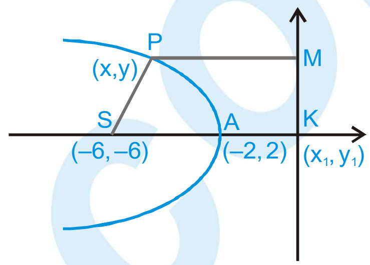
# SOLVED EXAMPLES

**Ex. 1** Find the equation of the parabola whose focus is (–6, –6) and vertex (–2, 2).

**Sol.** Let S(–6, –6) be the focus and A(–2, 2) is vertex of the parabola. On SA take a point $K(x_1, y_1)$ such that SA = AK. Draw KM perpendicular on SK. Then KM is the directrix of the parabola. Since A bisects SK,

$$\left(\frac{-6+x_1}{2}, \frac{-6+y_1}{2}\right) = (-2, 2)$$

$\Rightarrow$ $-6 + x_1 = -4$ and $-6 + y_1 = 4$ or $(x_1, y_1) = (2, 10)$




Hence the equation of the directrix KM is

$y - 10 = m(x - 2)$ ......... (i)

Also gradient of SK $= \frac{10-(-6)}{2-(-6)} = \frac{16}{8} = 2; \Rightarrow m = \frac{-1}{2}$

$y - 10 = \frac{-1}{2}(x-2)$ (from (i))

$\Rightarrow$ $x + 2y - 22 = 0$ is the directrix

Next, let PM be a perpendicular on the directrix KM from any point P(x, y) on the parabola. From SP = PM, the equation of the parabola is $\sqrt{\left\{(x+6)^2 + (y+6)^2\right\}} = \frac{|x+2y-22|}{\sqrt{(1^2+2^2)}}$

or $5(x^2 + y^2 + 12x + 12y + 72) = (x + 2y - 22)^2$

or $4x^2 + y^2 - 4xy + 104x + 148y - 124 = 0$

or $(2x - y)^2 + 104x + 148y - 124 = 0.$

**Ex. 2** Through the vertex O of a parabola $y^2 = 4x$ chords OP and OQ are drawn at right angles to one another. Show that for all position of P, PQ cuts the axis of the parabola at a fixed point.

**Sol.** The given parabola is $y^2 = 4x$ ...... (i)

Let $P \equiv (t_1^2, 2t_1)$, $Q \equiv (t_2^2, 2t_2)$

Slope of OP $= \frac{2t_1}{t_1^2} = \frac{2}{t_1}$ and slope of OQ $= \frac{2}{t_2}$

Since OP ⊥ OQ, $\frac{4}{t_1 t_2} = -1$ or $t_1 t_2 = -4$ ...... (ii)

The equation of PQ is $y(t_1 + t_2) = 2(x + t_1 t_2)$

$\Rightarrow$ $y\left(t_1 - \frac{4}{t_1}\right) = 2(x-4)$ [from (ii)]

$\Rightarrow$ $2(x-4) - y\left(t_1 - \frac{4}{t_1}\right) = 0$ $\Rightarrow$ $L_1 + \lambda L_2 = 0$

∴ variable line PQ passes through a fixed point which is point of intersection of $L_1 = 0$ & $L_2 = 0$

i.e. (4, 0) **Ans.**

**Ex. 3** Find the vertex, axis, focus, directrix, latusrectum of the parabola, also draw their rough sketches. $4y^2 + 12x - 20y + 67 = 0$

**Sol.** The given equation is

$4y^2 + 12x - 20y + 67 = 0 \quad \Rightarrow \quad y^2 + 3x - 5y + \frac{67}{4} = 0$

$\Rightarrow \quad y^2 - 5y = -3x - \frac{67}{4} \quad \Rightarrow \quad y^2 - 5y + \left(\frac{5}{2}\right)^2 = -3x - \frac{67}{4} + \left(\frac{5}{2}\right)^2$

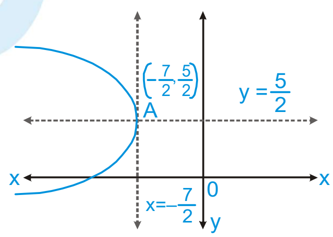
$\Rightarrow \quad \left(y - \frac{5}{2}\right)^2 = -3x - \frac{42}{4} \quad \Rightarrow \quad \left(y - \frac{5}{2}\right)^2 = -3\left(x + \frac{7}{2}\right)$ ....(i)

Let $\quad x = X - \frac{7}{2}, y = Y + \frac{5}{2}$ ....(ii)

Using these relations, equation (i) reduces to

$Y^2 = -3X$ ....(iii)

This is of the form $Y^2 = -4aX$. On comparing, we get $4a = 3 \Rightarrow a = 3/4$.




### Vertex

The coordinates of the vertex are (X = 0, Y = 0)

So, the coordinates of the vertex are

$\left(-\frac{7}{2}, \frac{5}{2}\right)$ [Putting X = 0, Y = 0 in (ii)]

### Axis

The equation of the axis of the parabola is Y = 0.

So, the equation of the axis is

$y = \frac{5}{2}$ [Putting Y = 0 in (ii)]

### Focus

The coordinates of the focus are (X = –a, Y = 0)

i.e. $\quad (X = -3/4, Y = 0)$.

So, the coordinates of the focus are

$(-17/4, 5/2)$ [Putting X = 3/4 in (ii)]

### Directrix

The equation of the directrix is X = a i.e. $X = \frac{3}{4}$.

So, the equation of the directrix is

$x = -\frac{11}{4}$ [Putting X = 3/4 in (ii)]

### Latusrectum

The length of the latusrectum of the given parabola is 4a = 3.

**Ex. 4** Find the locus of the point P from which tangents are drawn to the parabola $y^2 = 4ax$ having slopes $m_1$ and $m_2$ such that -

**(i)** $m_1^2 + m_2^2 = \lambda$ (constant) **(ii)** $\theta_1 - \theta_2 = \theta_0$ (constant)

where $\theta_1$ and $\theta_2$ are the inclinations of the tangents from positive x-axis.

**Sol.** Equation of tangent to $y^2 = 4ax$ is $y = mx + a/m$

Let it passes through P(h, k)

$\therefore$ $m^2h - mk + a = 0$

**(i)** $m_1^2 + m_2^2 = \lambda$

$(m_1 + m_2)^2 - 2m_1m_2 = \lambda$

$\frac{k^2}{h^2} - 2.\frac{a}{h} = \lambda$

$\therefore$ locus of P(h, k) is $y^2 - 2ax = \lambda x^2$

**(ii)** $\theta_1 - \theta_2 = \theta_0$

$\tan(\theta_1 - \theta_2) = \tan\theta_0$

$\frac{m_1 - m_2}{1 + m_1m_2} = \tan\theta_0$

$(m_1 + m_2)^2 - 4m_1m_2 = \tan^2\theta_0(1 + m_1m_2)^2$

$\frac{k^2}{h^2} - \frac{4a}{h} = \tan^2\theta_0\left(1 + \frac{a}{h}\right)^2$

$k^2 - 4ah = (h + a)^2 \tan^2\theta_0$

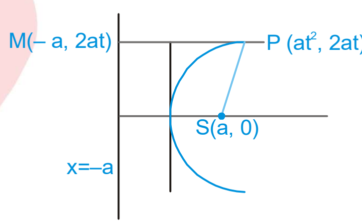
$\therefore$ locus of P(h, k) is $y^2 - 4ax = (x + a)^2\tan^2\theta_0$

**Ex. 5** Prove that focal distance of a point P($at^2$, 2at) on parabola $y^2 = 4ax$ (a > 0) is $a(1 + t^2)$.

**Sol.**

$\because$ PS = PM = $a + at^2$

PS = $a(1 + t^2)$.




**Ex. 6** If the endpoint $t_1$, $t_2$ of a chord satisfy the relation $t_1$ $t_2$ = k (const.) then prove that the chord always passes through a fixed point. Find the point?

**Sol.** Equation of chord joining ($at_1^2$, $2at_1$) and ($at_2^2$, $2at_2$) is

$y - 2at_1 = \frac{2}{t_1 + t_2}(x - at_1^2)$

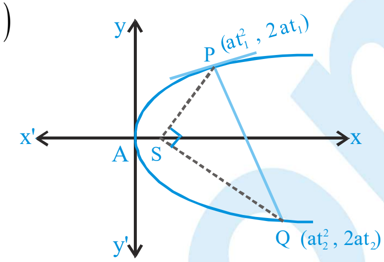
$(t_1 + t_2)y - 2at_1^2 - 2at_1t_2 = 2x - 2at_1^2$

$y = \frac{2}{t_1 + t_2}(x + ak)$ ($\because$ $t_1t_2 = k$)

$\therefore$ This line passes through a fixed point (– ak, 0).

**Ex. 7** Prove that the normal chord to a parabola $y^2 = 4ax$ at the point whose ordinate is equal to abscissa subtends a right angle at the focus.

**Sol.** Let the normal at $P\left(at_1^2, 2at_1\right)$ meet the curve at $Q\left(at_2^2, 2at_2\right)$




$\therefore$ PQ is a normal chord.

and $t_2 = -t_1 - \dfrac{2}{t_1}$ ......(i)

By given condition $2at_1 = at_1^2$

$\therefore$ $t_1 = 2$ from equation (i), $t_2 = -3$

then P(4a, 4a) and Q(9a, –6a)

but focus S(a, 0)

$\therefore$ Slope of SP $= \dfrac{4a-0}{4a-a} = \dfrac{4a}{3a} = \dfrac{4}{3}$

and Slope of SQ $= \dfrac{-6a-0}{9a-a} = \dfrac{-6a}{8a} = -\dfrac{3}{4}$

$\therefore$ Slope of SP $\times$ Slope of SQ $= \dfrac{4}{3} \times -\dfrac{3}{4} = -1$

$\therefore$ $\angle PSQ = \pi/2$

i.e. PQ subtends a right angle at the focus S.

**E.x 8** Three normals are drawn from the point (14, 7) to the curve $y^2 - 16x - 8y = 0$. Find the coordinates of the feet of the normals.

**Sol.** The given parabola is $y^2 - 16x - 8y = 0$ ......... (i)

Let the co-ordinates of the feet of the normal from (14, 7) be $P(\alpha, \beta)$. Now the equation of the tangent at $P(\alpha, \beta)$ to parabola (i) is

$y\beta - 8(x+\alpha) - 4(y+\beta) = 0$

or $(\beta - 4)y = 8x + 8a + 4\beta$ ......... (ii)

Its slope $= \dfrac{8}{\beta - 4}$

Equation of the normal to parabola (i) at $(\alpha, \beta)$ is $y - \beta = \dfrac{4-\beta}{8}(x - \alpha)$

It passes through (14, 7)

$\Rightarrow$ $7 - \beta = \dfrac{4-\beta}{8}(14 - \alpha)$ $\Rightarrow$ $\alpha = \dfrac{6\beta}{\beta - 4}$ .......... (iii)

Also $(\alpha, \beta)$ lies on parabola (i) i.e. $\beta^2 - 16\alpha - 8\beta = 0$ ......... (iv)

Putting the value of $\alpha$ from (iii) in (iv), we get $\beta^2 - \dfrac{96\beta}{\beta - 4} - 8\beta = 0$

$\Rightarrow$ $\beta^2(\beta - 4) - 96\beta - 8\beta(\beta - 4) = 0$ $\Rightarrow$ $\beta(\beta^2 - 4\beta - 96 - 8\beta + 32) = 0$

$\Rightarrow$ $\beta(\beta^2 - 12\beta - 64) = 0$ $\Rightarrow$ $\beta(\beta - 16)(\beta + 4) = 0$

$\Rightarrow$ $\beta = 0, 16, -4$

from (iii), $\alpha = 0$ when $\beta = 0$; $\alpha = 8$, when $\beta = 16$; $\alpha = 3$ when $\beta = -4$

Hence the feet of the normals are (0, 0), (8, 16) and (3, –4)

**Ex. 9** A tangent to the parabola $y^2 = 8x$ makes an angle of 45° with the straight line $y = 3x + 5$. Find its equation and its point of contact.

**Sol.** Slope of required tangent's are $m = \frac{3 \pm 1}{1 \mp 3}$

$$m_1 = -2, m_2 = \frac{1}{2}$$

✈ Equation of tangent of slope m to the parabola $y^2 = 4ax$ is $y = mx + \frac{a}{m}$.

