CHAPTER

# 17 Trigonometry

**1.** If sum of all the solutions of the equation

$$8\cos x\cdot\left(\cos\left(\frac{\pi}{6}+x\right)\cdot\cos\left(\frac{\pi}{6}-x\right)-\frac{1}{2}\right)=1$$

in $[0, \pi]$ is $k\pi$, then $k$ is equal to

(a) $\frac{20}{9}$ (b) $\frac{2}{3}$ (c) $\frac{13}{9}$ (d) $\frac{8}{9}$

*(2018)*

**2.** $PQR$ is a triangular park with $PQ = PR = 200$ m. A T.V. tower stands at the mid-point of $QR$. If the angles of elevation of the top of the tower at $P$, $Q$ and $R$ are respectively 45°, 30° and 30°, then the height of the tower (in m) is

(a) $50\sqrt{2}$ (b) 100 (c) 50 (d) $100\sqrt{3}$

*(2018)*

**3.** If $\tan A$ and $\tan B$ are the roots of the quadratic equation, $3x^2 - 10x - 25 = 0$, then the value of $3\sin^2(A + B) - 10\sin(A + B)\cdot\cos(A + B) - 25\cos^2(A + B)$ is

(a) 10 (b) –10 (c) 25 (d) –25

*(Online 2018)*

**4.** An aeroplane flying at a constant speed, parallel to the horizontal ground, $\sqrt{3}$ km above it, is observed at an elevation of 60° from a point on the ground. If, after five seconds, its elevation from the same point, is 30°, then the speed (in km/hr) of the aeroplane, is

(a) 750 (b) 1440 (c) 1500 (d) 720

*(Online 2018)*

**5.** Consider the following two statements :

Statement $p$ : The value of sin120° can be derived by taking $\theta = 240°$ in the equation $2\sin\frac{\theta}{2}=\sqrt{1+\sin\theta}-\sqrt{1-\sin\theta}$

Statement $q$ : The angles $A$, $B$, $C$ and $D$ of any quadrilateral $ABCD$ satisfy the equation

$$\cos\left(\frac{1}{2}(A+C)\right)+\cos\left(\frac{1}{2}(B+D)\right)=0$$

Then the truth value of $p$ and $q$ are respectively :

(a) $T, T$ (b) $F, F$ (c) $F, T$ (d) $T, F$

*(Online 2018)*

**6.** A tower $T_1$ of height 60 m is located exactly opposite to a tower $T_2$ of height 80 m on a straight road. From the top of $T_1$, if the angle of depression of the foot of $T_2$ is twice the angle of elevation of the top of $T_2$, then the width (in m) of the road between the feet of the towers $T_1$ and $T_2$ is

(a) $20\sqrt{3}$ (b) $10\sqrt{3}$ (c) $10\sqrt{2}$ (d) $20\sqrt{2}$

*(Online 2018)*

**7.** The number of solutions of $\sin 3x = \cos 2x$, in the interval $\left(\frac{\pi}{2},\pi\right)$ is

(a) 2 (b) 4 (c) 3 (d) 1

*(Online 2018)*

**8.** A man on the top of a vertical tower observes a car moving at a uniform speed towards the tower on a horizontal road. If it takes 18 min. for the angle of depression of the car to change from 30° to 45°; then after this, the time taken (in min.) by the car to reach the foot of the tower, is :

(a) $9(1+\sqrt{3})$ (b) $18(\sqrt{3}-1)$

(c) $\frac{9}{2}(\sqrt{3}-1)$ (d) $18(1+\sqrt{3})$

*(Online 2018)*

**9.** If an angle $A$ of a $\Delta ABC$ satisfies $5\cos A + 3 = 0$, then the roots of the quadratic equation, $9x^2 + 27x + 20 = 0$ are :

(a) $\sin A$, $\sec A$ (b) $\sec A$, $\cot A$

(c) $\sec A$, $\tan A$ (d) $\tan A$, $\cos A$ *(Online 2018)*

**10.** If $5(\tan^2 x - \cos^2 x) = 2\cos 2x + 9$, then the value of $\cos 4x$ is

(a) $\frac{1}{3}$ (b) $\frac{2}{9}$ (c) $-\frac{7}{9}$ (d) $-\frac{3}{5}$

*(2017)*

**11.** Let a vertical tower $AB$ have its end $A$ on the level ground. Let $C$ be the mid-point of $AB$ and $P$ be a point on the ground such that $AP = 2AB$. If $\angle BPC = \beta$, then $\tan\beta$ is equal to

(a) $\frac{1}{4}$ (b) $\frac{2}{9}$ (c) $\frac{4}{9}$ (d) $\frac{6}{7}$

*(2017)*

**12.** The value of $\tan^{-1}\left[\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right], |x|<\frac{1}{2}, x\neq 0,$ is equal to

(a) $\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x^2$ (b) $\frac{\pi}{4}+\frac{1}{2}\cos^{-1}x^2$

(c) $\frac{\pi}{4}-\cos^{-1}x^2$ (d) $\frac{\pi}{4}+\cos^{-1}x^2$

*(Online 2017)*

**13.** A value of $x$ satisfying the equation $\sin[\cot^{-1}(1 + x)] = \cos[\tan^{-1}x]$, is

(a) $\frac{1}{2}$ (b) 0 (c) –1 (d) $-\frac{1}{2}$

*(Online 2017)*

**14.** The two adjacent sides of a cyclic quadrilateral are 2 and 5 and the angle between them is 60°. If the area of the quadrilateral is $4\sqrt{3}$, then the perimeter of the quadrilateral is

(a) 12 (b) 12.5 (c) 13 (d) 13.2

*(Online 2017)*

**15.** If $0 \le x < 2\pi$, then the number of real values of $x$, which satisfy the equation
$\cos x + \cos 2x + \cos 3x + \cos 4x = 0$, is

(a) 3 (b) 5 (c) 7 (d) 9

*(2016)*

**16.** A man is walking towards a vertical pillar in a straight path, at a uniform speed. At a certain point $A$ on the path, he observes that the angle of elevation of the top of the pillar is 30°. After walking for 10 minutes from $A$ in the same direction, at a point $B$, he observes that the angle of elevation of the top of the pillar is 60°. Then the time taken (in minutes) by him, from $B$ to reach the pillar is

(a) 6 (b) 10 (c) 20 (d) 5

*(2016)*

**17.** The number of $x \in [0, 2\pi]$ for which
$\left|\sqrt{2\sin^4 x + 18\cos^2 x} - \sqrt{2\cos^4 x + 18\sin^2 x}\right| = 1$ is

(a) 2 (b) 6 (c) 4 (d) 8

*(Online 2016)*

**18.** The angle of elevation of the top of a vertical tower from a point $A$, due east of it is 45°. The angle of elevation of the top of the same tower from a point $B$, due south of $A$ is 30°. If the distance between $A$ and $B$ is $54\sqrt{2}$ m, then the height of the tower (in metres), is

(a) 108 (b) $36\sqrt{3}$ (c) $54\sqrt{3}$ (d) 54

*(Online 2016)*

**19.** If $A > 0$, $B > 0$ and $A + B = \frac{\pi}{6}$, then the minimum value of $\tan A + \tan B$ is

(a) $\sqrt{3} - \sqrt{2}$ (b) $4 - 2\sqrt{3}$

(c) $\frac{2}{\sqrt{3}}$ (d) $2 - \sqrt{3}$ *(Online 2016)*

**20.** Let $P = \{\theta : \sin\theta - \cos\theta = \sqrt{2}\cos\theta\}$ and $Q = \{\theta : \sin\theta + \cos\theta = \sqrt{2}\sin\theta\}$ be two sets. Then

(a) $P \subset Q$ and $Q - P \neq \phi$ (b) $Q \not\subset P$

(c) $P = Q$ (d) $P \not\subset Q$ *(Online 2016)*

**21.** If the angles of elevation of the top of a tower from three collinear points $A$, $B$ and $C$, on a line leading to the foot of the tower are 30°, 45° and 60° respectively, then the ratio, $AB : BC$ is

(a) $1 : \sqrt{8}$ (b) 2 : 3 (c) $\sqrt{8} : 1$ (d) $\sqrt{8}$ A$\sqrt{7}$

*(2015)*

**22.** Let μπα $^{-6}$ = μπα $^{-6}$ + μπα $^{-6}\left(\frac{7}{6- \;7}\right)$, where $< \frac{6}{\sqrt{8}}3$

Then a value of $y$ is

(a) $\frac{8 \;-\; ^8}{6+8\;^7}$ (b) $\frac{8 \;+\; ^8}{6+8\;^7}$

(c) $\frac{8 \;-\; ^8}{6-8\;^7}$ (d) $\frac{8 \;+\; ^8}{6-8\;^7}$ *(2015)*

**23.** In a $\Delta ABC$, $- = 7 + \sqrt{8}$ and $\angle C = 60°$. Then the ordered pair $(\angle A, \angle B)$ is equal to

(a) (15°, 105°) (b) (105°, 15°)

(c) (45°, 75°) (d) (75°, 45°) *(Online 2015)*

**24.** If - . = 7 μπα $^{-6}$ + -ιε $^{-6}\left(\frac{7}{6+\;^7}\right)$1 >6, then $f(5)$ is equal to

(a) $\pi/2$ (b) $\pi$

(c) $4\tan^{-1}(5)$ (d) μπα $^{-6}\left(\frac{;:}{6:;}\right)$

*(Online 2015)*

**25.** If ο{ −α + ο{ −β = $\frac{8}{7}$ πα p -ιε α + -ιε β = $\frac{6}{7}$ and θ is the arithmetic mean of α and β, then sin 2θ + cos 2θ is equal to

