1/4}$ + C

D-3. The value of $\int \frac{dx}{x\sqrt{1-x^3}}$ is equal to

(A) $\frac{1}{3}\ \ell n \left|\frac{\sqrt{1-x^3}-1}{\sqrt{1-x^3}+1}\right|$ + C

(B) $\frac{1}{3}\ \ell n \left|\frac{\sqrt{1-x^2}+1}{\sqrt{1-x^2}-1}\right|$ + C

(C) $\frac{1}{3}\ \ell n \left|\frac{1}{\sqrt{1-x^3}}\right|$ + C

(D) $\frac{1}{3}\ \ell n\ |1 - x^3|$ + C

D-4. The value of $\int \sqrt{\frac{e^x-1}{e^x+1}}$ dx is equal to

(A) $\ell n\ \left(e^x+\sqrt{e^{2x}-1}\right) - \sec^{-1}(e^x)$ + C

(B) $\ell n\ \left(e^x+\sqrt{e^{2x}-1}\right) + \sec^{-1}(e^x)$ + C

(C) $\ell n\ \left(e^x-\sqrt{e^{2x}-1}\right) - \sec^{-1}(e^x)$ + C

(D) $\ell n\ \left(e^x+\sqrt{e^{2x}-1}\right) - \sin^{-1}(e^x)$ + C

D-5. If $\int \frac{dx}{x^4 + x^3} = \frac{A}{x^2} + \frac{B}{x} + \ell n \left|\frac{x}{x+1}\right| + C$, then

(A) $A = \frac{1}{2}$, $B = 1$ (B) $A = 1$, $B = -\frac{1}{2}$ (C) $A = -\frac{1}{2}$, $B = 1$ (D) $A = -\frac{1}{2}$, $B = \frac{1}{2}$

Section (E) : Integration of trigonometric functions :

E-1. The value of $\int \frac{\cos 2x}{(\sin x + \cos x)^2}\, dx$ is equal to

(A) $\frac{-1}{\sin x + \cos x} + C$ (B) $\ell n\,(\sin x + \cos x) + C$

(C) $\ell n\,(\sin x - \cos x) + C$ (D) $\ell n\,(\sin x + \cos x)^2 + C$

E-2 The value of $\int [1 + \tan x \,.\, \tan(x + \alpha)]\, dx$ is equal to

(A) $\cos \alpha \,.\, \ell n \left|\frac{\sin x}{\sin(x+\alpha)}\right| + C$ (B) $\tan \alpha \,.\, \ell n \left|\frac{\sin x}{\sin(x+\alpha)}\right| + C$

(C) $\cot \alpha \,.\, \ell n \left|\frac{\sec(x+\alpha)}{\sec x}\right| + C$ (D) $\cot \alpha \,.\, \ell n \left|\frac{\cos(x+\alpha)}{\cos x}\right| + C$

E-3 The value of $\int \sqrt{\sec x - 1}\, dx$ is equal to

(A) $2\, \ell n \left(\cos\frac{x}{2} + \sqrt{\cos^2\frac{x}{2} - \frac{1}{2}}\right) + C$ (B) $\ell n \left(\cos\frac{x}{2} + \sqrt{\cos^2\frac{x}{2} - \frac{1}{2}}\right) + C$

(C) $-2\, \ell n \left(\cos\frac{x}{2} + \sqrt{\cos^2\frac{x}{2} - \frac{1}{2}}\right) + C$ (D) $-2\, \ell n \left(\sin\frac{x}{2} + \sqrt{\cos^2\frac{x}{2} - \frac{1}{2}}\right) + C$

E-4. The value of $\int \frac{dx}{\cos^3 x \sqrt{\sin 2x}}$ is equal to

(A) $\sqrt{2}\left(\sqrt{\cos x} + \frac{1}{5}\tan^{5/2} x\right) + C$ (B) $\sqrt{2}\left(\sqrt{\tan x} + \frac{1}{5}\tan^{5/2} x\right) + C$

(C) $\sqrt{2}\left(\sqrt{\tan x} - \frac{1}{5}\tan^{5/2} x\right) + C$ (D) $\sqrt{2}\left(\sqrt{\cos x} - \frac{1}{5}\tan^{5/2} x\right) + C$

E-5. Antiderivative of $\frac{\sin^2 x}{1 + \sin^2 x}$ w.r.t. x is :

(A) $x - \frac{\sqrt{2}}{2}\arctan\left(\sqrt{2}\tan x\right) + C$ (B) $x - \frac{1}{\sqrt{2}}\arctan\left(\frac{\tan x}{\sqrt{2}}\right) + C$

(C) $x - \sqrt{2}\arctan\left(\sqrt{2}\tan x\right) + C$ (D) $x - \sqrt{2}\arctan\left(\frac{\tan x}{\sqrt{2}}\right) + C$

E-6. Integrate $\frac{1}{1 - \cot x}$

(A) $\frac{1}{2}\log|\sin x - \cos x| + \frac{1}{2}x + C$ (B) $\frac{1}{2}\log|\sin x + \cos x| + \frac{1}{2}x + C$

(C) $\frac{1}{2}\log|\sin x + \cos x| - \frac{1}{2}x + C$ (D) $\frac{1}{2}\log|\sin x - \cos x| - \frac{1}{2}x + C$

E-7. $I = \int \frac{dx}{\sin x + \sec x}$ is equal to

(A) $\frac{1}{2\sqrt{3}}\log\left|\frac{\sqrt{3}+\sin x - \cos x}{\sqrt{3}-(\sin x - \cos x)}\right| + \tan^{-1}(\sin x + \cos x) + C$

(B) $\frac{1}{2\sqrt{3}}\log\left|\frac{\sqrt{3}+\sin x - \cos x}{\sqrt{3}-(\sin x - \cos x)}\right| + \tan^{-1}(\sin x - \cos x) + C$

(C) $\frac{1}{2\sqrt{3}}\log\left|\frac{\sqrt{3}+\sin x + \cos x}{\sqrt{3}-(\sin x - \cos x)}\right| + \tan^{-1}(\sin x + \cos x) + C$

(D) $\frac{1}{2\sqrt{3}}\log\left|\frac{\sqrt{3}+\sin x - \cos x}{\sqrt{3}-(\sin x + \cos x)}\right| + \tan^{-1}(\sin x + \cos x) + C$

Section (F) : Reduction formulae

F-1. If $I_n = \int \frac{e^x}{x^n} dx$ and $I_n = \frac{-e^x}{k_1 x^{n-1}} + \frac{1}{k_2 - 1} I_{n-1}$, then $(k_2 - k_1)$ is equal to :

(A) 0 (B) 1 (C) 2 (D) 3

F-2. If $I_n = \int \cot^n x \ dx$ and $I_0 + I_1 + 2(I_2 + \ldots + I_8) + I_9 + I_{10} = A\left(u + \frac{u^2}{2} + \ldots + \frac{u^9}{9}\right) + C$, where $u = \cot x$ and C is an arbitrary constant, then

(A) A = 2 (B) A = – 1 (C) A = 1 (D) A is dependent on x

PART - III : MATCH THE COLUMN

1. **Column – I** **Column – II**

(A) If $F(x) = \int \frac{x + \sin x}{1 + \cos x} dx$ and $F(0) = 0$, then the value of $F(\pi/2)$ is (p) $\frac{\pi}{2}$

(B) Let $F(x) = \int e^{\sin^{-1} x}\left(1 - \frac{x}{\sqrt{1-x^2}}\right) dx$ and $F(0) = 1$, (q) $\frac{\pi}{3}$

If $F(1/2) = \frac{k\sqrt{3}\ e^{\pi/6}}{\pi}$, then the value of k is

(C) Let $F(x) = \int \frac{dx}{(x^2+1)(x^2+9)}$ and $F(0) = 0$, (r) $\frac{\pi}{4}$

if $F(\sqrt{3}) = \frac{5}{36}k$, then the value of k is

(D) Let $F(x) = \int \frac{\sqrt{\tan x}}{\sin x \cos x} dx$ and $F(0) = 0$ (s) π

if $F(\pi/4) = \frac{2k}{\pi}$, then the value of k is

2. If I = $\int \frac{dx}{a+b\ \ \cos x}$, where a, b > 0 and a + b = u, a – b = v, then match the following column

Column – I		**Column – II**	
(A)	v = 0	(p)	$I = \frac{1}{\sqrt{uv}}\ \ell n \left|\frac{\sqrt{u}+\sqrt{v}\quad \tan\frac{x}{2}}{\sqrt{u}-\sqrt{v}\quad \tan\frac{x}{2}}\right| + C$
(B)	v > 0	(q)	$I = \frac{2}{\sqrt{uv}}\ \tan^{-1}\left(\sqrt{\frac{v}{u}}\quad \tan\ \frac{x}{2}\right) + C$
(C)	v < 0	(r)	$I = \frac{1}{\sqrt{-u\ \ v}}\ \ell n \left|\frac{\sqrt{u}+\sqrt{-v}\quad \tan\frac{x}{2}}{\sqrt{u}-\sqrt{-v}\quad \tan\frac{x}{2}}\right| + C$
		(s)	$\frac{2}{u}\ \tan\ \frac{x}{2} + C$

