CHAPTER 18

Mathematical Reasoning

1. The Boolean expression $\sim(p \vee q) \vee (\sim p \wedge q)$ is equivalent to
(a) $\sim q$ (b) $\sim p$
(c) p (d) q *(2018)*

2. If $(p \wedge \sim q) \wedge (p \wedge r) \to \sim p \vee q$ is false, then the truth values of p, q and r are respectively
(a) T, T, T (b) F, F, F
(c) T, F, T (d) F, T, F *(Online 2018)*

3. If $p \to (\sim p \vee \sim q)$ is false, then the truth values of p and q are respectively :
(a) F, F (b) F, T
(c) T, T (d) T, F *(Online 2018)*

4. The following statement $(p \to q) \to [(\sim p \to q) \to q]$ is
(a) equivalent to $\sim p \to q$ (b) equivalent to $p \to \sim q$
(c) a fallacy (d) a tautology *(2017)*

5. The proposition $(\sim p) \vee (p \wedge \sim q)$ is equivalent to
(a) $p \wedge \sim q$ (b) $p \vee \sim q$
(c) $p \to \sim q$ (d) $q \to p$
(Online 2017)

6. Contrapositive of the statement 'If two numbers are not equal, then their squares are not equal', is
(a) If the squares of two numbers are not equal, then the numbers are equal.
(b) If the squares of two numbers are equal, then the numbers are not equal.
(c) If the squares of two numbers are equal, then the numbers are equal.
(d) If the squares of two numbers are not equal, then the numbers are not equal.
(Online 2017)

7. The Boolean expression $(p \wedge \sim q) \vee q \vee (\sim p \wedge q)$ is equivalent to
(a) $\sim p \wedge q$ (b) $p \wedge q$
(c) $p \vee q$ (d) $p \vee \sim q$ *(2016)*

8. Consider the following two statements:
P : If 7 is an odd number, then 7 is divisible by 2.
Q : If 7 is a prime number, then 7 is an odd number.
If V_1 is the truth value of the contrapositive of P and V_2 is the truth value of contrapositive of Q, then the ordered pair (V_1, V_2) equals
(a) (F, F) (b) (F, T)
(c) (T, F) (d) (T, T)
(Online 2016)

9. The contrapositive of the following statement, "If the side of a square doubles, then its area increases four times", is
(a) If the area of a square increases four times, then its side is not doubled.
(b) If the area of a square increases four times, then its side is doubled.
(c) If the area of a square does not increase four times, then its side is not doubled.
(d) If the side of a square is not doubled, then its area does not increase four times.
(Online 2016)

10. The negation of $\sim s \vee (\sim r \wedge s)$ is equivalent to
(a) $s \vee (r \vee \sim s)$ (b) $s \wedge r$
(c) $s \wedge \sim r$ (d) $s \wedge (r \wedge \sim s)$ *(2015)*

11. The contrapositive of the statement
"If it is raining, then I will not come", is
(a) if I will come, then it is not raining
(b) if I will not come, then it is raining
(c) if I will not come, then it is not raining
(d) if I will come, then it is raining *(Online 2015)*

12. Consider the following statements :
P : Suman is brilliant.
Q : Suman is rich.
R : Suman is honest.
The negation of the statement,
"Suman is brilliant and dishonest if and only if Suman is rich" can be equivalently expressed as :
(a) $\sim Q \leftrightarrow \sim P \wedge R$ (b) $\sim Q \leftrightarrow \sim P \vee R$
(c) $\sim Q \leftrightarrow P \vee \sim R$ (d) $\sim Q \leftrightarrow P \wedge \sim R$
(Online 2015, 2011)

13. The statement $\sim (p \leftrightarrow \sim q)$ is
(a) equivalent to $\sim p \leftrightarrow q$
(b) a tautology
(c) a fallacy
(d) equivalent to $p \leftrightarrow q$ *(2014)*

14. Consider :
Statement-1 : $(p \wedge \sim q) \wedge (\sim p \wedge q)$ is a fallacy.
Statement-2 : $(p \to q) \leftrightarrow (\sim q \to \sim p)$ is a tautology.
(a) Statement-1 is true, Statement-2 is true, Statement-2 is not a correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is false.
(c) Statement-1 is false, Statement-2 is true.
(d) Statement-1 is true, Statement-2 is true, Statement-2 is a correct explanation for Statement-1. *(2013)*

15. The negation of the statement "If I become a teacher, then I will open a school", is

(a) Neither I will become a teacher nor I will open a school.
(b) I will not become a teacher or I will open a school.
(c) I will become a teacher and I will not open a school.
(d) Either I will not become a teacher or I will not open a school.

