

Exercise-1

Marked questions are recommended for Revision.

PART - I : SUBJECTIVE QUESTIONS

Section (A) : Definite Integration in terms of Indefinite Integration, using substitution and By parts

A-1. Evaluate :

$$(i) \int_0^1 \frac{\sqrt[3]{x^2} - \sqrt[4]{x}}{\sqrt{x}} dx$$

$$(ii) \int_0^1 x \cos(\tan^{-1} x) dx$$

A-2. Evaluate :

$$(i) \int_{-\infty}^{\infty} \frac{dx}{x^2 + 2x + 2}$$

$$(ii) \int_{\sqrt{2}}^{\infty} \frac{dx}{x\sqrt{x^2 - 1}}$$

$$(iii) \int_0^4 \frac{x^2}{1+x} dx$$

$$(iv) \int_0^{\pi/2} \sqrt{\cos \theta} \sin^3 \theta d\theta$$

A-3. Evaluate :

$$(i) \int_0^1 \sin^{-1} x dx$$

$$(ii) \int_1^2 \frac{\ln x}{x^2} dx$$

$$(iii) \int_0^1 x^2 \sin^{-1} x dx.$$

A-4. Evaluate

$$(i) \int_0^{\pi/3} f(x) dx \text{ where } f(x) = \text{Minimum} \{ \tan x, \cot x \} \forall x \in \left(0, \frac{\pi}{2} \right)$$

$$(ii) \int_{-1}^1 f(x) dx \text{ where } f(x) = \min \{ x+1, \sqrt{1-x} \}$$

$$(iii) \int_{-1}^1 f(x) dx \text{ where } f(x) = \text{minimum} (|x|, 1 - |x|, 1/4)$$

A-5. Evaluate

$$(i) \int_0^1 \sin^{-1} \left(\frac{2x}{1+x^2} \right) dx$$

$$(ii) \int_0^1 \frac{x \tan^{-1} x}{(1+x^2)^{3/2}} dx$$

$$(iii) \int_a^b \sqrt{(x-a)(b-x)} dx, a > b$$

$$(iv) \int_0^{\sqrt{3}} \tan^{-1} \left(\frac{2x}{1-x^2} \right) dx$$

A-6. Evaluate :

$$(i) \int_0^{\infty} \frac{dx}{e^x + e^{-x}}$$

$$(ii) \int_0^1 \frac{x}{1+\sqrt{x}} dx$$

$$(iii) \int_0^{\pi/2} \frac{\sin x \cos x}{\cos^2 x + 3 \cos x + 2} dx$$

$$(iv) \int_0^{\pi/2} \frac{\sin 2\theta d\theta}{\sin^4 \theta + \cos^4 \theta}$$

$$(v) \int_0^{\pi/4} \frac{\sin x + \cos x}{9 + 16 \sin 2x} dx$$

A-7 (i) Find the value of a such that $\int_0^a \frac{1}{e^x + 4e^{-x} + 5} dx = \ln \sqrt[3]{2}.$

(ii) Find the value of $\int_0^{(\pi/2)^{1/3}} x^5 \cdot \sin x^3 dx$

Section (B) : Definite Integration using Properties

B-1. Let $f(x) = \ln \left(\frac{1-\sin x}{1+\sin x} \right)$, then show that $\int_a^b f(x) dx = \int_b^a \ln \left(\frac{1+\sin x}{1-\sin x} \right) dx$.

B-2. Evaluate :

(i) $\int_0^2 [x^2] dx$ (where $[.]$ denotes greatest integer function)

(ii) $\int_0^\pi \sqrt{1+\sin 2x} dx$ (iii) $\int_0^2 f(x) dx$ where $f(x) = \begin{cases} 2x+1 & 0 \leq x < 1 \\ 3x^2 & 1 \leq x \leq 2 \end{cases}$

(iv) $\int_0^4 |x^2 - 4x + 3| dx$ (v) $\int_0^\infty [\cot^{-1} x] dx$ (where $[.]$ denotes greatest integer function)

(vi) $\int_{-5}^5 |x+2| dx$ (vii) $\int_{-1}^1 [\cos^{-1} x] dx$ (where $[.]$ denotes greatest integer function)

B-3. Evaluate :

(i) $\int_{-1}^1 e^{|x|} dx$ (ii) $\int_{-\pi/4}^{\pi/4} |\sin x| dx$ (iii) $\int_{-\pi/4}^{\pi/4} \frac{x + \pi/4}{2 - \cos 2x} dx$

(iv) $\int_{-1}^1 \sin^5 x \cos^4 x dx$ (v) $\int_{-\pi/2}^{\pi/2} \frac{g(x) - g(-x)}{f(-x) + f(x)} dx$

B-4. Evaluate

(i) $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$ (ii) $\int_0^{\pi/2} \frac{e^{\sin x}}{e^{\sin x} + e^{\cos x}} dx$ (iii) $\int_0^a \frac{\sqrt{x}}{\sqrt{x} + \sqrt{a-x}} dx$

(iv) $\int_0^{\pi/2} \frac{a \sin x + b \cos x}{\sin x + \cos x} dx$ (v) $\int_0^{\pi/2} \frac{\sin x - \cos x}{(\sin x + \cos x)^2} dx$

B-5. Evaluate :

(i) $\int_0^{2\pi} \{\sin(\sin x) + \sin(\cos x)\} dx$ (ii) $\int_0^\pi \frac{dx}{5 + 4 \cos 2x}$

(iii) $\int_0^{\pi/2} (2 \ln \sin x - \ln \sin 2x) dx$ (iv) $\int_0^\infty \ln \left(x + \frac{1}{x} \right) \cdot \frac{dx}{1+x^2}$

B-6. Evaluate :

(i) $\int_{-1}^2 \{2x\} dx$ (where function $\{.\}$ denotes fractional part function)

(ii) $\int_0^{10\pi} (|\sin x| + |\cos x|) dx$

(iii) $\frac{\int_0^n [x] dx}{\int_0^n \{x\} dx}$, where $[x]$ and $\{x\}$ are integral and fractional parts of x and $n \in \mathbb{N}$

(iv) $\int_0^{2n\pi} \left(|\sin x| - \left\lfloor \frac{|\sin x|}{2} \right\rfloor \right) dx$ (where $[.]$ denotes the greatest integer function and $n \in \mathbb{I}$)

B-7. If $f(x)$ is a function defined $\forall x \in \mathbb{R}$ and $f(x) + f(-x) = 0 \forall x \in \left[-\frac{T}{2}, \frac{T}{2}\right]$ and has period T , then prove that

$$\phi(x) = \int_a^x f(t) dt \text{ is also periodic with period } T.$$

Section (C) : Leibnitz formula and Wallis' formula

C-1. (i) If $f(x) = 5^{g(x)}$ and $g(x) = \int_2^{x^2} \frac{t}{\ln(1+t^2)} dt$, then find the value of $f'(\sqrt{2})$.

(ii) The value of $\lim_{x \rightarrow 0} \frac{\frac{d}{dx} \int_0^{x^3} \sqrt{\cos t} dt}{1 - \sqrt{\cos x}}$

(iii) Find the slope of the tangent to the curve $y = \int_x^{x^2} \cos^{-1} t^2 dt$ at $x = \frac{1}{\sqrt[4]{2}}$

C-2. (i) If $f(x) = \int_0^{\sin^2 x} \sin^{-1} \sqrt{t} dt + \int_0^{\cos^2 x} \cos^{-1} \sqrt{t} dt$, then prove that $f'(x) = 0 \forall x \in \mathbb{R}$.

(ii) Find the value of x for which function $f(x) = \int_{-1}^x t(e^t - 1)(t - 1)(t - 2)^3(t - 3)^5 dt$ has a local minimum

C-3. If $y = \int_1^x x \sqrt{\ln t} dt$

then find the value of $\frac{d^2 y}{dx^2}$ at $x = e$

C-4. $\lim_{n \rightarrow \infty} \frac{\int_{1/(n+1)}^{1/n} \tan^{-1}(nx) dx}{\int_{1/(n+1)}^{1/n} \sin^{-1}(nx) dx}$ is equal to

C-5. Let f be a differentiable function on $\mathbb{R}$ and satisfying the integral equation

$$x \int_0^x f(t) dt - \int_0^x t f(x-t) dt = e^x - 1 \quad \forall x \in \mathbb{R}, \text{ then } f(1) \text{ equals to}$$

C-6. Evaluate :

(i) $\int_{-\pi/2}^{\pi/2} \sin^2 x \cos^2 x (\sin x + \cos x) dx$

(ii) $\int_0^{\pi} x \sin^5 x dx$

(iii) $\int_0^2 x^{3/2} \sqrt{2-x} dx$

(iv) $\int_0^{2\pi} x (\sin^2 x \cos^2 x) dx$

SECTION (D) : ESTIMATION & MEAN VALUE THEOREM

D-1. Prove the following inequalities :

$$(i) \quad \frac{\sqrt{3}}{8} < \int_{\pi/4}^{\pi/3} \frac{\sin x}{x} dx < \frac{\sqrt{2}}{6} \quad (ii) \quad 4 \leq \int_1^3 \sqrt{(3+x^3)} dx \leq 2\sqrt{30}$$

D-2. Show that

$$(i) \quad \frac{1}{10\sqrt{2}} < \int_0^1 \frac{x^9}{\sqrt{1+x}} dx < \frac{1}{10} \quad (ii) \quad \frac{1}{2} \ln 2 < \int_0^1 \frac{\tan x}{1+x^2} dx < \frac{\pi}{2}$$

D-3. (i) Show that $\int_0^2 \sin x \cdot \cos \sqrt{x} dx = 2 \sin c \cdot \cos \sqrt{c}$ for some $c \in (0, 2)$

(ii) $f(x)$ is a continuous function $x \in \mathbb{R}$, then show that $\int_1^4 f(x) dx = 2\alpha f(\alpha^2)$ some $\alpha \in (1, 2)$

Section (E) : Integration as a limit of sum and reduction formula

E-1. Evaluate :

$$(i) \quad \lim_{n \rightarrow \infty} \sum_{r=0}^{n-1} \frac{1}{\sqrt{n^2 - r^2}} \quad (ii) \quad \lim_{n \rightarrow \infty} \frac{3}{n} \left[1 + \sqrt{\frac{n}{n+3}} + \sqrt{\frac{n}{n+6}} + \sqrt{\frac{n}{n+9}} + \dots + \sqrt{\frac{n}{n+3(n-1)}} \right]$$

$$(iii) \quad \lim_{n \rightarrow \infty} \frac{1}{n^4} \left(\sum_{r=1}^{2n} (3nr^2 + 2n^2r) \right)$$

E-2. (i) If $I_n = \int_0^{\pi/4} \tan^n x dx$, then show that $I_n + I_{n-2} = \frac{1}{n-1}$

(ii) $I_n = \int_0^{\pi/2} (\sin x)^n dx$, $n \in \mathbb{N}$. Show that $I_n = \frac{n-1}{n} I_{n-2} \quad \forall n \geq 2$

Section (F) : Area Under Curve

F-1. Find the area enclosed between the curve $y = x^3 + 3$, $y = 0$, $x = -1$, $x = 2$.

F-2. (i) Find the area bounded by $x^2 + y^2 - 2x = 0$ and $y = \sin \frac{\pi x}{2}$ in the upper half of the circle.

(ii) Find the area bounded by the curve $y = 2x^4 - x^2$, x -axis and the two ordinates corresponding to the minima of the function.

(iii) Find area of the curve $y^2 = (7-x)(5+x)$ above x -axis and between the ordinates $x = -5$ and $x = 1$.

F-3. Find the area of the region bounded by the curve $y^2 = 2y - x$ and the y -axis.

F-4. Find the area bounded by the y -axis and the curve $x = e^y \sin \pi y$, $y = 0$, $y = 1$.

F-5. (i) Find the area bounded in the first quadrant between the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ and the line $3x + 4y = 12$

(ii) Find the area of the region bounded by $y = \{x\}$ and $2x - 1 = 0$, $y = 0$, ($\{ \}$ stands for fraction part)

F-6. Compute the area of the figure bounded by straight lines $x = 0$, $x = 2$ and the curves $y = 2^x$ and $y = 2x - x^2$

F-7. Let $f(x) = \sqrt{\tan x}$. Show that area bounded by $y = f(x)$, $y = f(c)$, $x = 0$ and $x = a$, $0 < c < a < \frac{\pi}{2}$ is minimum when $c = \frac{a}{2}$

- F-8.** Find the area included between the parabolas $y^2 = x$ and $x = 3 - 2y^2$.
- F-9.** A tangent is drawn to the curve $x^2 + 2x - 4ky + 3 = 0$ at a point whose abscissa is 3. This tangent is perpendicular to $x + 3 = 2y$. Find the area bounded by the curve, this tangent and ordinate $x = -1$
- F-10.** (i) Draw graph of $y = (\tan x)^n$, $n \in \left[0, \frac{\pi}{4}\right]$, $x \in \left[0, \frac{\pi}{4}\right]$. Hence show
 $0 < (\tan x)^{n+1} < (\tan x)^n$, $x \in \left(0, \frac{\pi}{4}\right)$
- (ii) Let A_n be the area bounded by the curve $y = (\tan x)^n$ and the lines $x = 0$, $y = 0$ and $x = \pi/4$. Prove that for $n > 2$, $A_n + A_{n-2} = 1/(n-1)$ and deduce that $1/(2n+2) < A_n < 1/(2n-2)$.