∴ tangent's $y = -2x - 1$ at $\left(\frac{1}{2}, -2\right)$

$$y = \frac{1}{2}x + 4 \text{ at } (8, 8)$$

**Ex. 10** If the equation $m^2(x + 1) + m(y - 2) + 1 = 0$ represents a family of lines, where 'm' is parameter then find the equation of the curve to which these lines will always be tangents.

**Sol.** $m^2(x + 1) + m(y - 2) + 1 = 0$

The equation of the curve to which above lines will always be tangents can be obtained by equating its discriminant to zero.

∴ $(y - 2)^2 - 4(x + 1) = 0$

$y^2 - 4y + 4 - 4x - 4 = 0$

$y^2 = 4(x + y)$

**Ex. 11** The angle between the tangents drawn from a point $(-a, 2a)$ to $y^2 = 4ax$ is -

**Sol.** The given point $(-a, 2a)$ lies on the directrix $x = -a$ of the parabola $y^2 = 4ax$. Thus, the tangents are at right angle.

**Ex. 12** Find the equations to the common tangents of the parabolas $y^2 = 4ax$ and $x^2 = 4by$.

**Sol.** Equation of tangent to $y^2 = 4ax$ is

$$y = mx + \frac{a}{m} \qquad \text{........(i)}$$

Equation of tangent to $x^2 = 4by$ is

$$x = m_1 y + \frac{b}{m_1} \quad \Rightarrow \quad y = \frac{1}{m_1}x - \frac{b}{(m_1)^2} \qquad \text{........(ii)}$$

for common tangent, **(i)** & **(ii)** must represent same line.

$$\therefore \quad \frac{1}{m_1} = m \quad \& \quad \frac{a}{m} = -\frac{b}{m_1^2}$$

$$\Rightarrow \quad \frac{a}{m} = -bm^2 \quad \Rightarrow \quad m = \left(-\frac{a}{b}\right)^{1/3}$$

∴ equation of common tangent is $y = \left(-\frac{a}{b}\right)^{1/3} x + a\left(-\frac{b}{a}\right)^{1/3}$.

**Ex. 13** Find the locus of point whose chord of contact w.r.t. to the parabola $y^2 = 4bx$ is the tangent of the parabola $y^2 = 4ax$.

**Sol.** Equation of tangent to $y^2 = 4ax$ is $y = mx + \frac{a}{m}$ ......... **(i)**

Let it is chord of contact for parabola $y^2 = 4bx$ w.r.t. the point P(h, k)

$\therefore$ Equation of chord of contact is $yk = 2b(x + h)$

$$y = \frac{2b}{k}x + \frac{2bh}{k}$$ ......... **(ii)**

From **(i)** & **(ii)**

$$m = \frac{2b}{k}, \frac{a}{m} = \frac{2bh}{k} \Rightarrow a = \frac{4b^2h}{k^2}$$

locus of P is $y^2 = \frac{4b^2}{a}x$.

**Ex. 14** Find the locus of the middle point of a chord of a parabola $y^2 = 4ax$ which subtends a right angle at the vertex.

**Sol.** The equation of the chord of the parabola whose middle point is $(\alpha, \beta)$ is

$y\beta - 2a(x + \alpha) = \beta^2 - 4a\alpha$

$\Rightarrow$ $y\beta - 2ax = \beta^2 - 2a\alpha$

or $$\frac{y\beta - 2ax}{\beta^2 - 2a\alpha} = 1$$ ......... **(i)**

Now, the equation of the pair of the lines OP and OQ joining the origin O i.e. the vertex to the points of intersection P and Q of the chord with the parabola $y^2 = 4ax$ is obtained by making the equation homogeneous by means of **(i)**.

Thus the equation of lines OP and OQ is $y^2 = \frac{4ax(y\beta - 2ax)}{\beta^2 - 2a\alpha}$

$\Rightarrow$ $y^2(\beta^2 - 2a\alpha) - 4a\beta xy + 8a^2x^2 = 0$

If the lines OP and OQ are at right angles, then the coefficient of $x^2$ + the coefficient of $y^2 = 0$

Therefore, $\beta^2 - 2a\alpha + 8a^2 = 0$

$\Rightarrow$ $\beta^2 = 2a(\alpha - 4a)$

Hence the locus of $(\alpha, \beta)$ is $y^2 = 2a(x - 4a)$

**Ex. 15** Find the locus of the point N from which 3 normals are drawn to the parabola $y^2 = 4ax$ are such that

**(i)** Two of them are equally inclined to x-axis **(ii)** Two of them are perpendicular to each other

**Sol.** Equation of normal to $y^2 = 4ax$ is

$y = mx - 2am - am^3$

Let the normal passes through N(h, k)

$\therefore$ $k = mh - 2am - am^3$ $\Rightarrow$ $am^3 + (2a - h)m + k = 0$

For given value's of (h, k) it is cubic in 'm'.

Let $m_1$, $m_2$ & $m_3$ are root's of above equation

$\therefore$ $m_1 + m_2 + m_3 = 0$ ......**(i)**

$$m_1m_2 + m_2m_3 + m_3m_1 = \frac{2a-h}{a} \qquad ......(ii)$$

$$m_1m_2m_3 = -\frac{k}{a} \qquad ......(iii)$$

**(i)** If two normal are equally inclined to x-axis, then $m_1 + m_2 = 0$

$\therefore \quad m_3 = 0 \quad \Rightarrow \quad y = 0$

**(ii)** If two normal's are perpendicular

$\therefore \quad m_1\, m_2 = -1$

from (3) $\quad m_3 = \frac{k}{a} \qquad .....(iv)$

from (2) $\quad -1 + \frac{k}{a}(m_1 + m_2) = \frac{2a-h}{a} \qquad .....(v)$

from (1) $\quad m_1 + m_2 = -\frac{k}{a} \qquad .....(vi)$

from (5) & (6), we get

$$-1 - \frac{k^2}{a} = 2 - \frac{h}{a}$$

$$y^2 = a(x - 3a)$$

**Ex. 16** The common tangent of the parabola $y^2 = 8ax$ and the circle $x^2 + y^2 = 2a^2$ is -

**Sol.** Any tangent to parabola is $y = mx + \frac{2a}{m}$

Solving with the circle $\quad x^2 + (mx + \frac{2a}{m})^2 = 2a^2$

$\Rightarrow \quad x^2(1 + m^2) + 4ax + \frac{4a^2}{m^2} - 2a^2 = 0$

$B^2 - 4AC = 0$ gives $m = \pm 1$

Tangent $y = \pm x \pm 2a$

**Ex. 17** If the tangent to the parabola $y^2 = 4ax$ meets the axis in T and tangent at the vertex A in Y and the rectangle TAYG is completed, show that the locus of G is $y^2 + ax = 0$.

**Sol.** Let $P(at^2, 2at)$ be any point on the parabola $y^2 = 4ax$.

Then tangent at $P(at^2, 2at)$ is $ty = x + at^2$

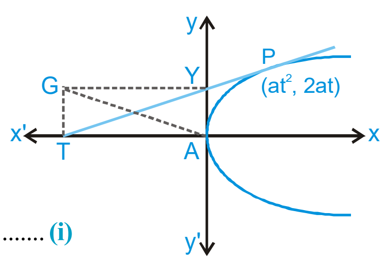
Since tangent meet the axis of parabola in T and tangent at the vertex in Y.

$\therefore$ Co-ordinates of T and Y are $(-at^2, 0)$ and $(0, at)$ respectively.

Let co-ordinates of G be $(x_1, y_1)$.

Since TAYG is rectangle.

$\therefore$ Mid-points of diagonals TY and GA is same




$\Rightarrow \quad \frac{x_1 + 0}{2} = \frac{-at^2 + 0}{2} \Rightarrow x_1 = -at^2 \qquad .......... (i)$

and $\frac{y_1+0}{2}=\frac{0+at}{2} \Rightarrow y_1 = at$ .......... **(ii)**

Eliminating t from **(i)** and **(ii)** then we get $x_1 = -a\left(\frac{y_1}{a}\right)^2$

or $y_1^2 = -ax_1$ or $y_1^2 + ax_1 = 0$

$\therefore$ The locus of $G(x_1, y_1)$ is $y^2 + ax = 0$

**Ex. 18** Find the focus of the point P from which tangents are drawn to parabola $y^2 = 4ax$ having slopes $m_1$, $m_2$ such that **(i)** $m_1 + m_2 = m_0$ (const) **(ii)** $\theta_1 + \theta_2 = \theta_0$ (const)

**Sol.** Equation of tangent to $y^2 = 4ax$, is

$$y = mx + \frac{a}{m}$$

Let it passes through P(h, k)

$\therefore$ $m^2h - mk + a = 0$

**(i)** $m_1 + m_2 = m_0 = \frac{k}{h}$ $\Rightarrow$ $y = m_0x$

**(ii)** $\tan\theta_0 = \frac{m_1+m_2}{1-m_1 m_2} = \frac{k/h}{1-a/h}$

$\Rightarrow$ $y = (x-a)\tan\theta_0$

**Ex. 19** If P(–3, 2) is one end of the focal chord PQ of the parabola $y^2 + 4x + 4y = 0$, then the slope of the normal at Q is -

**Sol.** The equation of the tangent at (–3, 2) to the parabola $y^2 + 4x + 4y = 0$ is

$$2y + 2(x-3) + 2(y+2) = 0$$

or $2x + 4y - 2 = 0$ $\Rightarrow$ $x + 2y - 1 = 0$

Since the tangent at one end of the focal chord is parallel to the normal at the other end, the slope of the normal at the other end of the focal chord is $-\frac{1}{2}$.

**Ex. 20** Prove that the two parabolas $y^2 = 4ax$ and $y^2 = 4c(x-b)$ cannot have common normal, other than the axis unless $b/(a-c) > 2$.

**Sol.** Given parabolas $y^2 = 4ax$ and $y^2 = 4c(x-b)$ have common normals. Then equation of normals in terms of slopes are $y = mx - 2am - am^3$ and $y = m(x-b) - 2cm - cm^3$ respectively then normals must be identical, compare the co-efficients

$$1 = \frac{2am + am^3}{mb + 2cm + cm^3}$$

$\Rightarrow$ $m[(c-a)m^2 + (b + 2c - 2a)] = 0, m \neq 0$ ($\because$ other than axis)

and $m^2 = \frac{2a-2c-b}{c-a}$, $m = \pm\sqrt{\frac{2(a-c)-b}{c-a}}$

or $\quad m = \pm\sqrt{\left(-2 - \dfrac{b}{c-a}\right)}$

$\therefore \quad -2 - \dfrac{b}{c-a} > 0$

or $\quad -2 + \dfrac{b}{a-c} > 0 \quad \Rightarrow \quad \dfrac{b}{a-c} > 2$

**Ex.21** If the line x – y – 1 = 0 intersect the parabola $y^2 = 8x$ at P & Q, then find the point of intersection of tangents at P & Q.

**Sol.** Let (h, k) be point of intersection of tangents then chord of contact is

$yk = 4(x + h)$

$4x - yk + 4h = 0 \quad$ .....(i)

But given is $x - y - 1 = 0$

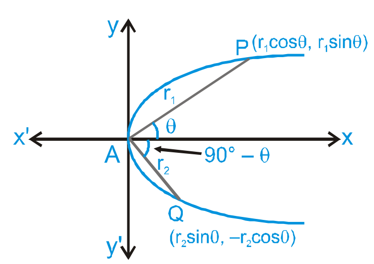
$\therefore \quad \dfrac{4}{1} = \dfrac{-k}{-1} = \dfrac{4h}{-1}$

$\Rightarrow \quad h = -1, k = 4$

$\therefore \quad$ point $\equiv (-1, 4)$

**Ex. 22** If $r_1, r_2$ be the length of the perpendicular chords of the parabola $y^2 = 4ax$ drawn through the vertex, then show that $(r_1 r_2)^{4/3} = 16a^2\left(r_1^{2/3} + r_2^{2/3}\right)$.

**Sol.** Since chord are perpendicular, therefore if one makes an angle θ then the other will make an angle (90° – θ) with x-axis




Let $\quad AP = r_1$ and $AQ = r_2$

If $\quad \angle PAX = \theta$

then $\quad \angle QAX = 90° - \theta$

$\therefore \quad$ Co-ordinates of P and Q are $(r_1 \cos\theta, r_1 \sin\theta)$

and $\quad (r_2 \sin\theta, -r_2 \cos\theta)$ respectively.