(2012)

16. Let S be a non-empty subset of R. Consider the following statement:
P : There is a rational number $x \in S$ such that $x > 0$.
Which of the following statements is the negation of the statement P ?

(a) There is a rational number $x \in S$ such that $x \leq 0$.
(b) There is no rational number $x \in S$ such that $x \leq 0$.
(c) Every rational number $x \in S$ satisfies $x \leq 0$.
(d) $x \in S$ and $x \leq 0 \Rightarrow x$ is not rational.

(2010)

17. **Statement-1 :** $\sim (p \leftrightarrow \sim q)$ is equivalent to $p \leftrightarrow q$.
Statement-2 : $\sim (p \leftrightarrow \sim q)$ is a tautology.

(a) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1
(b) Statement- 1 is true, Statement-2 is false
(c) Statement-1 is false, Statement-2 is true
(d) Statement-1 is true, Statement-2 is true; Statement-2 is correct explanation for Statement-1 *(2009)*

18. The statement $p \to (q \to p)$ is equivalent to

(a) $p \to (p \leftrightarrow q)$ (b) $p \to (p \to q)$
(c) $p \to (p \vee q)$ (d) $p \to (p \wedge q)$ *(2008)*

19. Let p be the statement "x is an irrational number", q be the statement "y is a transcendental number", and r be the statement "x is a rational number iff y is a transcendental number".
Statement-1 : r is equivalent to either q or p.
Statement-2 : r is equivalent to $\sim (p \leftrightarrow \sim q)$.

(a) Statement-1 is true, Statement-2 is false
(b) Statement-1 is false, Statement-2 is true
(c) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1
(d) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1 *(2008)*

ANSWER KEY

1. (b)	**2.** (c)	**3.** (c)	**4.** (d)	**5.** (c)	**6.** (c)	**7.** (c)	**8.** (b)	**9.** (c)	**10.** (b)	**11.** (a)	**12.** (d)
13. (d)	**14.** (a)	**15.** (c)	**16.** (c)	**17.** (b)	**18.** (c)	**19.** (a)					

Explanations

1. (b) : $\sim (p \vee q) \vee (\sim p \wedge q)$
$= (\sim p \wedge \sim q) \vee (\sim p \wedge q) = \sim p \wedge (\sim q \vee q) = \sim p$

2. (c) : Given $(p\wedge \sim q)\wedge (p\wedge r)\to \sim p\vee q$ is false

$\Rightarrow$ $(p\wedge \sim q\wedge r)\to \sim p\vee q$ is false

$\Rightarrow$ $\sim (p\wedge \sim q\wedge r)\vee(\sim p\vee q)$ is false

$\Rightarrow$ $(\sim p \vee q \vee \sim r) \vee (\sim p \vee q)$ is false

$\Rightarrow$ $\sim p\vee q\vee \sim r$ is false

So, truth values of $\sim p$, q and $\sim r$ must be F, F, F.
Thus, truth values of p, q and r must be T, F, T.

3. (c) :

p	q	$\sim p$	$\sim q$	$\sim p \vee \sim q$	$p \to (\sim p \vee \sim q)$
T	T	F	F	F	F
T	F	F	T	T	T
F	T	T	F	T	T
F	F	T	T	T	T

So, truth values of p and q are T, T.

4. (d) : We have
$(p \to q) \to [(\sim p \to q) \to q]$ simplifying as
$(p \to q) \to ((p \vee q) \to q)$
$(p \to q)((\sim p \wedge \sim q) \vee q)$
$(p \to q) \to ((\sim p \vee q) \wedge (\sim q \vee q))$
$(p \to q) \to (p \to q)$ which is a tautology.

5. (c) :

p	q	$\sim p$	$\sim q$	$p \wedge \sim q$	$(\sim p) \vee (p \wedge \sim q)$	$p \vee \sim q$	$p \to \sim q$	$q \to p$
T	T	F	F	F	F	T	F	T
T	F	F	T	T	T	T	T	T
F	T	T	F	F	T	F	T	F
F	F	T	T	F	T	T	T	T

6. (c) : Let p : Two numbers are not equal;
q : The squares of two numbers are not equal.
Then, given statement is $p \to q$
Contrapositive of $p \to q$ is $\sim q \to \sim p$
i.e., If the squares of two numbers are equal, then the numbers are equal.