PART - II : ONLY ONE OPTION CORRECT TYPE

SECTION (A) : D.I. IN TERMS OF INDEFINITE INTEGRATION, USING SUBSTITUTION AND BY PARTS

- A-1.** If $\int_1^x \frac{dt}{t|\sqrt{t^2-1}} = \frac{\pi}{6}$, then x can be equal to :
- (A) $\frac{2}{\sqrt{3}}$ (B) $\sqrt{3}$ (C) 2 (D) $\frac{4}{\sqrt{3}}$
- A-2.** The value of the integral $\int_0^1 \frac{dx}{x^2 + 2x \cos \alpha + 1}$, where $0 < \alpha < \frac{\pi}{2}$, is equal to:
- (A) $\sin \alpha$ (B) $\alpha \sin \alpha$ (C) $\frac{\alpha}{2 \sin \alpha}$ (D) $\frac{\alpha}{2} \sin \alpha$
- A-3.** If $f(x) = \begin{cases} x & x < 1 \\ x-1 & x \geq 1 \end{cases}$, then $\int_0^2 x^2 f(x) dx$ is equal to :
- (A) 1 (B) $\frac{4}{3}$ (C) $\frac{5}{3}$ (D) $\frac{5}{2}$
- A-4.** If $f(0) = 1$, $f(2) = 3$, $f'(2) = 5$ and $f'(0)$ is finite, then $\int_0^1 x \cdot f''(2x) dx$ is equal to
- (A) zero (B) 1 (C) 2 (D) 3
- A-5.** $\int_0^{\pi} |1 + 2 \cos x| dx$ is equal to :
- (A) $\frac{2\pi}{3}$ (B) π (C) 2 (D) $\frac{\pi}{3} + 2\sqrt{3}$
- A-6.** The value of $\int_{-1}^3 (|x-2| + [x]) dx$ is ($[x]$ stands for greatest integer less than or equal to x)
- (A) 7 (B) 5 (C) 4 (D) 3
- A-7.** $\int_0^{\infty} [2e^{-x}] dx$, where $[\cdot]$ denotes the greatest integer function, is equal to :
- (A) 0 (B) $\frac{1}{2}$ (C) e^2 (D) $2e^{-1}$

A-8. $\int_{\ell n \pi - \ell n 2}^{\ell n \pi} \frac{e^x}{1 - \cos\left(\frac{2}{3}e^x\right)} dx$ is equal to

- (A) $\sqrt{3}$ (B) $-\sqrt{3}$ (C) $\frac{1}{\sqrt{3}}$ (D) $-\frac{1}{\sqrt{3}}$

A-9. If $I_1 = \int_e^{e^2} \frac{dx}{\ell n x}$ and $I_2 = \int_1^2 \frac{e^x}{x} dx$, then

- (A) $I_1 = I_2$ (B) $2 I_1 = I_2$ (C) $I_1 = 2 I_2$ (D) $I_1 + I_2 = 0$

A-10. $\int_0^{\pi/4} \frac{x \cdot \sin x}{\cos^3 x} dx$ equals to :

- (A) $\frac{\pi}{4} + \frac{1}{2}$ (B) $\frac{\pi}{4} - \frac{1}{2}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{4} + 1$

A-11. The value of the definite integral $\int_{\frac{3}{2}}^{\frac{9}{4}} \left[\sqrt{2x - \sqrt{5(4x - 5)}} + \sqrt{2x + \sqrt{5(4x - 5)}} \right] dx$ is equal to

- (A) $4\sqrt{5} - \frac{2\sqrt{2}}{5}$ (B) $4\sqrt{5}$ (C) $4\sqrt{3} - \frac{4}{3}$ (D) $\frac{3\sqrt{5}}{\sqrt{8}}$

A-12. If $\int_{\ell n 2}^x \frac{dx}{\sqrt{e^x - 1}} = \frac{\pi}{6}$, then x is equal to

- (A) 4 (B) $\ln 8$ (C) $\ln 4$ (D) $\ln 2$

A-13. $\int_0^{\infty} \frac{x^2 + 1}{x^4 + 7x^2 + 1} dx =$

- (A) π (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{6}$

Section (B) : Definite Integration using Properties

B-1. Suppose for every integer n, $\int_n^{n+1} f(x) dx = n^2$. The value of $\int_{-2}^4 f(x) dx$ is :

- (A) 16 (B) 14 (C) 19 (D) 21

B-2. Let $f : \mathbb{R} \rightarrow \mathbb{R}$, $g : \mathbb{R} \rightarrow \mathbb{R}$ be continuous functions. Then the value of integral

$$\int_{\ell n \lambda}^{\ell n 1/\lambda} \frac{f\left(\frac{x^2}{4}\right)[f(x) - f(-x)]}{g\left(\frac{x^2}{4}\right)[g(x) + g(-x)]} dx \text{ is :}$$

- (A) depend on λ (B) a non-zero constant (C) zero (D) 2

B-3. $\int_{-1}^1 \cot^{-1}\left(\frac{x + x^3}{1 + x^4}\right) dx$ is equal to

- (A) 2π (B) $\frac{\pi}{2}$ (C) 0 (D) π

- B-4.** $\int_{-2}^0 \{x^3 + 3x^2 + 3x + 3 + (x+1)\cos(x+1)\} dx$ is equal to
 (A) -4 (B) 0 (C) 4 (D) 6
- B-5.** $\int_{-1}^1 x \ln(1+e^x) dx =$
 (A) 0 (B) $\ln(1+e)$ (C) $\ln(1+e) - 1$ (D) $1/3$
- B-6.** If $\int_{-1}^{3/2} |x \sin \pi x| dx = \frac{k}{\pi^2}$, then the value of k is :
 (A) $3\pi + 1$ (B) $2\pi + 1$ (C) 1 (D) 4
- B-7.** The value of definite integral is $\int_0^{\frac{\pi^2}{4}} \frac{dx}{1 + \sin \sqrt{x} + \cos \sqrt{x}}$
 (A) $\pi \ln 2$ (B) $\frac{\pi \ln 2}{2}$ (C) $\frac{\pi \ln 2}{4}$ (D) $2\pi \ln 2$
- B-8.** $\int_{2-\ln 3}^{3+\ln 3} \frac{\ln(4+x)}{\ln(4+x) + \ln(9-x)} dx$ is equal to :
 (A) cannot be evaluated (B) is equal to $\frac{5}{2}$
 (C) is equal to $1 + 2 \ln 3$ (D) is equal to $\frac{1}{2} + \ln 3$
- B-9.** The value of the definite integral $I = \int_0^{\pi} x \sqrt{1 + |\cos x|} dx$ is equal to
 (A) $2\sqrt{2} \pi$ (B) $\sqrt{2} \pi$ (C) 2π (D) 4π
- B-10.** The value of $\int_0^{\pi/2} \ln |\tan x + \cot x| dx$ is equal to :
 (A) $\pi \ln 2$ (B) $-\pi \ln 2$ (C) $\frac{\pi}{2} \ln 2$ (D) $-\frac{\pi}{2} \ln 2$
- B-11.** Let $I_1 = \int_0^1 \frac{e^x dx}{1+x}$ and $I_2 = \int_0^1 \frac{x^2 dx}{e^{x^3}(2-x^3)}$, then $\frac{I_1}{I_2}$ is
 (A) $3/e$ (B) $e/3$ (C) $3e$ (D) $1/3e$
- B-12.** The value of $\int_0^{[x]} \{x\} dx$ (where $[\cdot]$ and $\{ \cdot \}$ denotes greatest integer and fraction part function respectively) is
 (A) $\frac{1}{2} [x]$ (B) $2[x]$ (C) $\frac{1}{2[x]}$ (D) $[x]$
- B-13.** If $\int_0^{11} \frac{11^x}{11^{[x]}} dx = \frac{k}{\log 11}$, (where $[\cdot]$ denotes greatest integer function) then value of k is
 (A) 11 (B) 101 (C) 110 (D) 121

Section (C) : Leibnitz formula and Wallis' formula

C-1. $f(x) = \int_x^{x^2} \frac{e^t}{t} dt$, then $f'(1)$ is equal to:

- (A) e (B) $2e$ (C) $2e^2 - 2$ (D) $e^2 - e$

C-2. $f(x) = \int_0^x (t-1)(t-2)^2(t-3)^3(t-4)^5 dt$ ($x > 0$) then number of points of extremum of $f(x)$ is

- (A) 4 (B) 3 (C) 2 (D) 1

C-3. Limit $\lim_{h \rightarrow 0} \frac{\int_a^{x+h} \ln^2 t \, dt - \int_a^x \ln^2 t \, dt}{h}$ equals to :

- (A) 0 (B) $\ln^2 x$ (C) $\frac{2 \ln x}{x}$ (D) does not exist

C-4. The value of the function $f(x) = 1 + x + \int_1^x (\ln^2 t + 2 \ln t) dt$, where $f'(x)$ vanishes is:

- (A) e^{-1} (B) 0 (C) $2e^{-1}$ (D) $1 + 2e^{-1}$

C-5. If $\int_a^y \cos t^2 dt = \int_a^{x^2} \frac{\sin t}{t} dt$, then the value of $\frac{dy}{dx}$ is

- (A) $\frac{2 \sin^2 x}{x \cos^2 y}$ (B) $\frac{2 \sin x^2}{x \cos y^2}$ (C) $\frac{2 \sin x^2}{x \left(1 - 2 \sin \frac{y^2}{2}\right)}$ (D) $\frac{\sin x^2}{2y}$

C-6. If $\int_{\sin x}^1 t^2 (f(t)) dt = (1 - \sin x)$, then $f\left(\frac{1}{\sqrt{3}}\right)$ is

- (A) $1/3$ (B) $1/\sqrt{3}$ (C) 3 (D) $\sqrt{3}$

C-7. The value of $\lim_{a \rightarrow \infty} \frac{1}{a^2} \int_0^a \ln(1 + e^x) dx$ equals

- (A) 0 (B) 1 (C) $\frac{1}{2}$ (D) non-existent

C-8. $f(x) = \int_1^x \frac{\sin x \cos y}{y^2 + y + 1} dy$, then

- (A) $f'(x) = 0 \quad \forall x = \frac{n\pi}{2}, n \in \mathbb{Z}$ (B) $f'(x) = 0 \quad \forall x = (2n+1)\frac{\pi}{2}, n \in \mathbb{Z}$
(C) $f'(x) = 0 \quad \forall x = n\pi, n \in \mathbb{Z}$ (D) $f'(x) \neq 0 \quad \forall x \in \mathbb{R}$

C-9. $\int_0^{\pi/2} \sin^4 x \cos^3 x dx$ is equal to :

- (A) $\frac{6}{35}$ (B) $\frac{2}{21}$ (C) $\frac{2}{15}$ (D) $\frac{2}{35}$

C-10. $\int_0^1 x^2(1-x)^3 dx$ is equal to

(A) $\frac{1}{60}$

(B) $\frac{1}{30}$

(C) $\frac{2}{15}$

(D) $\frac{\pi}{120}$

SECTION (D) : ESTIMATION & MEAN VALUE THEOREM

D-1. Let $I = \int_1^3 \sqrt{x^4 + x^2} dx$, then

(A) $I > 6\sqrt{10}$

(B) $I < 2\sqrt{2}$

(C) $2\sqrt{2} < I < 6\sqrt{10}$

(D) $I < 1$

D-2. $I = \int_0^{2\pi} e^{\sin^2 x + \sin x + 1} dx$, then

(A) $\pi e^3 < I < 2\pi e^5$

(B) $2\pi e^{3/4} < I < 2\pi e^3$

(C) $2\pi e^3 < I < 2\pi e^4$

(D) $0 < I < 2\pi$

D-3. Let $f''(x) \geq 0$, $f'(x) > 0$, $f(0) = 3$ & $f(x)$ is defined in $[-2, 2]$. If $f(x)$ is non-negative, then

(A) $\int_{-1}^0 f(x) dx > 6$

(B) $\int_{-2}^2 f(x) dx > 12$

(C) $\int_{-2}^2 f(x) dx \geq 12$

(D) $\int_{-1}^1 f(x) dx > 12$

D-4. Let mean value of $f(x) = \frac{1}{x+c}$ over interval $(0, 2)$ is $\frac{1}{2} \ln 3$ then positive value of c is

(A) 1

(B) $\frac{1}{2}$

(C) 2

(D) $\frac{3}{2}$

SECTION (E) : INTEGRATION AS A LIMIT OF SUM AND REDUCTION FORMULA

E-1. $\lim_{n \rightarrow \infty} \sum_{r=1}^n \left(\frac{r^3}{r^4 + n^4} \right)$ equals to :

(A) $\frac{1}{n^2}$

(B) $\frac{1}{2} \ln 2$

(C) $\frac{1}{3} \ln 2$

(D) $\frac{1}{4} \ln 2$

E-2. $\lim_{n \rightarrow \infty} \sum_{r=2n+1}^{3n} \frac{n}{r^2 - n^2}$ is equal to :

(A) $\frac{1}{n} \sqrt{\frac{2}{3}}$

(B) $\frac{1}{n} \sqrt{\frac{3}{2}}$

(C) $\frac{1}{n} \frac{2}{3}$

(D) $\frac{1}{n} \frac{3}{2}$

E-3. $\lim_{n \rightarrow \infty} \left[\left(1 + \frac{1}{n^2} \right) \left(1 + \frac{2^2}{n^2} \right) \dots \left(1 + \frac{n^2}{n^2} \right) \right]^{1/n}$ is equal to :

(A) $\frac{e^{\pi/2}}{2e^2}$

(B) $2e^2 e^{\pi/2}$

(C) $\frac{2}{e^2} e^{\pi/2}$

(D) $2e^{\pi}$

E-4. $\lim_{n \rightarrow \infty} \frac{\pi}{n} \left[\sin \frac{\pi}{n} + \sin \frac{2\pi}{n} + \dots + \sin \frac{(n-1)\pi}{n} \right]$ is equals to :

(A) 0

(B) π

(C) 2

(D) 3

E-5. Let $I_n = \int_0^1 (1-x^3)^n dx$, ($n \in \mathbb{N}$) then

(A) $3n I_n = (3n-1) I_{n-1} \forall n \geq 2$

(B) $(3n-1) I_n = 3n I_{n-1} \forall n \geq 2$

(C) $(3n-1) I_n = (3n+1) I_{n-1} \forall n \geq 2$

(D) $(3n+1) I_n = 3n I_{n-1} \forall n \geq 2$

Section (F) : Area Under Curve

- F-1.** The area bounded by the x-axis and the curve $y = 4x - x^2 - 3$ is
 (A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) $\frac{4}{3}$ (D) $\frac{8}{3}$
- F-2.** The area of the figure bounded by right of the line $y = x + 1$, $y = \cos x$ and x-axis is:
 (A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) $\frac{5}{6}$ (D) $\frac{3}{2}$
- F-3.** Area bounded by curve $y^3 - 9y + x = 0$ and y-axis is
 (A) $\frac{9}{2}$ (B) 9 (C) $\frac{81}{2}$ (D) 81
- F-4.** Let $f: [0, \infty) \rightarrow \mathbb{R}$ be a continuous and strictly increasing function such that $f^3(x) = \int_0^x f^2(t) dt$, $x \geq 0$.
 The area enclosed by $y = f(x)$, the x-axis and the ordinate at $x = 3$ is _____
 (A) $\frac{3}{2}$ (B) $\frac{5}{2}$ (C) $\frac{7}{2}$ (D) $\frac{1}{2}$
- F-5.** The area bounded by the curve $y = e^x$ and the lines $y = |x - 1|$, $x = 2$ is given by:
 (A) $e^2 + 1$ (B) $e^2 - 1$ (C) $e^2 - 2$ (D) $e - 2$
- F-6.** The area bounded by $y = 2 - |2 - x|$ and $y = \frac{3}{|x|}$ is:
 (A) $\frac{4+3 \ln 3}{2}$ (B) $\frac{4-3 \ln 3}{2}$ (C) $\frac{3}{2} + \ln 3$ (D) $\frac{1}{2} + \ln 3$
- F-7.** The area bounded by the curve $y^2 = 4x$ and the line $2x - 3y + 4 = 0$ is
 (A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) $\frac{4}{3}$ (D) $\frac{5}{3}$
- F-8.** The area of the region bounded by $x = 0$, $y = 0$, $x = 2$, $y = 2$, $y \leq e^x$ and $y \geq \ln x$, is
 (A) $6 - 4 \ln 2$ (B) $4 \ln 2 - 2$ (C) $2 \ln 2 - 4$ (D) $6 - 2 \ln 2$
- F-9.** The area between two arms of the curve $|y| = x^3$ from $x = 0$ to $x = 2$ is
 (A) 2 (B) 4 (C) 8 (D) 16
- F-10.** The area bounded by the parabolas $y = (x + 1)^2$ and $y = (x - 1)^2$ and the line $y = \frac{1}{4}$ is
 (A) 4 sq. units (B) $\frac{1}{6}$ sq. units (C) $\frac{4}{3}$ sq. units (D) $\frac{1}{3}$ sq. units

PART - III : MATCH THE COLUMN

1. Let $\lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T (\sin x + \sin ax)^2 dx = L$ then

Column - I

- (A) for $a = 0$, the value of L is
 (B) for $a = 1$ the value of L is
 (C) for $a = -1$ the value of L is
 (D) $\forall a \in \mathbb{R} - \{-1, 0, 1\}$ the value of L is

Column- II

- (p) 0
 (q) $1/2$
 (r) $3/2$
 (s) 2
 (t) 1

2. Column – I

- (A) Area bounded by region $0 \leq y \leq 4x - x^2 - 3$ is
- (B) The area of figure formed by all the points satisfying the inequality $y^2 \leq 4(1 - |x|)$ is
- (C) The area bounded by $|x| + |y| \leq 1$ and $|x| \geq 1/2$ is
- (D) Area bounded by $x \leq 4 - y^2$ and $x \geq 0$ is

Column – II

- (p) $32/3$
- (q) $1/2$
- (r) $4/3$
- (s) $16/3$

Exercise-2

Marked questions are recommended for Revision.