Since P and Q lies on $y^2 = 4ax$

$\therefore \quad r_1^2 \sin^2\theta = 4ar_1 \cos\theta$ and $r_2^2 \cos^2\theta = 4ar_2 \sin\theta$

$\Rightarrow \quad r_1 = \dfrac{4a\cos\theta}{\sin^2\theta}$ and $r_2 = \dfrac{4a\sin\theta}{\cos^2\theta}$

$\therefore \quad (r_1 r_2)^{4/3} = \left(\dfrac{4a\cos\theta}{\sin^2\theta}.\dfrac{4a\sin\theta}{\cos^2\theta}\right)^{4/3} = \left(\dfrac{16a^2}{\sin\theta\cos\theta}\right)^{4/3} \quad$ ....... (i)

and $\quad 16a^2.\left(r_1^{2/3} + r_2^{2/3}\right) = 16a^2\left\{\left(\dfrac{4a\cos\theta}{\sin^2\theta}\right)^{2/3} + \left(\dfrac{4a\sin\theta}{\cos^2\theta}\right)^{2/3}\right\}$

$$=16a^2.(4a)^{2/3}\left\{\frac{(\cos\theta)^{2/3}}{(\sin\theta)^{4/3}}+\frac{(\sin\theta)^{2/3}}{(\cos\theta)^{4/3}}\right\}=16a^2.(4a)^{2/3}\left\{\frac{\cos^2\theta+\sin^2\theta}{(\sin\theta)^{4/3}(\cos\theta)^{4/3}}\right\}$$

$$=\frac{16a^2.(4a)^{2/3}}{(\sin\theta\cos\theta)^{4/3}}=\left(\frac{16a^2}{\cos\theta\cos\theta}\right)^{4/3}=(r_1r_2)^{4/3} \qquad \{\text{from } \textbf{(i)}\}$$

**Ex. 23** The area of the triangle formed by three points on a parabola is twice the area of the triangle formed by the tangents at these points.

**Sol.** Let the three points on the parabola be

$(at_1^2, 2at_1), (at_2^2, 2at_2)$ and $(at_3^2, 2at_3)$

The area of the triangle formed by these points

$$\Delta_1=\frac{1}{2}[at_1^2(2at_2-2at_3)+at_2^2(2at_3-2at_1)+at_2^2(2a_1-2at_2)]$$

$$=-a^2(t_2-t_3)(t_3-t_1)(t_1-t_2).$$

The points of intersection of the tangents at these points are

$(at_2t_3, a(t_2+t_3)), (at_3t_1, a(t_3+t_1))$ and $(at_1t_2, a(t_1+t_2))$

The area of the triangle formed by these three points

$$\Delta_2=\frac{1}{2}\{at_2t_3(at_3-at_2)+at_3t_1(at_1-at_3)+at_1t_2(at_2-at_1)\}$$

$$=\frac{1}{2}a^2(t_2-t_3)(t_3-t_1)(t_1-t_2)$$

Hence $\Delta_1=2\Delta_2$

**Ex. 24** Find the locus of point whose chord of contact w.r.t to the parabola $y^2 = 4bx$ is the tangents of the parabola $y^2 = 4ax$.

**Sol.** Equation of tangent to $y^2 = 4ax$ is $y = mx + \frac{a}{m}$ ......**(i)**

Let it is chord of contact for parabola $y^2 = 4bx$ w.r.t. the point P(h, k)

$\therefore$ Equation of chord of contact is $yk = 2b(x + h)$

$$y=\frac{2b}{k}x+\frac{2bh}{k} \qquad .....\textbf{(ii)}$$

From **(i)** & **(ii)**

$$m=\frac{2b}{k},\ \frac{a}{m}=\frac{2bh}{k} \quad\Rightarrow\quad a=\frac{4b^2h}{k^2}$$

locus of P is $y^2=\frac{4b^2}{a}x$ .

**Ex. 25** Prove that the orthocentre of any triangle formed by three tangents to a parabola lies on the directrix.

**Sol.** Let the equations of the three tangents be

$t_1 y = x + at_1^2$ ......**(i)**

$t_2 y = x + at_2^2$ ......**(ii)**

and $t_3 y = x + at_3^2$ ......**(iii)**

The point of intersection of **(ii)** and **(iii)** is found, by solving them, to be $(at_2t_3, a(t_2 + t_3))$

The equation of the straight line through this point & perpendicular to **(i)** is

$y - a(t_2 + t_3) = -t_1(x - at_2t_3)$

i.e. $y + t_1x = a(t_2 + t_3 + t_1t_2t_3)$ ......**(iv)**

Similarly, the equation of the straight line through the point of intersection of **(iii)** and **(i)** & perpendicular to **(ii)** is

$y + t_2x = a(t_3 + t_1 + t_1t_2t_3)$ ......**(v)**

and the equation of the straight line through the point of intersection of **(i)** and **(ii)** & perpendicular to **(iii)** is

$y + t_1x = a(t_1 + t_2 + t_1t_2t_3)$ ......**(vi)**

The point which is common to the straight lines **(iv)**, **(v)** and **(vi)**

i.e. the orthocentre of the triangle, is easily seen to be the point whose coordinates are

$x = -a,\ y = a(t_1 + t_2 + t_3 + t_1t_2t_3)$

and this point lies on the directrix.

# Exercise # 1

**[Single Correct Choice Type Questions]**

**1.** Directrix of a parabola is $x + y = 2$. If it's focus is origin, then latus rectum of the parabola is equal to -

**(A)** $\sqrt{2}$ units **(B)** 2 units **(C)** $2\sqrt{2}$ units **(D)** 4 units

**2.** The equation of the parabola whose vertex and focus lie on the axis of x at distances a and $a_1$ from the origin, respectively, is

**(A)** $y^2 = 4(a_1 - a)x$ **(B)** $y^2 = 4(a_1 - a)(x - a)$ **(C)** $y^2 = 4(a_1 - a)(x - a_1)$ **(D)** none of these

**3.** Maximum number of common chords of a parabola and a circle can be equal to

**(A)** 2 **(B)** 4 **(C)** 6 **(D)** 8

**4.** Which one of the following equations represent parametric equation to a parabolic curve ?

**(A)** $x = 3\cos t$ ; $y = 4\sin t$ **(B)** $x^2 - 2 = 2\cos t$ ; $y = 4\cos^2\frac{t}{2}$

**(C)** $\sqrt{x} = \tan t$ ; $\sqrt{y} = \sec t$ **(D)** $x = \sqrt{1-\sin t}$ ; $y = \sin\frac{t}{2} + \cos\frac{t}{2}$

**5.** If $(t^2, 2t)$ is one end of a focal chord of the parabola $y^2 = 4x$ then the length of the focal chord will be-

**(A)** $\left(t+\frac{1}{t}\right)^2$ **(B)** $\left(t+\frac{1}{t}\right)\sqrt{\left(t^2+\frac{1}{t^2}\right)}$ **(C)** $\left(t-\frac{1}{t}\right)\sqrt{\left(t^2+\frac{1}{t^2}\right)}$ **(D)** none

**6.** Through the focus of the parabola $y^2 = 2px$ $(p > 0)$ a line is drawn which intersects the curve at $A(x_1, y_1)$ and $B(x_2, y_2)$. The ratio $\frac{y_1y_2}{x_1x_2}$ equals

**(A)** 2 **(B)** – 1 **(C)** – 4 **(D)** some function of p

**7.** If the lines $(y - b) = m_1(x + a)$ and $(y - b) = m_2(x + a)$ are the tangents to the parabola $y^2 = 4ax$, then

**(A)** $m_1 + m_2 = 0$ **(B)** $m_1m_2 = 1$ **(C)** $m_1m_2 = -1$ **(D)** $m_1 + m_2 = 1$

**8.** The locus of the point $(\sqrt{3h}, \sqrt{3k+2})$ if it lies on the line $x - y - 1 = 0$ is

**(A)** a straight line **(B)** a circle **(C)** a parabola **(D)** none of these

**9.** From the focus of the parabola $y^2 = 8x$ as centre, a circle is described so that a common chord of the curves is equidistant from the vertex and focus of the parabola. The equation of the circle is -

**(A)** $(x - 2)^2 + y^2 = 3$ **(B)** $(x - 2)^2 + y^2 = 9$ **(C)** $(x + 2)^2 + y^2 = 9$ **(D)** $x^2 + y^2 - 4x = 0$

**10.** The ratio in which the line segment joining the points (4, –6) and (3, 1) is divided by the parabola $y^2 = 4x$ is

**(A)** $\frac{-20 \pm \sqrt{155}}{11} : 1$ **(B)** $\frac{-2 \pm 2\sqrt{155}}{11} : 2$

**(C)** $-20 \pm 2\sqrt{155} : 11$ **(D)** $-20 \pm \sqrt{155} : 11$

**11.** Length of the normal chord of the parabola, $y^2 = 4x$, which makes an angle of $\frac{\pi}{4}$ with the axis of x is:

**(A)** 8 **(B)** $8\sqrt{2}$ **(C)** 4 **(D)** $4\sqrt{2}$

12. The triangle PQR of area 'A' is inscribed in the parabola $y^2 = 4ax$ such that the vertex P lies at the vertex of the parabola and the base QR is a focal chord. The modulus of the difference of the ordinates of the points Q and R is -

(A) $\frac{A}{2a}$ (B) $\frac{A}{a}$ (C) $\frac{2A}{a}$ (D) $\frac{4A}{a}$

13. A set of parallel chords of the parabola $y^2 = 4ax$ have their midpoints on

(A) any stright line through the vertex (B) any straight line through the focus
(C) a straight line parallel to the axes (D) another parabola

14. Let A and B be two points on a parabola $y^2 = x$ with vertex V such that VA is perpendicular to VB and θ is the angle between the chord VA and the axis of the parabola. The value of $\frac{|VA|}{|VB|}$ is

(A) $\tan\theta$ (B) $\tan^3\theta$ (C) $\cot^2\theta$ (D) $\cot^3\theta$

15. The locus of a point such that two tangents drawn from it to the parabola $y^2 = 4ax$ are such that the slope of one is double the other is -

(A) $y^2 = \frac{9}{2}ax$ (B) $y^2 = \frac{9}{4}ax$ (C) $y^2 = 9ax$ (D) $x^2 = 4ay$

16. Two parabolas $y^2 = 4a(x - l_1)$ and $x^2 = 4a(y - l_2)$ always touch one another , the quantities $l_1$ and $l_2$ are both variable. Locus of their point of contact has the equation -

(A) $xy = a^2$ (B) $xy = 2a^2$ (C) $xy = 4a^2$ (D) none

17. If $y_1$, $y_2$ and $y_3$ are the ordinates of the vertices of a triangle inscribed in the parabola $y^2 = 4ax$, then its area is

(A) $\frac{1}{2a}(y_1 - y_2)(y_2 - y_3)(y_3 - y_1)$ (B) $\frac{1}{4a}(y_1 - y_2)(y_2 - y_3)(y_3 - y_1)$
(C) $\frac{1}{8a}(y_1 - y_2)(y_2 - y_3)(y_3 - y_1)$ (D) none of these

18. Tangents are drawn from the point (–1, 2) on the parabola $y^2 = 4x$ . The length , these tangents will intercept on the line $x = 2$ :

(A) 6 (B) $6\sqrt{2}$ (C) $2\sqrt{6}$ (D) none of these

19. C is the centre of the circle with centre (0, 1) and radius unity. P is the parabola $y = ax^2$. The set of values of 'a' for which they meet at a point other than the origin, is

(A) $a > 0$ (B) $a \in \left(0, \frac{1}{2}\right)$ (C) $\left(\frac{1}{4}, \frac{1}{2}\right)$ (D) $\left(\frac{1}{2}, \infty\right)$

20. The locus of the vertex of the family of parabolas $y = \frac{a^3x^2}{3} + \frac{a^2x}{2} - 2a$ is

(A) $xy = 105/64$ (B) $xy = 3/4$ (C) $xy = 35/16$ (D) $xy = 64/105$

21. Locus of the point of intersection of the perpendiculars tangent of the curve $y^2 + 4y - 6x - 2 = 0$ is :

(A) $2x - 1 = 0$ (B) $2x + 3 = 0$ (C) $2y + 3 = 0$ (D) $2x + 5 = 0$

**22.** An equilateral triangle SAB is inscribed in the parabola $y^2 = 4ax$ having its focus at S. If chord AB lies towards the left of S, then the side length of this triangle is

**(A)** $2a(2-\sqrt{3})$ **(B)** $4a(2-\sqrt{3})$ **(C)** $a(2-\sqrt{3})$ **(D)** $8a(2-\sqrt{3})$

**23.** If a normal to a parabola $y^2 = 4ax$ make an angle $\phi$ with its axis, then it will cut the curve again at an angle

