Construction with Given Sides and Angles (SAS & ASA Criteria)

A. Fill in the Blanks

1. To use the SAS criterion, the given angle must be the _____ angle.

2. If two angles of a triangle are 40° and 60°, the third angle must be _____.

3. In the ASA criterion, the given side must be _____ between the two given angles.

4. A minimum of _____ independent measurements are needed to define a unique triangle.

5. If you are given two sides and a non-included angle (SSA), you _____ always construct a unique triangle. (can / cannot)

B. Match the Following;

Column A (Given Information)	Column B (Criterion)
1. Side = 8 cm, Angle = 40°, Side = 6 cm (Angle is included)	A. SSS (Side-Side-Side)
2. Angle = 50°, Side = 10 cm, Angle = 60° (Side is included)	B. ASA (Angle-Side-Angle)
3. Side = 7 cm, Side = 7 cm, Side = 7 cm	C. SAS (Side-Angle-Side)
4. Angle = 90°, Angle = 30°, Side = 5 cm (Side is not included)	D. RHS (Right-angle-Hypotenuse-Side)
5. Angle = 90°, Hypotenuse = 13 cm, Side = 5 cm	E. AAS (Angle-Angle-Side)

C. Practice Problems

Construct the following triangles using the given measurements. State which criterion (SAS or ASA) you used for each construction.

1. Construct ΔABC where AB = 6 cm, ∠B = 60°, and BC = 5 cm.

2. Construct ΔPQR where QR = 7 cm, ∠Q = 50°, and ∠R = 70°.

3. Construct ΔLMN where LM = 5.5 cm, ∠L = 90°, and LN = 5.5 cm.

4. Construct ΔXYZ where XY = 8 cm, ∠X = 40°, and ∠Y = 100°.

5. Construct ΔDEF where DE = 7.2 cm, ∠E = 110°, and EF = 6 cm.
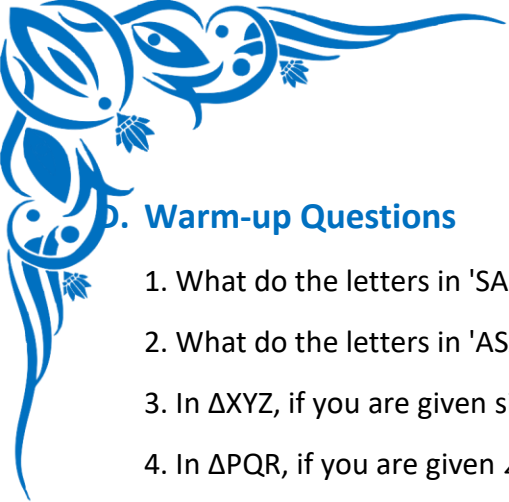
D. Warm-up Questions

1. What do the letters in 'SAS' stand for in triangle construction?

2. What do the letters in 'ASA' stand for in triangle construction?

3. In ΔXYZ, if you are given side XY and side YZ, which angle is the "included angle"?

4. In ΔPQR, if you are given ∠P and ∠Q, which side is the "included side"?

5. List the three essential tools you need from your geometry box for these constructions.

E. Challenge Questions

1. Can you construct ΔABC with AB = 8 cm, ∠A = 70°, and ∠C = 40°? If yes, construct it. (Hint: What is the sum of angles in a triangle?)

2. Construct an isosceles triangle PQR where the base QR = 6 cm and the equal base angles ∠Q and ∠R are both 50°.

3. Construct an equilateral triangle DEF with a side length of 6.5 cm using the ASA criterion. (Hint: What are the angles in an equilateral triangle?)

4. Explain why it is impossible to construct ΔXYZ with YZ = 7 cm, ∠Y = 120°, and ∠Z = 65°.

5. Construct a right-angled isosceles triangle ABC, where the two equal sides, AB and BC, are each 5 cm long. What are the measures of ∠A and ∠C?

F. Word Problems & Application

1. **Garden Plot:** A landscape designer is creating a triangular garden plot. One side of the plot is 8 meters long, and another side is 6 meters long. The angle between these two sides is 45°. Construct a scale drawing of this garden plot using a scale of 1 cm = 1 m.

2. **Surveying:** A surveyor stands at point A and measures the angle to a distant tree (T) as 50°. She then walks 100 meters to point B. From point B, she measures the angle to the same tree as 70°. Construct a scale drawing of the triangle ABT using a scale of 1 cm = 10 m and find the distance from point A to the tree.

3. **Support Beam:** A triangular metal brace is needed to support a shelf. The base of the brace (which sits on the shelf) is 7 cm long. The angle at one end of the base is 90° and the angle at the other end is 40°. Construct the brace.

4. **Sailboat Sail:** The main sail of a toy sailboat is a triangle. The boom (bottom edge) is 12 cm long and the mast (vertical edge) is 10 cm long. The angle where they meet is 90°. Construct the sail.

5. **Park Paths:** Two straight paths in a park diverge from a fountain at an angle of 120°. The first path is 50 m long, and the second path is 70 m long. Construct a scale drawing (1 cm = 10 m) of this section of the park. Measure the distance between the ends of the two paths on your drawing.
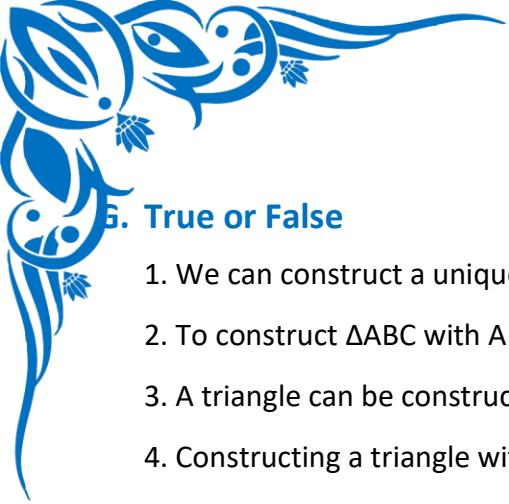
6. True or False

1. We can construct a unique triangle with angles 100°, 40°, and 40°. _____

2. To construct ΔABC with AB = 5 cm, BC = 7 cm, and ∠C = 50°, we use the SAS criterion. _____

3. A triangle can be constructed with sides 6 cm, 8 cm and an included angle of 180°. _____

4. Constructing a triangle with a side of 7 cm and angles of 90° and 45° on that side is an example of SAS. _____

5. All triangles with the same three angles are congruent. _____