APPLICATIONS OF DERIVATIVES

INCREASING & DECREASING FUNCTIONS

EXERCISE

Q.1 Determine the intervals of monotonic behavior for the given functions.

(i) $f(x) = -x^3 + 6x^2 - 9x - 2$ (ii) $f(x) = x + \frac{1}{x+1}$

(iii) $f(x) = x \cdot e^{x-x^2}$ (iv) $f(x) = x - \cos x$

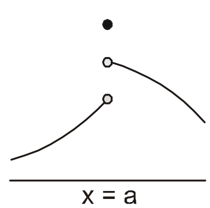
Q.2 Consider the function $f(x) = x - \tan^{-1}x$. Demonstrate that f(x) is monotonically increasing for all x in the set of real numbers.

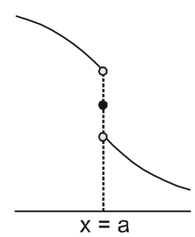
Q.3 If $f(x) = 2e^x - ae^{-x} + (2a + 1)x - 3$ monotonically increases for all x in the set of real numbers, determine the range of values for a.

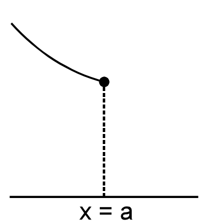
Q.4 Consider $f(x) = e^{2x} - ae^x + 1$. Show that f(x) cannot be monotonically decreasing for all x in the set of real numbers for any value of 'a'.

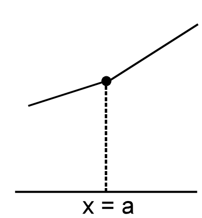
Q.5 Determine the values of 'a' for which the function $f(x) = (a + 2)x^3 - 3ax^2 + 9ax - 1$ is monotonically decreasing for all x in the set of real numbers.

Q.6 Provide comments on the monotonicity of f(x) at x = a. for each of the following graphs.

(i)


(ii)


(iii)


(iv)


ANSWER KEY

1. (i) I in $[1, 3]$; D in $(-\infty, 1] \cup (3, \infty)$

(ii) I in $(-\infty, -2] \cup [0, \infty)$; D in $[-2, -1) \cup (-1, 0]$

(iii) I in $\left[-\frac{1}{2}, 1\right]$; D in $\left(-\infty, -\frac{1}{2}\right] \cup [1, \infty)$

(iv) I for $x \in R$

3. $a \geq 0$

5. $-\infty < a \leq -3$

6. (i) neither M.I. nor M.D. (ii) M.D.

(iii) M.D (iv) M.I.

7. M.I. both at $x = 0$ and $x = 1$.

8. M.I. at $x = 0, 2$; neither M.I. nor M.D. at $x = 1$. No, $f(x)$ is not M.I. for $x \in [0, 2]$.