**(A)** $\tan^{-1}(2\tan\phi)$ **(B)** $\tan^{-1}\left(\frac{1}{2}\tan\phi\right)$ **(C)** $\cot^{-1}\left(\frac{1}{2}\tan\phi\right)$ **(D)** none

**24.** The straight lines joining any point P on the parabola $y^2 = 4ax$ to the vertex and perpendicular from the focus to the tangent at P intersect at R. Then the equation of the locus of R is

**(A)** $x^2 + 2y^2 - ax = 0$ **(B)** $2x^2 + y^2 - 2ax = 0$ **(C)** $2x^2 + 2y^2 - ay = 0$ **(D)** $2x^2 + y^2 - 2ay = 0$

**25.** The equation of a straight line passing through the point (3, 6) and cutting the curve $y = \sqrt{x}$ orthogonally is

**(A)** $4x + y - 18 = 0$ **(B)** $x + y - 9 = 0$ **(C)** $4x - y - 6 = 0$ **(D)** none

**26.** From the point (4, 6) a pair of tangent lines are drawn to the parabola, $y^2 = 8x$. The area of the triangle formed by these pair of tangent lines & the chord of contact of the point (4, 6) is

**(A)** 2 **(B)** 4 **(C)** 8 **(D)** none

**27.** A line of slope $\lambda(0 < \lambda < 1)$ touches the parabola $y + 3x^2 = 0$ at P. If S is the focus and M is the foot of the perpendicular of directrix from P, then tan ∠MPS equals

**(A)** $2\lambda$ **(B)** $\frac{2\lambda}{-1+\lambda^2}$ **(C)** $\frac{1-\lambda^2}{1+\lambda^2}$ **(D)** none of these

**28.** PQ is a normal chord of the parabola $y^2 = 4ax$ at P, A being the vertex of the parabola. Through P a line is drawn parallel to AQ meeting the x–axis in R. Then the length of AR is :

**(A)** equal to the length of the latus rectum

**(B)** equal to the focal distance of the point P

**(C)** equal to twice the focal distance of the point P

**(D)** equal to the distance of the point P from the directrix.

**29.** If a and c are the length of segments of any focal chord of the parabola $y^2 = 2bx$, $(b > 0)$, then the roots of the equation $ax^2 + bx + c = 0$ are

**(A)** real and distinct **(B)** real and equal

**(C)** imaginary **(D)** none of these

**30.** If the tangent at the point P $(x_1, y_1)$ to the parabola $y^2 = 4ax$ meets the parabola $y^2 = 4a(x + b)$ at Q & R, then the mid point of QR is -

**(A)** $(x_1 + b, y_1 + b)$ **(B)** $(x_1 - b, y_1 - b)$ **(C)** $(x_1, y_1)$ **(D)** $(x_1 + b, y_1)$

# Exercise # 2 Part # I [Multiple Correct Choice Type Questions]

**1.** The straight line $y + x = 1$ touches the parabola :

**(A)** $x^2 + 4y = 0$ **(B)** $x^2 - x + y = 0$ **(C)** $4x^2 - 3x + y = 0$ **(D)** $x^2 - 2x + 2y = 0$

**2.** The equations of the directrix of the parabola with vertex at the origin and having the axis along the a-axis and a common tangent of slope 2 with the circle $x^2 + y^2 = 5$ is

**(A)** $x = 10$ **(B)** $x = 20$ **(C)** $x = -10$ **(D)** $x = -20$

**3.** Tangent is drawn at any point $(x_1, y_1)$ other than the vertex on the parabola $y^2 = 4ax$. If tangents are drawn from any point on this tangent to the circle $x^2 + y^2 = a^2$ such that all the chords of contact pass through a fixed point $(x_2, y_2)$, then

**(A)** $x_1, a, x_2$ are in GP **(B)** $\frac{y_1}{2}, a, y_2$ are in G.P. **(C)** $-4, \frac{y_1}{y_2}, \frac{x_1}{x_2}$ are in GP **(D)** $x_1x_2 + y_1y_2 = a^2$

**4.** If the focus of the parabola $x^2 - ky + 3 = 0$ is (0, 2), then a values of k is (are)

**(A)** 4 **(B)** 6 **(C)** 3 **(D)** 2

**5.** Let P, Q and R are three co-normal points on the parabola $y^2 = 4ax$. Then the correct statement(s) is/are

**(A)** algebraic sum of the slopes of the normals at P, Q and R vanishes

**(B)** algebraic sum of the ordinates of the points P, Q and R vanishes

**(C)** centroid of the triangle PQR lies on the axis of the parabola

**(D)** circle circumscribing the triangle PQR passes through the vertex of the parabola

**6.** Let P be a point whose coordinates differ by unity and the point does not lie on any of the axes of reference. If the parabola $y^2 = 4x + 1$ passes through P, then the ordinate of P may be

**(A)** 3 **(B)** –1 **(C)** 5 **(D)** 1

**7.** If $y = 2$ is the directrix and (0, 1) is the vertex of the parabola $x^2 + \lambda y + \mu = 0$, then

**(A)** $\lambda = 4$ **(B)** $\mu = 8$ **(C)** $\lambda = -8$ **(D)** $\mu = 4$

**8.** The extremities of latus rectum of a parabola are (1, 1) and (1, –1). Then the equation of the parabola can be

**(A)** $y^2 = 2x - 1$ **(B)** $y^2 = 1 - 2x$ **(C)** $y^2 = 2x - 3$ **(D)** $y^2 = 2x - 4$

**9.** The tangent and normal at P (t), for all real positive t, to the parabola $y^2 = 4ax$ meet the axis of the parabola in T and G respectively, then the angle at which the tangent at P to the parabola is inclined to the tangent at P to the circle through the points P, T and G is -

**(A)** $\cot^{-1}t$ **(B)** $\cot^{-1}t^2$ **(C)** $\tan^{-1}t$ **(D)** $\sin^{-1}\left(\frac{t}{\sqrt{1+t^2}}\right)$

**10.** Variable chords of the parabola $y^2 = 4ax$ subtend a right angle at the vertex. Then :

**(A)** locus of the feet of the perpendiculars from the vertex on these chords is a circle

**(B)** locus of the middle points of the chords is a parabola

**(C)** variable chords passes through a fixed point on the axis of the parabola

**(D)** none of these

**11.** The parabola $y^2 = 4x$ and the circle having its center at (6, 5) intersect at right angle. The possible point of intersection of these curves can be

**(A)** (9, 6) **(B)** $(2, \sqrt{8})$ **(C)** (4, 4) **(D)** $(3, 2\sqrt{3})$

**12.** A normal drawn to the parabola $y^2 = 4ax$ meets the curves again at Q such that the angle subtended by PQ at the vertex is 90°. Then the coordinates of P can be

**(A)** $(8a, 4\sqrt{2}\,a)$ **(B)** (8a, 4a) **(C)** $(2a, -2\sqrt{2}\,a)$ **(D)** $(2a, 2\sqrt{2}\,a)$

**13.** A quadrilateral is inscribed in a parabola. Then,

**(A)** the quadrilateral may be cyclic

**(B)** diagonals of the quadrilateral may be equal

**(C)** all possible pairs of adjacent sides may be perpendicular

**(D)** none of these

**14.** Through a point P (– 2, 0), tangents PQ and PR are drawn to the parabola $y^2 = 8x$. Two circles each passing through the focus of the parabola and one touching at Q and other at R are drawn. Which of the following point(s) with respect to the triangle PQR lie(s) on the common chord of the two circles?

**(A)** centroid **(B)** orthocentre **(C)** incentre **(D)** circumcentre

**15.** The locus of the midpoint of the focal distance of a veriable point moving on the parabola $y^2 = 4ax$ is a parabola whose

**(A)** latus rectus is half the latus rectum of the original parabola

**(B)** vertex is (a/2, 0)

**(C)** directrix is y-axis

**(D)** focus has coordinates (a, 0)

**16.** A square has one vertex at the vertex of the parabola $y^2 = 4ax$ and the diagonal through the vertex lies along the axes of the parabola. If the ends of the other diagonal lie on the parabola, the coordinates of the vertices of the square are

**(A)** (4a, 4a) **(B)** (4a, – 4a) **(C)** (0, 0) **(D)** (8a, 0)

**17.** Two parabolas have the same focus. If their directrices are the x-axis & the y-axis respectively, then the slope of their common chord is -

**(A)** 1 **(B)** –1 **(C)** 4/3 **(D)** 3/4

**18.** If two distinct chords of a parabola $y^2 = 4ax$ passing through (a, 2a) are bisected on the line x + y = 1, then the length of the latus rectum can be

**(A)** 2 **(B)** 1 **(C)** 4 **(D)** 3

**19.** Let PQ be a chord of the parabola $y^2 = 4x$. A circle drawn with PQ as a diameter passes through the vertex V of the parabola. If ar(ΔPVQ) = 20 unit$^2$, then the coordinates of P are

**(A)** (16, 8) **(B)** (16, – 8) **(C)** (–16, 8) **(D)** (–16, –8)

**20.** The equation of the line that touches the curves y = x|x| and $x^2 + (y - 2)^2 = 4$, where x ≠ 0, is

**(A)** $y = 4\sqrt{5}\,x + 20$ **(B)** $y = 4\sqrt{3}\,x - 12$ **(C)** y = 0 **(D)** $y = -4\sqrt{5}\,x - 20$

## Part # II [Assertion & Reason Type Questions]

These questions contains, Statement I (assertion) and Statement II (reason).

**(A)** Statement-I is true, Statement-II is true ; Statement-II is correct explanation for Statement-I.

**(B)** Statement-I is true, Statement-II is true ; Statement-II is NOT a correct explanation for statement-I.

**(C)** Statement-I is true, Statement-II is false.

**(D)** Statement-I is false, Statement-II is true.

**1.** **Statement-I :** The line $y = x + 2a$ touches the parabola $y^2 = 4a(x + a)$.

**Statement-II :** The line $y = mx + am + a/m$ touches $y^2 = 4a(x + a)$ for all real values of m.

**2.** **Statement-I :** The values of a for which the point $(a, a^2)$ lies inside the triangle formed by the lines $x = 0$, $x + y = 2$, and $3y = x$ is (0, 1).

**Statement-II :** The parabola $y = x^2$ meets the line $x + y = 2$ at (1, 1).

**3.** **Statement-I :** If $P_1Q_1$ and $P_2Q_2$ are two focal chords of the parabola $y^2 = 4ax$, then the locus of point of intersection of chords $P_1P_2$ and $Q_1Q_2$ is directrix of the parabola. Here $P_1P_2$ and $Q_1Q_2$ are not parallel.

**Statement-II :** The locus of point of intersection of perpendicular tangents of parabola is directrix of parabola.

**4.** **Statement-I :** $(5x - 5)^2 + (5y + 10)^2 = (3x + 4y + 5)^2$ is a parabola.

**Statement-II :** If the distinct of the point from a given line and from a given point (not lying on the given line) is equal, then the locus of the variable point is a parabola.

**5.** **Statement-I :** If the endpoints of two normal chords AB and CD (normal at A and C) of a parabola $y^2 = 4ax$ are concyclic, then the tangents at A and C will intersect on the axis of the parabola.

**Statement-II :** If four ponits on the parabola $y^2 = 4ax$ are concyclic, then the sum of their ordinates is zero.

**6.** Consider a curve C : $y^2 - 8x - 2y - 15 = 0$ in which two tangents $T_1$ and $T_2$ are drawn from P(–4, 1).

**Statement-I :** $T_1$ and $T_2$ are mutually perpendicular tangents.

**Statement-II :** Point P lies on the axis of curve C.

# Exercise # 3 Part # I [Matrix Match Type Questions]

Following question contains statements given in two columns, which have to be matched. The statements in Column-I are labelled as A, B, C and D while the statements in Column-II are labelled as p, q, r and s. Any given statement in Column-I can have correct matching with ONE OR MORE statement(s) in Column-II.

