RELATIONS AND FUNCTIONS

TYPES OF RELATION

EXERCISE

Q.1 Let R be the relation on the set of natural numbers N, defined as follows:

R : $\{(x, y)\} : x + 3y = 12 \;\; x \in N, y \in N\}$ Determine

(i) R (ii) Domain of R (iii) Range of R

Q.2 If $X = \{x_1, x_2, x_3\}$ and $y = (x_1, x_2, x_3, x_4, x_5\}$ identify which of the following is a reflexive relation.

(a) $R_1 : \{(x_1, x_1), (x_2, x_2)$

(b) $R_1 : \{(x_1, x_1), (x_2, x_2), (x_3, x_3)$

(c) $R_3 :\{(x_1,x_1), (x_2, x_2),(x_3, x_3),(x_1, x_3),(x_2, x_4)$

(d) $R_3 : \{(x_1, x_1), (x_2, x_2),(x_3, x_3),(x_4, x_4)$

Q.3 If x = {a, b, c} and y = {a, b, c, d, e, f} determine which of the following relations is symmetric.

R_1: { } i.e. void relation

R_2: {(a, b)}

R_3: {(a, b), (b, a)(a, c)(c, a)(a, a)}

Q.4 If x = {a, b, c} and y = (a, b, c, d, e} identify the transitive relations among the following.

(a) $R_1 = \{\}$

(b) $R_2 = \{(a, a)\}$

(c) $R_3 = \{(a, a\}, (c, d)\}$

(d) $R_4 = \{(a, b), (b, c) (a, c), (a, a), (c, a)\}$

Q.5 Consider the relation R on the set N of natural numbers defined by x R y if and only if x divides y for all $x, y \in N$.

Q.6 Demonstrate that the relation R on the set Z of all integer numbers, defined by

$(x, y) \in R$ if and only if $x-y$ is divisible by n, is an equivalence relation on Z.

Q.7 Let R_1 be a relation on the set R of real numbers, defined as $(a, b) \in R_1$ if and only if $1+ab>0$ for all $a, b \in R$. Show that R_1 is reflexive and symmetric but not transitive.

Q.8 Consider the set A comprising the first ten natural numbers. Let R be a relation on A defined as $(x, y) \in R$ if and only if $x+2y=10$, expressed as $R = \{(x, y): x \in A, y \in A$ and $x + 2y = 10\}$. Represent R and R^{-1} both as sets of ordered pairs. Additionally, determine:

(i) Domains of R and R^{-1}

(ii) Range of R and R^{-1}

ANSWER KEY

1. (i) $R = \{(9, 1), (6, 2), (3, 3)\}$

 (ii) Domain of $R = \{9, 6, 3\}$

 (iii) Range of $R = \{1, 2, 3\}$

2. (b) Reflexive $R_1 : \{(x_1, x_1), (x_2, x_2), (x_3, x_3)$

3. R_1 exhibits symmetry as it contains no elements.

R_2 lacks symmetry due to the presence of $(b, a) \in R_2$,

While, R_3 is symmetric.

4. (a) R_1 qualifies as a transitive relation since it is a null relation.

(b) R_2 is transitive because all singleton relations are inherently transitive.

(c) R_3 demonstrates transitive behavior.

(d) R_4 also satisfies the conditions of transitivity.

5. we find that for any non zero integer a a R (a) and (–a) R a, but a ≠ – a.

8. Thus R = {(2, 4), (4, 3), (6, 2), (8, 1)}

R^{-1} = {(4, 2), (3, 4), (2, 6), (1, 8)}

Clearly, Dom (R) = {2, 4, 6, 8} = Range (R^{-1})

and Range (R) = {4, 3, 2, 1} = Dom (R^{-1})