Exercise-2

✍ **Marked questions are recommended for Revision.**

PART - I : ONLY ONE OPTION CORRECT TYPE

*** In each question C is arbitrary constant**

1.✍ Value of $\int \frac{1}{\sin(x-a)\cos(x-b)}\, dx$ is equal to

(A) $\frac{1}{\cos(a-b)}\ \ell n\left|\frac{\sin(x-a)}{\cos(x-b)}\right| + C$ (B) $\frac{1}{\cos(a-b)}\ \ell n\left|\frac{\cos(x-b)}{\sin(x-a)}\right| + C$

(C) $\frac{1}{\sin(a-b)}\ \ell n\left|\frac{\sin(x-a)}{\cos(x-b)}\right| + C$ (D) $\frac{1}{\sin(a+b)}\ \ell n\left|\frac{\cos(x-a)}{\sin(x-b)}\right| + C$

2. $\int$ tan x. tan 2x. tan 3x dx =

(A) $-\ell n|\cos x| - \frac{1}{2}\ell n|\sec 2x| + \frac{1}{3}\ell n|\sec 3x| + C$

(B) $-\ell n|\sec x| - \frac{1}{2}\ell n|\sec 2x| + \frac{1}{3}\ell n|\sec\ \ 3x| + C$

(C) $\ell n|\cos x| + \ell n|\cos 2x| + \ell n|\cos 3x| + C$

(D) $\ell n|\sec x| + \frac{1}{2}\ell n|\sec 2x| + \frac{1}{3}|\sec 3x| + C$

3. The value of $\int$ (sin x. cos x. cos 2x. cos 4x. cos 8x. cos 16 x) dx is equal to

(A) $\frac{\sin 16x}{1024} + C$ (B) $-\frac{\cos 32x}{1024} + C$ (C) $\frac{\cos 32x}{1096} + C$ (D) $-\frac{\cos 32x}{1096} + C$

4.✍ $\int x \sqrt{\dfrac{a^2 - x^2}{a^2 + x^2}}\, dx =$

(A) $\frac{1}{2} a^2 \cos^{-1}\left(\frac{x^2}{a^2}\right) + \frac{1}{2}\sqrt{a^4 + x^4} + C$

(B) $\frac{1}{2} \sin^{-1}\left(\frac{x^2}{a^2}\right) + \sqrt{a^4 + x^4} + C$

(C) $\frac{1}{2} a^2 \sin^{-1}\left(\frac{x^2}{a^2}\right) + \frac{1}{2}\sqrt{a^4 - x^4} + C$

(D) $\frac{1}{2} \cos^{-1}\left(\frac{x^2}{a^2}\right) + \frac{1}{2}\sqrt{a^4 - x^4} + C$

5.✍ The value of $\int \sqrt{\dfrac{x-1}{x+1}} \cdot \dfrac{1}{x^2}\, dx$ is equal to

(A) $\sin^{-1}\frac{1}{x} + \frac{\sqrt{x^2-1}}{x} + C$

(B) $\frac{\sqrt{x^2-1}}{x} + \cos^{-1}\frac{1}{x} + C$

(C) $\sec^{-1} x - \frac{\sqrt{x^2-1}}{x} + C$

(D) $\tan^{-1}\sqrt{x^2+1} - \frac{\sqrt{x^2-1}}{x} + C$

6. The value of $\int \dfrac{\ell n\,|x|}{x\sqrt{1+\ell n\,|x|}}\, dx$ equals :

(A) $\frac{2}{3}\sqrt{1+\ell n\,|x|}\,(\ell n\,|x| - 2) + C$

(B) $\frac{2}{3}\sqrt{1+\ell n\,|x|}\,(\ell n\,|x| + 2) + C$

(C) $\frac{1}{3}\sqrt{1+\ell n\,|x|}\,(\ell n\,|x| - 2) + C$

(D) $2\sqrt{1+\ell n\,|x|}\,(3\,\ell n\,|x| - 2) + C$

7. The value of $\int \dfrac{1}{[(x-1)^3(x+2)^5]^{1/4}}\, dx$ is equal to

(A) $\frac{4}{3}\left(\frac{x-1}{x+2}\right)^{1/4} + C$ (B) $\frac{4}{3}\left(\frac{x+2}{x-1}\right)^{1/4} + C$ (C) $\frac{1}{3}\left(\frac{x-1}{x+2}\right)^{1/4} + C$ (D) $\frac{1}{3}\left(\frac{x+1}{x-1}\right)^{1/4} + C$

8.✍ The value of $\int \sqrt{\dfrac{1-\sqrt{x}}{1+\sqrt{x}}}\, dx$ is equal to

(A) $\sqrt{x}\,\sqrt{1-x} - 2\sqrt{1-x} + \cos^{-1}\left(\sqrt{x}\right) + C$

(B) $\sqrt{x}\,\sqrt{1-x} + 2\sqrt{1-x} + \cos^{-1}\left(\sqrt{x}\right) + C$

(C) $\sqrt{x}\,\sqrt{1-x} - 2\sqrt{1-x} - \cos^{-1}\left(\sqrt{x}\right) + C$

(D) $\sqrt{x}\,\sqrt{1-x} + 2\sqrt{1-x} - \cos^{-1}\left(\sqrt{x}\right) + C$

9.✍ $\int \sin^{-1}\sqrt{\dfrac{x}{a+x}}\, dx$ is equal to

(A) $(a + x)$ arc tan $\sqrt{\frac{x}{a}} - \sqrt{ax} + C$

(B) $(a + x)$ arc tan $\sqrt{\frac{x}{a}} + \sqrt{ax} + C$

(C) $(a - x)$ arc tan $\sqrt{\frac{x}{a}} - \sqrt{ax} + C$

(D) $(a + x)$ arc cot $\sqrt{\frac{x}{a}} - \sqrt{ax} + C$

10. The value of $\int \dfrac{e^{\sqrt{x}}}{\sqrt{x}}\left(x + \sqrt{x}\right) dx$ is equal to :

(A) $2e^{\sqrt{x}}\,[\sqrt{x} - x + 1\,] + C$

(B) $2e^{\sqrt{x}}\,[x - 2\sqrt{x} + 1\,] + C$

(C) $2e^{\sqrt{x}}\,[x - \sqrt{x} + 1\,] + C$

(D) $2e^{\sqrt{x}}\,(x + \sqrt{x} + 1\,) + C$

11. If $I = \int \frac{2}{x}(x^{\ell nx})(\ell nx)^3 dx = Ax^{\ell nx}(\ell nx)^2 - B\,x^{\ell nx} + C$, then $\frac{A}{B}$ is equal to :

(A) 1 (B) –1 (C) 2 (D) –2

12. The value of $\int e^{\tan\theta}(\sec\theta - \sin\theta)\,d\theta$ is equal to

(A) $-e^{\tan\theta}\sin\theta + C$ (B) $e^{\tan\theta}\sin\theta + C$ (C) $e^{\tan\theta}\sec\theta + C$ (D) $e^{\tan\theta}\cos\theta + C$

13. The value of $\int \left\{\ln(1+\sin x) + x\tan\left(\frac{\pi}{4} - \frac{x}{2}\right)\right\}dx$ is equal to:

(A) $x\,\ell n\,(1 + \sin x) + C$
(B) $\ell n\,(1 + \sin x) + C$
(C) $-x\,\ell n\,(1 + \sin x) + C$
(D) $\ell n\,(1 - \sin x) + C$

14. The value of $\int x.\frac{\ell n\left(x + \sqrt{1+x^2}\right)}{\sqrt{1+x^2}}dx$ equals:

(A) $\sqrt{1+x^2}\ \ell n\left(x + \sqrt{1+x^2}\right) - x + C$

(B) $\frac{x}{2}.\ \ell n^2\left(x + \sqrt{1+x^2}\right) - \frac{x}{\sqrt{1+x^2}} + C$

(C) $\frac{x}{2}.\ \ell n^2\left(x + \sqrt{1+x^2}\right) + \frac{x}{\sqrt{1+x^2}} + C$

(D) $\sqrt{1+x^2}\ \ell n\left(x + \sqrt{1+x^2}\right) + x + C$

15. If $\int \frac{x\tan^{-1}x}{\sqrt{1+x^2}}dx = \sqrt{1+x^2}\ f(x) + A\ \ell n\left|x + \sqrt{x^2+1}\right| + C$, then

(A) $f(x) = \tan^{-1}x$, $A = -1$
(B) $f(x) = \tan^{-1}x$, $A = 1$
(C) $f(x) = 2\tan^{-1}x$, $A = -1$
(D) $f(x) = 2\tan^{-1}x$, $A = 1$