(a) $\frac{8}{:}$ (b) $\frac{9}{:}$ (c) $\frac{<}{:}$ (d) $\frac{=}{:}$

*(Online 2015)*

**26.** Let 10 vertical poles standing at equal distances on a straight line, subtend the same angle of elevation α at a point $O$ on this line and all the poles are on the same side of $O$. If the height of the longest pole is '$h$' and the distance of the foot of the smallest pole from $O$ is '$a$'; then the distance between two consecutive poles is

(a) $\frac{\text{-ιε α + ο\{ −α}}{\text{>-ιε α}}$ (b) $\frac{\text{ο\{ −α − -ιε α}}{\text{>ο\{ −α}}$

(c) $\frac{\text{ο\{ −α − -ιε α}}{\text{>-ιε α}}$ (d) $\frac{\text{-ιε α + ο\{ −α}}{\text{>ο\{ −α}}$

*(Online 2015)*

**27.** Let $f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x)$ where $x \in R$ and $k \ge 1$. Then $f_4(x) - f_6(x)$ equals

(a) $\frac{1}{3}$ (b) $\frac{1}{4}$ (c) $\frac{1}{12}$ (d) $\frac{1}{6}$

*(2014)*

**28.** A bird is sitting on the top of a vertical pole 20 m high and its elevation from a point $O$ on the ground is 45°. It flies off horizontally straight away from the point $O$. After one second, the elevation of the bird from $O$ is reduced to 30°. Then the speed (in m/s) of the bird is

(a) $40(\sqrt{3} - \sqrt{2})$ (b) $20\sqrt{2}$

(c) $20(\sqrt{3} - 1)$ (d) $40(\sqrt{2} - 1)$ *(2014)*

**29.** If $x$, $y$, $z$ are in A.P. and $\tan^{-1}x$, $\tan^{-1}y$ and $\tan^{-1}z$ are also in A.P., then

(a) $2x = 3y = 6z$ (b) $6x = 3y = 2z$

(c) $6x = 4y = 3z$ (d) $x = y = z$ *(2013)*

**30.** *ABCD* is a trapezium such that *AB* and *CD* are parallel and $BC \perp CD$. If $\angle ADB = \theta$, $BC = p$ and $CD = q$, then *AB* is equal to

(a) $\dfrac{p^2 + q^2\cos\theta}{p\cos\theta + q\sin\theta}$ (b) $\dfrac{p^2 + q^2}{p^2\cos\theta + q^2\sin\theta}$

(c) $\dfrac{(p^2 + q^2)\sin\theta}{(p\cos\theta + q\sin\theta)^2}$ (d) $\dfrac{(p^2 + q^2)\sin\theta}{p\cos\theta + q\sin\theta}$ *(2013)*

**31.** The expression $\dfrac{\tan A}{1-\cot A} + \dfrac{\cot A}{1-\tan A}$ can be written as

(a) $\sec A \operatorname{cosec} A + 1$ (b) $\tan A + \cot A$

(c) $\sec A + \operatorname{cosec} A$ (d) $\sin A \cos A + 1$ *(2013)*

**32.** In a $\Delta PQR$, if $3\sin P + 4\cos Q = 6$ and $4\sin Q + 3\cos P = 1$, then the angle *R* is equal to

(a) $\pi/4$ (b) $3\pi/4$ (c) $5\pi/6$ (d) $\pi/6$ *(2012)*

**33.** If $A = \sin^2 x + \cos^4 x$, then for all real *x*

(a) $1 \le A \le 2$ (b) $\dfrac{3}{4} \le A \le \dfrac{13}{16}$

(c) $\dfrac{3}{4} \le A \le 1$ (d) $\dfrac{13}{16} \le A \le 1$ *(2011)*

**34.** Let $\cos(\alpha + \beta) = \dfrac{4}{5}$ and let $\sin(\alpha - \beta) = \dfrac{5}{13}$, where $0 \le \alpha, \beta \le \dfrac{\pi}{4}$. Then $\tan 2\alpha =$

(a) $\dfrac{25}{16}$ (b) $\dfrac{56}{33}$

(c) $\dfrac{19}{12}$ (d) $\dfrac{20}{17}$ *(2010)*

**35.** For a regular polygon, let *r* and *R* be the radii of the inscribed and the circumscribed circles. A false statement among the following is

(a) there is a regular polygon with $r/R = 1/2$

(b) there is a regular polygon with $r/R = 1/\sqrt{2}$

(c) there is a regular polygon with $r/R = 2/\sqrt{3}$

(d) there is a regular polygon with $r/R = \sqrt{3}/2$ *(2010)*

**36.** Let *A* and *B* denote the statements

*A* : $\cos\alpha + \cos\beta + \cos\gamma = 0$

*B* : $\sin\alpha + \sin\beta + \sin\gamma = 0$

If $\cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta) = -\dfrac{3}{2}$, then

(a) *A* is false and *B* is true

(b) both *A* and *B* are true

(c) both *A* and *B* are false

(d) *A* is true and *B* is false *(2009)*

**37.** *AB* is a vertical pole with *B* at the ground level and *A* at the top. A man finds that the angle of elevation of the point *A* from a certain point *C* on the ground is 60°. He moves away from the pole along the line *BC* to a point *D* such that $CD = 7$ m. From *D* the angle of elevation of the point *A* is 45°. Then the height of the pole is

(a) $\dfrac{7\sqrt{3}}{2}\dfrac{1}{\sqrt{3}+1}$ m (b) $\dfrac{7\sqrt{3}}{2}\dfrac{1}{\sqrt{3}-1}$ m

(c) $\dfrac{7\sqrt{3}}{2}(\sqrt{3}+1)$ m (d) $\dfrac{7\sqrt{3}}{2}(\sqrt{3}-1)$ m *(2008)*

**38.** The value of $\cot\left(\operatorname{cosec}^{-1}\dfrac{5}{3} + \tan^{-1}\dfrac{2}{3}\right)$ is

(a) $\dfrac{5}{17}$ (b) $\dfrac{6}{17}$ (c) $\dfrac{3}{17}$ (d) $\dfrac{4}{17}$ *(2008)*

**39.** A tower stands at the centre of a circular park. *A* and *B* are two points on the boundary of the park such that *AB* (= *a*) subtends an angle of 60° at the foot of the tower, and the angle of elevation of the top of the tower from *A* or *B* is 30°. The height of the tower is

(a) $a/\sqrt{3}$ (b) $a\sqrt{3}$

(c) $2a/\sqrt{3}$ (d) $2a\sqrt{3}$ *(2007)*

**40.** The largest interval lying in $\left(\dfrac{-\pi}{2}, \dfrac{\pi}{2}\right)$ for which the function,

$f(x) = 4^{-x^2} + \cos^{-1}\left(\dfrac{x}{2} - 1\right) + \log(\cos x)$ is defined, is

(a) $\left[-\dfrac{\pi}{4}, \dfrac{\pi}{2}\right)$ (b) $\left[0, \dfrac{\pi}{2}\right)$

(c) $[0, \pi]$ (d) $\left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$ *(2007)*

**41.** If $\sin^{-1}\left(\dfrac{x}{5}\right) + \operatorname{cosec}^{-1}\left(\dfrac{5}{4}\right) = \dfrac{\pi}{2}$, then the value of *x* is

(a) 4 (b) 5 (c) 1 (d) 3. *(2007)*

**42.** If $0 < x < \pi$ and $\cos x + \sin x = 1/2$, then $\tan x$ is

(a) $\dfrac{(1-\sqrt{7})}{4}$ (b) $\dfrac{(4-\sqrt{7})}{3}$

(c) $-\dfrac{(4+\sqrt{7})}{3}$ (d) $\dfrac{(1+\sqrt{7})}{4}$ *(2006)*

**43.** The number of value of *x* in the interval $[0, 3\pi]$ satisfying the equation $2\sin^2 x + 5\sin x - 3 = 0$ is

(a) 4 (b) 6 (c) 1 (d) 2 *(2006)*

**44.** If in a $\Delta ABC$, the altitudes from the vertices *A, B, C* on opposite sides are in H.P., then $\sin A$, $\sin B$, $\sin C$ are in

(a) H.P.