PART - I : ONLY ONE OPTION CORRECT TYPE

- The value of $\int_0^1 (\{2x\} - 1)(\{3x\} - 1) dx$, (where $\{ \}$ denotes fractional part of x) is equal to :
 (A) $\frac{19}{36}$ (B) $\frac{19}{144}$ (C) $\frac{19}{72}$ (D) $\frac{19}{18}$
- If $\int_0^{100} f(x) dx = a$, then $\sum_{r=1}^{100} \left(\int_0^1 f(r-1+x) dx \right) =$
 (A) $100a$ (B) a (C) 0 (D) $10a$
- $\lim_{t \rightarrow \left(\frac{\pi}{2}\right)^-} \int_0^t \tan \theta \sqrt{\cos \theta} \ln(\cos \theta) d\theta$ is equal to:
 (A) -4 (B) 4 (C) -2 (D) Does not exists
- If $f(x) = \begin{cases} 0, & \text{where } x = \frac{n}{n+1}, n=1, 2, 3, \dots \\ 1, & \text{else where} \end{cases}$, then the value of $\int_0^2 f(x) dx$.
 (A) 1 (B) 0 (C) 2 (D) ∞
- If $\int_0^\infty e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$, then $\int_0^\infty e^{-ax^2} dx$ where $a > 0$ is :
 (A) $\frac{\sqrt{\pi}}{2}$ (B) $\frac{\sqrt{\pi}}{2a}$ (C) $2\frac{\sqrt{\pi}}{a}$ (D) $\frac{1}{2}\sqrt{\frac{\pi}{a}}$
- If $\sum_{i=1}^4 (\sin^{-1} x_i + \cos^{-1} y_i) = 6\pi$, then $\int_{\sum_{i=1}^4 x_i}^{\sum_{i=1}^4 y_i} x \ln(1+x^2) \left(\frac{e^x}{1+e^{2x}} \right) dx$ is equal to
 (A) 0 (B) $e^4 + e^{-4}$ (C) $\ln\left(\frac{17}{12}\right)$ (D) $e^4 - e^{-4}$
- The tangent to the graph of the function $y = f(x)$ at the point with abscissa $x = 1$ form an angle of $\pi/6$ and at the point $x = 2$, an angle of $\pi/3$ and at the point $x = 3$, an angle of $\pi/4$ with positive x -axis. The value of $\int_1^3 f'(x) f''(x) dx + \int_2^3 f''(x) dx$ ($f''(x)$ is supposed to be continuous) is :

(A) $\frac{4\sqrt{3}-1}{3\sqrt{3}}$ (B) $\frac{3\sqrt{3}-1}{2}$ (C) $\frac{4-\sqrt{3}}{3}$ (D) $\frac{4}{3}-\sqrt{3}$

8. Let $A = \int_0^1 \frac{e^t}{1+t} dt$, then $\int_{a-1}^a \frac{e^{-t}}{t-a-1} dt$ has the value :

(A) Ae^{-a} (B) $-Ae^{-a}$ (C) $-ae^{-a}$ (D) Ae^a

9. $\int_1^2 x^{2x^2+1}(1+2\ln x)dx$ is equal to

(A) 256 (B) 255 (C) $\frac{255}{2}$ (D) 128

10. If $f(x)$ is a function satisfying $f\left(\frac{1}{x}\right) + x^2 f(x) = 0$ for all non-zero x , then $\int_{\sin\theta}^{\operatorname{cosec}\theta} f(x) dx$ equals to :

(A) $\sin\theta + \operatorname{cosec}\theta$ (B) $\sin^2\theta$ (C) $\operatorname{cosec}^2\theta$ (D) none of these

11. If $\frac{C_0}{1} + \frac{C_1}{2} + \frac{C_2}{3} = 0$, where C_0, C_1, C_2 are all real, the equation $C_2x^2 + C_1x + C_0 = 0$ has:

(A) atleast one root in $(0, 1)$ (B) one root in $(1, 2)$ & other in $(3, 4)$
(C) one root in $(-1, 1)$ & the other in $(-5, -2)$ (D) both roots imaginary

12. If $f(x) = \int_0^x (2\cos^2 3t + 3\sin^2 3t) dt$, $f(x + \pi)$ is equal to :

(A) $f(x) + 2f(\pi)$ (B) $f(x) + 2f\left(\frac{\pi}{2}\right)$ (C) $f(x) + 4f\left(\frac{\pi}{4}\right)$ (D) $2f(x)$

13. Let $f(x) = \int_0^x \frac{dt}{\sqrt{1+t^3}}$ and $g(x)$ be the inverse of $f(x)$, then which one of the following holds good?

(A) $2g'' = g^2$ (B) $2g'' = 3g^2$ (C) $3g'' = 2g^2$ (D) $3g'' = g^2$

14. Let $f(x)$ is differentiable function satisfying $2\int_1^2 f(tx)dt = x + 2, \forall x \in \mathbb{R}$ Then $\int_0^1 (8f(8x) - f(x) - 21x) dx$

equals to

(A) 3 (B) 5 (C) 7 (D) 9

15. Let $I_n = \int_0^1 x^n (\tan^{-1} x) dx, n \in \mathbb{N}$, then

(A) $(n+1)I_n + (n-1)I_{n-2} = \frac{\pi}{4} + \frac{1}{n} \quad \forall n \geq 3$ (B) $(n+1)I_n + (n-1)I_{n-2} = \frac{\pi}{2} - \frac{1}{n} \quad \forall n \geq 3$
(C) $(n+1)I_n - (n-1)I_{n-2} = \frac{\pi}{4} + \frac{1}{n} \quad \forall n \geq 3$ (D) $(n+1)I_n - (n-1)I_{n-2} = \frac{\pi}{2} - \frac{1}{n} \quad \forall n \geq 3$

16. If $u_n = \int_0^{\pi/2} x^n \sin x dx$, then the value of $u_{10} + 90 u_8$ is :

(A) $9 \left(\frac{\pi}{2}\right)^8$ (B) $\left(\frac{\pi}{2}\right)^9$ (C) $10 \left(\frac{\pi}{2}\right)^9$ (D) $9 \left(\frac{\pi}{2}\right)^9$

17. The value of $\int_{1/e}^{\tan x} \frac{t}{1+t^2} dt + \int_{1/e}^{\cot x} \frac{1}{t(1+t^2)} dt$, where $x \in (\pi/6, \pi/3)$, is equal to :

- (A) 0 (B) 2 (C) 1 (D) cannot be determined

18. Let $A_1 = \int_0^x \left(\int_0^u f(t) dt \right) du$ and $A_2 = \int_0^x f(u) \cdot (x - u) du$ then $\frac{A_1}{A_2}$ is equal to :

- (A) $\frac{1}{2}$ (B) 1 (C) 2 (D) -1

19. $\lim_{n \rightarrow \infty} \left(\sin \frac{\pi}{2n} \cdot \sin \frac{2\pi}{2n} \cdot \sin \frac{3\pi}{2n} \cdots \sin \frac{(n-1)\pi}{2n} \right)^{1/n}$ is equal to :

- (A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{4}$ (D) $\frac{3}{4}$

20. Area bounded by the region consisting of points (x, y) satisfying $y \leq \sqrt{2-x^2}$, $y^2 \geq x$, $\sqrt{y} \geq -x$ is

- (A) $\frac{\pi}{2}$ (B) π (C) 2π (D) $\pi/4$

21. The area enclosed between the curves

$y = \log_e(x + e)$, $x = \log_e\left(\frac{1}{y}\right)$ and the x-axis is

- (A) 2 (B) 1 (C) 4 (D) 3

22. The area bounded by the curve $x = a \cos^3 t$, $y = a \sin^3 t$ is

- (A) $\frac{3\pi a^2}{8}$ (B) $\frac{3\pi a^2}{16}$ (C) $\frac{3\pi a^2}{32}$ (D) $3\pi a^2$

23. The area bounded by the curve $f(x) = x + \sin x$ and its inverse function between the ordinates $x = 0$ and $x = 2\pi$ is

- (A) 4π (B) 8π (C) 4 (D) 8

24. $P(2, 2)$, $Q(-2, 2)$, $R(-2, -2)$ & $S(2, -2)$ are vertices of a square. A parabola passes through P, S & its vertex lies on x-axis. If this parabola bisects the area of the square PQRS, then vertex of the parabola is

- (A) $(-2, 0)$ (B) $(0, 0)$ (C) $\left(-\frac{3}{2}, 0\right)$ (D) $(-1, 0)$

25. The ratio in which the curve $y = x^2$ divides the region bounded by the curve; $y = \sin\left(\frac{\pi x}{2}\right)$ and the x-axis as x varies from 0 to 1, is :

- (A) 2: π (B) 1: 3 (C) 3: π (D) $(6 - \pi)$: π

26. If $f(x) = \sin x$, $\forall x \in \left[0, \frac{\pi}{2}\right]$, $f(x) + f(\pi - x) = 2$. $\forall x \in \left(\frac{\pi}{2}, \pi\right]$ and $f(x) = f(2\pi - x)$, $x \in (\pi, 2\pi]$, then the area enclosed by $y = f(x)$ and x-axis is

- (A) π (B) 2π (C) 2 (D) 4

27. The area bounded by the curves $y = x e^x$, $y = x e^{-x}$ and the line $x = 1$

- (A) $\frac{2}{e}$ (B) $1 - \frac{2}{e}$ (C) $\frac{1}{e}$ (D) $1 - \frac{1}{e}$

28. Obtain the area enclosed by region bounded by the curves $y = x \sin x$ and $y = 2x - 2x^2$.

- (A) $7/6$ (B) $7/24$ (C) $12/7$ (D) $7/12$

29. The area of the region on plane bounded by $\max(|x|, |y|) \leq 1$ and $xy \leq \frac{1}{2}$ is
- (A) $\frac{1}{2} + \pi n^2$ (B) $3 + \pi n^2$ (C) $31/4$ (D) $1 + 2\pi n^2$

30. Consider the following statements :

S_1 : The value of $\int_0^{2\pi} \cos^{-1}(\cos x) dx$ is π^2

S_2 : Area enclosed by the curve $|x - 2| + |y + 1| = 1$ is equal to 3 sq. unit

S_3 : If $\frac{d}{dx} f(x) = g(x)$ for $a \leq x \leq b$, then $\int_a^b f(x)g(x)dx$ equals to $\frac{[f(b)]^2 - [f(a)]^2}{2}$.

S_4 : Area of the region $R \equiv \{(x, y) ; x^2 \leq y \leq x\}$ is $\frac{1}{6}$

State, in order, whether S_1, S_2, S_3, S_4 are true or false

- (A) TFTT (B) TTTT (C) FFFF (D) TFTF

PART - II : SINGLE AND DOUBLE VALUE INTEGER TYPE

1. $\int_2^4 \frac{3x^2 + 1}{(x^2 - 1)^3} dx = \frac{\lambda}{n^2}$ where $\lambda, n \in \mathbb{N}$ and $\gcd(\lambda, n) = 1$, then find the value of $\lambda + n$
2. Let $U = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \min. (\sqrt{3} \sin x, \cos x) dx$ and $V = \int_{-3}^5 x^2 \operatorname{sgn}(x - 1) dx$. If $V = \lambda U$, then find the value of λ .
- [Note : $\operatorname{sgn} k$ denotes the signum function of k .]
3. Let $f(x)$ be a function satisfying $f(x) = f\left(\frac{100}{x}\right) \forall x > 0$. If $\int_1^{10} \frac{f(x)}{x} dx = 5$ then find the value of $\int_1^{100} \frac{f(x)}{x} dx$
4. Evaluate
$$\frac{2005 \int_0^{1002} \frac{dx}{\sqrt{1002^2 - x^2} + \sqrt{1003^2 - x^2}} + \int_{1002}^{1003} \sqrt{1003^2 - x^2} dx}{\int_0^1 \sqrt{1 - x^2} dx} = k$$
, then find the sum of squares of digits of natural number k .
5. If $\int_0^{\pi/2} \sqrt{\sin 2\theta} \cdot \sin \theta d\theta = \frac{\pi}{n}$ then find n
6. Let $I_1 = \int_0^{\pi/4} (1 + \tan x)^2 dx$, $I_2 = \int_0^1 \frac{dx}{(1+x)^2(1+x^2)}$
- then find the value of $\frac{I_1}{I_2}$

7. Find the value of $\ell n \left(\int_0^1 e^{t^2+t} (2t^2 + t + 1) dt \right)$
8. If $\int_{-1}^0 \frac{x}{x+1+e^x} dx$ is equal to $-\ln k$, then find the value of k .
9. If f, g, h be continuous functions on $[0, a]$ such that $f(a-x) = f(x)$, $g(a-x) = -g(x)$ and $3h(x) - 4h(a-x) = 5$, then find the value of $\int_0^a f(x)g(x)h(x)dx$
10. If $f(x) = \frac{\sin x}{x} \quad \forall x \in (0, \pi]$, If $\frac{\pi}{k} \int_0^{\pi/2} f(x) f\left(\frac{\pi}{2} - x\right) dx = \int_0^{\pi} f(x) dx$ then find the value of k .
11. Evaluate: $3 \int_0^{\frac{\pi}{4}} \frac{\sin^2 x + b^2 \cos^2 x}{a^4 \sin^2 x + b^4 \cos^2 x} dx$, where $a^2 + b^2 = \frac{3\pi}{4}$, $a^2 \neq b^2$ and $ab \neq 0$.
12. $\int_0^{2\pi} |\sqrt{15} \sin x + \cos x| dx$
13. Let a be a real number in the interval $[0, 314]$ such that $\int_{-\pi+a}^{3\pi+a} |x-a-\pi| \sin\left(\frac{x}{2}\right) dx = -16$, then determine number of such values of a .
14. $\sum_{n=1}^{\infty} \left(\frac{1}{4n-3} - \frac{1}{4n-1} \right) = \frac{\pi}{n}$, find 'n' (Note that $\tan^{-1} x + c = \int \frac{1}{1+x^2} dx$)
15. If $f(x) = x + \int_0^1 t(x+t) f(t) dt$, then the value of the definite integral $\int_0^1 f(x) dx$ can be expressed in the form of rational as $\frac{p}{q}$ (where p and q are coprime). Find $(p+q)$.
16. If $f(x) = (ax+b)e^x$ satisfies the equation $f(x) = \int_0^x e^{x-y} f'(y) dy - (x^2 - x + 1)e^x$, find $(a^2 + b^2)$
17. If the minimum of the following function $f(x)$ defined at $0 < x < \frac{\pi}{2}$.
 $f(x) = \int_0^x \frac{d\theta}{\cos \theta} + \int_x^{\frac{\pi}{2}} \frac{d\theta}{\sin \theta}$ is equal to $\ell n(a + \sqrt{b})$ where $a, b \in \mathbb{N}$ and b is not a perfect square then find the value of $(a+b)$
18. If $f(\pi) = 2$ and $\int_0^{\pi} (f(x) + f''(x)) \sin x dx = 5$, then find the value of $f(0)$
 (it is given that $f(x)$ is continuous in $[0, \pi]$)