16. $\int \frac{x + \sqrt{x+1}}{x+2}dx$ is equal to

(A) $(x+1) - 2\sqrt{x+1} + 2\,\ell n|x+2| - 2\tan^{-1}\sqrt{x+1} + C$

(B) $(x+1) + 2\sqrt{x+2} - 2\,\ell n|x+2| - 2\tan^{-1}\sqrt{x+2} + C$

(C) $(x+1) + 2\sqrt{x+1} - 2\,\ell n|x+2| - 2\tan^{-1}\sqrt{x+1} + C$

(D) $(x+1) + 2\sqrt{x+2} - 2\,\ell n|x+1| + 2\tan^{-1}\sqrt{x+2} + C$

17. The value of $\int \sqrt{\frac{1-\cos x}{\cos\alpha - \cos x}}\,dx$, where $0 < \alpha < x < \pi$, is equal to

(A) $2\,\ell n\left(\cos\frac{\alpha}{2} - \cos\frac{x}{2}\right) + C$

(B) $\sqrt{2}\,\ell n\left(\cos\frac{\alpha}{2} - \cos\frac{x}{2}\right) + C$

(C) $2\sqrt{2}\,\ell n\left(\cos\frac{\alpha}{2} - \cos\frac{x}{2}\right) + C$

(D) $-2\sin^{-1}\left(\frac{\cos\frac{x}{2}}{\cos\frac{\alpha}{2}}\right) + C$

18. If $I = \int \frac{\sin x + \sin^3 x}{\cos 2x}dx = A\cos x + B\,\ell n\,|f(x)| + C$, then

(A) $A = \frac{1}{4}$, $B = \frac{-1}{\sqrt{2}}$, $f(x) = \frac{\sqrt{2}\cos x - 1}{\sqrt{2}\cos x + 1}$

(B) $A = -\frac{1}{2}$, $B = \frac{-3}{4\sqrt{2}}$, $f(x) = \frac{\sqrt{2}\cos x - 1}{\sqrt{2}\cos x + 1}$

(C) $A = -\frac{1}{2}$, $B = \frac{3}{\sqrt{2}}$, $f(x) = \frac{\sqrt{2}\cos x + 1}{\sqrt{2}\cos x - 1}$

(D) $A = \frac{1}{2}$, $B = \frac{-3}{4\sqrt{2}}$, $f(x) = \frac{\sqrt{2}\cos x - 1}{\sqrt{2}\cos x + 1}$

19. The value of $\int \frac{1}{\cos^6 x + \sin^6 x} dx$ is equal to

(A) $\tan^{-1}(\tan x + \cot x) + C$
(B) $-\tan^{-1}(\tan x + \cot x) + C$
(C) $\tan^{-1}(\tan x - \cot x) + C$
(D) $-\tan^{-1}(\tan x - \cot x) + C$

20. Consider the following statements :

S_1 : The antiderivative of every even function is an odd function.

S_2 : Primitive of $\frac{3x^4 - 1}{(x^4 + x + 1)^2}$ w.r.t. x is $\frac{x}{x^4 + x + 1}$ + C.

S_3 : $\int \frac{1}{\sqrt{\sin^3 x \cos x}} dx = \frac{-2}{\sqrt{\tan x}}$ + C.

S_4 : The value of $\int \left(\sqrt{\frac{a+x}{a-x}} - \sqrt{\frac{a-x}{a+x}}\right)$ dx is equal to $-2\sqrt{a^2 - x^2}$ + C

State, in order, whether S_1, S_2, S_3, S_4 are true or false

(A) FFTT (B) TTTT (C) FFFF (D) TFTF

21. If $I_n = \int (\sin x + \cos x)^n dx$, snd $I_n = \frac{1}{n}(\sin x + \cos x)^{n-1}(\sin x - \cos x) + \frac{2k}{n} I_{n-2}$ then k =

(A) (n+ 1) (B) (n – 1) (C) (2n+ 1) (A) (2n– 1)

PART - II : SINGLE AND DOUBLE VALUE INTEGER TYPE

*** In each question C is arbitrary constant**

1. If $f(x) = \int \frac{2\sin x - \sin 2x}{x^3}$ dx, where $x \neq 0$, then $\lim_{x \to 0} f'(x)$ has the value

2. If $\int \sin^4 x \cos^4 x\, dx = \frac{1}{128}\left[ax - \sin 4x + \frac{1}{8}.\sin 8x\right] + C$ then value of 'a' equal to :

3. Let f(x) be the primitive of $\frac{3x+2}{\sqrt{x-9}}$ w.r. to x. If f(10) = 60 then twice of sum of digits of the value of f(13) is.

4. If $\int \frac{\sqrt{4+x^2}}{x^6} dx = \frac{(a+x^2)^{3/2}.(x^2-b)}{120x^5}$ + C then a + b equals to :

5. If $\int \sqrt{\frac{x}{a^3 - x^3}} dx = \frac{d}{b}\sin^{-1}\left(\frac{x^{3/2}}{a^{3/2}}\right)$ + C, (where b & d are coprime integer) then b + d equals to.

6. If $\int \frac{x\ dx}{\sqrt{1 + x^2 + \sqrt{(1+x^2)^3}}} = k\sqrt{1 + \sqrt{1+x^2}}$ + C then k equals to :

7. If $\int e^{\sin x}.\frac{x\cos^3 x - \sin x}{\cos^2 x} dx = e^{\sin x} f(x) + C$ such that f(0) = –1 then $\frac{\pi}{3} - f\left(\frac{\pi}{3}\right)$ is equal to :

8. Let $g(x) = \int \frac{1 + 2\cos x}{(\cos x + 2)^2} dx$ and g(0) = 0 then value of 32 $g\left(\frac{\pi}{2}\right)$ is.

9. If $f(x) = \sqrt{x-1}$; $g(x) = e^x$ and $\int fog(x)dx = Afog(x) + B\tan^{-1}(fog(x)) + C$ then $A^3 + B^2$ equals

10. If $\int \frac{2\sin 2\phi - \cos\phi}{6 - \cos^2\phi - 4\sin\phi}\, d\phi = p\, \ell n \left|\sin^2\phi - 4\sin\phi + 5\right| + q\tan^{-1}(\sin\phi - r) + C$ then p + q + r equal to :

11. If $\int \frac{(x-1)^2}{x^4 + x^2 + 1}dx = \frac{1}{\sqrt{a}}\tan^{-1}\left(\frac{x^2-1}{x\sqrt{3}}\right) - \frac{b}{\sqrt{a}}\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right) + C$ then $a^2 + b^2$ equals to :

12. If $\int \frac{1 + x\cos x}{x\left(1 - x^2 e^{2\sin x}\right)}dx = k\, \ell n\sqrt{\frac{x^2e^{2\sin x}}{1 - x^2e^{2\sin x}}} + C$ then k is equal to :

13. If $\int \frac{x^4+1}{x\left(x^2+1\right)^2}dx = A\, \ell n\, |x| + \frac{B}{1+x^2} + C$, then A + B equals to :

14. If $\int \frac{1}{1-\sin^4 x}dx = \frac{1}{a\sqrt{b}}\tan^{-1}\left(\sqrt{a}\tan x\right) + \frac{1}{b}\tan x + C$ then $\frac{a}{b}$ is equal to :

15. If $\int \frac{\cos^3 x + \cos^5 x}{\sin^2 x + \sin^4 x}dx = p\sin x - \frac{q}{\sin x} - r\tan^{-1}(\sin x) + C$ then p + 2q + r is equal to :

16. If $\int \frac{dx}{\sqrt{\sin^3 x\cos^5 x}} = a\sqrt{\cot x} + b\sqrt{\tan^3 x} + C$, where C is an arbitrary constant of integration, then the values of $a^2 + 9b$ equals to :

PART - III : ONE OR MORE THAN ONE OPTIONS CORRECT TYPE

*** In each question C is arbitrary constant**

1. The value of $\int 2^{mx}.3^{nx}dx$ (when m, n $\in$ N) is equal to :

(A) $\frac{2^{mx}+3^{nx}}{m\ell n2 + n\ell n3} + C$ (B) $\frac{e^{(m\ell n2 + n\ell n3)x}}{m\ell n2 + n\ell n3} + C$ (C) $\frac{2^{mx}.3^{nx}}{\ell n\left(2^m.3^n\right)} + C$ (D) $\frac{(mn).2^x.3^x}{m\ell n2 + n\ell n3} + C$

2. If $f\left(\frac{1-x}{1+x}\right) = x$ and $g(x) = \int f(x)\,dx$ then

(A) g(x) is continuous in domain
(B) g(x) is discontinuous at two points in its domain
(C) $\lim\limits_{x\to\infty} g'(x) = -1$
(D) $\int g(x)dx = -\frac{x^2}{2} + (2x+1)\,\lambda n\left(\frac{1+x}{e}\right) + C$

3. If $= \int \tan^5 x dx = A\tan^4 x - \frac{1}{2}\tan^2 x + B\,\ell n|\sec x| + C$ then

(A) $A = \frac{1}{4}$ (B) $A = \frac{1}{2}$ (C) B = 1 (D) B = – 1

4. The value of $\int \{1+2\tan x(\tan x+\sec x)\}^{1/2}\, dx$ is equal to

(A) $\ell n\, |\sec x\, (\sec x - \tan x)| + C$
(B) $\ell n\, |\text{cosec}\, x\, (\sec x + \tan x)| + C$
(C) $\ell n\, |\sec x\, (\sec x + \tan x)| + C$
(D) $-\ell n\, |\cos x(\sec x - \tan x)| + C$