(b) Arithmetic-Geometric progression

(c) A.P. (d) G.P. *(2005)*

**45.** If $\cos^{-1} x - \cos^{-1}\dfrac{y}{2} = \alpha$, then $4x^2 - 4xy\cos\alpha + y^2$ is equal to

(a) 4 (b) $2\sin 2\alpha$

(c) $-4\sin^2\alpha$ (d) $4\sin^2\alpha$ *(2005)*

**46.** In a triangle *ABC*, let $\angle C = \dfrac{\pi}{2}$. If *r* is the inradius and *R* is the circumradius of the triangle *ABC*, then $2(r + R)$ equals

(a) $a + b$ (b) $b + c$

(c) $c + a$ (d) $a + b + c$ *(2005)*

**47.** In a triangle $PQR$, if $\angle R = \frac{\pi}{2}$. If $\tan\left(\frac{P}{2}\right)$ and $\tan\left(\frac{Q}{2}\right)$ are the roots of $ax^2 + bx + c = 0$, $a \neq 0$ then
(a) $b = a + c$ (b) $b = c$
(c) $c = a + b$ (d) $a = b + c$ *(2005)*

**48.** A person standing on the bank of a river observes that the angle of elevation of the top of a tree on the opposite bank of the river is 60º and when he retires 40 meters away from the tree the angle of elevation becomes 30º. The breadth of the river is
(a) 40 m (b) 30 m
(c) 20 m (d) 60 m *(2004)*

**49.** The sides of a triangle are $\sin\alpha$, $\cos\alpha$ and $\sqrt{1+\sin\alpha\cos\alpha}$ for some $0 < \alpha < \frac{\pi}{2}$. Then the greatest angle of the triangle is
(a) 120º (b) 90º
(c) 60º (d) 150º *(2004)*

**50.** If $u = \sqrt{a^2\cos^2\theta + b^2\sin^2\theta} + \sqrt{a^2\sin^2\theta + b^2\cos^2\theta}$ then the difference between the maximum and minimum values of $u^2$ is given by
(a) $(a + b)^2$ (b) $2\sqrt{a^2+b^2}$
(c) $2(a^2 + b^2)$ (d) $(a - b)^2$ *(2004)*

**51.** Let $\alpha$, $\beta$ be such that $\pi < \alpha - \beta < 3\pi$. If $\sin\alpha + \sin\beta = -21/65$, and $\cos\alpha + \cos\beta = -27/65$, then the value of $\cos\frac{\alpha-\beta}{2}$ is
(a) $\frac{6}{65}$ (b) $\frac{3}{\sqrt{130}}$ (c) $-\frac{3}{\sqrt{130}}$ (d) $\frac{-6}{65}$ *(2004)*

**52.** If in a triangle $ABC$, $a\cos^2\left(\frac{C}{2}\right) + c\cos^2\left(\frac{A}{2}\right) = \frac{3b}{2}$, then the sides $a$, $b$ and $c$
(a) are in G.P. (b) are in H.P.
(c) satisfy $a + b = c$ (d) are in A.P. *(2003)*

**53.** In a triangle $ABC$, medians $AD$ and $BE$ are drawn. If $AD = 4$, $\angle DAB = \pi/6$ and $\angle ABE = \pi/3$, then the area of the $\Delta ABC$ is
(a) 16/3 (b) 32/3
(c) 64/3 (d) 8/3 *(2003)*

**54.** The upper 3/4$^{th}$ portion of a vertical pole subtends an angle $\tan^{-1}(3/5)$ at a point in the horizontal plane through its foot and at a distance 40 m from the foot. A possible height of the vertical pole is
(a) 40 m (b) 60 m
(c) 80 m (d) 20 m *(2003)*

**55.** The sum of the radii of inscribed and circumscribed circles for an $n$ sides regular polygon of side $a$, is
(a) $\frac{a}{2}\cot\left(\frac{\pi}{2n}\right)$ (b) $a\cot\left(\frac{\pi}{2n}\right)$
(c) $\frac{a}{4}\cot\left(\frac{\pi}{2n}\right)$ (d) $a\cot\left(\frac{\pi}{n}\right)$ *(2003)*

**56.** The trigonometric equation $\sin^{-1}x = 2\sin^{-1}a$, has a solution for
(a) all real values (b) $|a| \le \frac{1}{\sqrt{2}}$
(c) $|a| \ge \frac{1}{\sqrt{2}}$ (d) $\frac{1}{2} < |a| < \frac{1}{\sqrt{2}}$ *(2003)*

**57.** In a triangle with sides $a, b, c$, $r_1 > r_2 > r_3$ (which are the ex-radii) then
(a) $a > b > c$ (b) $a < b < c$
(c) $a > b$ and $b < c$ (d) $a < b$ and $b > c$ *(2002)*

**58.** $\cot^{-1}\left[(\cos\alpha)^{\frac{1}{2}}\right] + \tan^{-1}\left[(\cos\alpha)^{\frac{1}{2}}\right] = x$
then $\sin x =$
(a) 1 (b) $\cot^2(\alpha/2)$
(c) $\tan\alpha$ (d) $\cot(\alpha/2)$ *(2002)*

**59.** The number of solutions of $\tan x + \sec x = 2\cos x$ in $[0, 2\pi)$ is
(a) 2 (b) 3 (c) 0 (d) 1 *(2002)*

## ANSWER KEY

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **1.** (c) | **2.** (b) | **3.** (d) | **4.** (b) | **5.** (c) | **6.** (a) | **7.** (d) | **8.** (a) | **9.** (c) | **10.** (c) | **11.** (b) | **12.** (b) |
| **13.** (d) | **14.** (a) | **15.** (c) | **16.** (d) | **17.** (d) | **18.** (d) | **19.** (b) | **20.** (c) | **21.** (c) | **22.** (c) | **23.** (b) | **24.** (b) |
| **25.** (c) | **26.** (c) | **27.** (c) | **28.** (c) | **29.** (d) | **30.** (d) | **31.** (a) | **32.** (d) | **33.** (c) | **34.** (b) | **35.** (c) | **36.** (b) |
| **37.** (c) | **38.** (b) | **39.** (a) | **40.** (b) | **41.** (d) | **42.** (c) | **43.** (a) | **44.** (c) | **45.** (d) | **46.** (a) | **47.** (c) | **48.** (c) |
| **49.** (a) | **50.** (d) | **51.** (c) | **52.** (d) | **53.** (b) | **54.** (a) | **55.** (a) | **56.** (b) | **57.** (a) | **58.** (a) | **59.** (b) | |

# Explanations

**1.** **(c) :** Note that $\cos(A+B)\cos(A-B) = \cos^2 A - \sin^2 B$

We have, $8\cos x\,\{\cos(\pi/6 + x)\cos(\pi/6 - x) - 1/2\} = 1$

$$\Rightarrow\ 8\cos x\left\{\cos^2\frac{\pi}{6} - \sin^2 x - 1/2\right\} = 1$$

$$\Rightarrow\ 8\cos x\left(\frac{3}{4} - 1 + \cos^2 x - \frac{1}{2}\right) = 1 \Rightarrow\ 8\cos x(\cos^2 x - 3/4) = 1$$

$$\Rightarrow\ 8\cos x\frac{(4\cos^2 x - 3)}{4} = 1 \Rightarrow (4\cos^3 x - 3\cos x) = \frac{1}{2}$$

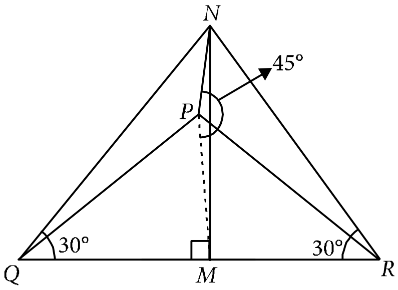
$$\Rightarrow\ \cos 3x = \frac{1}{2}$$

As $x \in [0, \pi] \Rightarrow 3x \in [0, 3\pi]$ which gives

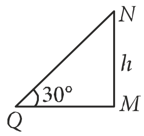
$3x = \pi/3, 5\pi/3, 7\pi/3 \ \therefore\ x = \pi/9, 5\pi/9, 7\pi/9$

which gives sum of all values of $x = \dfrac{13\pi}{9}$ $\quad\therefore\ k = \dfrac{13}{9}$

**2.** **(b) :**




Let the height of tower $MN$ be $h$.

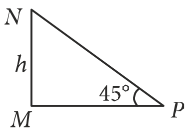


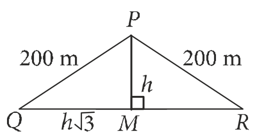

The triangle $NMQ$ gives

$QM = h\sqrt{3}$, as $\tan 30° = \dfrac{1}{\sqrt{3}}$

The triangle $NMP$ gives

$PM = h$




As $\Delta PQR$ is isosceles, $PM$ is also an altitude.

$\therefore\ PM^2 + QM^2 = PQ^2$ gives $4h^2 = (200)^2 \Rightarrow h = 100$

**3.** **(d) :** We have, $3x^2 - 10x - 25 = 0$ ...(i)

Since $\tan A$ and $\tan B$ are roots of (i)

$\therefore\ \tan A + \tan B = \dfrac{10}{3}$ and $\tan A\tan B = \dfrac{-25}{3}$

Now, $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A\tan B} = \dfrac{\frac{10}{3}}{1 + \frac{25}{3}} = \dfrac{5}{15}$

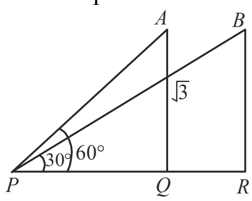
$\therefore\ \sin(A+B) = \dfrac{5}{\sqrt{221}}$ and $\cos(A+B) = \dfrac{14}{\sqrt{221}}$

Now, $3\sin^2(A+B) - 10\sin(A+B)\cos(A+B) - 25\cos^2(A+B)$

$$= \frac{3\times 25}{221} - \left(\frac{10\times 5\times 14}{221}\right) - \left(\frac{25\times 14\times 14}{221}\right)$$

$$= \frac{25}{221}(3 - 28 - 196) = -25$$

**4.** **(b) :** Let $A$ and $B$ be the two positions of aeroplane observed from point $P$.




Given, $AQ = \sqrt{3}$ km

In $\Delta APQ$, $\tan 60° = \dfrac{AQ}{PQ}$

$\Rightarrow\ \sqrt{3} = \dfrac{\sqrt{3}}{PQ} \Rightarrow PQ = 1$ km

In $\Delta BPR$, $\tan 30° = \dfrac{BR}{PR} \Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{\sqrt{3}}{PR}$ $(\because\ AQ = BR = \sqrt{3}$ km$)$

$\Rightarrow\ PR = 3$ km

Now, $QR = PR - PQ = (3 - 1)$ km $= 2$ km

Thus, aeroplane covers 2 km in 5 seconds.