19. ✎ If $f(x) = 2x^3 - 15x^2 + 24x$ and $g(x) = \int_0^x f(t) dt + \int_0^{5-x} f(t) dt$ ($0 < x < 5$). Find the number of integers for which $g(x)$ is increasing.
20. Let $f(x) = \begin{cases} 1-x & \text{if } 0 \leq x \leq 1 \\ 0 & \text{if } 1 < x \leq 2 \\ (2-x)^2 & \text{if } 2 < x \leq 3 \end{cases}$ and function $F(x) = \int_0^x f(t) dt$. If number of points of discontinuity in $[0, 3]$ and non-differentiability in $(0, 3)$ of $F(x)$ are α and β respectively, then $(\alpha - \beta)$ is equal to.
21. ✎ Find the value of m ($m > 0$) for which the area bounded by the line $y = mx + 2$ and $x = 2y - y^2$ is $9/2$ square units.
22. ✎ Find area bounded by $y = f^{-1}(x)$, $x = 10$, $x = 4$ and x -axis given that area bounded by $y = f(x)$, $x = 2$, $x = 6$ and x -axis is 30 sq. units, where $f(2) = 4$ and $f(6) = 10$. (given $f(x)$ is an invertible function)
23. Consider a line $\square : 2x - \sqrt{3}y = 0$ and a parameterized $C : x = \tan t, y = \frac{1}{\cos t} \left(0 \leq t < \frac{\pi}{2} \right)$. If the area of the part bounded by $\square$, C and the y -axis is equal to $\frac{1}{4} \ln(a + \sqrt{b})$, where $a, b, \in \mathbb{N}$, b is not perfect square then find the value of $(a + b)$.

PART - III : ONE OR MORE THAN ONE OPTIONS CORRECT TYPE

1. ✎ $\int_0^\infty \frac{x}{(1+x)(1+x^2)} dx$ equals to :
- (A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$
- (C) is same as $\int_0^\infty \frac{dx}{(1+x)(1+x^2)}$ (D) cannot be evaluated
2. The value of integral $\int_a^b \frac{|x|}{x} dx$, $a < b$ is :
- (A) $b - a$ if $a > 0$ (B) $a - b$ if $b < 0$ (C) $b + a$ if $a < 0 < b$ (D) $|b| - |a|$
3. ✎ If $I_n = \int_0^1 \frac{dx}{(1+x^2)^n}$; $n \in \mathbb{N}$, then which of the following statements hold good?
- (A) $2n I_{n+1} = 2^{-n} + (2n-1) I_n$ (B) $I_2 = \frac{\pi}{8} + \frac{1}{4}$
- (C) $I_2 = \frac{\pi}{8} - \frac{1}{4}$ (D) $I_3 = \frac{\pi}{16} - \frac{5}{48}$
4. The value of integral $\int_0^\pi x f(\sin x) dx$ is :
- (A) $\frac{\pi}{2} \int_0^\pi f(\sin x) dx$ (B) $\pi \int_0^{\pi/2} f(\sin x) dx$ (C) 0 (D) $2\pi \int_0^{\pi/4} f(\sin x) dx$
5. If $I = \int_0^{2\pi} \sin^2 x dx$, then

(A) $I = 2 \int_0^{\pi} \sin^2 x dx$ (B) $I = 4 \int_0^{\pi/2} \sin^2 x dx$ (C) $I = \int_0^{2\pi} \cos^2 x dx$ (D) $I = 8 \int_0^{\pi/4} \sin^2 x dx$

6. ✎ Given f is an odd function defined everywhere, periodic with period 2 and integrable on every interval.

Let $g(x) = \int_0^x f(t) dt$. Then :

- (A) $g(2n) = 0$ for every integer n (B) $g(x)$ is an even function
(C) $g(x)$ and $f(x)$ have the same period (D) $g(x)$ is an odd function

7. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined as $f(x) = \int_{-1}^x \frac{dt}{1+t^2} + \int_1^{e^{-x}} \frac{dt}{1+t^2}$, then

- (A) $f(x)$ is periodic (B) $f(f(x)) = f(x) \forall x \in \mathbb{R}$
(C) $f(1) = f'(1) = \frac{\pi}{2}$ (D) $f(x)$ is unbounded

8. ✎ If $a, b \in \mathbb{R}^+$ then $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{n}{(k+an)(k+bn)}$ is equal to

- (A) $\frac{1}{a-b} \ln \frac{b(b+1)}{a(a+1)}$ if $a \neq b$ (B) $\frac{1}{a-b} \ln \frac{a(b+1)}{b(a+1)}$ if $a \neq b$
(C) non existent if $a = b$ (D) $\frac{1}{a(1+a)}$ if $a = b$

9. Let $f(x) = \int_x^{x+\frac{\pi}{3}} |\sin \theta| d\theta$ ($x \in [0, \pi]$)

- (A) $f(x)$ is strictly increasing in this interval (B) $f(x)$ is differentiable in this interval
(C) Range of $f(x)$ is $[2 - \sqrt{3}, 1]$ (D) $f(x)$ has a maxima at $x = \frac{\pi}{3}$

10. If $f(x)$ is integrable over $[1, 2]$, then $\int_1^2 f(x) dx$ is equal to :

- (A) $\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=1}^n f\left(\frac{r}{n}\right)$ (B) $\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=n+1}^{2n} f\left(\frac{r}{n}\right)$
(C) $\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=1}^n f\left(\frac{r+n}{n}\right)$ (D) $\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=1}^{2n} f\left(\frac{r}{n}\right)$

11. ✎ Let $I_n = \int_0^{\frac{1}{2}} \frac{1}{\sqrt{1-x^n}} dx$ where $n > 2$, then

- (A) $I_n < \frac{\pi}{6}$ (B) $I_n > \frac{\pi}{6}$ (C) $I_n < \frac{1}{2}$ (D) $I_n > \frac{1}{2}$

12. If $f(x) = 2^{\{x\}}$, where $\{x\}$ denotes the fractional part of x . Then which of the following is true ?

- (A) f is periodic (B) $\int_0^1 2^{\{x\}} dx = \frac{1}{\ln 2}$ (C) $\int_0^1 2^{\{x\}} dx = \log_2 e$ (D) $\int_0^{100} 2^{\{x\}} dx = 100 \log_2 e$

13. Let $f(x) = \int_0^x |2t - 3| dt$, then f is

- (A) continuous at $x = 3/2$ (B) continuous at $x = 3$
(C) differentiable at $x = 3/2$ (D) differentiable at $x = 0$

14. Let $I_n = \int_0^{\pi} (\sin x)^n dx$, $n \in \mathbb{N}$, then
 (A) I_n is rational if n is odd
 (B) I_n is irrational if n is even
 (C) I_n is an increasing sequence
 (D) I_n is a decreasing sequence
15. Let $f(x)$ be a function satisfying $f(x) + f(x+2) = 10 \forall x \in \mathbb{R}$, then
 (A) $f(x)$ is a periodic function
 (B) $f(x)$ is aperiodic function
 (C) $\int_{-1}^7 f(x) dx = 20$
 (D) $\int_{-1}^7 f(x) dx = 40$
16. Let $I_n = \int_0^{\pi} \frac{\sin^2(nx)}{\sin^2 x} dx$, $n \in \mathbb{N}$, then
 (A) $I_{n+2} + I_n = 2I_{n+1}$
 (B) $I_n = I_{n+1}$
 (C) $I_n = n\pi$
 (D) $I_1, I_2, I_3, \dots, I_n$ are in Harmonic progression
17. Let $f(x)$ be a continuous function and $I = \int_1^9 \sqrt{x} f(x) dx$, then
 (A) There exists some $c \in (1, 9)$ such that $I = 8\sqrt{c} f(c)$
 (B) There exists some $p, q \in (1, 3)$ such that $I = 2[p^2 f(p^2) + q^2 f(q^2)]$
 (C) There exists some $\alpha \in (1, 9)$ such that $I = 9\sqrt{\alpha} f(\alpha)$
 (D) If $f(x) \geq 0 \forall x \in [1, 9] \Rightarrow I > 0$
18. Let $A = \int_1^{e^2} \frac{\ln x}{\sqrt{x}} dx$, then
 (A) $A > 2 \left(e - \frac{1}{e} \right)$
 (B) $A < (e-1) \left(2 + \frac{1}{\sqrt{e}} \right)$
 (C) $A > (e-1) \left(2 + \frac{1}{\sqrt{e}} \right)$
 (D) $A < (e^2 - 1) \frac{2}{e}$
19. Let $f(a, b) = \int_a^b (x^2 - 4x + 3) dx$, ($b > a$) then
 (A) $f(a, 3)$ is least when $a = 1$
 (B) $f(4, b)$ is an increasing function $\forall b \geq 4$
 (C) $f(0, b)$ is least for $b = 2$
 (D) $\min\{f(a, b)\} = -\frac{4}{3}$
20. Let $I = \int_2^{\infty} \left(\frac{\lambda x}{x^2 + 1} - \frac{1}{2x + 1} \right) dx$ & I is a finite real number, then
 (A) $\lambda = \frac{1}{2}$
 (B) $\lambda = 1$
 (C) $I = \frac{1}{2} \ln \left(\frac{5}{2} \right)$
 (D) $I = \frac{1}{4} \ln \left(\frac{5}{4} \right)$
21. Let $f(x)$ be a strictly increasing, non-negative function such that $f'(x) < 0 \forall x \in (\alpha, \beta)$ & $I = \int_{\alpha}^{\beta} f(x) dx$ ($\beta > \alpha$), then
 (A) $I < f \left(\frac{\alpha + \beta}{2} \right) (\beta - \alpha)$
 (B) $I > f \left(\frac{\alpha + \beta}{2} \right) (\beta - \alpha)$
 (C) $I > \frac{1}{2} (f(\alpha) + f(\beta)) (\beta - \alpha)$
 (D) $I < \frac{1}{2} (f(\alpha) + f(\beta)) (\beta - \alpha)$

22. $I_1 = \int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx$, $I_2 = \int_0^{\pi} \frac{x^3 \sin x}{(\pi^2 - 3\pi x + 3x^2)(1 + \cos^2 x)} dx$, then
 (A) $I_1 = \frac{\pi^2}{8}$ (B) $I_1 = \frac{\pi^2}{4}$ (C) $I_1 = I_2$ (D) $I_1 > I_2$
23. Let $L_1 = \lim_{x \rightarrow 0^+} \frac{\int_0^{x^2} \sin \sqrt{t} dt}{x - \sin x}$, $L_2 = \lim_{x \rightarrow 0^-} \frac{\int_0^{x^2} \sin \sqrt{t} dt}{x - \sin x}$, then identify the correct option(s).
 (A) $L_1 = 4$ (B) $L_1 + L_2 = 8$ (C) $L_1 + L_2 = 0$ (D) $|L_2| = |L_1|$
24. $\lim_{n \rightarrow \infty} \frac{(1^k + 2^k + 3^k + \dots + n^k)}{(1^2 + 2^2 + \dots + n^2)(1^3 + 2^3 + \dots + n^3)} = F(k)$, then ($k \in \mathbb{N}$)
 (A) $F(k)$ is finite for $k \leq 6$ (B) $F(5) = 0$ (C) $F(6) = \frac{12}{7}$ (D) $F(6) = \frac{5}{7}$
25. Let $T_n = \sum_{r=1}^n \frac{n}{r^2 - 2r.n + 2n^2}$, $S_n = \sum_{r=0}^{n-1} \frac{n}{r^2 - 2r.n + 2n^2}$, then
 (A) $T_n > S_n \forall n \in \mathbb{N}$ (B) $T_n > \frac{\pi}{4}$ (C) $S_n < \frac{\pi}{4}$ (D) $\lim_{n \rightarrow \infty} S_n = \frac{\pi}{4}$
26. $f(x) = \int_0^1 f(tx) dt$, where $f'(x)$ is a continuous function such that $f(1) = 2$, then
 (A) $f(x)$ is a periodic function (B) $f'(x) = 0$
 (C) $f(x)$ is an even function (D) $f(x)$ is an odd function
27. Area bounded by $y = \sin^{-1}x$, $y = \cos^{-1}x$, $y = 0$ in first quadrant is equal to :
 (A) $\int_0^{1/\sqrt{2}} (\sin^{-1}x) dx + \int_{1/\sqrt{2}}^1 (\cos^{-1}x) dx$ (B) $\int_{\pi/4}^{\pi/2} (\sin y - \cos y) dy$
 (C) $\int_0^{\pi/4} (\cos y - \sin y) dy$ (D) $(\sqrt{2} - 1)$ sq.unit
28. Let $f(x)$ be a non-negative, continuous and even function such that area bounded by x -axis, y -axis & $y = f(x)$ is equal to $(x^2 + x^3)$ sq. units $\forall x \geq 0$, then
 (A) $\sum_{r=1}^n f'(r) = 3n^2 + 5n \forall n \in \mathbb{N}$ (B) $\sum_{r=1}^n f'(r) = 6n^2 + 5n \forall n \in \mathbb{N}$
 (C) $f(x) = 3x^2 + 2x \forall x \leq 0$ (D) $f(x) = 3x^2 - 2x \forall x \leq 0$
29. Let 'c' be a positive real number such that area bounded by $y = 0$, $y = [\tan^{-1}x]$ from $x = 0$ to $x = c$ is equal to area bounded by $y = 0$, $y = [\cot^{-1}x]$, from $x = 0$ to $x = c$ (where $[*]$ represents greatest integer function), then
 (A) $c = \tan 1 + \cot 1$ (B) $c = 2 \operatorname{cosec} 2$ (C) $c = \tan 1 - \cot 1$ (D) $c = -2 \cot 2$
30. Area bounded by $y = x^2 - 2|x|$ and $y = -1$ is equal to
 (A) $2 \int_0^1 (2x - x^2) dx$

- (B) $\frac{2}{3}$ sq. units
- (C) $\frac{2}{3}$ (Area of rectangle ABCD) where points A, B, C, D are $(-1, -1)$, $(-1, 0)$, $(1, 0)$ & $(1, -1)$
- (D) $\frac{2}{3}$ (Area of rectangle ABCD) where points A, B, C, D are $(-1, -1)$, $(-1, 0)$, $(1, 0)$ & $(1, -1)$

PART - IV : COMPREHENSION

Comprehension # 1

If $y = \int_{u(x)}^{v(x)} f(t) dt$, let us define $\frac{dy}{dx}$ in a different manner as $\frac{dy}{dx} = v'(x) f^2(v(x)) - u'(x) f^2(u(x))$ and the

equation of the tangent at (a, b) as $y - b = \left(\frac{dy}{dx} \right)_{(a,b)} (x - a)$

- If $y = \int_x^{x^2} t^2 dt$, then equation of tangent at $x = 1$ is
 (A) $y = x + 1$ (B) $x + y = 1$ (C) $y = x - 1$ (D) $y = x$
- If $F(x) = \int_1^x e^{t^2/2} (1 - t^2) dt$, then $\frac{d}{dx} F(x)$ at $x = 1$ is
 (A) 0 (B) 1 (C) 2 (D) -1
- If $y = \int_{x^3}^{x^4} \ln t dt$, then $\lim_{x \rightarrow 0^+} \frac{dy}{dx}$ is
 (A) 0 (B) 1 (C) 2 (D) -1

Comprehension # 2

Let $g(t) = \int_{x_1}^{x_2} f(t, x) dx$. Then $g'(t) = \int_{x_1}^{x_2} \frac{\partial}{\partial t} (f(t, x)) dx$. Consider $f(x) = \int_0^\pi \frac{\ln(1 + x \cos \theta)}{\cos \theta} d\theta$.