5.✍ The value of $\int \frac{\ell n\left(\frac{x-1}{x+1}\right)}{x^2-1}\, dx$ is equal to

(A) $\frac{1}{2}\ \ell n^2\ \frac{x-1}{x+1} + C$
(B) $\frac{1}{4}\ \ell n^2\ \frac{x-1}{x+1} + C$
(C) $\frac{1}{2}\ \ell n^2\ \frac{x+1}{x-1} + C$
(D) $\frac{1}{4}\ \ell n^2\ \frac{x+1}{x-1} + C$

6. The value of $\int \frac{\ell n\,(\tan x)}{\sin x\ \cos x}\, dx$ is equal to

(A) $\frac{1}{2}\ \ell n^2\,(\cot x) + C$
(B) $\frac{1}{2}\ \ell n^2\,(\sec x) + C$
(C) $\frac{1}{2}\ \ell n^2\,(\sin x\ \sec x) + C$
(D) $\frac{1}{2}\ \ell n^2\,(\cos x\ \text{cosec}\, x) + C$

7. The value of $\int \frac{\cos^3 x}{\sin^2 x + \sin x}\, dx$ is equal to :

(A) $\ell n\ |\sin x| + \sin x + C$
(B) $\ell n\ |\sin x| - \sin x + C$
(C) $-\ \ell n\ |\text{cosec}\, x| - \sin x + C$
(D) $-\ \ell n\ |\sin x| + \sin x + C$

8. If $\int \frac{(x-1)\, dx}{x^2\sqrt{2x^2-2x+1}} = \frac{\sqrt{f(x)}}{g(x)}$ + C, where f(x) is a quadratic expression and g(x) is a monic linear expression.

(A) $f(x) = 2x^2 - 2x + 1$
(B) $g(x) = x + 1$
(C) $g(x) = x$
(D) $f(x) = 2x^2 - 2x$

9. If $\int e^{3x} \cos 4x\, dx = e^{3x}\,(A \sin 4x + B \cos 4x) + C$ then :

(A) $4A = 3B$ (B) $2A = 3B$ (C) $3A = 4B$ (D) $4A + 3B = 1$

10. $I = \int \frac{\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}}\, dx$ equals to

(A) $-x + \frac{2}{\pi}\,(2x-1)\sin^{-1}\sqrt{x} + \frac{2}{\pi}\sqrt{x-x^2} + C$

(B) $x - \frac{4x}{\pi}\cos^{-1}\sqrt{x} - \frac{2}{\pi}\sin^{-1}\sqrt{x} + \frac{2}{\pi}\sqrt{x}\ \sqrt{1-x} + C$

(C) $-x + \frac{2}{\pi}\,(2x+1)\cos^{-1}\sqrt{x} + \frac{2}{\pi}\sqrt{x}\ \sqrt{1-x} + C$

(D) $x - \frac{4x}{\pi}\sin^{-1}\sqrt{x} + C$

11.✍ If $\int \frac{x^2-x+1}{(1+x^2)^{3/2}}\, e^x dx = e^x f(x) + C$ then

(A) f(x) is a an even function
(B) f(x) is a bounded function
(C) Range of f(x) is (0, 1]
(D) f(x) has two points of exterma.

12. If $\int \frac{4e^x + 6e^{-x}}{9e^x - 4e^{-x}}\,dx = Ax + B\,\ell n\,|9e^{2x} - 4| + C$, then

(A) $A + 18B = 16$
(B) $18B - A = 19$
(C) $A - 18B = 17$
(D) $A + 18B = 32$

13. The value of $\int \frac{x^2 + \cos^2 x}{1 + x^2}\,\text{cosec}^2 x\,dx$ is equal to:

(A) $\cot x - \cot^{-1} x + C$
(B) $C - \cot x + \cot^{-1} x$
(C) $-\tan^{-1} x - \frac{\text{cosec}\,x}{\sec x} + C$
(D) $\frac{1}{\tan^{-1} x} - \cot x + C$

14. The value of $\int \frac{dx}{\sqrt{x - x^2}}$; $\left(x > \frac{1}{2}\right)$ is equal to

(A) $2\sin^{-1}\sqrt{x} + C$
(B) $\sin^{-1}(2x - 1) + C$
(C) $C - 2\cos^{-1}(2x - 1)$
(D) $\cos^{-1} 2\sqrt{x - x^2} + C$

15. $\int \frac{x^3 - 1}{x^3 + x}\,dx$ is equal to

(A) $x - \ell n\,|x| + \ell n\,(x^2 + 1) - \tan^{-1}x + C$
(B) $x - \ell n\,|x| + \frac{1}{2}\,\ell n\,(x^2 + 1) - \tan^{-1}x + C$
(C) $x + \ell n\,|x| + \frac{1}{2}\,\ell n\,(x^2 + 1) + \tan^{-1}x + C$
(D) $x + \ell n\sqrt{\frac{x^2 + 1}{x^2}} + \cot^{-1}x + C$

16. The value of $2\int \sin x \,.\, \text{cosec}\,4x\,dx$ is equal to

(A) $\frac{1}{2\sqrt{2}}\,\ell n\left|\frac{1 + \sqrt{2}\sin x}{1 - \sqrt{2}\sin x}\right| - \frac{1}{4}\,\ell n\left|\frac{1 + \sin x}{1 - \sin x}\right| + C$
(B) $\frac{1}{2\sqrt{2}}\,\ell n\left|\frac{1 + \sqrt{2}\sin x}{1 - \sqrt{2}\sin x}\right| - \frac{1}{2}\,\ell n\left|\frac{1 + \sin x}{\cos x}\right| + C$
(C) $\frac{1}{2\sqrt{2}}\,\ell n\left|\frac{1 - \sqrt{2}\sin x}{1 + \sqrt{2}\sin x}\right| - \frac{1}{4}\,\ell n\left|\frac{1 + \sin x}{1 - \sin x}\right| + C$
(D) $-\frac{1}{2\sqrt{2}}\,\ell n\left|\frac{1 - \sqrt{2}\sin x}{1 + \sqrt{2}\sin x}\right| + \frac{1}{4}\,\ell n\left|\frac{1 - \sin x}{1 + \sin x}\right| + C$

17. If $\int \frac{3\cot 3x - \cot x}{\tan x - 3\tan 3x}\,dx = p\,f(x) + q\,g(x) + C$, then which of the following may be correct?

(A) $p = 1$; $q = \frac{1}{\sqrt{3}}$; $f(x) = x$; $g(x) = \ell n\left|\frac{\sqrt{3} - \tan x}{\sqrt{3} + \tan x}\right|$
(B) $p = 1$; $q = -\frac{1}{\sqrt{3}}$; $f(x) = x$; $g(x) = \ell n\left|\frac{\sqrt{3} - \tan x}{\sqrt{3} + \tan x}\right|$
(C) $p = 1$; $q = -\frac{2}{\sqrt{3}}$; $f(x) = x$; $g(x) = \ell n\left|\frac{\sqrt{3} + \tan x}{\sqrt{3} - \tan x}\right|$
(D) $p = 1$; $q = -\frac{1}{\sqrt{3}}$; $f(x) = x$; $g(x) = \ell n\left|\frac{\sqrt{3} + \tan x}{\sqrt{3} - \tan x}\right|$

18. If $\int \frac{dx}{5 + 4\cos x} = P\tan^{-1}\left(m\tan\frac{x}{2}\right) + C$ then :

(A) $P = 2/3$
(B) $m = 1/3$
(C) $P = 1/3$
(D) $m = 2/3$

19. The value of $\int \frac{\sin 2x}{\sin^4 x + \cos^4 x} dx$ is equal to:

(A) $\cot^{-1}(\cot^2 x) + C$
(B) $-\cot^{-1}(\tan^2 x) + C$
(C) $\tan^{-1}(\tan^2 x) + C$
(D) $-\tan^{-1}(\cos 2x) + C$

PART - IV : COMPREHENSION

Comprehension # 1 (Q.No. 1 to 3)

Let $I_{n,m} = \int \sin^n x \cos^m x.dx$. Then we can relate $I_{n,m}$ with each of the following

(i) $I_{n-2,m}$ (ii) $I_{n+2,m}$ (iii) $I_{n,m-2}$
(iv) $I_{n,m+2}$ (v) $I_{n-2,m+2}$ (vi) $I_{n+2,m-2}$

Suppose we want to establish a relation between $I_{n,m}$ and $I_{n,m-2}$, then we set

$P(x) = \sin^{n+1}x \cos^{m-1}x$ ....(1)

In $I_{n,m}$ and $I_{n,m-2}$ the exponent of cosx is m and m – 2 respectively, the minimum of the two is m – 2, adding 1 to the minimum we get m – 2 + 1 = m – 1. Now choose the exponent m – 1 of cosx in P(x). Similarly choose the exponent of sin x for P (x)

Now differentiating both sides of (1), we get

$P'(x) = (n+1)\sin^n x \cos^m x - (m-1)\sin^{n+2}x \cos^{m-2}x$
$= (n+1)\sin^n x \cos^m x - (m-1)\sin^n x (1-\cos^2 x)\cos^{m-2}x$
$= (n+1)\sin^n x \cos^m x - (m-1)\sin^n x \cos^{m-2} x + (m-1)\sin^n x \cos^m x$
$= (n+m)\sin^n x \cos^m x - (m-1)\sin^n x \cos^{m-2}x$

Now integrating both sides, we get

$\sin^{n+1}x \cos^{m-1}x = (n+m)\, I_{n,m} - (m-1)\, I_{n,m-2}$.

