RELATIONS AND FUNCTIONS

BINARY OPERATIONS

EXERCISE

Q.1. Determine whether the following operation define a binary operation on the given set or not:

(i) The operation '*' on N defined by a * b = a^b for all a, b ∈ N.

(ii) The operation 'O' on Z defined by a O b = a^b for all a, b ∈ Z.

(iii) The operation '*' on N defined by a * b = a + b – 2 for all a, b ∈ N

(iv) The operation '×6' on S = {1, 2, 3, 4, 5} defined by a ×6b = Remainder when ab is divided by 6.

(v) The operation '+6' on S = {0, 1, 2, 3, 4, 5} defined by a +6b $= \begin{cases} a+b, & \text{if } a+b<6 \\ a+b-6, & \text{if } a+b\geq 6 \end{cases}$

(vi) The operation '⊙' on N defined by a ⊙ b= $a^b + b^a$ for all a, b ∈ N

(vii) The operation '*' on Q defined by $a*b=\frac{(a-1)}{(b+1)}$) for all a, b ∈ Q

Q.2. Determine whether the given definition of * provides a binary operation. If * does not constitute a binary operation, provide a justification for this.

(i) On Z^+, defined * by a * b = a – b

(ii) On Z^+, define * by a*b = ab

(iii) On R, define * by a*b = ab^2

(iv) On Z^+ define * by a * b = |a − b|

(v) On Z^+ define * by a * b = a

(vi) On R, define * by $a * b = a + 4b^2$

Here, Z^+ denotes the set of all non-negative integers.

Q.3. Consider the binary operation * on the set I of integers, defined by $a * b = 2a + b - 3$ Determine the value of $3 * 4$.

Q.4. For the binary operation * defined on the set {1, 2, 3, 4, 5} by LCM of $a * b$=LCM of a and b, is it a binary operation? Provide justification for your answer.

Q.5. Let '*' be a binary operation on N defined by $a * b$ = l.c.m. (a, b) for all $a, b \in N$

(i) Find $2 * 4$, $3 * 5$, $1 * 6$.

(ii) Verify the commutativity and associativity of '*' on N.

Q.6. Identify the binary operations from the following list that are associative and commutative:

(i) * on N defined by $a * b = 1$ for all $a, b \in N$

(ii) * on Q defined by $a * b = \frac{(a+b)}{2}$ for all $a, b \in Q$

Q.7 Consider any set A with more than one element. Let '*' be a binary operation on A defined by $a * b = b$ for all $a, b \in A$. Determine if '*' is commutative or associative on A.

Q.8. Verify the commutativity and associativity of each of the following binary operations:

(i) '*' on Z defined by $a * b = a + b + ab$ for all $a, b \in Z$

(ii) '*' on N defined by $a * b = 2^{ab}$ for all $a, b \in N$

(iii) '*' on Q defined by $a * b = a - b$ for all $a, b \in Q$

(iv) '$\odot$' on Q defined by $a \odot b = a^2 + b^2$ for all $a, b \in Q$

(v) 'o' on Q defined by $a \text{ o } b = \frac{ab}{2}$ for all $a, b \in Q$

(vi) ‘*’ on Q defined by $a * b = ab^2$ for all $a, b \in Q$

(vii) ‘*’ on Q defined by $a * b = a + a b$ for all $a, b \in Q$

(viii) ‘*’ on R defined by $a * b = a + b - 7$ for all $a, b \in R$

(ix) ‘*’ on Q defined by $a * b = (a - b)^2$ for all $a, b \in Q$

(x) ‘*’ on Q defined by $a * b = ab + 1$ for all $a, b \in Q$

(xi) ‘*’ on N defined by $a * b = a^b$ for all $a, b \in N$

(xii) ‘*’ on Z defined by $a * b = a - b$ for all $a, b \in Z$

(xiii) ‘*’ on Q defined by $a * b = \frac{ab}{4}$ for all $a, b \in Q$

(xiv) ‘*’ on Z defined by $a * b = a + b - ab$ for all $a, b \in Z$

(xv) ‘*’ on Q defined by $a * b = g\,c\,d\,(a, b)$ for all $a, b \in Q$

Q.9. Show that the binary operation, denoted as o, defined by $a \text{ o } b = a + b - ab$ for all rational numbers in the set $Q - \{-1\}$ (excluding 1), is commutative on $Q - \{1\}$.

Q.10. Demonstrate that the binary operation *, defined on the set of integers Z by

$a * b = 3a + 7b$, is non-commutative.

Q.11. For the set Z of integers a binary operation * is defined by $a * b = ab + 1$ for all

$a, b \in Z$. Establish that * is not associative on Z.

ANSWER KEY

1. (i) Thus, * is a binary operation on N.

(ii) Thus, * is not a binary operation on Z

(iii) Thus, there exist $a = 1$ and $b = 1$ such that $a * b \notin N$

(iv) Thus, $\times_6$ is not a binary operation on S.

(v)Thus $\times_6$ is not a binary operation on S.

(vi) Thus, $\odot$ is a binary operation on N.

(vii)So, * is not a binary operation in Q.

2. (i) Thus, * is not a binary operation on Z^+.

(ii) Thus, * is a binary operation on R.

(iii) Thus, * is a binary operation on R.

(iv) Thus, * is a binary operation on Z^+.

(v) Thus, * is a binary operation on Z^+.

(vi) Thus, * is a binary operation on R.

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4. Thus, * is not a binary operation on {1, 2, 3, 4, 5}.

5. (i) 2 * 4 = l.c.m. (2, 4)= 4

3 * 5 = l.c.m. (3, 5)= 15

1 * 6 = l.c.m. (1, 6)= 6

(ii) Thus, * is associative on N.

6. (i) Thus, * is associative on N.

(ii) Thus, * is not associative on N

7. Thus, * is associative on A

8. (i) Thus, * is associative on Z.

(ii) Thus, * is not associative on N

(iii) Thus, * is not associative on Q

(iv) Thus, $\odot$ is not associative on Q.

(v) Thus, o is associative on Q.

(vi) Thus, * is not associative on Q.

(vii) Thus, * is not associative on Q

(viii) Thus, * is associative on R.

(ix) Thus, * is not associative on Q.

(x) Thus, * is not associative on Q.

(xi) Thus, * is not associative on N

(xii) Thus, * is not associative on Z

(xiii) Thus, * is associative on Q

(xiv) Thus, * is associative on Z

(xv) Thus, * is associative on N

9. Thus, o is commutative on Q – {-1}

10. Thus, * is not commutative on Z.