**1.** Consider the parabola $(x-1)^2 + (y-2)^2 = \dfrac{(12x-5y+3)^2}{169}$

| Column-I | | Column-II | |
|---|---|---|---|
| **(A)** | Locus of point of intersection of perpendicular tangent | **(p)** | $12x - 5y - 2 = 0$ |
| **(B)** | Locus of foot of perpendicular from focus upon any tangent | **(q)** | $5x + 12y - 29 = 0$ |
| **(C)** | Line along which minimum length of focal chord occurs | **(r)** | $12x - 5y + 3 = 0$ |
| **(D)** | Line about which parabola is symmetrical | **(s)** | $24x - 10y + 1 = 0$ |

**2.**

| Column-I | | Column-II | |
|---|---|---|---|
| **(A)** | The normal chord at a point t on the parabola $y^2 = 4x$ subtends a right angle at the vertex, then $t^2$ is | **(p)** | 4 |
| **(B)** | The area of the triangle inscribed in the curve $y^2 = 4x$. If the parameter of vertices are 1, 2 and 4 is | **(q)** | 2 |
| **(C)** | The number of distinct normal possible from $\left(\frac{11}{4}, \frac{1}{4}\right)$ to the parabola $y^2 = 4x$ is | **(r)** | 3 |
| **(D)** | The normal at (a, 2a) on $y^2 = 4ax$ meets the curve again at $(at^2, 2at)$, then the value of $\lvert t-1 \rvert$ is | **(s)** | 6 |

**3.** Consider the parabola $y^2 = 12x$

| Column-I | | Column-II | |
|---|---|---|---|
| **(A)** | Tangent and normal at the extremities of the latus rectum intersect the x axis at T and G respectively. The coordinates of the middle point of T and G are | **(p)** | (0, 0) |
| **(B)** | Variable chords of the parabola passing through a fixed point K on the axis, such that sum of the squares of the reciprocals of the two parts of the chords through K, is a constant. The coordinate of the point K are | **(q)** | (3, 0) |
| | | **(r)** | (6, 0) |
| **(C)** | All variable chords of the parabola subtending a right angle at the origin are concurrent at the point | | |
| **(D)** | AB and CD are the chords of the parabola which intersect at a point E on the axis. The radical axis of the two circles described on AB and CD as diameter always passes through | **(s)** | (12, 0) |

**4.** **Column-I** **Column-II**

| | Column-I | | Column-II |
|---|---|---|---|
| **(A)** | Area of a triangle formed by the tangents drawn from a point (–2, 2) to the parabola $y^2 = 4(x + y)$ and their corresponding chord of contact is | **(p)** | 8 |
| **(B)** | Length of the latus rectum of the conic $25\{(x - 2)^2 + (y - 3)^2\} = (3x + 4y - 6)^2$ is | **(q)** | $4\sqrt{3}$ |
| **(C)** | If focal distance of a point on the parabola $y = x^2 - 4$ is 25/4 and points are of the form $(\pm\sqrt{a}, b)$ then value of a + b is | **(r)** | 4 |
| **(D)** | Length of side of an equilateral triangle inscribed in a parabola $y^2 - 2x - 2y - 3 = 0$ whose one angular point is vertex of the parabola, is | **(s)** | 24/5 |

**5.** **Column-I** **Column-II**

| | Column-I | | Column-II |
|---|---|---|---|
| **(A)** | Tangents are dranw from point (2, 3) to the parabola $y^2 = 4x$. Then the points of contact are | **(p)** | (9, –6) |
| **(B)** | From a point P on the circle $x^2 + y^2 = 5$, the equation of chord of contact to the parabola $y^2 = 4x$ is $y = 2(x - 2)$. Then the coordinate of point P will be | **(q)** | (1, 2) |
| **(C)** | P(4, –4) and Q are points on the parabola $y^2 = 4x$ such that the area of ΔPOQ is 6 sq. units, where O is the vertex. Then the coordinates of Q may be | **(r)** | (–2, 1) |
| **(D)** | The common chord of the circle $x^2 + y^2 = 5$ and the parabola $6y = 5x^2 + 7x$ will pass through point(s). | **(s)** | (4, 4) |

## Part # II [Comprehension Type Questions]

### Comprehension # 1

Tangent to the parabola $y = x^2 + ax + 1$ at the point of intersection of the y-axis also touches the circle $x^2 + y^2 = r^2$. Also, no point of the parabola is below the x-axis.

**1.** The radius of circle when a attains its maximum value is

**(A)** $1/\sqrt{10}$ **(B)** $1/\sqrt{5}$ **(C)** 1 **(D)** $\sqrt{5}$

**2.** The slope of the tangent when the radius of the circle is maximum is

**(A)** –1 **(B)** 1 **(C)** 0 **(D)** 2

**3.** The minimum area bounded by the tangent and the coordinates axes is

**(A)** 1 **(B)** 1/3 **(C)** 1/2 **(D)** 1/4

## Comprehension # 2

Observe the following facts for a parabola :

(i) Axis of the parabola is the only line which can be the perpendicular bisector of the two chords of the parabola.

(ii) If AB and CD are two parallel chords of the parabola and the normals at A and B intersect at P and the normals at C and D intersect at Q, then PQ is a normal to the parabola.

Let a parabola is passing through (0, 1), (–1, 3), (3, 3) & (2, 1)

**On the basis of above information, answer the following questions :**

**1.** The vertex of the parabola is -

**(A)** $\left(1, \frac{1}{3}\right)$ **(B)** $\left(\frac{1}{3}, 1\right)$ **(C)** (1, 3) **(D)** (3, 1)

**2.** The directrix of the parabola is -

**(A)** $y - \frac{1}{24} = 0$ **(B)** $y + \frac{1}{2} = 0$ **(C)** $y + \frac{1}{24} = 0$ **(D)** $y + \frac{1}{12} = 0$

**3.** For the parabola $y^2 = 4x$, AB and CD are any two parallel chords having slope 1. $C_1$ is a circle passing through O, A and B and $C_2$ is a circle passing through O, C and D, where O is origin. $C_1$ and $C_2$ intersect at -

**(A)** (4, –4) **(B)** (–4, 4) **(C)** (4, 4) **(D)** (–4, –4)

## Comprehension # 3

If l and m are variable real numbers such that $5l^2 + 6m^2 - 4lm + 3l = 0$, then the variable line $lx + my = 1$ always touches a fixed parabola, whose axes is parallel to the x-axis.

**1.** The vertex of the parabola is

**(A)** (–5/3, 4/3) **(B)** (–7/4, 3/4) **(C)** (5/6, –7/6) **(D)** (1/2, –3/4)

**2.** The focus of the parabola is

**(A)** (1/6, –7/6) **(B)** (1/3, 4/3) **(C)** (3/2, –3/2) **(D)** (–3/4, (3/4)

**3.** The directrix of the parabola is

**(A)** $6x + 7 = 0$ **(B)** $4x + 11 = 0$ **(C)** $3x + 11 = 0$ **(D)** none of these

## Comprehension # 4

PQ is the double ordinate of the parabola $y^2 = 4x$ which passes through the focus S. ΔPQA is an isosceles right angle triangle, where A is on the axis of the parabola. Line PA meets the parabola at C and QA meets the parabola at B.

**1.** The area of trapezium PBCQ is

**(A)** 96 sq. units **(B)** 64 sq. units **(C)** 72 sq. units **(D)** 48 sq. units

**2.** The circumradius of trapezium PBCQ is

**(A)** $6\sqrt{5}$ **(B)** $3\sqrt{6}$ **(C)** $2\sqrt{10}$ **(D)** $5\sqrt{3}$

**3.** The ratio of the inradius of ΔABC and that of ΔPAQ is

**(A)** 2 : 1 **(B)** 3 : 2 **(C)** 4 : 3 **(D)** 3 : 1

# Exercise # 4 [Subjective Type Questions]

**1.** If the end points $P(t_1)$ and $Q(t_2)$ of a chord of a parabola $y^2 = 4ax$ satisfy the relation $t_1t_2 = k$ (constant) then prove that the chord always passes through a fixed point. Find that point also ?

**2.** If from the vertex of a parabola a pair of chords be drawn at right angles to one another, & with these chords as adjacent sides a rectangle be constructed , then find the locus of the outer corner of the rectangle.

**3.** Prove that the line joining the orthocenter to the centroid of a triangle formed by the focal chord of a parabola and tangents drawn an its extremities is parallel to the axis of the parabola.

**4.** Find the condition on 'a' & 'b' so that the two tangents drawn to the parabola $y^2 = 4ax$ from a point are normals to the parabola $x^2 = 4by$.

**5.** Two perpendicular chords are drawn from the origin 'O' to the parabola $y = x^2$, which meet the parabola at P and Q. Rectangle POQR is completed. Find the locus of vertex R.

**6.** Show that the common tangents to the parabola $y^2 = 4x$ and the circle $x^2 + y^2 + 2x = 0$ form an equilateral triangle.

**7.** TP & TQ are tangents to the parabola and the normals at P & Q meet at a point R on the curve. Prove that the centre of the circle circumscribing the triangle TPQ lies on the parabola $2y^2 = a(x - a)$.

**8.** Find the set of values of $\alpha$ in the interval $[\pi/2, 3\pi/2]$, for which the point $(\sin\alpha, \cos\alpha)$ does not lie outside the parabola $2y^2 + x - 2 = 0$.

**9.** If a leaf of a book is folded so that one corner moves along an opposite side, then prove that the line of crease will always touch parabola.

**10.** If 'm' varies then find the range of c for which the line $y = mx + c$ touches the parabola $y^2 = 8(x + 2)$.

**11.** Prove that on the axis of any parabola $y^2 = 4ax$ there is a certain point K which has the property that , if a chord PQ of the parabola be drawn through it , then $\frac{1}{(PK)^2} + \frac{1}{(QK)^2}$ is same for all positions of the chord. Find also the coordinates of the point K.

**12.** A parabola of latus rectum $l$ touches a fixed equal parabola. The axes of two parabolas are parallel. Then find the locus of the vertex of the moving parabola.

**13.** Show that a circle circumscribing the triangle formed by three co-normal points passes through the vertex of the parabola and its equation is, $2(x^2+y^2) - 2(h+2a)x - ky = 0$, where (h, k) is the point from where three concurrent normals are drawn.

**14.** Prove that the locus of the middle points of all tangents drawn from points on the directrix to the parabola $y^2 = 4ax$ is $y^2(2x + a) = a(3x + a)^2$.

**15.** Find the area of the trapezium whose vertices lie on the parabola $y^2 = 4x$ and diagonals pass through (1, 0) and have length 25/4 units each.

**16.** The normal at a point P to the parabola $y^2 = 4ax$ meets its axis at G. Q is another point on the parabola such that QG is perpendicular to the axis of the parabola. Prove that $QG^2 - PG^2 =$ constant.

**17.** In the parabola $y^2 = 4ax$, the tangent at the point P, whose abscissa is equal to the latus rectum meets the axis in T & the normal at P cuts the parabola again in Q. Prove that PT : PQ = 4 : 5.

**18.** A tangent is drawn to the parabola $y^2 = 4ax$ at P such that it cuts the y-axis at Q. A line perpendicular to this tangent is drawn through Q which cuts the axis of the parabola at R. If the rectangle PQRS is completed, then find the locus of S.

**19.** If $(x_1, y_1)$, $(x_2, y_2)$ and $(x_3, y_3)$ be three points on the parabola $y^2 = 4ax$ and the normals at these points meet in a point, then prove that $\frac{x_1 - x_2}{y_3} + \frac{x_2 - x_3}{y_1} + \frac{x_3 - x_1}{y_2} = 0$

**20.** Show that the locus of a point, such that two of the three normals drawn from it to the parabola $y^2 = 4ax$ are perpendicular is $y^2 = a(x - 3a)$.

**21.** A series of chord are drawn so that their projections on the straight line, which is inclined at an angle $\alpha$ to the axis, are of constant length c. Prove that the locus of their middle point is the curve.

**22.** Prove that the locus of the middle point of portion of a normal to $y^2 = 4ax$ intercepted between the curve & the axis is another parabola. Find the vertex & the latus rectum of the second parabola.

**23.** P & Q are the points of contact of the tangents drawn from the point T to the parabola $y^2 = 4ax$. If PQ be the normal to the parabola at P, prove that TP is bisected by the directrix.

**24.** Prove that for a suitable point P on the axis of the parabola, a chord AB through the point P can be drawn such that $[(1/AP^2) + (1/BP^2)]$ is the same for all positions of the chord.

**25.** A ray of light is coming along the line y = b from the positive direction of x-axis & strikes a concave mirror whose intersection with the xy-plane is a parabola $y^2 = 4\,ax$. Find the equation of the reflected ray & show that it passes through the focus of the parabola. Both a & b are positive.

