Angle Sum Property and Exterior Angle Property

A. Fill in the Blanks

1. The sum of the angles in any triangle is always _____.
2. An exterior angle of a triangle is equal to the sum of its two _____ interior angles.
3. Each angle in an equilateral triangle measures _____.
4. In a right-angled triangle, the two acute angles are _____ (their sum is 90°).
5. A triangle cannot have more than one _____ angle.

B. Match the Following;

Column A	Column B
1. A triangle with angles 30°, 90°, x.	A. 120°
2. An isosceles triangle with a vertex angle of 80°. The base angle is x.	B. 60°
3. An exterior angle is x, and the opposite interior angles are 50° and 70°.	C. 40°
4. A triangle with angles in the ratio 1:1:1. One angle is x.	D. 50°
5. An exterior angle is 100°. One opposite interior angle is 60°. The other is .	E. 70°

C. Practice Problems

Solve for the unknown angle(s) in each problem. Show your work.

1. In ΔXYZ, ∠X = x, ∠Y = 2x, and ∠Z = 3x. Find the value of x and the measure of each angle.
2. Find the value of x in the triangle below. (A triangle with angles: (x + 10)°, (x + 20)°, and 30°)
3. One angle of a triangle is 100°. The other two angles are equal. Find the measure of each of the equal angles.
4. Find the value of the exterior angle 'y'. (A triangle with interior angles 75° and 40°. 'y' is the exterior angle opposite to these two.)

5. In the figure below, find the value of x. (A triangle with an exterior angle of 125°. The two opposite interior angles are x and 80°.)

D. Warm-up Questions

Find the value of 'x' in each of the following figures.

1. A triangle has angles measuring 50° and 70°. What is the third angle, x?
2. In a right-angled triangle, one of the acute angles is 45°. What is the other acute angle, x?
3. An exterior angle of a triangle is x. The two opposite interior angles are 60° and 65°. Find x.
4. An exterior angle of a triangle is 110°. One of the opposite interior angles is 50°. Find the other opposite interior angle, x.
5. In ΔPQR, ∠P = 80°, ∠Q = 40°. Find ∠R (represented by x).

E. Challenge Questions

1. In the figure, line AB is parallel to line DE. Find the value of x. (A diagram showing parallel lines AB and DE. A transversal intersects AB at C and DE at D, forming ΔCDE. ∠BAC is given as 60°. ∠CDE = 70°. 'x' is ∠DCE.)
2. Find the values of x, y, and z in the given figure. (A diagram showing two intersecting lines, forming two vertically opposite triangles. In the top triangle, angles are 45°, 65°, and x. 'x' and 'y' are vertically opposite angles. In the bottom triangle, angles are y, 80°, and z.)
3. In ΔPQR, the angle bisector of ∠P and ∠Q meet at point O. If ∠R = 70°, find ∠POQ.
4. Find the value of x in the complex figure below. (A large triangle ABC. A point D is on BC. A line segment AD is drawn. In ΔABD, angles are 40° and x. In ΔADC, angles are 30° and y. ∠ADB and ∠ADC are on a straight line. The angle ∠BAC is (40+30)=70°. We need to find x in ΔABC where angles are 70°, x, and y.) Let's rephrase this for clarity: In ΔABC, ∠B = 60° and ∠C = 40°. AD is a line segment from A to BC such that AD is perpendicular to BC. Find ∠BAD.
5. The exterior angle at vertex C of ΔABC is 130°. If the interior opposite angles are in the ratio 2:3, find all the angles of the triangle.

F. Word Problems & Application

1. A triangular park has two angles measuring 55° and 65°. What is the measure of the third angle?
2. A ladder is leaning against a wall. The ladder makes an angle of 60° with the ground. Assuming the wall is perpendicular to the ground (forms a 90° angle), what is the angle the ladder makes with the wall?
3. Sarah is flying a kite. The kite string, the ground, and the vertical line from the kite to the ground form a right-angled triangle. If the string makes a 35° angle with the ground, what is the angle at the kite?

4. Three roads intersect to form a triangular plot of land. The angles at the vertices of the plot are $(3x)°$, $(4x - 10)°$, and $(2x + 10)°$. Find the measure of the largest angle of the plot.

5. A surveyor is mapping a triangular piece of land. He stands at one corner and measures the angle to be 40°. He walks to the next corner and finds the exterior angle of his path is 110°. What is the third angle of the triangular land?

G. True or False

1. A triangle can have two obtuse angles. _____
2. The sum of the two interior opposite angles is always less than the exterior angle. _____
3. The sum of the three exterior angles of a triangle is 360°. _____
4. If the angles of a triangle are 2x, 3x, and 4x, then the value of x is 20°. _____
5. An exterior angle of a triangle can be a right angle. _____