- Range of $f(x)$ is
 (A) $(0, \pi)$ (B) $(0, \pi^2)$ (C) $\left(-\frac{\pi}{2}, \frac{\pi}{2} \right)$ (D) $\left(-\frac{\pi^2}{2}, \frac{\pi^2}{2} \right)$
- The number of critical points of $f(x)$, in the interior of its domain, is
 (A) 0 (B) 1 (C) 2 (D) infinitely many
- $f(x)$ is
 (A) discontinuous at $x = 0$ (B) differentiable at $x = 1$
 (C) continuous at $x = 0$ (D) None of these

Comprehension # 3

If length of perpendicular drawn from points of a curve to a straight line approaches zero along an infinite branch of the curve, the line is said to be an asymptote to the curve. For example, y-axis is an asymptote to $y = \frac{1}{x}$ & x-axis is an asymptote to $y = e^{-x}$.

Asymptotes parallel to x-axis :

If $\lim_{x \rightarrow \infty} f(x) = e$ (a finite number) then $y = e$ is an asymptote to $y = f(x)$. Similarly if $\lim_{x \rightarrow -\infty} f(x) = \alpha$, then $y = \alpha$ is also an asymptote.

Asymptotes parallel to y-axis :

If $\lim_{x \rightarrow a} f(x) = \infty$ or $\lim_{x \rightarrow a} f(x) = -\infty$, then $x = a$ is an asymptote to $y = f(x)$.

7. Number of asymptotes parallel to co-ordinate axes for the function $f(x) = \frac{(x+1)(x+2)}{(x-1)(x-2)}$ is equal to :
 (A) 1 (B) 2 (C) 3 (D) 4
8. Area bounded by $y = \frac{2x}{x^2 + 1}$, its asymptote and ordinates at points of extremum is equal to (in square unit)
 (A) π^2 (B) $2\pi^2$ (C) π^3 (D) $2\pi^3$
9. Area bounded by $y = x^2 e^{-x}$ and its asymptote in first quadrant is equal to (in square unit)
 (A) $2e$ (B) e (C) 1 (D) 2

Exercise-3

Marked questions are recommended for Revision.

PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

* Marked Questions may have more than one correct option.

1*. If $I_n = \int_{-\pi}^{\pi} \frac{\sin nx}{(1 + \pi^x) \sin x} dx$, $n = 0, 1, 2, \dots$, then [IIT-JEE - 2009, Paper-2, (4, -1), 80]

- (A) $I_n = I_{n+2}$ (B) $\sum_{m=1}^{10} I_{2m+1} = 10\pi$ (C) $\sum_{m=1}^{10} I_{2m} = 0$ (D) $I_n = I_{n+1}$

2. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a continuous function which satisfies $f(x) = \int_0^x f(t) dt$. Then the value of $f(\pi/5)$ is [IIT-JEE - 2009, Paper-2, (4, -1), 80]

3*. Area of the region bounded by the curve $y = e^x$ and lines $x = 0$ and $y = e$ is [IIT-JEE 2009, P-1, (4, -1), 80]

- (A) $e - 1$ (B) $\int_1^e \ln(e + 1 - y) dy$ (C) $e - \int_0^1 e^x dx$ (D) $\int_1^e \ln y dy$

4. The value of $\lim_{x \rightarrow 0} \frac{1}{x^3} \int_0^x \frac{t \ln(1+t)}{t^4 + 4} dt$ is [IIT-JEE-2010, Paper-1 (3, -1)/84]

- (A) 0 (B) $\frac{1}{12}$ (C) $\frac{1}{24}$ (D) $\frac{1}{64}$

5. The value(s) of $\int_0^1 \frac{x^4(1-x)^4}{1+x^2} dx$ is (are) [IIT-JEE-2010, Paper-1 (3, 0)/84]

- (A) $\frac{22}{7} - \pi$ (B) $\frac{2}{105}$ (C) 0 (D) $\frac{71}{15} - \frac{3\pi}{2}$

6*. Let f be a real-valued function defined on the interval $(0, \infty)$ by $f(x) = \int_0^x \sqrt{1 + \sin t} dt$. Then which of the following statement(s) is (are) true? [IIT-JEE-2010, Paper-1 (3, 0)/84]

- (A) $f''(x)$ exists for all $x \in (0, \infty)$
 (B) $f'(x)$ exists for all $x \in (0, \infty)$ and f' is continuous on $(0, \infty)$, but not differentiable on $(0, \infty)$
 (C) there exists $\alpha > 1$ such that $|f'(x)| < |f(x)|$ for all $x \in (\alpha, \infty)$
 (D) there exists $\beta > 0$ such that $|f(x)| + |f'(x)| \leq \beta$ for all $x \in (0, \infty)$

7. For any real number, let $[x]$ denote the largest integer less than or equal to x . Let f be a real valued function defined on the interval $[-10, 10]$ by [IIT-JEE-2010, Paper-1 (3, 0)/84]

$$f(x) = \begin{cases} x - [x] & \text{if } [x] \text{ is odd,} \\ 1 + [x] - x & \text{if } [x] \text{ is even} \end{cases}$$

Then the value of $\frac{\pi^2}{10} \int_{-10}^{10} f(x) \cos \pi x dx$ is

8. Let f be a real-valued function defined on the interval $(-1, 1)$ such that $e^{-x} f(x) = 2 + \int_0^x \sqrt{t^4 + 1} dt$, for all $x \in (-1, 1)$ and let f^{-1} be the inverse function of f . Then $(f^{-1})'(2)$ is equal to

[IIT-JEE-2010, Paper-2 (5, -2)/84]

- (A) 1 (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{1}{e}$

Comprehension (9 to 11)

Consider the polynomial

$$f(x) = 1 + 2x + 3x^2 + 4x^3$$

 Let s be the sum of all distinct real roots of $f(x)$ and let $t = |s|$

9. The real number s lies in the interval.

[IIT-JEE 2010, Paper-2, (3, -1), 79]

- (A) $\left(-\frac{1}{4}, 0\right)$ (B) $\left(-11, -\frac{3}{4}\right)$ (C) $\left(-\frac{3}{4}, -\frac{1}{2}\right)$ (D) $\left(0, \frac{1}{4}\right)$

10. The area bounded by the curve $y = f(x)$ and the lines $x = 0$, $y = 0$ and $x = t$, lies in the interval

[IIT-JEE 2010, Paper-2, (3, -1), 79]

- (A) $\left(\frac{3}{4}, 3\right)$ (B) $\left(\frac{21}{64}, \frac{11}{16}\right)$ (C) (9, 10) (D) $\left(0, \frac{21}{64}\right)$

11. The function $f'(x)$ is

[IIT-JEE 2010, Paper-2, (3, -1), 79]

- (A) increasing in $\left(-t, \frac{1}{4}\right)$ and decreasing in $\left(-\frac{1}{4}, t\right)$
 (B) decreasing in $\left(-t, -\frac{1}{4}\right)$ and increasing in $\left(-\frac{1}{4}, t\right)$
 (C) increasing in $(-t, t)$
 (D) decreasing in $(-t, t)$

12. The value of $\int_{\sqrt{n^2}}^{\sqrt{n^3}} \frac{x \sin x^2}{\sin x^2 + \sin(n^6 - x^2)} dx$ is

[IIT-JEE 2011, Paper-1, (3, -1), 80]

- (A) $\frac{1}{4} \leq \frac{3}{2}$ (B) $\frac{1}{2} \leq \frac{3}{2}$ (C) $\leq \frac{3}{2}$ (D) $\frac{1}{6} \leq \frac{3}{2}$

13. Let the straight line $x = b$ divide the area enclosed by $y = (1 - x)^2$, $y = 0$, and $x = 0$ into two parts R_1 ($0 \leq x \leq b$) and R_2 ($b \leq x \leq 1$) such that $R_1 - R_2 = \frac{1}{4}$. Then b equals

[IIT-JEE 2011, Paper-1, (3, -1), 80]

- (A) $\frac{3}{4}$ (B) $\frac{1}{2}$ (C) $\frac{1}{3}$ (D) $\frac{1}{4}$

14. Let $f: [-1, 2] \rightarrow [0, \infty)$ be a continuous function such that $f(x) = f(1 - x)$ for all $x \in [-1, 2]$.

Let $R_1 = \int_{-1}^2 x f(x) dx$, and R_2 be the area of the region bounded by $y = f(x)$, $x = -1$, $x = 2$, and the x -axis. Then

[IIT-JEE 2011, Paper-2, (3, -1), 80]

- (A) $R_1 = 2R_2$ (B) $R_1 = 3R_2$ (C) $2R_1 = R_2$ (D) $3R_1 = R_2$

- 15*. If S be the area of the region enclosed by $y = e^{-x^2}$, $y = 0$, $x = 0$, and $x = 1$. Then

[IIT-JEE 2012, Paper-1, (4, 0), 70]

- (A) $S \geq \frac{1}{e}$ (B) $S \geq 1 - \frac{1}{e}$ (C) $S \leq \frac{1}{4} \left(1 + \frac{1}{\sqrt{e}}\right)$ (D) $S \leq \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{e}} \left(1 - \frac{1}{\sqrt{2}}\right)$

16. The value of the integral $\int_{-\pi/2}^{\pi/2} \left(x^2 + \ln \frac{\pi + x}{\pi - x}\right) \cos x dx$ is

[IIT-JEE 2012, Paper-2, (3, -1), 66]

- (A) 0 (B) $\frac{\pi^2}{2} - 4$ (C) $\frac{\pi^2}{2} + 4$ (D) $\frac{\pi^2}{2}$

Paragraph for Question Nos. 17 to 18

Let $f(x) = (1-x)^2 \sin^2 x + x^2$ for all $x \in \mathbb{R}$ and let $g(x) = \int_1^x \left(\frac{2(t-1)}{t+1} - \ln t \right) f(t) dt$ for all $x \in (1, \infty)$.

17. Which of the following is true ?

[IIT-JEE 2012, Paper-2, (3, -1), 66]

- (A) g is increasing on $(1, \infty)$
- (B) g is decreasing on $(1, \infty)$
- (C) g is increasing on $(1, 2)$ and decreasing on $(2, \infty)$
- (D) g is decreasing on $(1, 2)$ and increasing on $(2, \infty)$

18. Consider the statements :

P : There exists some $x \in \mathbb{R}$ such that $f(x) + 2x = 2(1 + x^2)$

Q : There exists some $x \in \mathbb{R}$ such that $2f(x) + 1 = 2x(1 + x)$

Then

- (A) both P and Q are true
- (B) P is true and Q is false
- (C) P is false and Q is true
- (D) both P and Q are false

19.* If $f(x) = \int_0^x e^{t^2} (t-2)(t-3) dt$ for all $x \in (0, \infty)$, then

[IIT-JEE 2012, Paper-2, (4, 0), 66]

- (A) f has a local maximum at $x = 2$
- (B) f is decreasing on $(2, 3)$
- (C) there exists some $c \in (0, \infty)$ such that $f''(c) = 0$
- (D) f has a local minimum at $x = 3$

20. The area enclosed by the curves $y = \sin x + \cos x$ and $y = |\cos x - \sin x|$ over the interval $\left[0, \frac{\pi}{2}\right]$ is

[JEE (Advanced) 2013, Paper-1, (2, 0)/60]

- (A) $4(\sqrt{2}-1)$
- (B) $2\sqrt{2}(\sqrt{2}-1)$
- (C) $2(\sqrt{2}+1)$
- (D) $2\sqrt{2}(\sqrt{2}+1)$

21. Let $f : \left[\frac{1}{2}, 1\right] \rightarrow \mathbb{R}$ (the set of all real numbers) be a positive, non-constant and differentiable function such that $f'(x) < 2f(x)$ and $f\left(\frac{1}{2}\right) = 1$. Then the value of $\int_{1/2}^1 f(x) dx$ lies in the interval

[JEE (Advanced) 2013, Paper-1, (2, 0)/60]

- (A) $(2e-1, 2e)$
- (B) $(e-1, 2e-1)$
- (C) $\left(\frac{e-1}{2}, e-1\right)$
- (D) $\left(0, \frac{e-1}{2}\right)$

22. For $a \in \mathbb{R}$ (the set of all real numbers), $a \neq -1$, $\lim_{n \rightarrow \infty} \frac{(1^a + 2^a + \dots + n^a)}{(n+1)^{a-1}[(na+1) + (na+2) + \dots + (na+n)]} = \frac{1}{60}$. Then $a =$

[JEE (Advanced) 2013, Paper-2, (3, -1)/60]

- (A) 5
- (B) 7
- (C) $\frac{-15}{2}$
- (D) $\frac{-17}{2}$

23*. Let $f: [a, b] \rightarrow [1, \infty)$ be a continuous function and let $g: \mathbb{R} \rightarrow \mathbb{R}$ be defined as

$$g(x) = \begin{cases} 0 & \text{if } x < a, \\ \int_a^x f(t) dt & \text{if } a \leq x \leq b, \\ \int_a^b f(t) dt & \text{if } x > b. \end{cases}, \text{ Then}$$

[JEE (Advanced) 2014, Paper-1, (3, 0)/60]

- (A) $g(x)$ is continuous but not differentiable at a
 (B) $g(x)$ is differentiable on $\mathbb{R}$
 (C) $g(x)$ is continuous but not differentiable at b
 (D) $g(x)$ is continuous and differentiable at either a or b but not both

24*. Let $f: (0, \infty) \rightarrow \mathbb{R}$ be given by $f(x) = \int_{\frac{1}{x}}^x e^{-\left(t+\frac{1}{t}\right)} \frac{dt}{t}$. Then

- (A) $f(x)$ is monotonically increasing on $[1, \infty)$ [JEE (Advanced) 2014, Paper-1, (3, 0)/60]
 (B) $f(x)$ is monotonically decreasing on $(0, 1)$
 (C) $f(x) + f\left(\frac{1}{x}\right) = 0$, for all $x \in (0, \infty)$
 (D) $f(2^x)$ is an odd function of x on $\mathbb{R}$

