THREE DIMENSIONAL GEOMETRY

SHORTEST DISTANCE BETWEEN TWO LINE

EXERCISE

Q.1 Determine the shortest distance between two lines l_1 and l_2 with the given vector equations.

$$\vec{r} = 3\hat{i} - 4\hat{j} + 2\hat{k} + \lambda(4\hat{i} + \hat{j} + \hat{k})$$
$$\vec{r} = 5\hat{i} + \hat{j} - \hat{k} + \mu(2\hat{i} - \hat{j} - 3\hat{k})$$

(A) $\frac{11}{\sqrt{12}}$ (B) $\frac{23}{\sqrt{10}}$ (C) $\frac{18}{\sqrt{10}}$ (D) $\frac{10}{\sqrt{11}}$

Q.2 Calculate the shortest distance between two lines l_1 and l_2 based on their vector equations provided below.

$$\vec{r} = 3\hat{i} + 2\hat{j} - \hat{k} + \lambda(3\hat{i} - 2\hat{j} + \hat{k})$$
$$\vec{r} = 2\hat{i} - \hat{j} + \hat{k} + \mu(3\hat{i} - 2\hat{j} + \hat{k})$$

(A) $\sqrt{\frac{172}{14}}$ (B) $\sqrt{\frac{145}{14}}$ (C) $\sqrt{\frac{171}{14}}$ (D) $\sqrt{\frac{171}{134}}$

Q.3 Determine the shortest distance between the given lines.

$$I_1 : \frac{x-5}{2} = \frac{y-2}{5} = \frac{z-1}{4}$$
$$I_2 : \frac{x+4}{3} = \frac{y-7}{6} = \frac{z-3}{7}$$

(A) $\frac{115}{\sqrt{134}}$ (B) $\frac{115}{\sqrt{184}}$ (C) $\frac{115}{134}$ (D) $\frac{\sqrt{115}}{134}$

Q.4 Determine the shortest distance between the given set of parallel lines.

$$\vec{r} = 6\hat{i} + 2\hat{j} - \hat{k} + \lambda(\hat{i} + 2\hat{j} - 4\hat{k})$$
$$\vec{r} = \hat{i} + \hat{j} + \hat{k} + \mu(\hat{i} + 2\hat{j} - 4\hat{k})$$

(A) $d = \sqrt{\frac{324}{45}}$ (B) $d = \sqrt{\frac{405}{21}}$ (C) $d = \sqrt{\frac{24}{21}}$ (D) $d = \sqrt{\frac{21}{567}}$

Q.5 Determine the distance between lines l_1 and l_2 based on the provided vector equations.

$$\vec{r} = 2\hat{i} + 2\hat{j} - 2\hat{k} + \lambda(3\hat{i} + 2\hat{j} + 5\hat{k})$$
$$\vec{r} = 4\hat{i} - \hat{j} + 5\hat{k} + \mu(3\hat{i} - 2\hat{j} + 4\hat{k})$$

(A) $\frac{57}{\sqrt{47}}$ (B) $\frac{57}{\sqrt{77}}$ (C) $\frac{7}{\sqrt{477}}$ (D) $\frac{57}{\sqrt{477}}$

Q.6 Demonstrate that the lines $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$ given by the equations:

$\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5}$ and intersect each other. Additionally, determine their point of intersection.

Q.7 Show that the lines: $\frac{x-a+d}{\alpha-\delta} = \frac{y-a}{\alpha} = \frac{z-a-d}{\alpha+\delta}$ and $\frac{x-b+c}{\beta-\gamma} = \frac{y-b}{\beta} = \frac{z-b-c}{\beta+\gamma}$ are coplanar.

Q.8 Determine the shortest distance and the equation representing the shortest distance between the following pair of lines:

$$\vec{r} = (-\hat{i} + \hat{j} + 9\hat{k}) + \lambda(2\hat{i} + \hat{j} - 3\hat{k}) \text{ and } \vec{r} = (3\hat{i} - 15\hat{j} + 9\hat{k}) + \mu(2\hat{i} - 7\hat{j} + 5\hat{k})$$

ANSWER KEY

1. (C) $\frac{18}{\sqrt{10}}$

2. (C) $\sqrt{\frac{171}{14}}$

3. (A) $\frac{115}{\sqrt{134}}$

4. (B) $d = \sqrt{\frac{405}{21}}$

5. (D) $\frac{57}{\sqrt{477}}$

8. $4\sqrt{3}; \vec{r} = (3\hat{i} + 3\hat{j} - 3\hat{k}) - \mu(4\hat{i} + 4\hat{j} + 4\hat{k})$