## Exercise # 5 Part # I [Previous Year Questions] [AIEEE/JEE-MAIN]

**1.** The length of the latus rectum of the parabola $x^2 - 4x - 8y + 12 = 0$ is- **[AIEEE-2002]**

(1) 4 (2) 6 (3) 8 (4) 10

**2.** The equation of tangents to the parabola $y^2 = 4ax$ at the ends of its latus rectum is- **[AIEEE-2002]**

(1) $x - y + a = 0$ (2) $x + y + a = 0$ (3) $x + y - a = 0$ (4) both (1) and (2)

**3.** The normal at the point $(bt_1^2, 2bt_1)$ on a parabola meets the parabola again in the point $(bt_2^2, 2bt_2)$, then- **[AIEEE-2003]**

(1) $t_2 = t_1 + \frac{2}{t_1}$ (2) $t_2 = -t_1 - \frac{2}{t_1}$ (3) $t_2 = -t_1 + \frac{2}{t_1}$ (4) $t_2 = t_1 - \frac{2}{t_1}$

**4.** If $a \neq 0$ and the line $2bx + 3cy + 4d = 0$ passes through the points of intersection of the parabolas $y^2 = 4ax$ and $x^2 = 4ay$, then- **[AIEEE-2004]**

(1) $d^2 + (2b + 3c)^2 = 0$ (2) $d^2 + (3b + 2c)^2 = 0$ (3) $d^2 + (2b - 3c)^2 = 0$ (4) $d^2 + (3b - 2c)^2 = 0$

**5.** The locus of the vertices of the family of parabolas $y = \frac{a^3x^2}{3} + \frac{a^2x}{2} - 2a$ is- **[AIEEE-2006]**

(1) $xy = \frac{3}{4}$ (2) $xy = \frac{35}{16}$ (3) $xy = \frac{64}{105}$ (4) $xy = \frac{105}{64}$

**6.** The equation of a tangent to the parabola $y^2 = 8x$ is $y = x + 2$. The point on this line from which the other tangent to the parabola is perpendicular to the given tangents is- **[AIEEE-2007]**

(1) (–1, 1) (2) (0, 2) (3) (2, 4) (4) (–2, 0)

**7.** A parabola has the origin as its focus and the line $x = 2$ as the directrix. Then the vertex of the parabola is at - **[AIEEE-2008]**

(1) (0, 2) (2) (1, 0) (3) (0, 1) (4) (2, 0)

**8.** If two tangents drawn from a point P to the parabola $y^2 = 4x$ are at right angles then the locus of P is :- **[AIEEE-2010]**

(1) $x = 1$ (2) $2x + 1 = 0$ (3) $x = -1$ (4) $2x - 1 = 0$

**9.** Given : A circle, $2x^2 + 2y^2 = 5$ and a parabola, $y^2 = 4\sqrt{5}\,x$. **[JEE (Main)-2013]**

**Statement–I :** An equation of a common tangent to these curves is $y = x + \sqrt{5}$.

**Statement–II :** If the line, $y = mx + \frac{\sqrt{5}}{m}$ $(m \neq 0)$ is their common tangent, then m satisfies $m^4 - 3m^2 + 2 = 0$.

(1) Statement-I is true, Statement-II is true; statement-II is a correct explanation for Statement-I.
(2) Statement-I is true, Statement-II is true; statement-II is not a correct explanation for Statement-I.
(3) Statement-I is true, Statement-II is false.
(4) Statement-I is false, Statement-II is true.

**10.** Let O be the vertex and Q be any point on the parabola, $x^2 = 8y$. If the point P divides the line segment OQ internally in the ratio 1 : 3, then the locus of P is **[Main 2015]**

**(1)** $y^2 = 2x$ **(2)** $x^2 = 2y$ **(3)** $x^2 = y$ **(4)** $y^2 = x$

**11.** Let P be the point on the parabola, $y^2 = 8x$ which is at a minimum distance from the centre C of the circle. $x^2 + (y + 6)^2 = 1$. Then the equation of the circle, passing through C and having its centre at P is : **[Main 2016]**

**(1)** $x^2 + y^2 - x + 4y - 12 = 0$ **(2)** $x^2 + y^2 - \frac{x}{4} + 2y - 24 = 0$

**(3)** $x^2 + y^2 - 4x + 9y + 18 = 0$ **(4)** $x^2 + y^2 - 4x + 8y + 12 = 0$

## Part # II [Previous Year Questions][IIT-JEE ADVANCED]

**1. (a)** If the line $x - 1 = 0$ is the directrix of the parabola $y^2 - kx + 8 = 0$, then one of the values of 'k' is:

**(A)** 1/8 **(B)** 8 **(C)** 4 **(D)** 1/4

**(b)** If $x + y = k$ is normal to $y^2 = 12x$, then 'k' is - **[JEE 2000 ( Screening)]**

**(A)** 3 **(B)** 9 **(C)** $-9$ **(D)** $-3$

**2.(a)** The equation of the common tangent touching the circle $(x - 3)^2 + y^2 = 9$ and the parabola $y^2 = 4x$ above the x- axis is -

**(A)** $\sqrt{3}y = 3x + 1$ **(B)** $\sqrt{3}y = -(x + 3)$ **(C)** $\sqrt{3}y = x + 3$ **(D)** $\sqrt{3}y = -(3x + 1)$

**(b)** The equation of the directrix of the parabola $y^2 + 4y + 4x + 2 = 0$ is - **[JEE 2001 ( Screening)]**

**(A)** $x = -1$ **(B)** $x = 1$ **(C)** $x = -3/2$ **(D)** $x = 3/2$

**3.** The locus of the mid-point of the line segment joining the focus to a moving point on the parabola $y^2 = 4ax$ is another parabola with directrix **[JEE 2002 ( Screening)]**

**(A)** $x = -a$ **(B)** $x = -\frac{a}{2}$ **(C)** $x = 0$ **(D)** $x = \frac{a}{2}$

**4.** The equation of the common tangent to the curves $y^2 = 8x$ and $xy = -1$ is - **[JEE 2002 (Scr)]**

**(A)** $3y = 9x + 2$ **(B)** $y = 2x + 1$ **(C)** $2y = x + 8$ **(D)** $y = x + 2$

**5.** If a focal chord of the parabola $y^2 = 16x$ is a tangent to the circle $(x - 6)^2 + y^2 = 2$. then the set of possible values of the slope of this chord, are - **[JEE 2003 (Scr)]**

**(A)** $\{-1, 1\}$ **(B)** $\{-2, 2\}$ **(C)** $\left\{-2, \frac{1}{2}\right\}$ **(D)** $\left\{2, -\frac{1}{2}\right\}$

**6.** Normals with slopes $m_1$, $m_2$, $m_3$ are drawn from the point P to the parabola $y^2 = 4x$. If locus of P with $m_1 m_2 = \alpha$ is a part of the parabola itself, find $\alpha$. **[JEE 2004 (Mains)]**

**7.** Two tangents are drawn from point (1, 4) to the parabola $y^2 = 4x$. Angles between tangents is -

**(A)** $\pi/6$ **(B)** $\pi/4$ **(C)** $\pi/3$ **(D)** $\pi/2$ **[JEE 2004 (Screening)]**

**8.** At any point P on the parabola $y^2 - 2y - 4x + 5 = 0$, a tangent is drawn which meets the directrix at Q. Find the locus of point R which divides QP externally in the ratio $\frac{1}{2} : 1$. **[JEE 2004 (Mains)]**

**9.** Tangent to the curve $y = x^2 + 6$ at point P (1, 7) touches the circle $x^2 + y^2 + 16x + 12y + c = 0$ at a point Q. Then coordinate of Q is - **[JEE 2005 (Screening)]**

**(A)** (–6, 11) **(B)** (6, –11) **(C)** (–6, –7) **(D)** (–6, –11)

**10.** The axis of a parabola is along the line y = x and the distance of its vertex from origin is $\sqrt{2}$ and that of origin from its focus is $2\sqrt{2}$. If vertex and focus both lie in the first quadrant, then the equation of the parabola is - **[JEE 2006]**

**(A)** $(x + y)^2 = (x - y - 2)$ **(B)** $(x - y)^2 = (x + y - 2)$

**(C)** $(x - y)^2 = 4(x + y - 2)$ **(D)** $(x - y)^2 = 8(x + y - 2)$

**11.** The equations of the common tangents to the parabola $y = x^2$ and $y = - x^2 + 4x - 4$ is/are- **[JEE 2006]**

**(A)** $y = 4(x - 1)$ **(B)** $y = 0$ **(C)** $y = -4(x - 1)$ **(D)** $y = -30x - 50$

**12.** **Match the following** **[JEE 2006]**

Normals are drawn at points P, Q and R lying on the parabola $y^2 = 4x$ which intersect at (3, 0). Then

| | | | |
|---|---|---|---|
| **(i)** | Area of ΔPQR | **(A)** | 2 |
| **(ii)** | Radius of circumcircle of ΔPQR | **(B)** | 5/2 |
| **(iii)** | Centroid of ΔPQR | **(C)** | (5/2, 0) |
| **(iv)** | Circumcentre of ΔPQR | **(D)** | (2/3, 0) |

## 13 to 15 are based on this paragraph **[JEE 2006]**

Let ABCD be a square of side length 2 units. $C_2$ is the circle through vertices A, B, C, D and $C_1$ is the circle touching all the sides of the square ABCD. L is a line through A.

**13.** If P is a point on $C_1$ and Q in another point on $C_2$, then $\dfrac{PA^2 + PB^2 + PC^2 + PD^2}{QA^2 + QB^2 + QC^2 + QD^2}$ is equal to -

**(A)** 0.75 **(B)** 1.25 **(C)** 1 **(D)** 0.5

**14.** A circle touches the line L and circle $C_1$ externally such that both the circles are on the same side of the line, then the locus of centre of the circle is -

**(A)** ellipse **(B)** hyperbola **(C)** parabola **(D)** pair of straight line

**15.** A line M through A is drawn parallel to BD. Point S moves such that its distances from the line BD and the vertex A are equal. If locus of S cuts M at $T_2$ and $T_3$ and AC at $T_1$ then area of $\Delta T_1T_2T_3$ is

**(A)** 1/2 sq. units **(B)** 2/3 sq. units **(C)** 1 sq. units **(D)** 2 sq. units

## 16 to 18 are based on this paragraph

Consider the circle $x^2 + y^2 = 9$ and the parabola $y^2 = 8x$. They intersect at P and Q in the first and the fourth quadrants, respectively. Tangents to the circle at P and Q intersect the x-axis at R and tangents to the parabola at P and Q intersect the x-axis at S.

**16.** The ratio of the areas of the triangle PQS and PQR is :- **[JEE 2007]**

**(A)** $1 : \sqrt{2}$ **(B)** 1 : 2 **(C)** 1 : 4 **(D)** 1 : 8

**17.** The radius of the circumcircle of the triangle PRS is :- **[JEE 2007]**

**(A)** 2 **(B)** $3\sqrt{3}$ **(C)** $3\sqrt{2}$ **(D)** $2\sqrt{3}$

18. The radius of the incircle of the triangle PQR is :- **[JEE 2007]**

(A) 4 (B) 3 (C) $\frac{8}{3}$ (D) 2

## Assertion and Reason

19. **Statement-I :** The curve $y = \frac{-x^2}{2} + x + 1$ is symmetric with respect to the line $x = 1$ because

**Statement-II :** A parabola is symmetric about its axis. **[JEE 2007]**

(A) Statement-I is True, Statement-II is True ; Statement-II is a correct explanation for Statement-I.

(B) Statement-I is True, Statement-II is True ; Statement-II is NOT a correct explanation for Statement-I.

(C) Statement-I is True, Statement-II is False.

(D) Statement-I is False, Statement-II is True.