25. The value of $\int_0^1 4x^3 \left\{ \frac{d^2}{dx^2} (1-x^2)^5 \right\} dx$ is [JEE (Advanced) 2014, Paper-1, (3, 0)/60]

26. The following integral $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (2 \operatorname{cosec} x)^{17} dx$ is equal to [JEE (Advanced) 2014, Paper-2, (3, -1)/60]

(A) $\int_0^{\log(1+\sqrt{2})} 2(e^u + e^{-u})^{16} du$

(B) $\int_0^{\log(1+\sqrt{2})} (e^u + e^{-u})^{17} du$

(C) $\int_0^{\log(1+\sqrt{2})} (e^u - e^{-u})^{17} du$

(D) $\int_0^{\log(1+\sqrt{2})} 2(e^u - e^{-u})^{16} du$

Paragraph For Questions 27 and 28

Given that for each $a \in (0, 1)$

$$\lim_{h \rightarrow 0^+} \int_h^{1-h} t^{-a} (1-t)^{a-1} dt$$

exists. Let this limit be $g(a)$. In addition, it is given that the function $g(a)$ is differentiable on $(0, 1)$. [JEE (Advanced) 2014, Paper-2, (3, -1)/60]

27. The value of $g\left(\frac{1}{2}\right)$ is

- (A) π (B) 2π (C) $\frac{\pi}{2}$ (D) $\frac{\pi}{4}$

28. The value of $g'\left(\frac{1}{2}\right)$ is

- (A) $\frac{\pi}{2}$ (B) π (C) $-\frac{\pi}{2}$ (D) 0

29. List I

List II

[JEE (Advanced) 2014, Paper-2, (3, -1)/60]

- P.** The number of polynomials $f(x)$ with non-negative integer coefficients of degree ≤ 2 , satisfying $f(0) = 0$ and $\int_0^1 f(x)dx = 1$, is **1. 8**
- Q.** The number of points in the interval $[-\sqrt{13}, \sqrt{13}]$ at which $f(x) = \sin(x^2) + \cos(x^2)$ attains its maximum value, is **2. 2**
- R.** $\int_{-2}^2 \frac{3x^2}{(1+e^x)} dx$ equals **3. 4**
- S.** $\frac{\left(\int_{-1/2}^{1/2} \cos 2x \log\left(\frac{1+x}{1-x}\right) dx\right)}{\left(\int_0^{1/2} \cos 2x \log\left(\frac{1+x}{1-x}\right) dx\right)}$ equals **4. 0**

- 30.** Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function defined by $f(x) = \begin{cases} [x], & x \leq 2 \\ 0, & x > 2 \end{cases}$ where $[x]$ is the greatest integer less than or equal to x . If $I = \int_{-1}^2 \frac{xf(x^2)}{2+f(x+1)} dx$, then the value of $(4I - 1)$ is

[JEE (Advanced) 2015, P-1 (4, 0) / 88]

- 31.** If $\alpha = \int_0^1 \left(e^{9x+3\tan^{-1}x} \right) \left(\frac{12+9x^2}{1+x^2} \right) dx$ where $\tan^{-1} x$ takes only principal values, then the value of $\left(\log_e |1+\alpha| - \frac{3\pi}{4} \right)$ is

[JEE (Advanced) 2015, P-2 (4, 0) / 80]

- 32.** Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a continuous odd function, which vanishes exactly at one point and $f(1) = \frac{1}{2}$. Suppose that $F(x) = \int_{-1}^x f(t) dt$ for all $x \in [-1, 2]$ and $G(x) = \int_{-1}^x t |f(f(t))| dt$ for all $x \in [-1, 2]$. If $\lim_{x \rightarrow 1} \frac{F(x)}{G(x)} = \frac{1}{14}$, then the value of $f\left(\frac{1}{2}\right)$ is.

[JEE (Advanced) 2015, P-2 (4, 0) / 80]

- 33*.** Let $f(x) = 7\tan^8 x + 7\tan^6 x - 3\tan^4 x - 3\tan^2 x$ for all $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Then the correct expression(s) is (are)

[JEE (Advanced) 2015, P-2 (4, -2) / 80]

- (A) $\int_0^{\pi/4} xf(x) dx = \frac{1}{12}$ (B) $\int_0^{\pi/4} f(x) dx = 0$
- (C) $\int_0^{\pi/4} xf(x) dx = \frac{1}{6}$ (D) $\int_0^{\pi/4} f(x) dx = 1$

- 34.** Let $f'(x) = \frac{192x^3}{2+\sin^4 \pi x}$ for all $x \in \mathbb{R}$ with $f\left(\frac{1}{2}\right) = 0$. If $m \leq \int_{1/2}^1 f(x) dx \leq M$, then the possible values of m and M are

[JEE (Advanced) 2015, P-2 (4, -2) / 80]

- (A) $m = 13, M = 24$ (B) $m = \frac{1}{4}, M = \frac{1}{2}$
- (C) $m = -11, M = 0$ (D) $m = 1, M = 12$

- 35*.** The option(s) with the values of a and L that satisfy the following equation is(are)

$$\frac{\int_0^{4\pi} e^t (\sin^6 at + \cos^4 at) dt}{\int_0^{\pi} e^t (\sin^6 at + \cos^4 at) dt} = L ?$$

[JEE (Advanced) 2015, P-2 (4, -2)/ 80]

(A) $a = 2, L = \frac{e^{4\pi} - 1}{e^{\pi} - 1}$

(B) $a = 2, L = \frac{e^{4\pi} + 1}{e^{\pi} + 1}$

(C) $a = 4, L = \frac{e^{4\pi} - 1}{e^{\pi} - 1}$

(D) $a = 4, L = \frac{e^{4\pi} + 1}{e^{\pi} + 1}$

Paragraph For Questions 36 and 37

Let $F : \mathbb{R} \rightarrow \mathbb{R}$ be a thrice differentiable function. Suppose that $F(1) = 0$, $F(3) = -4$ and $F'(x) < 0$ for all $x \in (1/2, 3)$. Let $f(x) = xF(x)$ for all $x \in \mathbb{R}$.

[JEE (Advanced) 2015, P-2 (4, -2)/ 80]

36*. The correct statement(s) is(are)

(A) $f'(1) < 0$

(B) $f(2) < 0$

(C) $f'(x) \neq 0$ for any $x \in (1, 3)$

(D) $f'(x) = 0$ for some $x \in (1, 3)$

37*. If $\int_1^3 x^2 F'(x) dx = -12$ and $\int_1^3 x^3 F''(x) dx = 40$, then the correct expression(s) is(are)

(A) $9f'(3) + f'(1) - 32 = 0$

(B) $\int_1^3 f(x) dx = 12$

(C) $9f'(3) - f'(1) + 32 = 0$

(D) $\int_1^3 f(x) dx = -12$

38. Let $F(x) = \int_x^{x^2 + \frac{\pi}{6}} 2\cos^2 t \, dt$ for all $x \in \mathbb{R}$ and $f: \left[0, \frac{1}{2}\right] \rightarrow [0, \infty)$ be a continuous function. For $a \in \left[0, \frac{1}{2}\right]$ if $F'(a) + 2$ is the area of the region bounded by $x = 0$, $y = 0$, $y = f(x)$ and $x = a$, then $f(0)$ is

[JEE (Advanced) 2015, P-1 (4, 0) /88]

39. The total number of distinct $x \in (0, 1]$ for which $\int_0^x \frac{t^2}{1+t^4} dt = 2x - 1$ is

[JEE (Advanced) 2016, Paper-1, (3, 0)/62]

40. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{x^2 \cos x}{1 + e^x} dx$ is equal to

[JEE (Advanced) 2016, Paper-2, (3, -1)/62]

(A) $\frac{\pi^2}{4} - 2$

(B) $\frac{\pi^2}{4} + 2$

(C) $\pi^2 - e^{\pi/2}$

(D) $\pi^2 + e^{\pi/2}$

41. Area of the region $\{(x, y) \in \mathbb{R}^2 : y \geq \sqrt{x+3}, 5y \leq x+9 \leq 15\}$ is equal to

[JEE (Advanced) 2016, Paper-2, (3, 1)/62]

(A) $\frac{1}{6}$

(B) $\frac{4}{3}$

(C) $\frac{3}{2}$

(D) $\frac{5}{3}$

42. Let $f(x) = \lim_{n \rightarrow \infty} \left(\frac{n^n (x+n) \left(x + \frac{n}{2}\right) \dots \left(x + \frac{n}{n}\right)}{n! \left(x^2 + n^2\right) \left(x^2 + \frac{n^2}{4}\right) \dots \left(x^2 + \frac{n^2}{n^2}\right)} \right)^{\frac{x}{n}}$, for all $x > 0$. Then
 [JEE (Advanced) 2016, Paper-2, (4, -2)/62]
- (A) $f\left(\frac{1}{2}\right) \geq f(1)$ (B) $f\left(\frac{1}{3}\right) \leq f\left(\frac{2}{3}\right)$ (C) $f'(2) \leq 0$ (D) $\frac{f'(3)}{f(3)} \geq \frac{f'(2)}{f(2)}$
- 43*. Let $f : \mathbb{R} \rightarrow (0, 1)$ be a continuous function. Then, which of the following function(s) has (have) the value zero at some point in the interval $(0, 1)$?
 [JEE(Advanced) 2017, Paper-1, (4, -2)/61]
- (A) $e^x - \int_0^x f(t) \sin t \, dt$ (B) $f(x) + \int_0^{\frac{\pi}{2}} f(t) \sin t \, dt$
 (C) $x - \int_0^{\frac{\pi}{2}-x} f(t) \cos t \, dt$ (D) $x^9 - f(x)$
44. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a differentiable function such that $f(0) = 0$, $f\left(\frac{\pi}{2}\right) = 3$ and $f'(0) = 1$. If
 $g(x) = \int_x^{\frac{\pi}{2}} [f'(t) \operatorname{cosec} t - \cot t \operatorname{cosec} t f(t)] \, dt$ for $x \in \left(0, \frac{\pi}{2}\right]$, then $\lim_{x \rightarrow 0} g(x) =$
 [JEE(Advanced) 2017, Paper-1, (3, 0)/61]
- 45*. If $I = \sum_{k=1}^{98} \int_k^{k+1} \frac{k+1}{x(x+1)} \, dx$, then
 [JEE(Advanced) 2017, Paper-2, (4, -2)/61]
- (A) $I > \log_e 99$ (B) $I < \log_e 99$ (C) $I < \frac{49}{50}$ (D) $I > \frac{49}{50}$
- 46*. If the line $x = \alpha$ divides the area of region $R = \{(x, y) \in \mathbb{R}^2 : x^3 \leq y \leq x, 0 \leq x \leq 1\}$ into two equal parts, then
 [JEE(Advanced) 2017, Paper-2, (4, -2)/61]
- (A) $2\alpha^4 - 4\alpha^2 + 1 = 0$ (B) $\alpha^4 + 4\alpha^2 - 1 = 0$ (C) $\frac{1}{2} < \alpha < 1$ (D) $0 < \alpha \leq \frac{1}{2}$
47. If $g(x) = \int_{\sin x}^{\sin(2x)} \sin^{-1}(t) \, dt$, then
 [JEE(Advanced) 2017, Paper-2, (4, -2)/61]
- (A) $g'\left(-\frac{\pi}{2}\right) = 2\pi$ (B) $g'\left(-\frac{\pi}{2}\right) = -2\pi$ (C) $g'\left(\frac{\pi}{2}\right) = 2\pi$ (D) $g'\left(\frac{\pi}{2}\right) = -2\pi$
48. For each positive integer n , let $y_n = \frac{1}{n} ((n+1)(n+2) \dots (n+n))^{1/n}$.
 For $x \in \mathbb{R}$, let $[x]$ be the greatest integer less than or equal to x . If $\lim_{n \rightarrow \infty} y_n = L$, then the value of $[L]$ is _____.
 [JEE(Advanced) 2018, Paper-1, (3, 0)/60]
49. A farmer F_1 has a land in the shape of a triangle with vertices at $P(0, 0)$, $Q(1, 1)$ and $R(2, 0)$. From this land, a neighbouring farmer F_2 takes away the region which lies between the side PQ and a curve of the form $y = x^n$ ($n > 1$). If the area of the region taken away by the farmer F_2 is exactly 30% of the area of ΔPQR , then the value of n is
 [JEE(Advanced) 2018, Paper-1, (3, 0)/60]
50. The value of the integral $\int_0^{\frac{1}{2}} \frac{1 + \sqrt{3}}{((x+1)^2(1-x)^6)^{\frac{1}{4}}} \, dx$ is _____. [JEE(Advanced) 2018, Paper-2, (3, 0)/60]

1. $\int_0^{\pi} [\cot x] dx$, where $[\cdot]$ denotes the greatest integer function, is equal to : **[AIEEE 2009 (4, -1), 144]**
 (1) 1 (2) -1 (3) $-\frac{\pi}{2}$ (4) $\frac{\pi}{2}$
2. The area of the region bounded by the parabola $(y - 2)^2 = x - 1$, the tangent to the parabola at the point (2, 3) and the x-axis is **[AIEEE 2009 (8, -2), 144]**
 (1) 6 sq unit (2) 9 sq unit (3) 12 sq unit (4) 3 sq unit
3. Let $p(x)$ be a function defined on $\mathbf{R}$ such that $p'(x) = p'(1 - x)$, for all $x \in [0, 1]$, $p(0) = 1$ and $p(1) = 41$. Then $\int_0^1 p(x) dx$ equals **[AIEEE 2010 (8, -2), 144]**
 (1) 21 (2) 41 (3) 42 (4) $\sqrt{41}$
4. The area bounded by the curves $y = \cos x$ and $y = \sin x$ between the ordinates $x = 0$ and $x = \frac{3\pi}{2}$ is **[AIEEE 2010 (4, -1), 144]**
 (1) $4\sqrt{2} + 2$ (2) $4\sqrt{2} - 1$ (3) $4\sqrt{2} + 1$ (4) $4\sqrt{2} - 2$
5. For $x \in \left(0, \frac{5\pi}{2}\right)$, define $f(x) = \int_0^x \sqrt{t} \sin t dt$. Then f has : **[AIEEE 2011, I, (4, -1), 120]**
 (1) local maximum at π and 2π .
 (2) local minimum at π and 2π
 (3) local minimum at π and local maximum at 2π .
 (4) local maximum at π and local minimum at 2π .
6. Let $[.]$ denote the greatest integer function then the value of $\int_0^{1.5} x [x^2] dx$ is : **[AIEEE 2011, II, (4, -1), 120]**
 (1) 0 (2) $\frac{3}{2}$ (3) $\frac{3}{4}$ (4) $\frac{5}{4}$
7. The area of the region enclosed by the curves $y = x$, $x = e$, $y = \frac{1}{x}$ and the positive x-axis is **[AIEEE 2011, I, (4, -1), 120]**
 (1) $\frac{1}{2}$ square units (2) 1 square units (3) $\frac{3}{2}$ square units (4) $\frac{5}{2}$ square units
8. The area bounded by the curves $y^2 = 4x$ and $x^2 = 4y$ is : **[AIEEE 2011, II, (4, -1), 120]**
 (1) $\frac{32}{3}$ (2) $\frac{16}{3}$ (3) $\frac{8}{3}$ (4) 0
9. The area bounded between the parabolas $x^2 = \frac{y}{4}$ and $x^2 = 9y$ and the straight line $y = 2$ is : **[AIEEE-2012, (4, -1)/120]**
 (1) $20\sqrt{2}$ (2) $\frac{10\sqrt{2}}{3}$ (3) $\frac{20\sqrt{2}}{3}$ (4) $10\sqrt{2}$
- 10.* If $g(x) = \int_0^x \cos 4t dt$, then $g(x + \pi)$ equals **[AIEEE-2012, (4, -1)/120]**

- (1) $\frac{g(x)}{g(\pi)}$ (2) $g(x) + g(\pi)$ (3) $g(x) - g(\pi)$ (4) $g(x) \cdot g(\pi)$

11. Statement-I : The value of the integral $\int_{\pi/6}^{\pi/3} \frac{dx}{1 + \sqrt{\tan x}}$ is equal to $\pi/6$. [AIEEE - 2013, (4, -1), 360]

Statement-II : $\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$

- (1) Statement-I is true; Statement-II is true; Statement-II is a **correct** explanation for Statement-I.
 (2) Statement-I is true; Statement-II is true; Statement-II is **not** a correct explanation for Statement-I.
 (3) Statement-I is true; Statement-II is false.
 (4) Statement-I is false; Statement-II is true.