$\therefore$ Speed of aeroplane $= \dfrac{2}{5}$ km/sec.

$$= \left(\frac{2}{5}\times 3600\right)\text{km/hr} = 1440 \text{ km/hr}$$

**5.** **(c) :** Statement $p$ : Given, $2\sin\dfrac{\theta}{2} = \sqrt{1+\sin\theta} - \sqrt{1-\sin\theta}$

Putting $\theta = 240°$ in R.H.S, we get $\sqrt{1+\sin\theta} - \sqrt{1-\sin\theta}$

$$= \sqrt{1+\sin(180°+60°)} - \sqrt{1-\sin(180°+60°)}$$

$$= \sqrt{1-\sin 60°} - \sqrt{1+\sin 60°}$$

$$= \sqrt{1-\frac{\sqrt{3}}{2}} - \sqrt{1+\frac{\sqrt{3}}{2}} = \frac{\sqrt{2-\sqrt{3}}}{\sqrt{2}} - \frac{\sqrt{2+\sqrt{3}}}{\sqrt{2}}$$

$$= \frac{\sqrt{2-\sqrt{3}} - \sqrt{2+\sqrt{3}}}{\sqrt{2}} = \frac{\sqrt{2-\sqrt{3}} - \frac{1}{\sqrt{2-\sqrt{3}}}}{\sqrt{2}}$$

$$= \frac{2-\sqrt{3}-1}{\sqrt{4-2\sqrt{3}}} = \frac{1-\sqrt{3}}{\sqrt{(\sqrt{3}-1)^2}} = \frac{1-\sqrt{3}}{\sqrt{3}-1} = -1$$

$$\text{L.H.S.} = 2\sin\frac{\theta}{2} = 2\sin 120° = \sqrt{3}$$

Thus, L.H.S $\neq$ R.H.S. $\therefore$ Statement $p$ is false.

Statement $q$ :

We know that $A + B + C + D = 360°$ (Angle sum property of a quadrilateral)

$$\Rightarrow\ A + C = 360° - (B + D) \Rightarrow\ \frac{1}{2}(A+C) = 180° - \frac{1}{2}(B+D)$$

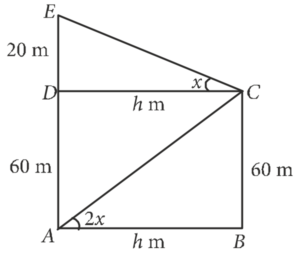
$$\Rightarrow\ \cos\left(\frac{1}{2}(A+C)\right) = \cos\left(180° - \frac{1}{2}(B+D)\right)$$

$$\Rightarrow\ \cos\left(\frac{1}{2}(A+C)\right) + \cos\left(\frac{1}{2}(B+D)\right) = 0$$

Hence, statement $q$ is true.

**6. (a) :** Let $AE$ be the tower $T_2$ of height 80 m and $BC$ be the tower $T_1$ of height 60 m.

Let $AB = CD = h$ m




In $\Delta EDC$, we have

$$\frac{ED}{CD} = \tan x \Rightarrow \frac{20}{h} = \tan x$$

$$\Rightarrow h = \frac{20}{\tan x} \quad \ldots \text{(i)}$$

In $\Delta ABC$, we have $\frac{BC}{AB} = \tan 2x$

$$\Rightarrow \frac{60}{h} = \tan 2x \Rightarrow \frac{60}{h} = \frac{2\tan x}{1-\tan^2 x}$$

$$\Rightarrow \frac{30}{h} = \frac{\tan x}{1-\tan^2 x} \Rightarrow h = \frac{30(1-\tan^2 x)}{\tan x} \quad \ldots \text{(ii)}$$

From (i) and (ii), we have $\frac{30(1-\tan^2 x)}{\tan x} = \frac{20}{\tan x}$

$\Rightarrow$ $3 - 3\tan^2 x = 2$ $\Rightarrow$ $1 = 3\tan^2 x$

$$\Rightarrow \tan x = \pm\frac{1}{\sqrt{3}} \Rightarrow x = \frac{\pi}{6} \text{ or } \frac{5\pi}{6}$$

So putting the value of $x$ in (i), we have

$$h = \frac{20}{\tan(\pi/6)} \text{ or } h = 20\sqrt{3} \text{ m.}$$

**7. (d) :** We have $\sin 3x = \cos 2x$

$\Rightarrow$ $3\sin x - 4\sin^3 x = 1 - 2\sin^2 x$

$\Rightarrow$ $4\sin^3 x - 2\sin^2 x - 3\sin x + 1 = 0$

$\Rightarrow$ $(\sin x - 1)(4\sin^2 x + 2\sin x - 1) = 0$

Now, $\sin x \neq 1$ $\left[\because x \in \left(\frac{\pi}{2}, \pi\right)\right]$

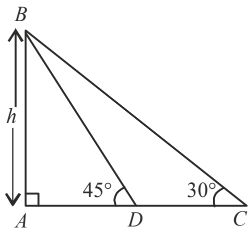
$\therefore$ $4\sin^2 x + 2\sin x - 1 = 0$

$$\Rightarrow \sin x = \frac{-2 \pm \sqrt{4+16}}{2\times 4} = \frac{-2\pm 2\sqrt{5}}{8} = \frac{-1\pm\sqrt{5}}{4}$$

But $\sin x \neq \frac{-1-\sqrt{5}}{4}$ $\left[\because x \in \left(\frac{\pi}{2}, \pi\right)\right]$

$\therefore$ $\sin x = \frac{-1+\sqrt{5}}{4}$, which is the only solution.

**8. (a) :** Let the length of tower be $h$ *i.e.*, $AB = h$




In $\Delta BAD$, we have $\frac{AB}{AD} = \tan 45°$

$\Rightarrow$ $AB = AD \Rightarrow AD = h$

Now, in $\Delta BAC$

$$\frac{AB}{AC} = \tan 30°$$

$$\Rightarrow \frac{h}{AD+CD} = \tan 30° \Rightarrow \frac{h}{h+CD} = \frac{1}{\sqrt{3}}$$

$\Rightarrow$ $CD = (\sqrt{3}-1)h$

Time taken by car to reach point $D$ from $C$ = 18 min

$$\therefore \text{ Speed of car } = \frac{CD}{\text{Time taken to cover } CD} = \frac{(\sqrt{3}-1)h}{18}$$

$$\text{So, time taken to cover } DA = \frac{DA}{\text{Speed of car}}$$

$$= \frac{h}{\left(\frac{\sqrt{3}-1}{18}\right)h} = \left(\frac{18}{\sqrt{3}-1}\right) = 9(\sqrt{3}+1) \text{ min.}$$

**9. (c) :** $5\cos A + 3 = 0$

$\Rightarrow$ $\cos A = \frac{-3}{5}$ [Clearly $A \in (90°, 180°)$] $\Rightarrow \sec A = \frac{-5}{3}$

Now, $9x^2 + 27x + 20 = 0$

$\Rightarrow$ $9x^2 + 15x + 12x + 20 = 0 \Rightarrow 3x(3x+5) + 4(3x+5) = 0$

$\Rightarrow$ $(3x+4)(3x+5) = 0 \Rightarrow x = \frac{-4}{3}$ or $x = \frac{-5}{3}$

Now, $\tan^2 A + 1 = \sec^2 A \Rightarrow \tan^2 A = \frac{25}{9} - 1$

$$\Rightarrow \tan^2 A = \frac{16}{9} \Rightarrow \tan A = \pm\frac{4}{3} \Rightarrow \tan A = -\frac{4}{3}$$

So, the roots are $\tan A$ and $\sec A$.