Similarly we can establish the other relations.

1. The relation between $I_{4,2}$ and $I_{2,2}$ is

(A) $I_{4,2} = \frac{1}{6}(-\sin^3 x \cos^3 x + 3I_{2,2})$
(B) $I_{4,2} = \frac{1}{6}(\sin^3 x \cos^3 x + 3I_{2,2})$
(C) $I_{4,2} = \frac{1}{6}(\sin^3 x \cos^3 x - 3I_{2,2})$
(D) $I_{4,2} = \frac{1}{6}(-\sin^3 x \cos^3 x + 2I_{2,2})$

2. The relation between $I_{4,2}$ and $I_{6,2}$ is

(A) $I_{4,2} = \frac{1}{5}(\sin^5 x \cos^3 x + 8I_{6,2})$
(B) $I_{4,2} = \frac{1}{5}(-\sin^5 x \cos^3 x + 8I_{6,2})$
(C) $I_{4,2} = \frac{1}{5}(\sin^5 x \cos^3 x - 8I_{6,2})$
(D) $I_{4,2} = \frac{1}{5}(\sin^5 x \cos^3 x + 8I_{6,2})$

3. The relation between $I_{4,2}$ and $I_{4,4}$ is

(A) $I_{4,2} = \frac{1}{3}(\sin^5 x \cos^3 x + 8\, I_{4,4})$
(B) $I_{4,2} = \frac{1}{3}(-\sin^5 x \cos^3 x + 8\, I_{4,4})$
(C) $I_{4,2} = \frac{1}{3}(\sin^5 x \cos^3 x - 8\, I_{4,4})$
(D) $I_{4,2} = \frac{1}{3}(\sin^5 x \cos^3 x + 6\, I_{4,4})$

Comprehension # 2 (Q. No. 4 to 6)

It is known that

$$\sqrt{\tan x} + \sqrt{\cot x} = \begin{cases} \dfrac{\sqrt{\sin x}}{\sqrt{\cos x}} + \dfrac{\sqrt{\cos x}}{\sqrt{\sin x}} & \text{if } \quad 0 < x < \dfrac{\pi}{2} \\ \dfrac{\sqrt{-\sin x}}{\sqrt{-\cos x}} + \dfrac{\sqrt{-\cos x}}{\sqrt{-\sin x}} & \text{if } \quad \pi < x < \dfrac{3\pi}{2} \end{cases},$$

$$\frac{d}{dx}\left(\sqrt{\tan x} - \sqrt{\cot x}\right) = \frac{1}{2}\left(\sqrt{\tan x} + \sqrt{\cot x}\right)(\tan x + \cot x),\ \forall\ x \in \left(0, \frac{\pi}{2}\right) \cup \left(\pi, \frac{3\pi}{2}\right)$$

and $\dfrac{d}{dx}\left(\sqrt{\tan x} + \sqrt{\cot x}\right) = \dfrac{1}{2}\left(\sqrt{\tan x} - \sqrt{\cot x}\right)(\tan x + \cot x),\ \forall\ x \in \left(0, \dfrac{\pi}{2}\right) \cup \left(\pi, \dfrac{3\pi}{2}\right)$.

4. Value of integral $I = \int(\sqrt{\tan x} + \sqrt{\cot x})\ dx$, where $x \in \left(0, \dfrac{\pi}{2}\right) \cup \left(\pi, \dfrac{3\pi}{2}\right)$ is

(A) $\sqrt{2}\ \tan^{-1}\left(\dfrac{\sqrt{\tan x} - \sqrt{\cot x}}{\sqrt{2}}\right) + C$ (B) $\sqrt{2}\ \tan^{-1}\left(\dfrac{\sqrt{\tan x} + \sqrt{\cot x}}{\sqrt{2}}\right) + C$

(C) $-\sqrt{2}\ \tan^{-1}\left(\dfrac{\sqrt{\tan x} - \sqrt{\cot x}}{\sqrt{2}}\right) + C$ (D) $-\sqrt{2}\ \tan^{-1}\left(\dfrac{\sqrt{\tan x} + \sqrt{\cot x}}{\sqrt{2}}\right) + C$

5. Value of the integral $I = \int(\sqrt{\tan x} + \sqrt{\cot x})\ dx$, where $x \in \left(0, \dfrac{\pi}{2}\right)$, is

(A) $\sqrt{2}\ \sin^{-1}(\cos x - \sin x) + C$ (B) $\sqrt{2}\ \sin^{-1}(\sin x - \cos x) + C$

(C) $\sqrt{2}\ \sin^{-1}(\sin x + \cos x) + C$ (D) $-\sqrt{2}\ \sin^{-1}(\sin x + \cos x) + C$

6. Value of the integral $I = \int(\sqrt{\tan x} + \sqrt{\cot x})\ dx$, where $x \in \left(\pi, \dfrac{3\pi}{2}\right)$, is

(A) $\sqrt{2}\ \sin^{-1}(\cos x - \sin x) + C$ (B) $\sqrt{2}\ \sin^{-1}(\sin x - \cos x) + C$

(C) $\sqrt{2}\ \sin^{-1}(\sin x + \cos x) + C$ (D) $-\sqrt{2}\ \sin^{-1}(\sin x + \cos x) + C$

Exercise-3

✍ Marked questions are recommended for Revision.

*** Marked Questions may have more than one correct option.**

PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

1.✍ Integrate, $\displaystyle\int \frac{x^3 + 3x + 2}{(x^2+1)^2\,(x+1)}\ dx.$ **[IIT-JEE 1999, Part-2, (7, 0), 120]**

2. Let $f(x) = \int e^x (x-1)(x-2)\,dx$ then f decreases in the interval : **[IIT-JEE 2000, Scr, (1, 0), 35]**

(A) $(-\infty, 2)$ (B) $(-2, -1)$ (C) $(1, 2)$ (D) $(2, +\infty)$

3. Evaluate, $\displaystyle\int \sin^{-1}\left(\frac{2x+2}{\sqrt{4x^2 + 8x + 13}}\right) dx.$ **[IIT-JEE 2001, Main, (5, 0), 100]**

4. For any natural number m, evaluate,

$\int \left(x^{3m}+x^{2m}+x^{m}\right)\left(2x^{2m}+3x^{m}+6\right)^{1/m} dx,\ x>0.$ **[IIT-JEE 2002, Main, (5, 0), 60]**

5. $\int \dfrac{x^2-1}{x^3\sqrt{2x^4-2x^2+1}}$ dx is equal to **[IIT-JEE 2006, (3, –1), 184]**

(A) $\dfrac{\sqrt{2x^4-2x^2+1}}{x^2}+C$

(B) $\dfrac{\sqrt{2x^4-2x^2+1}}{x^3}+C$

(C) $\dfrac{\sqrt{2x^4-2x^2+1}}{x}+C$

(D) $\dfrac{\sqrt{2x^4-2x^2+1}}{2x^2}+C$

6. Let f(x) = $\dfrac{x}{(1+x^n)^{1/n}}$ for $n \ge 2$ and g(x) = $\underbrace{(f\circ f\circ ....\circ f)}_{\text{f occurs n times}}$ (x). Then $\int x^{n-2}g(x)\,dx$ equals

[IIT-JEE 2007, Paper-2, (3, – 1), 81]

(A) $\dfrac{1}{n(n-1)}\left(1+nx^n\right)^{1-\frac{1}{n}}+K$

(B) $\dfrac{1}{(n-1)}\left(1+nx^n\right)^{1-\frac{1}{n}}+K$

(C) $\dfrac{1}{n(n+1)}\left(1+nx^n\right)^{1+\frac{1}{n}}+K$

(D) $\dfrac{1}{(n+1)}\left(1+nx^n\right)^{1+\frac{1}{n}}+K$

7. Let F(x) be an indefinite integral of $\sin^2 x$. **[IIT-JEE 2007, Paper-1, (3, – 1), 81]**

STATEMENT-1 : The function F(x) satisfies $F(x+\pi) = F(x)$ for all real x.

because

STATEMENT-2 : $\sin^2(x+\pi) = \sin^2 x$ for all real x.

