CONTINUITY AND DIFFERENTIABILITY

DIFFERENTIABILITY

EXERCISE

Q.1 If $f(x) = \begin{cases} [2x] + x, & x < 1 \\ \{x\} + 1, & x \geq 1 \end{cases}$, provide remarks on the continuity and differentiability at $x = 1$, where [.] represents the greatest integer function, and {.} denotes the fractional part function.

Q.2 If $f(x) = \begin{cases} x \tan^{-1}\dfrac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$, provide observation on the derivability of f(x) at x = 0.

Q.3 If feasible, determine the equation of the tangent to the specified curves at the given points.

(i) $y = x^3 + 3x^2 + 28x + 1$ at $x = 0$.

(ii) $y = (x - 8)^{2/3}$ at $x = 8$.

Q.4 If $f(x) = \begin{cases} \left(\dfrac{e^{[x]} + |x| - 1}{[x] + \{2x\}}\right) & x \neq 0 \\ \dfrac{1}{2} & x = 0 \end{cases}$, provide observations on the continuity at x = 0 and differentiability at x = 0, where [.] represents the greatest integer function and {.} denotes the fractional part function.

Q.5 If $f(x) = [x] + [1 - x]$, $-1 \leq x \leq 3$, plot its graph and provide insights into the continuity and differentiability of f(x), where [.] denotes the greatest integer function.

Q.6 If $f(x) = \begin{cases} |1 - 4x^2|, & 0 \leq x < 1 \\ [x^2 - 2x], & 1 \leq x \leq 2 \end{cases}$, sketch the graph of f(x) and discuss the differentiability and continuity of f(x), where [.] represents the greatest integer function.

Q.7 For all x, y $\in R^+$ and $f'(1) = 1$, if $f\left(\frac{x}{y}\right) = f(x) - f(y)$ demonstrate that $f(x) = \ln x$.

Q.8 If f(x) and g(x) are both differentiable, then establish that $f(x) \pm g(x)$ will also be differentiable.

Q.9 If $f'(2) = 4$, determine the value of $\lim_{h\to 0} \frac{f(2+h)-f(2+\sinh)}{h.\ \sinh.\tanh}$.

Q.10 If f(x) is a polynomial function that fulfills $f(x)\ .\ f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right)$ for all $x \in R - \{0\}$ and $f(3) = -8$, then determine the value of f(4)

Q.11 If $f(x + y) = f(x)\ .\ f(y)$ holds for all real x, y and $f(0) \neq 0$, then demonstrate that the function, $g(x) = \frac{f(x)}{1+f^2(x)}$ is an even function.

ANSWER KEY

1. Discontinuous and non-differentiable at $x = 1$

2. non-differentiable at $x = 0$

3. (i) $y = 28x + 1$

(ii) $x = 8$

4. discontinuous hence non-differentiable at $x = 0$

5. f(x) is discontinuous at $x = -1, 0, 1, 2, 3$ hence non-differentiable.

6. f(x) is discontinuous at $x = 1, 2$ & non differentiable at $x = \frac{1}{2}$, 1, 2.

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