**10. (c):** $5(\tan^2 x - \cos^2 x) = 2\cos 2x + 9$

Let $u = \tan^2 x$, we have $5\left(u - \frac{1}{1+u}\right) = 2\left(\frac{1-u}{1+u}\right) + 9$

$\Rightarrow$ $5(u^2 + u - 1) = 2 - 2u + 9 + 9u$

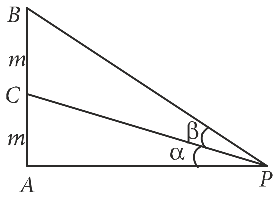
$\therefore$ $5u^2 - 2u - 16 = 0 \Rightarrow (5u+8)(u-2) = 0$

But $u$ is positive $\therefore$ $u = 2$

Now, $\tan^2 x = 2 \Rightarrow \cos 2x = \frac{1-\tan^2 x}{1+\tan^2 x} = \frac{1-2}{1+2} = \frac{-1}{3}$

$$\Rightarrow \cos 4x = 2\cos^2 2x - 1 = 2\left(\frac{1}{9}\right) - 1 = \frac{-7}{9}$$

**11. (b):**




Let $\angle APC = \alpha$, we have $\tan(\alpha+\beta) = \frac{AB}{AP} = \frac{1}{2}$

Now, $\tan\alpha = \frac{m}{4m} = \frac{1}{4}$

Now, $\tan\beta = \tan(\alpha + \beta - \alpha)$

$$= \frac{\tan(\alpha+\beta) - \tan\alpha}{1+\tan(\alpha+\beta)\tan\alpha} = \frac{\frac{1}{2}-\frac{1}{4}}{1+\frac{1}{2}\cdot\frac{1}{4}} = \frac{\frac{1}{4}}{\frac{9}{8}} = \frac{2}{9}$$

**12. (b):** We have, $\tan^{-1}\left[\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right]$

Put $x^2 = \cos 2\theta$ $\Rightarrow \theta = \frac{1}{2}\cos^{-1}(x^2)$

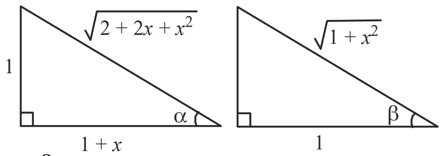
$$\therefore \tan^{-1}\left[\frac{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}\right] = \tan^{-1}\left[\frac{1+\tan\theta}{1-\tan\theta}\right]$$

$$= \tan^{-1}\left[\tan\left(\frac{\pi}{4}+\theta\right)\right] = \frac{\pi}{4} + \frac{1}{2}\cos^{-1}(x^2)$$

**13. (d):** We have, $\sin[\cot^{-1}(1+x)] = \cos[\tan^{-1} x]$

Let $\cot^{-1}(1+x) = \alpha$ and $\tan^{-1} x = \beta$

$\Rightarrow$ $1 + x = \cot\alpha$ and $x = \tan\beta$




$\therefore \quad \sin\alpha = \cos\beta$

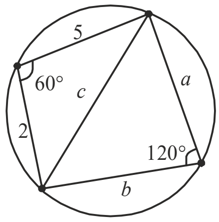
$$\Rightarrow \sin\left[\sin^{-1}\left(\frac{1}{\sqrt{2+x^2+2x}}\right)\right] = \cos\left[\cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)\right]$$

$$\Rightarrow \frac{1}{\sqrt{x^2+2x+2}} = \frac{1}{\sqrt{1+x^2}}$$

$\Rightarrow \; x^2 + 2x + 2 = x^2 + 1 \;\Rightarrow\; x = -1/2$

**14. (a):** $\cos 60° = \dfrac{4+25-c^2}{2\cdot 2\cdot 5} \Rightarrow \dfrac{1}{2} = \dfrac{29-c^2}{20}$

$\Rightarrow \; 10 = 29 - c^2 \Rightarrow c^2 = 19 \Rightarrow c = \sqrt{19}$




Now, $\cos 120° = \dfrac{a^2+b^2-c^2}{2ab}$

$\Rightarrow \; -\dfrac{1}{2} = \dfrac{a^2+b^2-19}{2ab}$

$\Rightarrow \; a^2 + b^2 - 19 = -ab$

$\Rightarrow \; a^2 + b^2 + ab = 19$

Area of quadrilateral $= \dfrac{1}{2}\times 2\times 5\times \sin 60° + \dfrac{1}{2}ab\sin 120° = 4\sqrt{3}$

$\Rightarrow \; \dfrac{5\sqrt{3}}{2} + \dfrac{ab\sqrt{3}}{4} = 4\sqrt{3} \;\Rightarrow\; \dfrac{ab}{4} = 4 - \dfrac{5}{2} = \dfrac{3}{2}$

$\Rightarrow \; ab = 6 \;\therefore\; a^2 + b^2 = 13 \;\therefore\; a = 2, b = 3$

Perimeter of quadrilateral = 2 + 5 + 2 + 3 = 12

**15. (c):** $\cos x + \cos 2x + \cos 3x + \cos 4x = 0$

Using sum – product formula, we have

$(\cos x + \cos 3x) + (\cos 2x + \cos 4x) = 0$

$\Rightarrow \; 2\cos x \cos 2x + 2\cos x \cos 3x = 0$

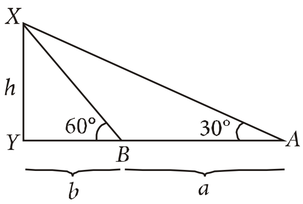
$\Rightarrow \; 2\cos x(\cos 2x + \cos 3x) = 0 \;\Rightarrow\; 2\cos x\cdot 2\cos\dfrac{5x}{2}\cos\dfrac{x}{2} = 0$

If $x \in [0, 2\pi)$ we have the solution as

$x = \dfrac{\pi}{2}, \dfrac{3\pi}{2}, \pi, \dfrac{\pi}{5}, \dfrac{3\pi}{5}, \dfrac{7\pi}{5}, \dfrac{9\pi}{5}$

Thus we have 7 solutions.

**16. (d) :**




We have $\tan 30° = \dfrac{h}{a+b}$ and $\tan 60° = \dfrac{h}{b}$

Eliminating $h$, we have $= \dfrac{\sqrt{3}}{1/\sqrt{3}} = \dfrac{a+b}{b} \;\Rightarrow\; a + b = 3b \;\therefore a = 2b$

As the man covers distance $a$ in 10 minutes, he will take 5 minutes to reach the pillar, as he has to travel half the distance.

**17. (d) :** Let $a = 2\sin^4 x + 18\cos^2 x$ and $b = 2\cos^4 x + 18\sin^2 x$

Now, $a - b = 2(\sin^4 x - \cos^4 x) + 18(\cos^2 x - \sin^2 x)$

$= 2(\sin^2 x - \cos^2 x) + 18(\cos^2 x - \sin^2 x) = 16\cos 2x$

$a + b = 2(\sin^4 x + \cos^4 x) + 18(\cos^2 x + \sin^2 x)$

$= 2\{(1 - 2\sin^2 x\cos^2 x)\} + 18 = 20 - \sin^2 2x = 19 + \cos^2 2x$

The given equation becomes, $\sqrt{\;} - \sqrt{\;} = 6$

On squaring both sides, we get $a + b - 7\sqrt{\;\;} = 6$

$\Rightarrow \; (a + b - 1)^2 = 4ab \Rightarrow (a + b)^2 - 2(a + b) + 1 = 4ab$

$\Rightarrow \; (a - b)^2 - 2(a + b) + 1 = 0$

$\Rightarrow \; 256\cos^2 2x - 2(19 + \cos^2 2x) + 1 = 0$

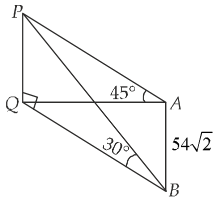
$\Rightarrow \; 254\cos^2 2x - 37 = 0$

$\cos^2 2x = \dfrac{8<}{7:9} = \lambda$--m. where $|\lambda| \le 1$ So, $\cos 2x = \pm\sqrt{\lambda}$

We have 4 values in the first cycle and the four again in the next cycle.

Recall that $n \in [0, 2\pi]$, $2x \in [0, 4\pi]$

**18. (d) :** Let the tower $PQ$ be $H$




Now, in right triangle $PQA$

$\tan 45° = \dfrac{H}{QA} \Rightarrow H = QA$

In right triangle $PQB$

$\tan 30° = \dfrac{H}{BQ} \Rightarrow BQ = \sqrt{3}H$

In right triangle $QAB$

$QA^2 + (54\sqrt{2})^2 = QB^2$

$\Rightarrow \; H^2 + (54\sqrt{2})^2 = 3H^2 \;\Rightarrow\; 54\sqrt{2} = \sqrt{2}H \;\Rightarrow\; H = 54$ m

**19. (b) :** We know that, $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A\tan B}$

$\Rightarrow \; \dfrac{1}{\sqrt{3}} = \dfrac{y}{1 - \tan A\tan B}$, where $y = \tan A + \tan B$

$\Rightarrow \; \tan A\tan B = 1 - \sqrt{3}y$

Also A.M. ≥ G.M. $\Rightarrow \; \dfrac{\tan A + \tan B}{2} \ge \sqrt{\tan A\tan B}$

$\Rightarrow \; y \ge 2\sqrt{1-\sqrt{3}y} \Rightarrow y^2 \ge 4 - 4\sqrt{3}y \;\Rightarrow\; y^2 + 4\sqrt{3}y - 4 \ge 0$

$\Rightarrow \; y \le -2\sqrt{3} - 4$ or $y \ge -2\sqrt{3} + 4$

($y \le -2\sqrt{3} - 4$ is not possible as $\tan A$; $\tan B > 0$)

**20. (c) :** We have, $\sin\theta - \cos\theta = \sqrt{2}\cos\theta$

$\Rightarrow \; \sin\theta = (\sqrt{2}+1)\cos\theta = \dfrac{2-1}{\sqrt{2}-1}\cos\theta$

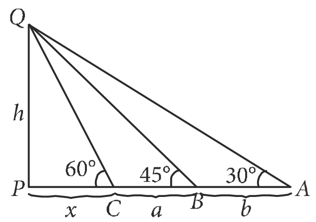
$\Rightarrow \; \sin\theta = \dfrac{1}{\sqrt{2}-1}\cos\theta \Rightarrow (\sqrt{2}-1)\sin\theta = \cos\theta$ …(i)

Also, $\sin\theta + \cos\theta = \sqrt{2}\sin\theta$

$\Rightarrow \; (\sqrt{2} - 1)\sin\theta = \cos\theta$ …(ii)

