APPLICATIONS OF DERIVATIVES

TANGENTS & NORMALS

EXERCISE (FOR COMPETITIVE EXAM)

Q.1 Determine the slope of the normal to the curve given by $x = 1 - a \sin \theta$, $y = b \cos^2 \theta$ at $\theta = \frac{\pi}{2}$.

Q.2 Determine the equations of the tangent and normal lines to the specified curves at the provided points.

(i) $y = x^4 - 6x^3 + 13x^2 - 10x + 5$ at $(1, 3)$

(ii) $y^2 = \frac{x^3}{4-x}$ at $(2, -2)$.

Q.3 Demonstrate that the area of the triangle formed by any tangent to the curve $xy=c^2$ and the coordinate axes remains constant.

Q.4 The curve is defined by the equation $x = at^2$. And $y = at^3$. Two variable perpendicular lines passing through the origin 'O' intersect the curve at point p and Q. demonstrate that the locus of the point of the tangents at P and Q is $4y^2 = 3ax - a^2$.

Q.5 Determine the number of tangents that can be drawn from the point (1, 1) to the curve described by $y-1=x^3$. Additionally, find the equations of these tangents.

Q.6 Determine the equation of the tangent to the hyperbola defined by $y = \frac{x+9}{x+5}$ that passes through the origin (0, 0).

Q.7 For the curve by $xm + n = am - ny^{2n}$ where a is a positive constant and m and n are positive integers demonstrate that the mth power of the subagent changes in proportion to the nth power of the subnormal.

Q.8 Demonstrate that the portion of the tangent to the curve $y = \frac{a}{2} \text{In} \frac{a+\sqrt{a^2-x^2}}{a-\sqrt{a^2-x^2}} - \sqrt{a^2-x^2}$ contained between the y-axis and the point of tangency maintains a constant length.

Q.9 Determine the length of the subnormal to the curve $y^2 = x^3$ at the point (4, 8).

ANSWER KEY

1. $-\frac{a}{2b}$

2. (i) Tangent : $y = 2x + 1$, Normal :$x + 2y = 7$

   (ii) Tangent: $2x + y = 2$, Normal: $x - 2y = 6$

5. $y = 1$, $4y = 27x - 23$

6. $x + y = 0$; $25y + x = 0$

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