12. The area (in square units) bounded by the curves $y = \sqrt{x}$, $2y - x + 3 = 0$, x-axis, and lying in the first quadrant is : [AIEEE - 2013, (4, -1), 360]

- (1) 9 (2) 36 (3) 18 (4) $\frac{27}{4}$

13. The integral $\int_0^{\pi} \sqrt{1 + 4\sin^2 \frac{x}{2} - 4\sin \frac{x}{2}} dx$ equals : [JEE(Main)2014, (4, -1), 120]

- (1) $4\sqrt{3} - 4$ (2) $4\sqrt{3} - 4 - \frac{\pi}{3}$ (3) $\pi - 4$ (4) $\frac{2\pi}{3} - 4 - 4\sqrt{3}$

14. The area of the region described by $A = \{(x, y) : x^2 + y^2 \leq 1 \text{ and } y^2 \leq 1 - x\}$ is : [JEE(Main)2014, (4, -1), 120]

- (1) $\frac{\pi}{2} - \frac{2}{3}$ (2) $\frac{\pi}{2} + \frac{2}{3}$ (3) $\frac{\pi}{2} + \frac{4}{3}$ (4) $\frac{\pi}{2} - \frac{4}{3}$

15. The integral $\int_2^4 \frac{\log x^2}{\log x^2 + \log(36 - 12x + x^2)} dx$ is equal to [JEE(Main)2015, (4, -1), 120]

- (1) 2 (2) 4 (3) 1 (4) 6

16. The area (in sq. units) of the region described by $\{(x, y) : y^2 \leq 2x \text{ and } y \geq 4x - 1\}$ is [JEE(Main)2015, (4, -1), 120]

- (1) $\frac{7}{32}$ (2) $\frac{5}{64}$ (3) $\frac{15}{64}$ (4) $\frac{9}{32}$

17. The area (in sq. units) of the region $\{(x, y) : y^2 \geq 2x \text{ and } x^2 + y^2 \leq 4x, x \geq 0, y \geq 0\}$ is [JEE(Main)2016, (4, -1), 120]

- (1) $\pi - \frac{8}{3}$ (2) $\pi - \frac{4\sqrt{2}}{3}$ (3) $\frac{\pi}{2} - \frac{2\sqrt{2}}{3}$ (4) $\pi - \frac{4}{3}$

18. $\lim_{n \rightarrow \infty} \left(\frac{(n+1)(n+2)\dots 3n}{n^{2n}} \right)^{1/n}$ is equal to : [JEE(Main)2016, (4, -1), 120]

- (1) $\frac{27}{e^2}$ (2) $\frac{9}{e^2}$ (3) $3 \log 3 - 2$ (4) $\frac{18}{e^4}$

19. The area (in sq. units) of the region $\{(x, y) : x \geq 0, x + y \leq 3, x^2 \leq 4y \text{ and } y \leq 1 + \sqrt{x}\}$ is :
[JEE(Main) 2017, (4, - 1), 120]

(1) $\frac{59}{12}$ (2) $\frac{3}{2}$ (3) $\frac{7}{3}$ (4) $\frac{5}{2}$

20. The integral $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1 + \cos x}$ is equal to
[JEE(Main) 2017, (4, - 1), 120]

(1) -2 (2) 2 (3) 4 (4) -1

21. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 x}{1 + 2^x} dx$ is :
[JEE(Main) 2018, (4, - 1), 120]

(1) 4π (2) $\frac{\pi}{4}$ (3) $\frac{\pi}{8}$ (4) $\frac{\pi}{2}$

22. Let $g(x) = \cos x^2$, $f(x) = \sqrt{x}$, and α, β ($\alpha < \beta$) be the roots of the quadratic equation $18x^2 - 9\pi x + \pi^2 = 0$. Then the area (in sq. units) bounded by the curve $y = (g \circ f)(x)$ and the lines $x = \alpha$, $x = \beta$ and $y = 0$, is
[JEE(Main) 2018, (4, - 1), 120]

(1) $\frac{1}{2}(\sqrt{3} - \sqrt{2})$ (2) $\frac{1}{2}(\sqrt{2} - 1)$ (3) $\frac{1}{2}(\sqrt{3} - 1)$ (4) $\frac{1}{2}(\sqrt{3} + 1)$

23. Let $I = \int_a^b (x^4 - 2x^2) dx$. If I is minimum then the ordered pair (a, b) is :
[JEE(Main) 2019, Online (10-01-19), P-1 (4, - 1), 120]

(1) $(0, \sqrt{2})$ (2) $(\sqrt{2}, -\sqrt{2})$ (3) $(-\sqrt{2}, \sqrt{2})$ (4) $(-\sqrt{2}, 0)$

24. The value of $\int_{-\pi/2}^{\pi/2} \frac{dx}{[x] + [\sin x] + 4}$, where $[t]$ denotes the greatest less than or equal to t , is :
[JEE(Main) 2019, Online (10-01-19), P-2 (4, - 1), 120]

(1) $\frac{1}{12}(7\pi - 5)$ (2) $\frac{3}{10}(4\pi - 3)$ (3) $\frac{3}{20}(4\pi - 3)$ (4) $\frac{1}{12}(7\pi + 5)$

25. The integral $\int_1^e \left\{ \left(\frac{x}{e} \right)^{2x} - \left(\frac{e}{x} \right)^x \right\} \log_e x dx$ is equal to
[JEE(Main) 2019, Online (12-01-19), P-2 (4, - 1), 120]

(1) $\frac{3}{2} - \frac{1}{e} - \frac{1}{2e^2}$ (2) $\frac{3}{2} - e - \frac{1}{2e^2}$ (3) $\frac{1}{2} - e - \frac{1}{e^2}$ (4) $-\frac{1}{2} + \frac{1}{e} - \frac{1}{2e^2}$

Answers

EXERCISE - 1

PART - I

Section (A) :

- A-1.** (i) $-\frac{10}{21}$ (ii) $\sqrt{2} - 1$ **A-2.** (i) π (ii) $\frac{\pi}{4}$ (iii) $4 + \ln 5$ (iv) $\frac{8}{21}$
- A-3.** (i) $\frac{\pi - 2}{2}$ (ii) $\frac{1}{2} \ln \left(\frac{e}{2} \right)$ (iii) $\frac{\pi}{6} - \frac{2}{9}$ **A-4.** (i) $\ln(\sqrt{3})$ (ii) $7/6$ (iii) $\frac{3}{8}$
- A-5.** (i) $\frac{\pi}{2} - \ln 2$ (ii) $\frac{4 - \pi}{4\sqrt{2}}$ (iii) $-\frac{\pi}{8}$ (b - a)² (iv) $\pi \left(1 - \frac{1}{\sqrt{3}} \right) - \ln 4$
- A-6.** (i) $\frac{\pi}{4}$ (ii) $\frac{5}{3} - 2 \ln 2$ (iii) $\ln \left(\frac{9}{8} \right)$ (iv) $\frac{\pi}{2}$ (v) $\frac{1}{20} \ln 3$
- A-7.** (i) $\ln 11$ (ii) $\frac{1}{3}$

Section (B) :

- B-2.** (i) $5 - \sqrt{2} - \sqrt{3}$ (ii) $2\sqrt{2}$ (iii) 9 (iv) 4 (v) $\cot 1$ (vi) 29
(vii) $\cos 1 + \cos 2 + \cos 3 + 3$
- B-3.** (i) $2e - 2$ (ii) $2 - \sqrt{2}$ (iii) $\frac{\pi^2}{6\sqrt{3}}$ (iv) 0 (v) 0
- B-4.** (i) $\frac{\pi}{4}$ (ii) $\frac{\pi}{4}$ (iii) $\frac{a}{2}$ (iv) $(a + b) \frac{\pi}{4}$ (v) 0
- B-5.** (i) 0 (ii) $\frac{\pi}{3}$ (iii) $-\frac{\pi}{2} \ln 2$ (iv) $\pi \ln 2$
- B-6.** (i) $\frac{3}{2}$ (ii) 40 (iii) $n - 1$ (iv) $4n$

Section (C) :

- C-1.** (i) $4\sqrt{2}$ (ii) 12 (iii) $\left(\frac{\sqrt[4]{8}}{3} - \frac{1}{4} \right) \pi$ **C-2.** (ii) 1, 3 **C-3.** $5/2$ **C-4.** $\frac{1}{2}$
- C-5.** e **C-6.** (i) $\frac{4}{15}$ (ii) $\frac{8\pi}{15}$ (iii) $\frac{\pi}{2}$ (iv) $\frac{\pi^2}{4}$

Section (E) :

- E-1.** (i) $\frac{\pi}{2}$ (ii) 2 (iii) 12

Section (F) :

- F-1.** $\frac{51}{4}$ sq. unit **F-2.** (i) $\frac{\pi}{2} - \frac{4}{\pi}$ (ii) $\frac{7}{120}$ (iii) 9π

- F-3.** $4/3$ sq. units **F-4.** $\frac{(e+1)\pi}{1+\pi^2}$ **F-5.** (i) $3(\pi-2)$ (ii) $\frac{1}{8}$ **F-6.** $\left(\frac{3}{\log_e 2} - \frac{4}{3}\right)$ sq. units
- F-8.** 4 sq. units. **F-9.** $\frac{16}{3}$ sq. units.

PART - II

SECTION (A) :

- A-1.** (A) **A-2.** (C) **A-3.** (C) **A-4.** (C) **A-5.** (D) **A-6.** (A) **A-7.** (B)
- A-8.** (A) **A-9.** (A) **A-10.** (B) **A-11.** (D) **A-12.** (C) **A-13.** (C)

Section (B) :

- B-1.** (C) **B-2.** (C) **B-3.** (D) **B-4.** (C) **B-5.** (D) **B-6.** (A) **B-7.** (B)
- B-8.** (D) **B-9.** (C) **B-10.** (A) **B-11.** (C) **B-12.** (A) **B-13.** (C)

Section (C) :

- C-1.** (A) **C-2.** (B) **C-3.** (B) **C-4.** (D) **C-5.** (B) **C-6.** (C) **C-7.** (C)
- C-8.** (B) **C-9.** (D) **C-10.** (A)

SECTION (D) :

- D-1.** (C) **D-2.** (B) **D-3.** (C) **D-4.** (A)

SECTION (E) :

- E-1.** (D) **E-2.** (B) **E-3.** (C) **E-4.** (C) **E-5.** (D)

Section (F) :

- F-1.** (C) **F-2.** (D) **F-3.** (C) **F-4.** (A) **F-5.** (C) **F-6.** (B) **F-7.** (A)
- F-8.** (A) **F-9.** (C) **F-10.** (D)

PART - III

- 1.** A-q, B-s, C-p, D-t **2.** (A) $\rightarrow$ (r), (B) $\rightarrow$ (s), (C) $\rightarrow$ (q), (D) $\rightarrow$ (p)

EXERCISE - 2

PART - I

- 1.** (C) **2.** (B) **3.** (A) **4.** (C) **5.** (D) **6.** (A) **7.** (D)
- 8.** (B) **9.** (C) **10.** (D) **11.** (A) **12.** (B) **13.** (B) **14.** (C)
- 15.** (B) **16.** (C) **17.** (C) **18.** (B) **19.** (C) **20.** (A) **21.** (A)
- 22.** (A) **23.** (D) **24.** (D) **25.** (D) **26.** (B) **27.** (A) **28.** (D)
- 29.** (B) **30.** (A)

PART - II

- 1.** 61 **2.** 64 **3.** 10 **4.** 29 **5.** 4 **6.** 4 **7.** 2
- 8.** 2 **9.** 0 **10.** 2 **11.** 8 **12.** 16 **13.** 25 **14.** 4
- 15.** 65 **16.** 5 **17.** 11 **18.** 3 **19.** 2 **20.** 0 **21.** 1
- 22.** 22 sq.unit **23.** 55

PART - III

1. (AC) 2. (ABCD) 3. (AB) 4. (AB) 5. (ABC) 6. (ABC) 7. (AB)
 8. (BD) 9. (BCD) 10. (BC) 11. (AD) 12. (ABCD) 13. (ABCD)
 14. (ABD) 15. (AD) 16. (AC) 17. (AB) 18. (BD) 19. (ABD) 20. (AD)
 21. (AC) 22. (BC) 23. (ACD) 24. (ABC) 25. (ABCD) 26. (ABC)
 27. (ABCD) 28. (AD) 29. (AB) 30. (BD)

PART - IV

1. (C) 2. (A) 3. (A) 4. (D) 5. (A) 6. (BC) 7. (C)
 8. (B) 9. (D)

EXERCISE - 3

PART - I

- 1*. (ABC) 2. 0 3*. (BCD) 4. (B) 5. (A) 6*. (BC)
 7. 4 8. (B) 9. (C) 10. (A) 11. (B) 12. (A) 13. (B)
 14. (C) 15.* (ABD) 16. (B) 17. (B) 18. (C) 19. (ABCD) 20. (B)
 21. (D) 22. (B) 23.* (AC) 24.* (ACD) 25. 2 26. (A) 27. (A)
 28. (D) 29. (D) 30. 0 31. 9 32. 7 33*. (AB) 34*. (D)
 35*. (AC) 36*. (ABC) 37*. (CD) 38. 3 39. 1 40. (A) 41. (C)
 42*. (BC) 43. (CD) 44. (2) 45. (BD) 46. (AC) 47. (BONUS)
 48. (1) 49. (4) 50. (2)

PART - II

1. (3) 2. (2) 3. (1) 4. (4) 5. (4) 6. (3) 7. (3)
 8. (2) 9. (3) 10.* (2) or (3) 11. (4) 12. (1) 13. (2) 14. (3)
 15. (3) 16. (4) 17. (1) 18. (1) 19. (4) 20. (2) 21. (2)
 22. (3) 23. (3) 24. (3) 25. (2)