From (i) and (ii), we get $P = Q$

**21. (c) :**




Using basic trigonometry in appropriate triangles

$\mu nz\ ;5° = — \Rightarrow \quad = \frac{}{\sqrt{8}} \qquad \mu nz\ 9{:}° = \frac{}{+} \Rightarrow \quad + \ =$

and $\mu nz\ 85° = \frac{}{+\ +} \Rightarrow \quad + \ + \ = \sqrt{8}$

We have, $\ = \left(6 - \frac{6}{\sqrt{8}}\right) = \frac{-\sqrt{8} - 6.}{\sqrt{8}}$

$= \sqrt{8} - \ = -\sqrt{8} - 6.$

$\therefore \frac{WX}{XY} = — = \sqrt{8} \quad \therefore WX\ AXY = \sqrt{8}\ A6$

**22. (c) :** As $\ < \frac{6}{\sqrt{8}} 1$ in this range using principal branch of tangent function, we have

$8\mu nz^{-6} \quad = \mu nz^{-6}\left(\frac{8 \ - \ ^{8}}{6 - 8\ ^{7}}\right)$ Also, $\mu nz^{-6}\left(\frac{7}{6 - \ ^{7}}\right) = 7\mu nz^{-6}$

Thus, $\tan^{-1}y = \tan^{-1}x + 2\tan^{-1}x = 3\tan^{-1}x$

$= \mu nz^{-6}\left(\frac{8 \ - \ ^{8}}{6 - 8\ ^{7}}\right) \quad \therefore \quad = \frac{8 \ - \ ^{8}}{6 - 8\ ^{7}}$

**23. (b) :** Since $— = \frac{7 + \sqrt{8}}{6} \Rightarrow \angle W > \angle X$

So only option (b) & (d) can be correct.

$— = \frac{-ız\ 65{:}°}{-ız\ 6{:}°} = \frac{\sqrt{8} + 6}{\sqrt{8} - 6} = 7 + \sqrt{8}$, which is true.

**24. (b) :** We have, $f(x) = \ 7\mu nz^{-6} \quad + -ız^{-6}\left(\frac{7}{6 + \ ^{7}}\right)$

$= 2\tan^{-1}x + \pi - 2\tan^{-1}x$

$\Rightarrow f(x) = \pi$. So, $f(5) = \pi$

**25. (c) :** Since $\cos\alpha + \cos\beta = \frac{8}{7}$

$\Rightarrow \ 7o\{-\left(\frac{\alpha+\beta}{7}\right)o\{-\left(\frac{\alpha-\beta}{7}\right) = \frac{8}{7}$ ...(i)

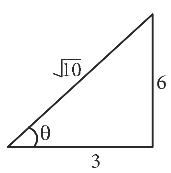
Also, $\sin\alpha + \sin\beta = \frac{6}{7}$

$\Rightarrow \ 7-ız\left(\frac{\alpha+\beta}{7}\right)o\{-\left(\frac{\alpha-\beta}{7}\right) = \frac{6}{7}$ ...(ii)

Dividing (ii) by (i), $\mu nz\left(\frac{\alpha+\beta}{7}\right) = \frac{6}{8}$

Since θ is the arithmetic mean of α and β

$\Rightarrow \theta = \frac{\alpha+\beta}{7}$



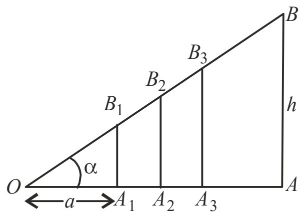

$\Rightarrow \mu nz\ \theta = \frac{6}{8}$

$\Rightarrow -ız\ \theta = \frac{6}{\sqrt{65}}\ nz\ p\ o\{-\theta = \frac{8}{\sqrt{65}}$

Now, $\sin 2\theta + \cos 2\theta = 2\sin\theta\cos\theta + 2\cos^2\theta - 1$

$= 7 \times \frac{6}{\sqrt{65}} \times \frac{8}{\sqrt{65}} + 7\left(\frac{>}{65}\right) - 6 = \frac{;}{65} + \frac{6=}{65} - 6 = \frac{<}{:}$

**26. (c) :** Let the distance between two consecutive poles be $x$. In $\Delta OAB$




$\frac{}{+>} = \frac{\mu nz\ \alpha}{} \Rightarrow \quad +> \ = \frac{}{\mu nz\ \alpha}$

$\Rightarrow \quad = \frac{- \ \ \mu nz\ \alpha}{>\mu nz\ \alpha}$

$= \frac{-\ \ o\{-\alpha - \ \ -ız\ \alpha.}{>-ız\ \alpha}$

**27. (c) :** $f_4(x) - f_6(x) = \frac{1}{4}(\sin^4 x + \cos^4 x) \ -\frac{1}{6}(\cos^6 x + \sin^6 x)$

$= \frac{1}{4}\{(\sin^2 x + \cos^2 x)^2 - 2\sin^2 x\cos^2 x\}$

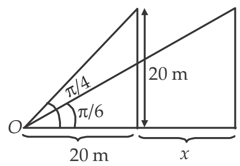
$-\frac{1}{6}\{(\cos^2 x + \sin^2 x)^3 - 3\cos^2 x\sin^2 x\}$

$= \frac{1}{4}(1 - 2\sin^2 x\cos^2 x) - \frac{1}{6}(1 - 3\sin^2 x\cos^2 x) \quad = \frac{1}{4} - \frac{1}{6} = \frac{1}{12}$

**Remark :** As the given expression is independent of $x$, as suggested by choices, one can simply put a convenient value to obtain the result at $x = 0$.

Hence $f_4(0) - f_6(0) = \frac{1}{4} - \frac{1}{6} = \frac{1}{12}$ etc.

**28. (c) :**




We have $\tan 30° = \frac{20}{20 + x} = \frac{1}{\sqrt{3}}$

$\Rightarrow 20 + x = 20\sqrt{3} \Rightarrow x = 20(\sqrt{3} - 1)$

The speed of bird is $20(\sqrt{3} - 1)$ m/s

**29. (d) :** As $x, y, z$ are in A.P. $\Rightarrow 2y = x + z$ … (i)

$\tan^{-1}x$, $\tan^{-1}y$ and $\tan^{-1}z$ are in A.P., then

$2\tan^{-1}y = \tan^{-1}x + \tan^{-1}z$

$2\tan^{-1}y = \tan^{-1}\left(\frac{x+z}{1-xz}\right) \Rightarrow \tan^{-1}\left(\frac{2y}{1-y^2}\right) = \tan^{-1}\left(\frac{x+z}{1-xz}\right)$

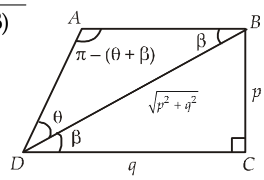
Thus $y^2 = xz$ … (ii)

From (i) and (ii), we get $x = y = z$.

**Remark :** $y \neq 0$ is implicit to make any of the choice correct.

**30. (d) :** Using sine rule in triangle $ABD$, we get

$\frac{AB}{\sin\theta} = \frac{BD}{\sin(\theta+\beta)} \Rightarrow AB = \frac{\sqrt{p^2+q^2}\sin\theta}{\sin(\theta+\beta)}$




As $\tan\beta = \frac{p}{q}$, we have

$\sin(\theta + \beta) = \sin\theta\cos\beta + \cos\theta\sin\beta$

$= \sin\theta \cdot \frac{q}{\sqrt{p^2+q^2}} + \cos\theta \cdot \frac{p}{\sqrt{p^2+q^2}} = \frac{p\cos\theta + q\sin\theta}{\sqrt{p^2+q^2}}$

We then get $AB = \frac{(p^2+q^2)\sin\theta}{p\cos\theta + q\sin\theta}$

**31. (a) :** $\dfrac{\tan A}{1-\cot A}+\dfrac{\cot A}{1-\tan A}$

$=\dfrac{\sin^2 A}{\cos A(\sin A-\cos A)}+\dfrac{\cos^2 A}{\sin A(\cos A-\sin A)}$

$=\dfrac{\sin^3 A-\cos^3 A}{(\sin A-\cos A)\cos A\sin A}$

$=\dfrac{(\sin A-\cos A)(\sin^2 A+\sin A\cos A+\cos^2 A)}{(\sin A-\cos A)\sin A\cos A}$

$=\dfrac{1+\sin A\cos A}{\sin A\cos A}=1+\sec A\operatorname{cosec} A$

**32. (d) :** $3\sin P + 4\cos Q = 6$

$4\sin Q + 3\cos P = 1$

$\Rightarrow$ $16 + 9 + 24(\sin(P+Q)) = 37 \Rightarrow 24(\sin(P+Q)) = 12$

$\Rightarrow \sin(P+Q)=\dfrac{1}{2} \Rightarrow \sin R=\dfrac{1}{2} \Rightarrow R=\dfrac{5\pi}{6}$ or $\dfrac{\pi}{6}$

But if $R=\dfrac{5\pi}{6}$ then $P<\dfrac{\pi}{6}$ and then $3\sin P<\dfrac{1}{2}$

and so $3\sin P + 4\cos Q < \dfrac{1}{2}+4\,(\neq 6)$ Thus, $R=\dfrac{\pi}{6}$.

**33. (c) :** $A = \sin^2 x + \cos^4 x$

We have $\cos^4 x \le \cos^2 x$

$\sin^2 x = \sin^2 x$

Adding $\sin^2 x + \cos^4 x \le \sin^2 x + \cos^2 x$ $\therefore A \le 1$.