(A) Statement-1 is True, Statement-2 is True ; Statement-2 **is** a correct explanation for Statement-1

(B)Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

8. Let I = $\int \dfrac{e^x}{e^{4x}+e^{2x}+1}$ dx, J = $\int \dfrac{e^{-x}}{e^{-4x}+e^{-2x}+1}$ dx . Then, for an arbitrary constant C, the value of J – I is equal to : **[IIT-JEE 2008, Paper-2, (3, – 1), 81]**

(A) $\dfrac{1}{2}\ell n\left|\dfrac{e^{4x}-e^{2x}+1}{e^{4x}+e^{2x}+1}\right|+C$

(B) $\dfrac{1}{2}\ell n\left|\dfrac{e^{2x}+e^{x}+1}{e^{2x}-e^{x}+1}\right|+C$

(C) $\dfrac{1}{2}\ell n\left|\dfrac{e^{2x}-e^{x}+1}{e^{2x}+e^{x}+1}\right|+C$

(D) $\dfrac{1}{2}\ell n\left|\dfrac{e^{4x}+e^{2x}+1}{e^{4x}-e^{2x}+1}\right|+C$

9. The integral $\int \dfrac{\sec^2 x}{(\sec x+\tan x)^{9/2}}dx$ equals (for some arbitrary constant K)

(A) $\dfrac{-1}{(\sec x+\tan x)^{11/2}}\left\{\dfrac{1}{11}-\dfrac{1}{7}(\sec x+\tan x)^2\right\}+K$ **[IIT-JEE 2012, Paper-1, (3, –1), 70]**

(B) $\dfrac{1}{(\sec x+\tan x)^{11/2}}\left\{\dfrac{1}{11}-\dfrac{1}{7}(\sec x+\tan x)^2\right\}+K$

(C) $\dfrac{-1}{(\sec x+\tan x)^{11/2}}\left\{\dfrac{1}{11}+\dfrac{1}{7}(\sec x+\tan x)^2\right\}+K$

(D) $\dfrac{1}{(\sec x+\tan x)^{11/2}}\left\{\dfrac{1}{11}+\dfrac{1}{7}(\sec x+\tan x)^2\right\}+K$

PART - II : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. If the integral $\int \frac{5\tan x}{\tan x - 2}\,dx = x + a$ □n |sin x – 2 cos x| + k, then a is equal to :

[AIEEE-2012, (4, –1)/120]

(1) – 1 (2) – 2 (3) 1 (4) 2

2. If $\int f(x)\ \ dx = \psi(x)$, then $\int x^5 f(x^3)\ \ dx$ is equal to **[AIEEE - 2013, (4, –1),360]**

(1) $\frac{1}{3}\left[x^3\psi(x^3) - \int x^2\psi(x^3)\ \ dx\right] + C$ (2) $\frac{1}{3}x^3\psi(x^3) - 3\int x^3\psi(x^3)\ \ dx + C$

(3) $\frac{1}{3}x^3\psi(x^3) - \int x^2\psi(x^3)\ \ dx + C$ (4) $\frac{1}{3}\left[x^3\psi(x^3) - \int x^3\psi(x^3)\ \ dx\right] + C$

3.✍ The integral $\int \left(1 + x - \frac{1}{x}\right) e^{x + \frac{1}{x}} dx$ is equal to : **[JEE(Main) 2014, (4, – 1), 120]**

(1) (x + 1) $e^{x+\frac{1}{x}}$ + c (2) –x $e^{x+\frac{1}{x}}$ + c (3) (x – 1) $e^{x+\frac{1}{x}}$ + c (4) x $e^{x+\frac{1}{x}}$ + c

4. The integral $\int \frac{dx}{x^2\left(x^4+1\right)^{3/4}}$ equals **[JEE(Main) 2015, (4, – 1), 120]**

(1) $\left(\frac{x^4+1}{x^4}\right)^{1/4}$ + c (2) $(x^4 + 1)^{1/4}$ + c (3) $-(x^4 + 1)^{1/4}$ + c (4) $-\left(\frac{x^4+1}{x^4}\right)^{1/4}$ + c

5. The integral $\int \frac{2x^{12}+5x^9}{(x^5+x^3+1)^3}dx$ is equal to **[JEE(Main) 2016, (4, –1), 120]**

(1) $\frac{x^{10}}{2(x^5+x^3+1)^2} + C$ (2) $\frac{x^5}{2(x^5+x^3+1)^2} + C$ (3) $\frac{-x^{10}}{2(x^5+x^3+1)^2} + C$ (4) $\frac{-x^5}{(x^5+x^3+1)^2} + C$

where C is an arbitrary constant

6. Let $I_n = \int \tan^n x\,dx$, (n > 1). If $I_4 + I_6 = a\tan^5 x + bx^5 + C$, where C is a constant of integration, then the ordered pair (a, b) is equal to **[JEE(Main) 2017, (4, –1), 120]**

(1) $\left(-\frac{1}{5}, 1\right)$ (2) $\left(\frac{1}{5}, 0\right)$ (3) $\left(\frac{1}{5}, -1\right)$ (4) $\left(-\frac{1}{5}, 0\right)$

7.✍ The integral $\int \frac{\sin^2 x\cos^2 x}{(\sin^5 x + \cos^3 x\sin^2 x + \sin^3 x\cos^2 x + \cos^5 x)^2}dx$ is equal to :

[JEE(Main) 2018, (4, –1), 120]

(1) $\frac{1}{1+\cot^3 x} + C$ (2) $\frac{-1}{1+\cot^3 x} + C$ (3) $\frac{1}{3(1+\tan^3 x)} + C$ (4) $\frac{-1}{3(1+\tan^3 x)} + C$

(where C is a constant of integration)

8.✍ Let n ≥ 2 be a natural number and 0 < θ < π / 2. Then $\int \frac{(\sin^n\theta - \sin\theta)^{\frac{1}{n}}\cos\theta}{\sin^{n+1}\theta}\,d\theta$ is equal to :

(where C is a constant of integration) **[JEE(Main) 2019, Online (10-01-19),P-1 (4, – 1), 120]**

(1) $\frac{n}{n^2-1}\left(1 - \frac{1}{\sin^{n+1}\theta}\right)^{\frac{n+1}{n}}$ + C (2) $\frac{n}{n^2-1}\left(1 - \frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}}$ + C

(3) $\frac{n}{n^2+1}\left(1 - \frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}}$ + C (4) $\frac{n}{n^2-1}\left(1 + \frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}}$ + C

9. The integral $\int \cos(\log_e x)dx$ is equal to : (where C is a constant of integration

[JEE(Main) 2019, Online (12-01-19),P-1 (4, – 1), 120]

(1) $x\left[\cos(\log_e x) - \sin(\log_e x)\right] + C$

(2) $\frac{x}{2}\left[\sin(\log_e x) - \cos(\log_e x)\right] + C$

(3) $x[\cos(\log_e x) + \sin(\log_e x)] + C$

(4) $\frac{x}{2}\left[\cos(\log_e x) + \sin(\log_e x)\right] + C$

Answers

EXERCISE - 1

PART - I

Section (A) :

A-1. (i) $\frac{(2x+3)^6}{12} + C$ (ii) $-\frac{\cos 2x}{2} + C$ (iii) $\frac{\tan(4x+5)}{4} + C$

(iv) $\frac{1}{3}\ \ell n\ |\sec(3x+2) + \tan(3x+2)| + C$ (v) $\frac{1}{2}\ \ell n\ |\sec(2x+1)| + C$

(vi) $\frac{2^{3x+4}}{3\ \ell n\ 2} + C$ (vii) $\frac{1}{2}\ \ell n\ |2x+1| + C$ (viii) $\frac{e^{4x+5}}{4} + C$

A-2. (i) $\frac{x}{2} - \frac{1}{4}\sin 2x + C$ (ii) $\frac{\sin 3x}{12} + \frac{3}{4}\sin x + C$

(iii) $-\frac{1}{10}\cos 5x + \frac{1}{2}\cos x + C$ (iv) $\cos x - \frac{1}{2}\cos 2x - \frac{1}{3}\cos 3x + C$

(v) $\frac{2}{3}\left((x+3)^{3/2} + (x+2)^{3/2}\right) + C$

Section (B) :

B-1. (i) $-\frac{1}{2}\cos x^2 + C$ (ii) $\frac{1}{2}\ \ell n\ |x^2+1| + C$

(iii) $\frac{1}{2}(\tan x)^2 + C$ or $\frac{\sec^2 x}{2} + C$ (iv) $\ell n\ |e^x + x| + C$

(v) $\ell n\ |x + \cos x| + C$ (vi) $\frac{1}{2}\ \ell n\ |e^{2x} - 2| + C$

(vii) $\frac{1}{2}\ \ell n\ |x^2 + \sin 2x + 2x| + C$ (viii) $\ell n\ |\ \ell n(\sec x + \tan x)\ | + C$

(ix) $\frac{2}{3}(x+2)^{3/2} - 4(x+2)^{1/2} + C$ (x) $\frac{1}{2}(e^{2x} - e^{-2x}) + 2x + C$