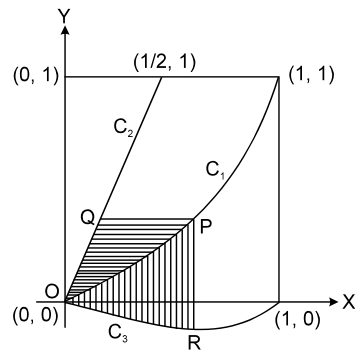
Advance Level Problems (ALP):-

- Find the integral value of a for which $\int_0^{\frac{\pi}{2}} (\sin x + a \cos x)^3 dx - \frac{4a}{\pi-2} \int_0^{\frac{\pi}{2}} x \cos x dx = 2$
- Evaluate : $\int_0^{\pi} \sqrt{(\cos x + \cos 2x + \cos 3x)^2 + (\sin x + \sin 2x + \sin 3x)^2} dx$
- Let α & β be distinct positive roots of the equation $\tan x = 2x$, then evaluate $\int_0^1 \sin(\alpha x) \cdot \sin(\beta x) dx$
- Evaluate % $\lim_{a \rightarrow \left(\frac{\pi}{2}\right)^-} \int_0^a (\cos x) \ln(\cos x) dx$
- Find the value of a ($0 < a < 1$) for which the following definite integral is minimized.
 $\int_0^{\pi} |\sin x - ax| dx$
- Find the $\lim_{n \rightarrow \infty} \left(\frac{{}^{3n}C_n}{{}^{2n}C_n} \right)^{\frac{1}{n}}$
where iC_j is a binomial coefficient which means $\frac{i \cdot (i-1) \cdot \dots \cdot (i-j+1)}{j \cdot (j-1) \cdot \dots \cdot 2 \cdot 1}$
- Show that $\int_0^{\infty} f\left(\frac{a}{x} + \frac{x}{a}\right) \cdot \frac{\ln x}{x} dx = \ln a \cdot \int_0^{\infty} f\left(\frac{a}{x} + \frac{x}{a}\right) \cdot \frac{dx}{x}$
- Evaluate $\lim_{n \rightarrow \infty} n^2 \int_{-\frac{1}{n}}^{\frac{1}{n}} (2014 \sin x + 2015 \cos x) |x| dx$
- Let sequence $\{a_n\}$ be defined as
 $a_1 = \frac{\pi}{4}$, $a_n = \int_0^{\frac{1}{2}} (\cos(\pi x) + a_{n-1}) \cos \pi x dx$, ($n = 2, 3, 4, \dots$)
then evaluate $\lim_{n \rightarrow \infty} a_n$
- Find $f(x)$ if it satisfies the relation $f(x) = e^x + \int_0^1 (x + ye^x) f(y) dy$.
- Evaluate : $\int_{-1/\sqrt{3}}^{1/\sqrt{3}} \frac{x^4}{1-x^4} \cos^{-1} \left(\frac{2x}{1+x^2} \right) dx$.

12. Evaluate $\int_0^1 \frac{1}{(5+2x-2x^2)(1+e^{(2-4x)})} dx$
13. Prove that for any positive integer k ;
 $\frac{\sin 2kx}{\sin x} = 2 [\cos x + \cos 3x + \dots + \cos (2k-1)x]$. Hence prove that;
 $\int_0^{\pi/2} \sin (2kx) \cdot \cot x \, dx = \frac{\pi}{2}$.
14. Prove that $\lim_{n \rightarrow \infty} \frac{1}{n} \left[\cos^{2p} \frac{\pi}{2n} + \cos^{2p} \frac{2\pi}{2n} + \cos^{2p} \frac{3\pi}{2n} + \dots + \cos^{2p} \frac{\pi}{2} \right] = \prod_{r=1}^p \frac{2r-1}{2r}$,
 where $\prod$ denotes the continued product and $p \in \mathbb{N}$.
15. If $n > 1$, evaluate $\int_0^{\infty} \frac{dx}{(x + \sqrt{1+x^2})^n}$
16. Let $f(x)$ be a continuous function $\forall x \in \mathbb{R}$, except at $x = 0$ such that $\int_0^a f(x) dx$, $a \in \mathbb{R}^+$ exists. If
 $g(x) = \int_x^a \frac{f(t)}{t} dt$, prove that $\int_0^a g(x) dx = \int_0^a f(x) dx$
17. Given that $\lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{\log_e(n^2 + r^2) - 2\log_e n}{n} = \log_e 2 + \frac{\pi}{2} - 2$, then
 evaluate : $\lim_{n \rightarrow \infty} \frac{1}{n^{2m}} [(n^2 + 1^2)^m (n^2 + 2^2)^m \dots (2n^2)^m]^{1/n}$.
18. For a natural number n , let $a_n = \int_0^{\pi/4} (\tan x)^{2n} dx$
 Now answer the following questions :
 (1) Express a_{n+1} in terms of a_n
 (2) Find $\lim_{n \rightarrow \infty} a_n$
 (3) Find $\lim_{n \rightarrow \infty} \sum_{k=1}^n (-1)^{k-1} (a_k + a_{k-1})$
19. Given that $\lim_{x \rightarrow 0} \frac{\int_0^x \frac{t^2}{\sqrt{a+t}} dt}{bx - \sin x} = 1$, then find the values of a and b
20. Prove that $m \sin x + \int_0^x \sec^m t \, dt > (m+1)x \quad \forall x \in \left(0, \frac{\pi}{2}\right) \quad m \in \mathbb{N}$
21. $f(x)$ is differentiable function: $g(x)$ is double differentiable function such that $|f(x)| \leq 1$ and $g(x) = f'(x)$. If $f^2(0) + g^2(0) = 9$ then show that there exists some $C \in (-3, 3)$ such that $g(C) g''(C) < 0$
22. Draw a graph of the function $f(x) = \cos^{-1}(4x^3 - 3x)$, $x \in [-1, 1]$ and find the area enclosed between the graph of the function and the x -axis as x varies from 0 to 1.
23. Consider a square with vertices at $(1, 1)$, $(-1, 1)$, $(-1, -1)$ and $(1, -1)$. Let S be the region consisting of all points inside the square which are nearer to the origin than to any edge. Sketch the region S and find its area.

24. If $[x]$ denotes the greatest integer function. Draw a rough sketch of the portions of the curves $x^2 = 4[\sqrt{x}]y$ and $y^2 = 4[\sqrt{y}]x$ that lie within the square $\{(x, y) \mid 1 \leq x \leq 4, 1 \leq y < 4\}$. Find the area of the part of the square that is enclosed by the two curves and the line $x + y = 3$.
25. Find the area of the region bounded by $y = f(x)$, $y = |g(x)|$ and the lines $x = 0$, $x = 2$, where 'f', 'g' are continuous functions satisfying $f(x+y) = f(x) + f(y) - 8xy \forall x, y \in \mathbb{R}$ and $g(x+y) = g(x) + g(y) + 3xy(x+y)$, $x, y \in \mathbb{R}$ also $f'(0) = 8$ and $g'(0) = -4$.
26. Let $f(x) = \begin{cases} -2 & , -3 \leq x \leq 0 \\ x-2 & , 0 < x \leq 3 \end{cases}$, where $g(x) = \min \{f(|x|) + |f(x)|, f(|x|) - |f(x)|\}$. Find the area bounded by the curve $g(x)$ and the x-axis between the ordinates $x = 3$ and $x = -3$.
27. Find the area of region $\{(x, y) : 0 \leq y \leq x^2 + 1, 0 \leq y \leq x + 1, 0 \leq x \leq 2\}$.
28. A curve $y = f(x)$ passes through the point $P(1, 1)$, the normal to the curve at P is $a(y - 1) + (x - 1) = 0$. If the slope of the tangent at any point on the curve is proportional to the ordinate of that point, determine the equation of the curve. Also obtain the area bounded by the y-axis, the curve and the normal to the curve at P .
29. Find the area bounded by $y = [-0.01x^4 - 0.02x^2]$, (where $[.]$ G.I.F.) and curve $3x^2 + 4y^2 = 12$, which lies below $y = -1$.
30. Let ABC be a triangle with vertices $A(6, 2(\sqrt{3} + 1))$, $B(4, 2)$ and $C(8, 2)$. If R be the region consisting of all these points and point P inside $\triangle ABC$ which satisfy $d(P, BC) \geq \max \{d(P, AB), d(P, AC)\}$ where $d(P, L)$ denotes the distance of the point P from the line L . Sketch the region R and find its area.
31. Find the area of the region which contains all the points satisfying the condition $|x - 2y| + |x + 2y| \leq 8$ and $xy \geq 2$.
32. Find the area of the region which is inside the parabola $y = -x^2 + 6x - 5$, outside the parabola $y = -x^2 + 4x - 3$ and left of the straight line $y = 3x - 15$.
33. Consider the curve $C: y = \sin 2x - \sqrt{3} |\sin x|$, C cuts the x-axis at $(a, 0)$, $x \in (-\pi, \pi)$.
 A_1 : The area bounded by the curve C and the positive x-axis between the origin and the line $x = a$.
 A_2 : The area bounded by the curve C and the negative x-axis between the line $x = a$ and the origin.
 Prove that $A_1 + A_2 + 8A_1A_2 = 4$.
34. Area bounded by the line $y = x$, curve $y = f(x)$, $(f(x) > x \forall x > 1)$ and the lines $x = 1$, $x = t$ is $(t + \sqrt{1+t^2} - (1 + \sqrt{2})) \forall t > 1$. Find $f(x)$ for $x > 1$.
35. Consider the two curves $y = 1/x^2$ and $y = 1/[4(x-1)]$.
 (i) At what value of 'a' ($a > 2$) is the reciprocal of the area of the figure bounded by the curves, the lines $x = 2$ and $x = a$ equal to 'a' itself?
 (ii) At what value of 'b' ($1 < b < 2$) the area of the figure bounded by these curves, the lines $x = b$ and $x = 2$ equal to $1 - 1/b$.
36. Let C_1 and C_2 be the graphs of the functions $y = x^2$ and $y = 2x$, $0 \leq x \leq 1$ respectively. Let C_3 be the graph of a function $y = f(x)$, $0 \leq x \leq 1$, $f(0) = 0$. For a point P on C_1 , let the lines through P , parallel to the axes, meet C_2 and C_3 at Q and R respectively (see figure). If for every position of P (on C_1), the areas of

the shaded regions OPQ and ORP are equal, determine the function $f(x)$.



37. Given the parabola $C : y = x^2$. If the circle centred at y axis with radius 1 touches parabola C at two distinct points, then find the coordinate of the center of the circle K and the area of the figure surrounded by C and K .

38. If $\begin{bmatrix} 4a^2 & 4a & 1 \\ 4b^2 & 4b & 1 \\ 4c^2 & 4c & 1 \end{bmatrix} \begin{bmatrix} f(-1) \\ f(1) \\ f(2) \end{bmatrix} = \begin{bmatrix} 3a^2 + 3a \\ 3b^2 + 3b \\ 3c^2 + 3c \end{bmatrix}$, $f(x)$ is a quadratic function and its maximum value occurs at a point V . A is a point of intersection of $y = f(x)$ with x -axis and point B is such that chord AB subtends a right angle at V . Find the area enclosed by $f(x)$ and chord AB .

39. $f(x)$ and $g(x)$ are polynomials of degree 2 such that $\left| \int_{a_1}^{a_2} (f(x) - 1) dx \right| = \left| \int_{b_1}^{b_2} (g(x) - 1) dx \right|$

where a_1, a_2 ($a_2 > a_1$) are roots of equation $f(x) = 1$ and b_1, b_2 ($b_2 > b_1$) are roots of equation $g(x) = 1$. If $f''(x)$ and $g''(x)$ are positive constant and

$$\left| \int_{a_1}^{a_2} f(x) dx \right| = (a_2 - a_1) - \left| \int_{b_1}^{b_2} (f(x) - 1) dx \right| \text{ but } \left| \int_{b_1}^{b_2} g(x) dx \right| \neq (b_2 - b_1) - \left| \int_{b_1}^{b_2} (g(x) - 1) dx \right| \text{ then}$$

- (A) $|f''(x)| < |g''(x)|$ (B) $|f''(x)| > |g''(x)|$ (C) $a_2 - a_1 > b_2 - b_1$ (D) $a_2 - a_1 > b_2 - b_1$

40. Let $L = 4x - 5y$, $L_i = \frac{x}{10} + \frac{y}{8} - \frac{i}{n}$, $L_i = \frac{x}{10} + \frac{y}{8} + \frac{i}{n}$, and $E = \frac{x^2}{50} + \frac{y^2}{32} - 1$.

Let A_i represents the area of region common between $L_{i-1} > 0$, $L_i < 0$, $E < 0$ and $L < 0$;

A'_i represents the area of region common between $L'_{i-1} < 0$, $L'_i > 0$, $E < 0$ and $L < 0$;

B_i represents the area of region common between $L_{i-1} > 0$, $L_i < 0$, $E < 0$ and $L > 0$;

B'_i represents the area of region common between $L'_{i-1} < 0$, $L'_i > 0$, $E < 0$ and $L > 0$, then value of

$(A_1 + A'_2 + A_3 + A'_4 + \dots) + (B_1 + B'_2 + B_3 + B'_4 + \dots)$ is equal to.

Answers

1. -1 2. $\frac{\pi}{3} + 2\sqrt{3}$ 3. 0 4. $\frac{1}{n^2}$ 5. $a = \frac{\sqrt{2}}{\pi} \sin\left(\frac{\pi}{\sqrt{2}}\right)$
6. $\frac{27}{16}$ 8. 2015 9. $\frac{\pi}{4(\pi-1)}$ 10. $\frac{-3e^x}{2(e-1)} - 3x$
11. $\frac{\pi}{4} \sum_{n=1}^{\infty} (2+\sqrt{3})^n + \frac{\pi^2}{12} - \frac{\pi}{\sqrt{3}}$ 12. $\frac{1}{2} \sum_{n=1}^{\infty} \frac{1}{\sqrt{11}^n} \sum_{n=1}^{\infty} \frac{\sqrt{11}+1}{\sqrt{11}-1}$ 15. $\frac{n}{n^2-1}$
17. $\left(\frac{2\sqrt{e^\pi}}{e^2}\right)^m$ 18. (1) $\frac{1}{2n+1} - a_n$ (2) 0 (3) $\frac{\pi}{4}$
19. $a = 4, b = 1$ 22. $3(\sqrt{3}-1)$ sq. units 23. $\frac{1}{3}(16\sqrt{2}-20)$
24. $\frac{19}{6}$ 25. $\frac{4}{3}$ 26. $\frac{23}{2}$ 27. $\frac{23}{6}$ 28. $y = e^{a(x-1)}, \left(1 + \frac{e^{-a}}{a} - \frac{1}{2a}\right)$
29. $2\sqrt{3} \sin^{-1} \sqrt{\frac{2}{3}} - \frac{2\sqrt{2}}{\sqrt{3}}$ 30. $\frac{4\sqrt{3}}{3}$ 31. $2(6-2 \log 4)$ 32. $\frac{73}{6}$ 34. $1+x + \frac{x}{\sqrt{1+x^2}}$
35. (i) $a = 1 + e^2$ (ii) $b = 1 + e^{-2}$ 36. $f(x) = x^3 - x^2$
37. centre $\left(0, \frac{5}{4}\right)$ and area $= \frac{3\sqrt{3}}{4} - \frac{\pi}{3}$ 38. $\frac{125}{3}$ square units. 39. (AC) 40. 20π