20. Consider the two curves $C_1 : y^2 = 4x$ ; $C_2 : x^2 + y^2 - 6x + 1 = 0$. Then **[JEE 2008]**

(A) $C_1$ and $C_2$ touch each other only at one point

(B) $C_1$ and $C_2$ touch each other exactly at two points

(C) $C_1$ and $C_2$ intersect (but do not touch) at exactly two points

(D) $C_1$ and $C_2$ neither intersect nor touch each other

21. The tangent PT and the normal PN to the parabola $y^2 = 4ax$ at a point P on it meet its axis at points T and N, respectively. The locus of the centroid of the triangle PTN is a parabola whose **[JEE 2009]**

(A) vertex is $\left(\frac{2a}{3}, 0\right)$ (B) directrix is $x = 0$ (C) latus rectum is $\frac{2a}{3}$ (D) focus is (a, 0)

22. Let A and B be two distinct points on the parabola $y^2 = 4x$. If the axis of the parabola touches a circle of radius r having AB as its diameter, then the slope of the line joining A and B can be - **[JEE 2010]**

(A) $-1/r$ (B) $1/r$ (C) $2/r$ (D) $-2/r$

23. Consider the parabola $y^2 = 8x$. Let $\Delta_1$ be the area of the triangle formed by the end points of its latus rectum and the point $P\left(\frac{1}{2}, 2\right)$ on the parabola, and $\Delta_2$ be the area of the triangle formed by drawing tangents at P and at the end points of the latus rectum. Then $\frac{\Delta_1}{\Delta_2}$ is **[JEE 2011]**

24. Let (x,y) be any point on the parabola $y^2 = 4x$. Let P be the point that divides the line segment from (0,0) to (x,y) in the ratio 1 : 3. Then the locus of P is - **[JEE 2011]**

(A) $x^2 = y$ (B) $y^2 = 2x$ (C) $y^2 = x$ (D) $x^2 = 2y$

25. Let L be a normal to the parabola $y^2 = 4x$. If L passes through the point (9,6), then L is given by - **[JEE 2011]**

(A) $y - x + 3 = 0$ (B) $y + 3x - 33 = 0$ (C) $y + x - 15 = 0$ (D) $y - 2x + 12 = 0$

26. Let S be the focus of the parabola $y^2 = 8x$ & let PQ be the common chord of the circle $x^2 + y^2 - 2x - 4y = 0$ and the given parabola. The area of the triangle PQS is **[JEE 2012]**

### Paragraph for Question 27 and 28

Let PQ be a focal chord of the parabolas $y^2 = 4ax$. The tangents to the parabola at P and Q meet at a point lying on the line $y = 2x + a$, $a > 0$.

**27.** If chord PQ subtends an angle $\theta$ at the vertex of $y^2 = 4ax$, then $\tan\theta =$ **[JEE Ad. 2013]**

**(A)** $\frac{2}{3}\sqrt{7}$ **(B)** $\frac{-2}{3}\sqrt{7}$ **(C)** $\frac{2}{3}\sqrt{5}$ **(D)** $\frac{-2}{3}\sqrt{5}$

**28.** Length of chord PQ is **[JEE Ad. 2013]**

**(A)** 7a **(B)** 5a **(C)** 2a **(D)** 3a

**29.** A line L : $y = mx + 3$ meets y-axis at E(0,3) and the arc of the parabola $y^2 = 16x$, $0 \le y \le 6$ at the point $F(x_0,y_0)$. The tangent to the parabola at $F(x_0,y_0)$ intersects the y-axis at $G(0,y_1)$. The slope m of the line L is chosen such that the area of the triangle EFG has a local maximum.

Match List-I with List-II and select the correct answer using the code given below the lists.

| | List-I | | List-II |
|---|---|---|---|
| **P.** | m = | 1. | $\frac{1}{2}$ |
| **Q.** | Maximum area of $\Delta$EFG is | 2. | 4 |
| **R.** | $y_0$ = | 3. | 2 |
| **S.** | $y_1$ = | 4. | 1 |

**Codes**

| | P | Q | R | S |
|---|---|---|---|---|
| (A) | 4 | 1 | 2 | 3 |
| (B) | 3 | 4 | 1 | 2 |
| (C) | 1 | 3 | 2 | 4 |
| (D) | 1 | 3 | 4 | 2 |

**[JEE Ad. 2013]**

### Comprehension

Given that for each $a \in (0, 1)$, $\lim_{h\to 0^+} \int_h^{1-h} t^{-a}(1-t)^{a-1}\,dt$ exists. Let this limit be g(a). In addition, it is given that the function g(a) is differentiable on (0, 1).

**30.** The value of $g\left(\frac{1}{2}\right)$ is **[JEE Ad. 2014]**

**(A)** $\pi$ **(B)** $2\pi$ **(C)** $\frac{\pi}{2}$ **(D)** $\frac{\pi}{4}$

**31.** The value of $g'\left(\frac{1}{2}\right)$ is **[JEE Ad. 2014]**

**(A)** $\frac{\pi}{2}$ **(B)** $\pi$ **(C)** $-\frac{\pi}{2}$ **(D)** 0

**32.** If the normals of the parabola $y^2 = 4x$ drawn at the end points of its latus rectum are tangents to the circle $(x - 3)^2 + (y + 2)^2 = r^2$, then the value of $r^2$ is **[JEE Ad. 2015]**

**33.** Let the curve C be the mirror image of the parabola $y^2 = 4x$ with respect to the line $x + y + 4 = 0$. If A and B are the points of intersection of C with the line $y = -5$, then the distance between A and B is . **[JEE Ad. 2015]**

**34.** Let P and Q be distinct points on the parabola $y^2 = 2x$ such that a circle with PQ as diameter passes through the veriex O and the area of the triangle $\Delta OPQ$ is $3\sqrt{2}$, then which of the following is (are) the coordiantes of P ? **[JEE Ad. 2015]**

**(A)** $\left(4, 2\sqrt{2}\right)$ **(B)** $\left(9, 3\sqrt{2}\right)$ **(C)** $\left(\frac{1}{4}, \frac{1}{\sqrt{2}}\right)$ **(D)** $\left(1, \sqrt{2}\right)$

**35.** The circle $C_1 : x^2 + y^2 = 3$, with centre at O, intersects the parabola $x^2 = 2y$ at the point P in the first quadrant. Let the tangent to the circle $C_1$ at P touches other two circles $C_2$ and $C_3$ at $R_2$ and $R_3$, respectively. Suppose $C_2$ and $C_3$ have equal radii $2\sqrt{3}$ and centres $Q_2$ and $Q_3$, respectively. If $Q_2$ and $Q_3$ lie on the y-axis, then **[JEE Ad. 2015]**

**(A)** $Q_2Q_3 = 12$ **(B)** $R_2R_3 = 4\sqrt{6}$

**(C)** area of the triangle $OR_2R_3$ is $6\sqrt{2}$ **(D)** area of triangle $PQ_2Q_3$ is $4\sqrt{2}$

**36.** Let P be the point on the parabola $y^2 = 4x$ which is at the shortest distance from the center S of the circle $x^2 + y^2 - 4x - 16y + 64 = 0$. Let Q be the point on the circle dividing the line segment SP internally. Then **[JEE Ad. 2016]**

**(A)** $SP = 2\sqrt{5}$

**(B)** $SQ : QP = (\sqrt{5} + 1) : 2$

**(C)** the x-intercept of the normal to the parabola at P is 6

**(D)** the slope of the tangent to the circle at Q is $\frac{1}{2}$

## MOCK TEST

### SECTION - I : STRAIGHT OBJECTIVE TYPE

**1.** A ray of light travels along a line y = 4 and strikes the surface of a curve $y^2 = 4(x + y)$ then equation of the line along reflected ray travels, is

**(A)** x = 0 **(B)** x = 2 **(C)** x + y = 4 **(D)** 2x + y = 4

**2.** Set of values of m for which a chord of slope m of the circle $x^2 + y^2 = 4$ touches parabola $y^2 = 4x$, is

**(A)** $\left(-\infty, -\sqrt{\frac{\sqrt{2}-1}{2}}\right) \cup \left(\sqrt{\frac{\sqrt{2}-1}{2}}, \infty\right)$ **(B)** $(-\infty, -1) \cup (1, \infty)$

**(C)** $(-1, 1)$ **(D)** $(-\infty, \infty)$

**3.** Normals at three points P, Q, R at the parabola $y^2 = 4ax$ meet in a point A and S be its focus, if $|SP| . |SQ| . |SR| = \lambda(SA)^2$, then $\lambda$ is equal to

**(A)** $a^3$ **(B)** $a^2$ **(C)** a **(D)** 1

**4.** In a parabola the angle $\theta$ that the latus rectum subtends at the vertex of the parabola is:

**(A)** dependent on the length of the latus rectum

**(B)** independent of the latus rectum and lies between $\frac{5\pi}{6}$ & $\pi$

**(C)** independent of the latus rectum and lies between $\frac{3\pi}{4}$ & $\frac{5\pi}{6}$

**(D)** independent of the latus rectum and lies between $\frac{2\pi}{3}$ & $\frac{3\pi}{4}$

**5.** Let P and Q be points (4, – 4) and (9, 6) of the parabola $y^2 = 4a(x - b)$. Let R be a point on the arc of the parabola between P & Q. Then the area of $\Delta PRQ$ is largest when

**(A)** $\angle PRQ = 90°$ **(B)** the point R is (4, 4)

**(C)** the point R is $\left(\frac{1}{4}, 1\right)$ **(D)** None of these

**6.** A parabola $y = ax^2 + bx + c$ crosses the x – axis at $(\alpha, 0)$ $(\beta, 0)$ both to the right of the origin. A circle also passes through these two points. The length of a tangent from the origin to the circle is:

**(A)** $\sqrt{\frac{bc}{a}}$ **(B)** $ac^2$ **(C)** $\frac{b}{a}$ **(D)** $\sqrt{\frac{c}{a}}$

**7.** Normals AO, $AA_1$, $AA_2$ are drawn to parabola $y^2 = 8x$ from the point A (h, 0). If triangle $OA_1A_2$ (O being the origin) is equilateral, then possible value of 'h' is

**(A)** 26 **(B)** 24 **(C)** 28 **(D)** 22

**8.** Locus of a point P if the three normals drawn from it to the parabola $y^2 = 4ax$ are such that two of them make complementry angles with the axis of the parabola is:

**(A)** $y^2 = a (x + a)$ **(B)** $y^2 = 2a (x - a)$ **(C)** $y^2 = a (x - 2a)$ **(D)** $y^2 = a (x - a)$

**9.** **$S_1$ :** Vertex of a parabola bisects the subtangent.

**$S_2$ :** Subnormal of a parabola is equal to its latusrectum.

**$S_3$ :** Circle with focal radius of a point on parabola as diameter touches the tangent drawn at the vertex of the parabola.

**$S_4$ :** Directrix of a parabola is the tangent of a circle drawn its focal chord as diameter.

**(A)** FTTT **(B)** FFTT **(C)** TTTT (D TFTT

**10.** From an external point P, pair of tangent lines are drawn to the parabola, $y^2 = 4x$. If $\theta_1$ & $\theta_2$ are the inclinations of these tangents with the axis of x such that, $\theta_1 + \theta_2 = \frac{\pi}{4}$, then the locus of P is:

**(A)** $x - y + 1 = 0$ **(B)** $x + y - 1 = 0$ **(C)** $x - y - 1 = 0$ **(D)** $x + y + 1 = 0$

## SECTION - II : MULTIPLE CORRECT ANSWER TYPE

**11.** If equation of tangent at P, Q and vertex A of a parabola are $3x + 4y - 7 = 0$, $2x + 3y - 10 = 0$ and $x - y = 0$ respectively, then

**(A)** focus is (4, 5) **(B)** length of latusrectum is $2\sqrt{2}$

**(C)** axis is $x + y - 9 = 0$ **(D)** vertex is $\left(\frac{9}{2}, \frac{9}{2}\right)$

**12.** The tangent to the hyperbola, $x^2 - 3y^2 = 3$ at the point $\left(\sqrt{3}, 0\right)$ when associated with two asymptotes constitutes :

**(A)** isosceles triangle but not equilateral **(B)** an equilateral triangle

**(C)** a triangles whose area is $\sqrt{3}$ sq. units **(D)** a right isosceles triangle.

**13.** If A & B are points on the parabola $y^2 = 4ax$ with vertex O such that OA perpendicular to OB & having lengths $r_1$ & $r_2$ respectively, then the value of $\frac{r_1^{4/3} r_2^{4/3}}{r_1^{2/3} + r_2^{2/3}}$ is

**(A)** $16a^2$ **(B)** $a^2$ **(C)** 4a **(D)** None of these

**14.** A point moves such that the sum of the squares of its distances from the two sides of length 'a' of a rectangle is twice the sum of the squares of its distances from the other two sides of length 'b'. The locus of the point can be :

**(A)** a circle **(B)** an ellipse **(C)** a hyperbola **(D)** a pair of lines

**15.** Let P, Q and R are three co-normal points on the parabola $y^2 = 4ax$. Then the correct statement(s) is/are

**(A)** algebraic sum of the slopes of the normals at P, Q and R vanishes

**(B)** algebraic sum of the ordinates of the points P, Q and R vanishes

**(C)** centroid of the triangle PQR lies on the axis of the parabola

**(D)** circle circumscribing the triangle PQR passes through the vertex of the parabola

## SECTION - III : ASSERTION AND REASON TYPE

16. **Statement-I :** Circumcircle of a triangle formed by the lines $x = 0$, $x + y + 1 = 0$ & $x - y + 1 = 0$ also passes through the point (1, 0)

**Statement-II :** Circumcircle of a triangle formed by three tangents of a parabola passes through its focus.

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I.