Again $A = t + (1-t)^2 = t^2 - t + 1$, $t \ge 0$, where minimum is 3/4

Thus $\dfrac{3}{4}\le A\le 1$.

**34. (b) :** $\cos(\alpha+\beta) = 4/5$ gives $\tan(\alpha+\beta) = 3/4$

Also $\sin(\alpha-\beta) = 5/13$ gives $\tan(\alpha-\beta) = 5/12$

$=\dfrac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha+\beta)\tan(\alpha-\beta)}=\dfrac{\frac{3}{4}+\frac{5}{12}}{1-\frac{3}{4}\times\frac{5}{12}}=\dfrac{36+20}{48-15}=\dfrac{56}{33}$

**35. (c) :** We have $\dfrac{r}{R}=\cos\dfrac{\pi}{n}$, let $\cos\dfrac{\pi}{n}=\dfrac{1}{\sqrt{2}}$.

Thus we get $\dfrac{\pi}{n}=\dfrac{\pi}{4}$ *i.e.,* $n = 4$, acceptable.

$\cos\dfrac{\pi}{n}=\dfrac{1}{2} \Rightarrow \dfrac{\pi}{n}=\dfrac{\pi}{3}$. $\therefore$ $n = 3$, acceptable.

$\cos\dfrac{\pi}{n}=\dfrac{\sqrt{3}}{2} \Rightarrow \dfrac{\pi}{n}=\dfrac{\pi}{6}$. $\therefore$ $n = 6$, acceptable.

But $\cos\dfrac{\pi}{n}=\dfrac{2}{3}$ will produce no value of $n$.

As $\dfrac{1}{2}<\dfrac{2}{3}<\dfrac{1}{\sqrt{2}} \Rightarrow \cos\dfrac{\pi}{3}<\cos\dfrac{\pi}{n}<\cos\dfrac{\pi}{4}$

$\Rightarrow \dfrac{\pi}{3}>\dfrac{\pi}{n}>\dfrac{\pi}{4} \Rightarrow 3<n<4$ (impossible)

**36. (b) :** $\cos(\beta-\gamma) + \cos(\gamma-\alpha) + \cos(\alpha-\beta) = -3/2$

$\Rightarrow$ $(\cos\beta\cos\gamma + \sin\beta\sin\gamma) + (\cos\gamma\cos\alpha + \sin\gamma\sin\alpha)$
$+ (\cos\alpha\cos\beta + \sin\alpha\sin\beta) = -3/2$

$\Rightarrow$ $2(\cos\beta\cos\gamma + \cos\gamma\cos\alpha + \cos\alpha\cos\beta)$
$+ 2(\sin\beta\sin\gamma + \sin\gamma\sin\alpha + \sin\alpha\sin\beta) + 3 = 0$

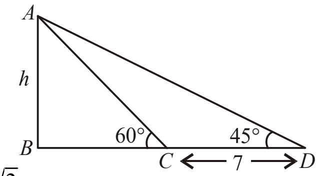
$\Rightarrow$ $\{\cos^2\alpha + \cos^2\beta + \cos^2\gamma$
$+ 2(\cos\alpha\cos\beta + \cos\beta\cos\gamma + \cos\gamma\cos\alpha)\}$
$+ \{\sin^2\alpha + \sin^2\beta + \sin^2\gamma$
$+ 2(\sin\alpha\sin\beta + \sin\beta\sin\gamma + \sin\gamma\sin\alpha)\} = 0$

$\Rightarrow$ $(\cos\alpha + \cos\beta + \cos\gamma)^2 + (\sin\alpha + \sin\beta + \sin\gamma)^2 = 0$

Which yields simultaneously

$\cos\alpha + \cos\beta + \cos\gamma = 0$ and $\sin\alpha + \sin\beta + \sin\gamma = 0$

**37. (c) : 1st Solution :** Let height of the pole $AB$ be $h$. Then




$BC = h\cot 60° = h/\sqrt{3}$

$BD = h\cot 45° = h$

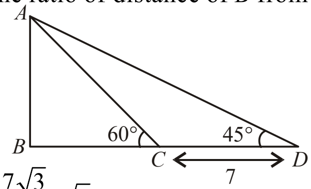
As $BD - BC = CD$

$\Rightarrow h-\dfrac{h}{\sqrt{3}}=7 \Rightarrow h(\sqrt{3}-1)=7\sqrt{3}$

$\Rightarrow h=\dfrac{7\sqrt{3}}{\sqrt{3}-1}=\dfrac{7\sqrt{3}(\sqrt{3}+1)}{2}=\dfrac{7\sqrt{3}}{2}(\sqrt{3}+1)\text{m}$

**2nd Solution :** We use the fact that the ratio of distance of $B$ from $D$ and that of $B$ from $C$

*i.e.* $BD$ to $BC$ is $\sqrt{3}:1$




$\dfrac{BD}{BC}=\sqrt{3}$, so that $\dfrac{BD}{CD}=\dfrac{\sqrt{3}}{\sqrt{3}-1}$

Then $BD=\dfrac{\sqrt{3}}{\sqrt{3}-1}CD=\dfrac{\sqrt{3}}{\sqrt{3}-1}\cdot 7=\dfrac{7\sqrt{3}}{2}(\sqrt{3}+1)$

As $AB = BD$, the height of the pole $=\dfrac{7\sqrt{3}}{2}(\sqrt{3}+1)\text{m}$

**38. (b) :** $\cot\left(\operatorname{cosec}^{-1}\dfrac{5}{3}+\tan^{-1}\dfrac{2}{3}\right)=\cot\left(\tan^{-1}\dfrac{3}{4}+\tan^{-1}\dfrac{2}{3}\right)$

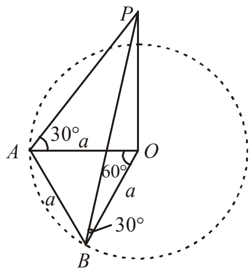
$=\cot\left(\tan^{-1}\dfrac{\frac{3}{4}+\frac{2}{3}}{1-\frac{3}{4}\cdot\frac{2}{3}}\right)=\cot\left(\tan^{-1}\dfrac{17}{6}\right)=\dfrac{6}{17}$

**39. (a) :** $OP$ = Tower

$OAB$ is equilateral triangle

$\therefore OA = OB = AB = a$

In $\Delta AOP$,




$\tan 30° = \dfrac{OP}{OA}$

$\Rightarrow OP=\dfrac{a}{\sqrt{3}}$

**40. (b) :** $f(x)$ is defined if $-1\le\dfrac{x}{2}-1\le 1$ and $\cos x>0$

or $0\le x\le 4$ and $-\dfrac{\pi}{2}<x<\dfrac{\pi}{2}$ $\therefore$ $0\le x<\dfrac{\pi}{2}$.

**41. (d) :** $\sin^{-1}\left(\dfrac{x}{5}\right)+\sin^{-1}\left(\dfrac{4}{5}\right)=\dfrac{\pi}{2}\Rightarrow \sin^{-1}\left(\dfrac{x}{5}\right)=\dfrac{\pi}{2}-\sin^{-1}\left(\dfrac{4}{5}\right)$

$\Rightarrow \dfrac{x}{5}=\sin\left(\dfrac{\pi}{2}-\sin^{-1}\left(\dfrac{4}{5}\right)\right)$

$\Rightarrow \dfrac{x}{5}=\cos\left(\sin^{-1}\dfrac{4}{5}\right)=\cos\left(\cos^{-1}\dfrac{3}{5}\right)=\dfrac{3}{5} \Rightarrow x = 3.$

**42. (c) :** $0 < x < \pi$, Given $\cos x + \sin x = \frac{1}{2}$

$\Rightarrow \quad 1 + \sin 2x = \frac{1}{4}$ (By squaring both sides)

$\frac{2\tan x}{1+\tan^2 x} = \frac{-3}{4}$

$\Rightarrow \quad 3\tan^2 x + 8\tan x + 3 = 0$

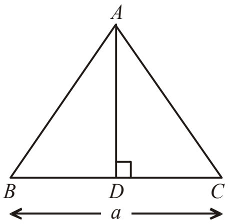
$\tan x = \frac{-8 \pm \sqrt{64-36}}{6} = \frac{-4 \pm \sqrt{7}}{3}$ $\because \tan x < 0 \Rightarrow \tan x = \frac{-4-\sqrt{7}}{8}$

**43. (a) :** $2\sin^2 x + 5\sin x - 3 = 0 \Rightarrow \sin x = \frac{1}{2}, \sin x \neq -3$

therefore $\sin x = \frac{1}{2}$, we know that each trigonometrical function assumes same value twice in $0 \le x \le 360°$.
In our problem $0° \le x \le 540°$. So number of values are 4 like 30°, 150°, 390°, 510°.

**44. (c) :** Altitude from $A$ to $BC$ is $AD$

Area of $\Delta ABC = \Delta = \frac{1}{2} AD \times BC$




$\therefore \frac{2 \cdot \Delta}{a} = AD$

$\therefore$ Altitudes are in H.P.

$\therefore \frac{2\Delta}{a}, \frac{2\Delta}{b}, \frac{2\Delta}{c} \in$ H.P.

$\Rightarrow \frac{1}{a}, \frac{1}{b}, \frac{1}{c} \in$ H.P. $\Rightarrow a, b, c \in$ A.P.

