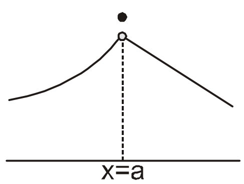
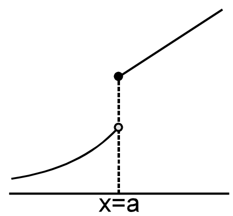
APPLICATIONS OF DERIVATIVES

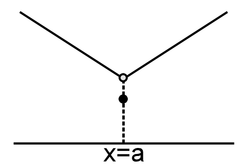
MAXIMA & MINIMA

EXERCISE

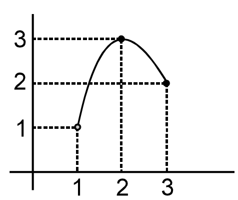
Q.1 In each of the following graphs, determine whether (x = a) represents a point of local maximum, minimum, or neither.

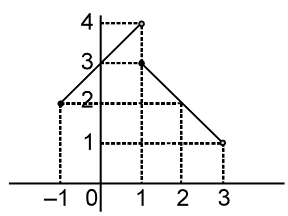
(i) 

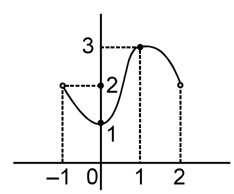
(ii) 

(iii) 

Q.2 Analyze the graphs of the given functions, and in each case, identify the points corresponding to global maximum/minimum and local maximum/minimum.

(i) 

(ii) 

(iii) 

Q.3 Determine the points of local maxima or minima for the given functions.

(i) $f(x) = (x - 1)^3 (x + 2)^2$ (ii) $f(x) = x^3 + x^2 + x + 1$.

Q.4 Let $f(x) = 2x^3 + 12x + x + 6$:

(i) Identify the potential points of maxima / minima for f (x) across all real value of x.

(ii) Determine the number of critical point of f (x) within the interval [0, 2]

(iii) Examine the absolute (global) maxima/minima value of f (x) for $x \in [0, 2]$

(iv) Prove that for $x \in (1, 3)$ the function does not possess a global maximum.

Q.5 Consider $f(x) = \frac{x}{2} + \frac{2}{x}$. Determine the local maximum and local minimum values of f(x). Can you provide an explanation for the discrepancy where the locally minimum value is greater than the locally maximum value?

Q.6 If $f(x) = \begin{cases} (x+\lambda)^2 & x < 0 \\ \cos x & x \geq 0 \end{cases}$, Determine the values of λ for which f(x) exhibits local maxima at $x = 0$.

Q.7 Consider the function $f(x) = \sin x\,(1 + \cos x)$ for x in the interval $(0, 2\pi)$. Determine the count of critical points of f(x). And ascertain which among these critical points correspond to maxima or minima.

Q.8 Discover two positive numbers x and y such that their sum is 35 and the product $x^2 y^5$ maximum.

Q.9 A square tin sheet with a side length of 18 cm is to be transformed into a box without a top by removing squares from each corner and folding up the resulting flaps to create the box. Determine the side length of the square to be cut off to maximize the volume of the box.

Q.10 Demonstrate that for a given surface area, a right circular cylinder attains its maximum volume when the height equals the diameter of the base.

Q.11 A normal extended to the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$. Determine the maximum distance of this normal from the centre.

Q.12 A line passing through the point P(1, 2) intersects the positive coordinate axes at points A and B. Calculate the minimum area of ΔPAB.

Q.13 Two towns, A and B, are located on the same side of a straight road at distances (a) and (b) respectively. Perpendiculars drawn from A and B meet the road at points C and D respectively, with a distance between C and D denoted as (c). The objective is to construct a hospital at a point P on the road to minimize the distance APB. Determine the optimal position for P.

ANSWER KEY

1. (i) Maxima

(ii) Neither maxima nor minima

(iii) Minima

2. (i) Local maxima at $x = 2$, Local minima at $x = 3$, Global maximum at $x = 2$. No global minimum

(ii) Local minima at $x = -1$, No point of Global minimum, no point of local or Global maxima

(iii) Local & Global maximum at $x = 1$, Local & Global minimum at $x = 0$.

3. (i) Maxima at $x = -2$, Minima at $x = -\frac{4}{5}$

(ii) No point of local maxima or minima.

4. (i) $x = 1,.2$ (ii) one

(iii) $f(0) = 6$ is global maximum, $f(1) = 11$ is global maximum

5. Local maxima at $x = -2$, $f(-2) = -2$;Local minima at $x = 2$, $f(2) = 2$

6. $\lambda \in [-1, 1)$

7. Three $x = \frac{\pi}{3}$ is point of maxima.

$x = \pi$ is not a point of extrema.

$x = \frac{5\pi}{3}$ is point of minima

8. $x = 25$, $y = 10$.

9. 3 cm

11. 1 unit

12. 4 units

13. P is at distance of $\frac{ac}{a+b}$ from C.