(xi) $\frac{1}{3}e^{3x} + e^{2x} + e^x + C$ (xii) $-\frac{1}{5}\ \ell n\ \left|1 + \frac{1}{x^5}\right| + C$

(xiii) $-\frac{1}{4}\left(1 + \frac{1}{x^5}\right)^{4/5} + C$ (xiv) $\frac{(x^2 - 8)^{3/2}}{24\ x^3} + C$

B-2. $2\sqrt{(x^2+2)} + C$ **B-3.** (i) $\ln\left(\frac{\sin x}{x}\right) + C$ (ii) $\ln\left(\frac{\ln(x+1)}{x}\right) + C$

Section (C) :

C-1. (i) $\frac{x^2}{2}\ \ell n\, x - \frac{x^2}{4} + C$

(ii) $\frac{x^2}{4} - \frac{x}{4}\sin 2x - \frac{1}{8}\cos 2x + C$

(iii) $\frac{x^2}{2}\tan^{-1}x - \frac{x}{2} + \frac{1}{2}\tan^{-1}x + C$

(iv) $x(\ell nx - 1) + C$

(v) $\frac{\sec x \tan x}{2} + \frac{1}{2}\ \ell n\,|\sec x + \tan x| + C$

(vi) $(x^2 - 1)\ e^{x^2} + C$

(vii) $x\sin^{-1}\sqrt{x} + \frac{\sqrt{x}\sqrt{1-x}}{2} - \frac{1}{2}\sin^{-1}\sqrt{x} + C$

(viii) $x\tan^{-1}x - \frac{1}{2}\ell n(1 + x^2) - \frac{(\tan^{-1}x)^2}{2} + C$

(ix) $\frac{e^x}{2}(\sin x - \cos x) + C$

(x) $e^x \tan x + C$

C-2. $y = x\left[\ell n(\ell nx) - \frac{1}{\ell nx}\right] + 2e$

Section (D) :

D-1. (i) $\frac{1}{2}\tan^{-1}\frac{x}{2} + C$

(ii) $\frac{1}{\sqrt{5}}\tan^{-1}\frac{x}{\sqrt{5}} + C$

(iii) $\frac{1}{2}\tan^{-1}\left(\frac{(x+1)}{2}\right) + C$

(iv) $\ell n\,|x^2 + 3x + 4| - \frac{4}{\sqrt{7}}\tan^{-1}\frac{2x+3}{\sqrt{7}} + C$

(v) $x - \arctan x + \ell n\frac{\sqrt{1+x^2}}{x} + C$

(vi) $\ell n\,|x + \sqrt{x^2 - 4}| + C$

(vii) $\frac{x}{2}\sqrt{x^2+4} + 2\ \ell n\,|x + \sqrt{x^2+4}| + C$

(viii) $\frac{x+1}{2}\sqrt{x^2+2x+5} + 2\ \ell n\left|x+1+\sqrt{x^2+2x+5}\right| + C$

(ix) $-\frac{(1-x-x^2)^{3/2}}{3} - \frac{3}{8}(2x+1)\sqrt{1-x-x^2} - \frac{15}{16}\sin^{-1}\left(\frac{2x+1}{\sqrt{5}}\right) + C$

(x) $\frac{2}{15}(a^3 + x^3)^{5/2} - \frac{2a^3}{9}(a^3 + x^3)^{3/2} + C$

D-2. (i) $\ell n\left|\frac{x+1}{x+2}\right| + C$

(ii) $\frac{1}{10}\ell n\,|x + 3| - \frac{1}{20}\ell n\,|x^2 + 1| + \frac{3}{10}\tan^{-1}x + C$

(iii) $4\ell n|x + 1| + \frac{1}{(x+1)} - 4\ell n|x + 2| + C$

(iv) $\frac{1}{2}\ell n\,|x + 1| - \ell n\,|x + 2| + \frac{1}{2}\ell n\,|x + 3| + C$

D-3. (i) $\frac{1}{2\sqrt{3}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{3}x}\right) - \frac{1}{4}\ell n\left|\frac{x+\frac{1}{x}-1}{x+\frac{1}{x}+1}\right| + C$

(ii) $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{2}\ x}\right) + C$

(iii) $-\frac{1}{2\sqrt{3}}\ell n\left|\frac{x+\frac{1}{x}-\sqrt{3}}{x+\frac{1}{x}+\sqrt{3}}\right| + C$

D-4. (i) $\ell n\left|\frac{\sqrt{x+2}-1}{\sqrt{x+2}+1}\right| + C$

(ii) $\frac{1}{4\sqrt{3}}\ell n\left|\frac{t-\sqrt{3}}{t+\sqrt{3}}\right| - \frac{1}{2}\tan^{-1}(t) + C$, where $t = \sqrt{x+1}$

(iii) $-\frac{1}{\sqrt{3}}\ell n\left|\left(t-\frac{1}{3}\right) + \sqrt{\left(t-\frac{1}{3}\right)^2 + \frac{2}{9}}\right| + C$, where $t = \frac{1}{x+1}$

(iv) $-\tan^{-1}\sqrt{\frac{x^2+2}{x^2}} + C$

D-5. (i) $\frac{1}{2}\ln\left|\left(x+\frac{1}{2}\right)+\sqrt{x^2+x}\right| + \sqrt{x^2+x} + C$

(ii) $\sqrt{x^2-1} - \ln\left|x+\sqrt{x^2-1}\right| + C$ (iii) $\frac{1}{2}\sin^{-1}x - \frac{x}{2}\sqrt{1-x^2} - \sqrt{1-x^2} + C$

Section (E) :

E-1. (i) $\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{\tan x/2}{\sqrt{3}}\right) + C$ (ii) $\frac{2}{\sqrt{3}}\tan^{-1}\left(\sqrt{3}\ \tan\ \frac{x}{2}\right) + C$

(iii) $\frac{10}{13}x - \frac{2}{13}\ell n\ |3\cos x + 2\sin x| + C$ (iv) $\ell n\left|1+\tan\frac{x}{2}\right| + C$

(v) $\frac{1}{\sqrt{6}}\tan^{-1}\left(\frac{\sqrt{3}\tan x}{\sqrt{2}}\right) + C$ (vi) $\ell n\,|1-\cot x| + C$

(vii) $\tan x + \frac{1}{4}\sin 2x - \frac{3x}{2} + C$

E-2. (i) $\frac{1}{40}\ell n\left(\frac{4(\sin x-\cos x)+5}{4(\sin x+\cos x)-5}\right)+C$ (ii) $\sin^{-1}\left(\frac{\sin x+\cos x}{3}\right) + C$

E-3. $A = \frac{1}{9}$, $B = \frac{1}{5}$

PART - II

Section (A) :

A-1. (D) **A-2** (A) **A-3.** (A) **A-4.** (B) **A-5.** (A) **A-6.** (B)

Section (B) :

B-1. (B) **B-2.** (C) **B-3.** (A) **B-4.** (D) **B-5.** (D) **B-6.** (C)
B-7. (A) **B-8.** (C)

Section (C) :

C-1. (C) **C-2.** (A) **C-3.** (C) **C-4.** (A) **C-5.** (C)

Section (D) :

D-1. (B) **D-2.** (D) **D-3.** (A) **D-4.** (A) **D-5.** (C)

Section (E) :

E-1. (B) **E-2** (C) **E-3.** (C) **E-4.** (B) **E-5.** (A) **E-6.** (A) **E-7.** (A)

Section (F) :

F-1. (B) **F-2.** (B)

PART - III

1. (A) $\rightarrow$ (p), (B) $\rightarrow$ (p), (C) $\rightarrow$ (r), (D) $\rightarrow$ (s) **2.** (A) $\rightarrow$ (s) ; (B) $\rightarrow$ (q) ; (C) $\rightarrow$ (r)

EXERCISE - 2

PART - I

1.	(A)	**2.**	(B)	**3.**	(B)	**4.**	(C)	**5.**	(C)	**6.**	(A)	**7.**	(A)
8.	(A)	**9.**	(A)	**10.**	(C)	**11.**	(A)	**12.**	(D)	**13.**	(A)	**14.**	(A)
15.	(A)	**16.**	(C)	**17.**	(D)	**18.**	(D)	**19.**	(C)	**20.**	(A)	**21.**	(B)

PART - II

1.	1	**2.**	a = 3	**3.**	12	**4.**	10	**5.**	5	**6.**	2	**7.**	2
8.	16	**9.**	12	**10.**	11	**11.**	13	**12.**	1	**13.**	2	**14.**	1
15.	11	**16.**	10										

PART - III

1.	(BC)	**2.**	(AC)	**3.**	(AC)	**4.**	(CD)	**5.**	(BD)	**6.**	(ACD)	**7.**	(BC)
8.	(AC)	**9.**	(CD)	**10.**	(AB)	**11.**	(ABC)	**12.**	(AB)	**13.**	(BC)	**14.**	(ABD)
15.	(BD)	**16.**	(ABD)	**17.**	(AD)	**18.**	(AB)	**19.**	(ABCD)				

PART - IV

1.	(A)	**2.**	(A)	**3.**	(B)	**4.**	(A)	**5.**	(B)	**6.**	(A)

