NUMBER SYSTEMS

SURDS AND RADICALS

EXERCISE

Q.1 Simplify and express the results in simplest form : $\frac{\sqrt{x^2-y^2}+x}{\sqrt{x^2+y^2}+y} \div \frac{\sqrt{x^2+y^2}-y}{x-\sqrt{x^2-y^2}}$.

Q.2 Simplify the following expressions :

(i) $(2\sqrt{2}+5\sqrt{3})+(\sqrt{2}-3\sqrt{3})$

(ii) $(3+\sqrt{3})(2+\sqrt{2})$

(iii) $(3+\sqrt{5})(3-\sqrt{5})$

Q.3 If $a=2+\sqrt{3}+\sqrt{5}$ and $b=3+\sqrt{3}-\sqrt{5}$, prove that $a^2 + b^2 - 4a - 6b - 3 = 0$.

Q.4 If $x=\sqrt{3}+2\sqrt{2}$ and $y=\sqrt{3}-2\sqrt{2}$, evaluate $x^4 + y^4 + 6x^2y^2$.

Q.5 If $x=1-\sqrt{2}$, find the value of

(i) $x+\frac{1}{x}$ (ii) $x-\frac{1}{x}$

(iii) $x^2+\frac{1}{x^2}$ (iv) $x^2-\frac{1}{x^2}$

(v) $x^4+\frac{1}{x^4}$ (vi) $x^4-\frac{1}{x^4}$

Q.6 For the identity $\frac{7+\sqrt{5}}{7-\sqrt{5}}-\frac{7-\sqrt{5}}{7+\sqrt{5}}=a+7\sqrt{5}b$, determine the rational numbers a and b.

Q.7 Simplify the following expressions :

(i) $\frac{1}{\sqrt{2}+1}+\frac{1}{\sqrt{3}+\sqrt{2}}+\frac{1}{\sqrt{4}+\sqrt{3}}+\frac{1}{\sqrt{5}+\sqrt{4}}$

(ii) $\frac{1}{\sqrt{2}+1}+\frac{1}{\sqrt{3}+2}+\frac{2}{\sqrt{5}+3}+\frac{2}{\sqrt{5}-3}$

Q.8 Simplify :

(i) $(9)^{\frac{9}{2}}$ (ii) $(9)^{-\frac{3}{2}}$ (iii) $(25)^{\frac{3}{2}}$

(iv) $(36)^{\frac{3}{2}}$ (v) $(49)^{-\frac{3}{2}}$ (vi) $(.0001)^{-\frac{3}{4}}$

Q.9 Simplify :

(i) $\left(\frac{243}{32}\right)^{-\frac{4}{5}}$ (ii) $\sqrt[3]{(343)^{-2}}$

Q.10 If $a^x = b$, $b^y = c$ and $c^z = a$, then prove that $xyz = 1$. Here a, b, c are positive real numbers and x, y, z are rational numbers.

ANSWER KEY

1. $\frac{y^2}{x^2}$

2. (i) $3\sqrt{2}+2\sqrt{3}$

   (ii) $6+3\sqrt{2}+2\sqrt{3}+\sqrt{6}$

   (iii) 4

4. 585

5. (i) $-2\sqrt{2}$, (ii) 2, (iii) 6,

   (iv) $-4\sqrt{2}$, (v) 34 (vi) $-24\sqrt{2}$

6. $a=0, b=\frac{1}{11}$

7. (i) $\sqrt{5}-1$,

   (ii) $1+\sqrt{2}-\sqrt{3}-\sqrt{5}$

8. (i) 27, (ii) $\frac{1}{27}$, (iii) 125,

   (iv) 216, (v) $\frac{1}{343}$, (vi) 1000

9. (i) $\frac{16}{81}$, (ii) $\frac{1}{49}$