(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I

(C) Statement-I is True, Statement-II is False

(D) Statement-I is False, Statement-II is True

17. Consider a curve $C : y^2 - 8x - 2y - 15 = 0$ in which two tangents $T_1$ and $T_2$ are drawn from $P(-4, 1)$.

**Statement-I :** $T_1$ and $T_2$ are mutually perpendicular tangents.

**Statement-II :** Point P lies on the axis of curve C.

(A) Statement-I is true, statement-II is true and statement-II is correct explanation for statement-I.

(B) Statement-I is true, statement-II is true and statement-II is NOT the correct explanation for statement-I.

(C) Statement-I is true, statement-II is false.

(D) Statement-I is false, statement-II is true.

18. **Statement-I :** Area of triangle formed by pair of tangents drawn from a point (12, 8) to the parabola $y^2 = 4x$ and their corresponding chord of contact is 32 sq. units.

**Statement-II :** If from a point $P(x_1, y_1)$ tangents are drawn to a parabola $y^2 = 4ax$ then area of triangle formed by these tangents and their corresponding chord of contact is $\dfrac{(y_1^2 - 4ax_1)^{\frac{3}{2}}}{4|a|}$ sq. units.

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I.

(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I

(C) Statement-I is True, Statement-II is False

(D) Statement-I is False, Statement-II is True

19. **Statement-I :** Consider the parabola $(y - 2)^2 = 8(x - 1)$ and circle $(x + 5)^2 + (y - 2)^2 = 8$. There is exactly one point such that tangents to both parabola and circle drawn from it are perpendicular.

**Statement-II :** Director circles of parabola and circle can intersect in atmost two points.

(A) Statement-I is true, statement-II is true and statement-II is correct explanation for statement-I.

(B) Statement-I is true, statement-II is true and statement-II is NOT the correct explanation for statement-I.

(C) Statement-I is true, statement-II is false.

(D) Statement-I is false, statement-II is true.

20. **Statement-I :** Normal chord drawn at the point (8, 8) of the parabola $y^2 = 8x$ subtends a right angle at the vertex of the parabola.

**Statement-II :** Every chord of the parabola $y^2 = 4ax$ passing through the point (4a, 0) subtends a right angle at the vertex of the parabola.

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I.

(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I

(C) Statement-I is True, Statement-II is False

(D) Statement-I is False, Statement-II is True

## SECTION - IV : MATRIX - MATCH TYPE

**21.** Consider the parabola $y^2 = 12x$

| | Column-I | | Column-II |
|---|---|---|---|
| **(A)** | Tangent and normal at the extremities of the latus rectum intersect the x axis at T and G respectively. The coordinates of the middle point of T and G are | **(p)** | (0, 0) |
| **(B)** | Variable chords of the parabola passing through a fixed point K on the axis, such that sum of the squares of the reciprocals of the two parts of the chords through K, is a constant. The coordinate of the point K are | **(q)** | (3, 0) |
| | | **(r)** | (6, 0) |
| **(C)** | All variable chords of the parabola subtending a right angle at the origin are concurrent at the point | | |
| **(D)** | AB and CD are the chords of the parabola which intersect at a point E on the axis. The radical axis of the two circles described on AB and CD as diameter always passes through | **(s)** | (12, 0) |

**22.**

| | Column – I | | Column – II |
|---|---|---|---|
| **(A)** | Parabola $y^2 = 4x$ and the circle having its centre at (6, 5) intersects at right angle, at the point (a, a) then one value of a is equal to | **(p)** | 13 |
| **(B)** | The angle between the tangents drawn to $(y - 2)^2 = 4(x + 3)$ at the points where it is intersected by the line $3x - y + 8 = 0$ is $\frac{4\pi}{p}$, then p has the value equal to | **(q)** | 8 |
| **(C)** | If the line $x - 1 = 0$ is the directrix of the parabola $y^2 - kx + 8 = 0$, then one of the value of k is | **(r)** | $10\sqrt{5}$ |
| **(D)** | Length of the normal chord of the parabola $y^2 = 8x$ at the point where abscissa & ordinate are equal is | **(s)** | 4 |
| | | **(t)** | 12 |

## SECTION - V : COMPREHENSION TYPE

**23.** **Read the following comprehensions carefully and answer the questions.**

Tangents are drawn to the parabola $y^2 = 4x$ from the point $P(6, 5)$ to touch the parabola at Q and R. $C_1$ is a circle which touches the parabola at Q and $C_2$ is a circle which touches the parabola at R. Both the circles $C_1$ and $C_2$ pass through the focus of the parabola.

**1.** Area of the $\Delta PQR$ equals

**(A)** $\frac{1}{2}$ **(B)** 1 **(C)** 2 **(D)** $\frac{1}{4}$

**2.** Radius of the circle $C_2$ is

**(A)** $5\sqrt{5}$ **(B)** $5\sqrt{10}$ **(C)** $10\sqrt{2}$ **(D)** $\sqrt{210}$

**3.** The common chord of the circles $C_1$ and $C_2$ passes through the

**(A)** incentre of the $\Delta PQR$. **(B)** circumcentre of the $\Delta PQR$.
**(C)** centroid of the $\Delta PQR$. **(D)** othocentre of the $\Delta PQR$.

**24. Read the following comprehensions carefully and answer the questions.**

If the locus of the circumcentre of a variable triangle having sides y-axis, $y = 2$ and ●$x + my = 1$, where (●, m) lies on the parabola $y^2 = 4ax$ is a curve C, then

**1.** Coordinates of the vertex of this curve C is

**(A)** $\left(2a, \frac{3}{2}\right)$ **(B)** $\left(-2a, -\frac{3}{2}\right)$ **(C)** $\left(-2a, \frac{3}{2}\right)$ **(D)** $\left(-2a, -\frac{3}{2}\right)$

**2.** The length of smallest focal chord of this curve C is :

**(A)** $\frac{1}{12a}$ **(B)** $\frac{1}{4a}$ **(C)** $\frac{1}{16a}$ **(D)** $\frac{1}{8a}$

**3.** The curve C is symmetric about the line :

**(A)** $y = -\frac{3}{2}$ **(B)** $y = \frac{3}{2}$ **(C)** $x = -\frac{3}{2}$ **(D)** $x = \frac{3}{2}$

**25. Read the following comprehensions carefully and answer the questions.**

In general, three normals can be drawn from a point to a parabola and the point where they meet the parabola are called co-normal points.

The equation of any normal to $y^2 = 4ax$ is $y = mx - 2am - am^3$. If it passes through (h, k), then $k = mh - 2am - am^3$ or $am^3 + m(2a - h) + k = 0$

This is cubic in m, it has three roots $m_1, m_2, m_3$

$$\therefore\ m_1 + m_2 + m_3 = 0,\ m_1m_2m_3 = \frac{-k}{a},\ m_1m_2 + m_2m_3 + m_3m_1 = \frac{2a - h}{a}$$

**1.** Minimum distance between the curves $y^2 = x - 1$ and $x^2 = y - 1$ is equal to

**(A)** $\frac{3\sqrt{2}}{4}$ **(B)** $\frac{5\sqrt{2}}{4}$ **(C)** $\frac{7\sqrt{2}}{4}$ **(D)** $\frac{\sqrt{2}}{4}$

**2.** If the normals from any point to the parabola $x^2 = 4y$ cuts the line $y = 2$ in points whose abscissaes are in A.P., then the slopes of the tangents at the three co-normal points are in

**(A)** A.P. **(B)** G.P. **(C)** H.P. **(D)** None of these

**3.** If the normals at three points. P, Q, R of the parabola $y^2 = 4ax$ meet in a point O and S be its focus, then |SP|.|SQ|.|SR| is equal to

**(A)** $a^2$ **(B)** $a(SO)^3$ **(C)** $a(SO)^2$ **(D)** None of these

## SECTION - VI : INTEGER TYPE

**26.** The two parabolas $y^2 = 4ax$ and $y^2 = 4(a - 1)(x - b)$ can not have common normal other than axis unless $b > \lambda$, then find $\lambda$.

**27.** Let $L_1 : x + y = 0$ and $L_2 : x - y = 0$ are tangent to a parabola whose focus is S(1, 2).
If the length of latus-rectum of the parabola can be expressed as $\frac{m}{\sqrt{n}}$ (where m and n are coprime) then find the value of $(m + n)$.

**28.** Points A, B & C lie on the parabola $y^2 = 4ax$. The tangents to the parabola at A, B & C, taken in pairs, intersect at points P, Q & R. Determine the ratio of the areas of the triangles ABC & PQR.

**29.** From a point A common tangents are drawn to the circle $x^2 + y^2 = a^2/2$ and the parabola $y^2 = 4a\,x$. Find the area of the quadrilateral formed by the common tangents, the chords of contact of the point A, w.r.t. the circle and the parabola is $\frac{\lambda a^2}{4}$, then find $\lambda$

**30.** The chord of the parabola $y^2 = 4\,a\,x$, whose equation is $y - x\sqrt{2} + 4a\sqrt{2} = 0$, is a normal to the curve, and its length is $\lambda\sqrt{3}a$, then find $\lambda$.

# ANSWER KEY

## EXERCISE - 1

**1.** C **2.** B **3.** C **4.** B **5.** A **6.** C **7.** C **8.** C **9.** B **10.** C **11.** B **12.** C **13.** C
**14.** D **15.** A **16.** C **17.** C **18.** B **19.** D **20.** A **21.** D **22.** B **23.** B **24.** B **25.** A **26.** A
**27.** B **28.** C **29.** C **30.** C

## EXERCISE - 2 : PART # I

**1.** ABC **2.** AC **3.** BCD **4.** BD **5.** ABCD **6.** AC **7.** AD **8.** AC **9.** CD
**10.** ABC **11.** AC **12.** CD **13.** AB **14.** ABCD **15.** ABCD **16.** ABCD **17.** AB **18.** ABD
**19.** AB **20.** ABC

## PART - II

**1.** C **2.** A **3.** B **4.** D **5.** A **6.** B

## EXERCISE - 3 : PART # I

**1.** $A \to r$ $B \to s$ $C \to p$ $D \to q$ **2.** $A \to q$ $B \to s$ $C \to q$ $D \to p$ **3.** $A \to q$ $B \to r$ $C \to s$ $D \to p$
**4.** $A \to r$ $B \to s$ $C \to p$ $D \to q$ **5.** $A \to q,s$ $B \to r$ $C \to p,q$ $D \to q,r$

## PART - II

**Comprehension # 1 : 1.** B **2.** C **3.** D **Comprehension # 2 : 1.** A **2.** C **3.** C
**Comprehension # 3 : 1.** A **2.** B **3.** C **Comprehension # 4 : 1.** B **2.** C **3.** D

## EXERCISE - 5 : PART # I

**1.** 3 **2.** 4 **3.** 2 **4.** 1 **5.** 4 **6.** 4 **7.** 2 **8.** 3 **9.** 2 **10.** 2 **11.** 4

## PART - II

**1.** (a) C (b) B **2.** (a) C (b) D **3.** C **4.** D **5.** A **6.** 2 **7.** C
**8.** $(x+1)(y-1)^2+4=0$ **9.** C **10.** D **11.** A, B **12.** **(i)** A, **(ii)** B, **(iii)** D, **(iv)** C **13.** A **14.** C **15.** C
**16.** C **17.** B **18.** D **19.** A **20.** B **21.** A, D **22.** C,D **23.** 2 **24.** C **25.** A,B,D **26.** 4 **27.** D
**28.** B **29.** A **30.** D **31.** B **32.** 2 **33.** 4 **34.** A **35.** A,B,C **36.** A,C,D

## MOCK TEST

**1.** A  **2.** A  **3.** C  **4.** D  **5.** C  **6.** D  **7.** C  **8.** D  **9.** D

**10.** C  **11.** A,B,C,D  **12.** B,C  **13.** A  **14.** C,D  **15.** A,B,C,D  **16.** A  **17.** B  **18.** C

**19.** B  **20.** D  **21.** $A \to q$ $B \to r$ $C \to s$ $D \to p$  **22.** $A \to s$ $B \to q$ $C \to s$ $D \to r$

**23.** **1.** A **2.** B **3.** C  **24.** **1.** C **2.** D **3.** B  **25.** **1.** A **2.** B **3.** C

**26.** 2  **27.** 11  **28.** 2 : 1  **29.** 15  **30.** 6