7. (c) : We have $(p \wedge \sim q) \vee q \vee (\sim p \wedge q)$
$\equiv ((p \vee q) \wedge (\sim q \vee q)) \vee (\sim p \wedge q)$
$\equiv (p \vee q) \wedge (t) \vee (\sim p \wedge q) \equiv (p \vee q) \vee (\sim p \wedge q) \equiv p \vee q$

8. (b) : We have,
Contrapositive of P : If 7 is not divisible by 2, then 7 is not an odd number. $T \Rightarrow F : F(V_1)$
Contrapositive of Q : If 7 is not an odd number, then 7 is not a prime number. $F \Rightarrow F : T\ (V_2)$

9. (c) : Contrapositive of $p \to q$ is given by $\sim q \to \sim p$

10. (b) : Using the rules of logic, we have $\sim s \vee (\sim r \wedge s)$
$= (\sim s \vee \sim r) \wedge (\sim s \wedge s) = (\sim s \vee \sim r) \wedge t = \sim s \vee \sim r$
Now the negation of above is $\sim (\sim s \vee \sim r) = s \wedge r$

11. (a) : The contrapositive of the statement is "If I will come, then it is not raining".

12. (d) : The negation of the statement "Suman is brilliant and dishonest iff suman is rich" is $\sim Q \leftrightarrow P \wedge \sim R$

13. (d): See the following truth table.

p	q	$\sim q$	$p \leftrightarrow \sim q$	$\sim (p \leftrightarrow \sim q)$	$p \leftrightarrow q$
T	T	F	F	T	T
T	F	T	T	F	F
F	T	F	T	F	F
F	F	T	F	T	T

As the truth table matches, we have the statement $\sim (p \leftrightarrow \sim q)$ is equivalent to $p \leftrightarrow q$

14. (a) : 1st solution : Let's prepare the truth table for the statements.

p	q	$\sim p$	$\sim q$	$p \wedge \sim q$	$\sim p \wedge q$	$(p \wedge \sim q) \wedge (\sim p \wedge q)$
T	T	F	F	F	F	F
T	F	F	T	T	F	F
F	T	T	F	F	T	F
F	F	T	T	F	F	F

Then Statement-1 is fallacy.

p	q	$\sim p$	$\sim q$	$p \to q$	$\sim q \to p$	$(p \to q) \to (\sim q \to p)$
T	T	F	F	T	T	T
T	F	F	T	F	F	T
F	T	T	F	T	T	T
F	F	T	T	T	T	T

Then Statement-2 is tautology.

2nd solution : $\sim (\sim p \vee q) \wedge \sim (\sim q \vee p)$
$\equiv \sim ((\sim p \vee q) \vee (\sim q \vee p)) \equiv \sim ((p \to q) \vee (q \to p)) \equiv \sim T$
Thus Statement-1 is true because its negation is false.
$((p \to q) \to (\sim q \to \sim p)) \wedge ((\sim q \to \sim p) \to (p \to q))$
$= ((\sim p \vee q) \to (q \vee \sim p)) \wedge ((q \vee \sim p) \to (\sim p \vee q))$
$\equiv T \wedge T \equiv T.$ Then Statement-2 is true.

15. (c) : The given statement is
"If I become a teacher, then I will open a school"
Negation of the given statement is
" I will become a teacher and I will not open a school"
$(\because \sim (p \to q) = p \wedge \sim q)$

16. (c) : The given statement is
P : at least one rational $x \in S$ such that $x > 0$.
The negation would be : There is no rational number $x \in S$ such that $x > 0$ which is equivalent to all rational numbers $x \in S$ satisfy $x \le 0$.

17. (b) : Let's prepare the truth table

p	q	$\sim q$	$p \leftrightarrow q$	$p \leftrightarrow \sim q$	$\sim (p \leftrightarrow \sim q)$
T	T	F	T	F	T
T	F	T	F	T	F
F	T	F	F	T	F
F	F	T	T	F	T

As the column for $\sim (p \leftrightarrow \sim q)$ and $(p \leftrightarrow q)$ is the same, we conclude that $\sim (p \leftrightarrow \sim q)$ is equivalent to $(p \leftrightarrow q)$.
$\sim (p \leftrightarrow \sim q)$ is NOT a tautology because it's statement value is not always true.

18. (c) : Let's simplify the statement
$p \to (q \to p) = \sim p \vee (q \to p) = \sim p \vee (\sim q \vee p)$
$= \sim p \vee p \vee \sim q = p \to (p \vee q)$

19. (a) : The given statement $r \equiv \sim p \leftrightarrow q$
The Statement-1 is $r_1 \equiv (p \wedge \sim q) \vee (\sim p \wedge q)$
The Statement-2 is $r_2 \equiv \sim (p \leftrightarrow \sim q) = (p \wedge q) \vee (\sim q \wedge \sim p)$
we can establish that $r = r_1$