**45. (d) :** Using $\cos^{-1} A - \cos^{-1} B$
$= \cos^{-1}\left(AB + \sqrt{(1-A^2)}\sqrt{(1-B^2)}\right)$

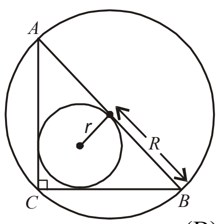
$\therefore \quad \cos^{-1} x - \cos^{-1}\frac{y}{2} = \alpha \Rightarrow \frac{xy}{2} + \sqrt{1-x^2}\sqrt{1-\frac{y^2}{4}} = \cos\alpha$

$\Rightarrow \quad \left(\cos\alpha - \frac{xy}{2}\right)^2 = (1-x^2)\left(1-\frac{y^2}{4}\right)$

$\Rightarrow \quad 4x^2 - 4xy\cos\alpha + y^2 = 4(1-\cos^2\alpha) = 4\sin^2\alpha.$

**46. (a) :** $\frac{c}{\sin C} = 2R$

$\therefore c = 2R$ ...(A) ($\because C = 90°$)




and $\tan\frac{C}{2} = \frac{r}{s-c}$

$\therefore r = (s-c)\left(\tan\frac{C}{2} = \tan 45° = 1\right)$

$= \frac{a+b+c}{2} - c$ ; $2r$

$= a + b - c$ ...(B)

adding (A) and (B) we get $2(r + R) = a + b$.

**47. (c) :** $\angle R = 90° \quad \therefore \quad \angle P + \angle Q = 90°$

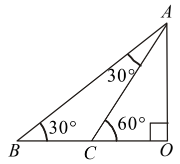
$\therefore \quad \frac{P}{2} = \frac{90}{2} - \frac{Q}{2}, \quad \frac{P}{2} = 45 - \frac{Q}{2} \Rightarrow \tan\frac{P/2}{1} = \frac{1-\tan Q/2}{1+\tan Q/2}$

$\Rightarrow \quad \tan\frac{P}{2} + \tan\frac{Q}{2} = 1 - \tan\frac{P}{2}\cdot\tan\frac{Q}{2} \Rightarrow -\frac{b}{a} = 1 - \frac{c}{a}$

$\left(\because \tan\frac{P}{2}, \tan\frac{Q}{2} \text{ are roots of } ax^2 + bx + c = 0\right)$

$\Rightarrow \quad \frac{c-b}{a} = 1 \Rightarrow c = a + b.$

**48. (c) :**




Breadth of river $OC = AC \cos 60° = 40 \cos 60°$

**49. (a) :** If $a^2 = \sin^2\alpha$, $b^2 = \cos^2\alpha$, $c^2 = 1 + \sin\alpha\cos\alpha$

then $\cos c = \frac{-\sin\alpha\cos\alpha}{2\sin\alpha\cos\alpha}$ $\quad \therefore \cos c = -1/2$

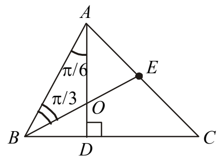
**50. (d) :** $u^2 = a^2(\cos^2\theta + \sin^2\theta) + b^2(\sin^2\theta + \cos^2\theta) +$
$2\sqrt{\left(a^4+b^4\right)\sin^2\theta\cos^2\theta + a^2b^2\left(\sin^4\theta + \cos^4\theta\right)}$

$= a^2 + b^2 + 2\sqrt{a^2b^2 + \left(a^2-b^2\right)^2\sin^2\theta\cos^2\theta}$

$= a^2 + b^2 + 2\sqrt{a^2b^2 + \left(\frac{a^2-b^2}{2}\right)^2\sin^2 2\theta}$

$\therefore u^2$ will be maximum or minimum according as $\theta = \pi/4$ or $\theta = 0°$

$\therefore$ Max. $u^2 = 2(a^2 + b^2)$ and




Min. $u^2 = a^2 + b^2 + 2ab = (a + b)^2$

Now Maximum $u^2$ – Minimum $u^2$
$= 2(a^2 + b^2) - (a^2 + b^2 + 2ab)$
$= a^2 + b^2 - 2ab = (a - b)^2$

**51. (c) :** $\sin\alpha + \sin\beta = -\frac{21}{65}$ and $\cos\alpha + \cos\beta = -\frac{27}{65}$

by squaring and adding we get
$2(1 + \cos\alpha\cos\beta + \sin\alpha\sin\beta) = \frac{(21)^2 + (27)^2}{(65)^2}$

$2[1 + \cos(\alpha - \beta)] = \frac{1170}{(65)^2}$

$\cos^2\frac{(\alpha-\beta)}{2} = \frac{1170}{4 \times 65 \times 65} = \frac{130 \times 9}{(130) \times (130)} = \frac{9}{130}$

$\therefore \cos\frac{\alpha-\beta}{2} = \frac{3}{\sqrt{130}}$

As $\pi < \alpha - \beta < 3\pi$ then $\cos\left(\frac{\alpha-\beta}{2}\right)$ = negative

**52. (d) :** $2a\cos^2\frac{C}{2} + 2c\cos^2 A/2 = 3b$ (from given)

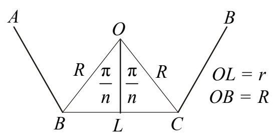
$\Rightarrow \quad a(1 + \cos C) + c(1 + \cos A) = 3b$

$\Rightarrow \quad a + c + a\cos C + c\cos A = 3b$

$(a\cos C + c\cos A = b$ projection formula)

$\Rightarrow \quad a + c + b = 3b \Rightarrow \quad a + c = 2b$

**53. (b) :** $\frac{OB}{AO} = \tan 30°$




$\Rightarrow \quad \overrightarrow{OB} = \frac{OA}{\sqrt{3}} = \frac{8\sqrt{3}}{9}$

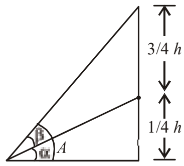
Area of triangle $ADB = \frac{1}{2} \times \frac{8\sqrt{3}}{9} \times 4 = \frac{16\sqrt{3}}{9}$

Area of triangle $ABC = 2 \times \frac{16\sqrt{3}}{9} = \frac{32\sqrt{3}}{9}$

**54. (a) :** $\alpha = A + \beta$

$\therefore \quad \beta = A - \alpha$

$$\tan\beta = \frac{\tan A - \tan\alpha}{1 - \tan A \tan\alpha}$$




$$\Rightarrow \quad \frac{3}{5} = \frac{\frac{h}{40} + \left(-\frac{h}{160}\right)}{1 - \left(\frac{h}{40}\right)\left(-\frac{h}{160}\right)} \Rightarrow h^2 - 200h + 6400 = 0$$

$\Rightarrow \quad (h - 40)(h - 160) = 0 \Rightarrow \quad h = 40$ or $h = 160$

**55. (a) :** If $R$ be the radius of circumcircle of regular polygon of $n$ sides, and $r$ be the radius of inscribed circle then

$$R = \frac{a}{2}\operatorname{cosec}\frac{\pi}{2n} \text{ and } r = \frac{a}{2}\cot\frac{\pi}{n}$$

$$\therefore\ R + r = \frac{a}{2}\left(\operatorname{cosec}\frac{\pi}{n} + \cot\frac{\pi}{n}\right) = \frac{a}{2}\left(\frac{1 + \cos\frac{\pi}{n}}{\sin\frac{\pi}{n}}\right) = \frac{a}{2}\cot\frac{\pi}{2n}$$

**56. (b) :** $\sin^{-1}x = 2\sin^{-1}a$

$$\Rightarrow -\frac{\pi}{2} \le 2\sin^{-1}a \le \frac{\pi}{2} \quad \left[\begin{array}{l}\because\ \sin^{-1}x = 2\sin^{-1}a \\ \text{and } -\frac{\pi}{2} \le \sin^{-1}x \le \frac{\pi}{2}\end{array}\right]$$

$$\Rightarrow -\frac{\pi}{4} \le \sin^{-1}a \le \frac{\pi}{4} \quad \Rightarrow \sin\left(-\frac{\pi}{4}\right) \le a \le \sin\left(\frac{\pi}{4}\right)$$

$$\Rightarrow -\frac{1}{\sqrt{2}} \le a \le \frac{1}{\sqrt{2}} \quad \Rightarrow |a| \le \frac{1}{\sqrt{2}}$$

**57. (a) :** As $r_1 > r_2 > r_3 \Rightarrow \frac{\Delta}{s-a} > \frac{\Delta}{s-b} > \frac{\Delta}{s-c}$

$$\Rightarrow \quad \frac{s-a}{\Delta} < \frac{s-b}{\Delta} < \frac{s-c}{\Delta} \quad \Rightarrow \quad a > b > c$$

**58. (a) :** Using $\tan^{-1}\theta + \cot^{-1}\theta = \frac{\pi}{2} = x$

$\therefore \quad \sin x = \sin\frac{\pi}{2} = 1$

**59. (b) :** $\tan x + \sec x = 2\cos x$

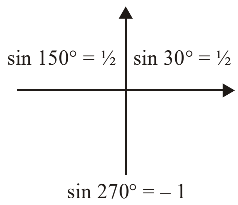
$1 + \sin x = 2\cos^2 x$

$1 + \sin x = 2(1 - \sin^2 x)$

$\Rightarrow \quad 2\sin^2 x + \sin x - 1 = 0$

$\Rightarrow \quad (2\sin x - 1)(1 + \sin x) = 0$

$\Rightarrow \quad \sin x = \frac{1}{2},\ \sin x = -1$




so there are three solutions which are $x = 30°, 150°, 270°$