EXERCISE - 3

PART - I

1. $\frac{3}{2}\tan^{-1}x - \frac{1}{2}\ell n(1 + x) + \frac{1}{4}\ell n(1 + x^2) + \frac{x}{1+x^2} + C$ **2.** (C)

3. $(x + 1)\tan^{-1}\left(\frac{2x+2}{3}\right) - \frac{3}{4}\ell n(4x^2 + 8x + 13) + C$ **4.** $\frac{(2x^{3m}+3x^{2m}+6x^m)^{\frac{m+1}{m}}}{6(m+1)} + C$

5.	(D)	**6.**	(A)	**7.**	(D)	**8.**	(C)	**9.**	(C)

PART - II

1.	(4)	**2.**	(3)	**3.**	(4)	**4.**	(4)	**5.**	(1)	**6.**	(2)	**7.**	(4)
8.	(2)	**9.**	(4)										

Advance Level Problems (ALP)

1. Evaluate : $\int \frac{\sin^8 x - \cos^8 x}{1 - 2\sin^2 x \cos^2 x}\, dx$

2. Evaluate : $\int \frac{\cos 5x + \cos 4x}{1 - 2\cos 3x} dx$

3. Evaluate : $\int \sqrt{x + \sqrt{x^2 + 2}}\ \ dx$

4. Evaluate : $\int \frac{dx}{\left(x^3 + 3x^2 + 3x + 1\right)\sqrt{x^2 + 2x - 3}}$

5. Evaluate : $\int \frac{(\cos\ \ 2x - 3)}{\cos^4 x\sqrt{4 - \cot^2 x}}\ \ dx$

6. Evaluate : $\left[\frac{\sqrt{x^2+1}\,\{\ell n(x^2+1) - 2\ell n x\}}{x^4}\right] dx$

7. Evaluate : $\int \frac{x}{(7x - 10 - x^2)^{3/2}}\ dx$

8. If $\int \frac{x\cos\alpha + 1}{\left(x^2 + 2x\cos\alpha + 1\right)^{3/2}}\ dx = \frac{f(x)}{\sqrt{g(x)}}$ + C then find f(x) and g(x).

9. Evaluate : cos x. e^x. x^2 dx

10. Evaluate : $\int e^x \left(\frac{x^3 - x + 2}{(x^2+1)^2}\right)\ dx$

11. Evaluate : $\int \frac{x^2}{(x\sin x + \cos x)^2}\ dx$

12. Evaluate : $\int \sin 4x . e^{\tan^2 x} dx$

13. Evaluate : $\int$ $\tan^{-1} x$. $\ell n\ (1 + x^2)$ dx.

14. Evaluate : $\int$ $e^x \frac{1 + n x^{n-1} - x^{2n}}{\left(1 - x^n\right)\sqrt{1 - x^{2n}}} dx$

15. Evaluate : $\int$ cos 2x ℓn (1 + tan x) dx

16. Evaluate : $\int \frac{dx}{\left(a + b\cos x\right)^2}$, (a > b)

17. Evaluate : $\int \frac{\sqrt{2-x-x^2}}{x^2}\,dx$

18. Integrate: $\int \frac{(5x^2-12)\,dx}{(x^2-6x+13)^2}$

19. If $\int \frac{3x^2+2x}{x^6+2x^5+x^4+2x^3+2x^2+5}\,dx = F(x)$, then find the value of [F(1) – F(0)], where [.] represents greatest integer function.

20. Evaluate : $\int \frac{\ell n\ (1+\sin^2 x)}{\cos^2 x}\,dx$

21. Evaluate : $\int \frac{1+\cos\alpha\ \cos x}{\cos\alpha+\cos x}\,dx$

22. Evaluate : $\int \frac{a+b\sin x}{(b+a\sin x)^2}\,dx$

23. Evaluate : $\int \frac{dx}{(x-\alpha)\sqrt{(x-\alpha)(x-\beta)}}$

24. Evaluate $\int \frac{(\cos 2x)^{1/2}}{\sin x}\,dx$

25. Evaluate $\int \frac{\sin^3 \frac{x}{2}}{\cos\frac{x}{2}\sqrt{\cos^3 x+\cos^2 x+\cos x}}\,dx$

26. If $\int \frac{x^2}{x^4+3x^2+9}\,dx = A\tan^{-1}\left(\frac{x^2-3}{3x}\right)+\frac{B}{\sqrt{3}}\ \ell n\left|\frac{x^2-\sqrt{3}\ x+3}{x^2+\sqrt{3}\ x+3}\right|$ + c, then find the value of 12(A + B).

27. Evaluate $\int \frac{3\cos x+2}{\sin x+2\cos x+3}\,dx$

28. Evaluate $\int \sqrt[3]{\tan x}\,dx$

29. Evaluate : $\int \sqrt{\frac{\operatorname{cosec} x-\cot x}{\operatorname{cosec} x+\cot x}}\cdot\frac{\sec x}{\sqrt{1+2\sec x}}\,dx$

Answers

1. $-\frac{1}{2}\sin 2x + C$

2. $-\left(\sin x + \frac{\sin 2x}{2}\right) + C$

3. $\frac{1}{3}\left(x+\sqrt{x^2+2}\right)^{3/2} - \frac{2}{\left(x+\sqrt{x^2+2}\right)^{1/2}} + C$

4. $\frac{\sqrt{x^2+2x-3}}{8(x+1)^2} + \frac{1}{16}.\cos^{-1}\left(\frac{2}{x+1}\right) + C$

5. $C - \frac{1}{3}\tan x.(2+\tan^2 x).\sqrt{4-\cot^2 x}$

6. $\frac{2\ (x^2+1)\sqrt{x^2+1}}{9x^3}.\left[1-\frac{3}{2}\ell n\left(1+\frac{1}{x^2}\right)\right] + C$

7. $\frac{2(7x-20)}{9\sqrt{7x-10-x^2}}+C$

8. x; $x^2+2x\cos\alpha+1$

9. $\frac{1}{2}e^x[(x^2-1)\cos x + (x-1)^2.\sin x] + C$

10. $e^x\left(\frac{x+1}{x^2+1}\right)+C$

11. $\frac{\sin x - x\cos x}{x\sin x+\cos x}+C$

12. $-2\cos^4 x.\ e^{\tan^2 x} + C$

13. $x\tan^{-1}x.\ \ell n(1+x^2) + (\tan^{-1}x)^2 - 2x\tan^{-1}x + \ell n(1+x^2) - \left(\ell n\sqrt{1+x^2}\right)^2 + C$

14. $e^x\sqrt{\frac{1+x^n}{1-x^n}} + C$

15. $\frac{1}{2}[\sin 2x.\ell n(1+\tan x) - x + \ell n|\sin x+\cos x|] + C$

16. $-\frac{b\sin x}{(a^2-b^2)(a+b\cos x)} + \frac{2a}{(a^2-b^2)^{3/2}}\tan^{-1}\sqrt{\frac{a-b}{a+b}}\tan\frac{x}{2}+C$

17. $-\frac{\sqrt{2-x-x^2}}{x} + \frac{\sqrt{2}}{4}\ln\left|\frac{4-x+2\sqrt{2}\sqrt{2-x-x^2}}{x}\right| - \sin^{-1}\left(\frac{2x+1}{3}\right)+K$

18. $\frac{13x-159}{8(x^2-6x+13)} + \frac{53}{16}\tan^{-1}\frac{x-3}{2}+C$

19. 0

20. $\tan x\ln(1+\sin^2 x) - 2x + \sqrt{2}\tan^{-1}(\sqrt{2}.\tan x)+C.$

21. $x\cos\alpha + \sin\alpha\ \ell n\left|\frac{\cos\frac{1}{2}(\alpha-x)}{\cos\frac{1}{2}(\alpha+x)}\right|+C$

22. $-\frac{\cos x}{b+a\sin x}+C$

23. $\frac{-2}{\alpha-\beta}.\sqrt{\frac{x-\beta}{x-\alpha}}+C$

24. $\sqrt{2}\log\left[\frac{\sqrt{\cot^2x-1}+\sqrt{2\cot^2 x}}{\sqrt{\cot^2x+1}}\right] - \log\left[\cot x+\sqrt{\cot^2x-1}\right]+c$

25. $\sec^{-1}\left(\sqrt{\cos x}+\frac{1}{\sqrt{\cos x}}\right)+c.$

26. 5

27. $\frac{6}{5}x + \frac{3}{5}\log|\sin x+2\cos x+3| - \frac{8}{5}\tan^{-1}\left(\frac{\tan\frac{x}{2}+1}{2}\right)+C$

28. $-\frac{1}{2}\log(1+\tan^{2/3}x)+\frac{1}{4}\log(\tan^{4/3}x-\tan^{2/3}x+1)+\frac{\sqrt{3}}{2}\tan^{-1}\frac{2\tan^{2/3}x-1}{\sqrt{3}}+c$

29. $\sin^{-1}\left(\frac{1}{2}\sec^2\frac{x}{2}\right)+C$